“Maintaining Capital in the Presence of Obsolescence”

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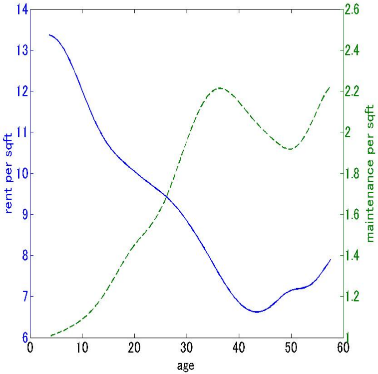
Objective

- How to maintain a capital asset that is subject to wear and tear and obsolescence
- A dynamic tradeoff:
  - A smaller expenditure on maintenance may raise short-run receipts
  - But it may lead to lower profits due to increased wear and tear in the long run
- The incentive for maintenance is larger, the larger is the forgone profit from reduced maintenance
- How should a maintenance expenditure pattern vary with asset types and market conditions?

U.S. Office Building Data



Kernel estimates of rent and maintenance expenditure

- **While the rent steadily declines, the maintenance initially increases and then decreases. Why?**

Optimal-Control Literature

- Early contributions
  - Naslund (1966), Swedish Journal of Economics
  - Thompson (1968), Management Science
  - Kamien and Schwartz (1971), Management Science
- Deterministic maintenance
- Probabilistic maintenance
- Subsequent researchers
  - Virtanen (1982), Mehrez and Berman (1994)
  - Dogramaci and Fraiman (2004), Bensoussan and Sethi (2007)

This Paper

This paper . . .

- Studies a deterministic maintenance problem
- Presents a nonlinear extension of the Thompson's model (1968)
  - Our solution is not bang-bang
- Distinguishes between maintenance and partial replacement
  - Simulation
- Applies an optimal-control model to data

The Model: Outline

- **Time: continuous, indexed by $t \in (0, Z]$**
- **An individual capital asset is:**
  - **owned at $t = 0$**
  - **used for productive purposes for a length of time**
  - **and then sold at $t = T \leq Z$**
- **An owner receives:**
  - **a flow of nonnegative production revenue over $(0, T)$**
  - **a lump-sum resale profit at $t = T$**
  - **these are larger, the more relatively capable is the asset**
- **Specifically, . . .**

The Model: Asset

- **Asset's relative capability at time t:**

$$\begin{aligned}\bar{c}(t) - c(t) &= [\bar{c}(0) - c(t)] + [\bar{c}(t) - \bar{c}(0)] \\ &\equiv a(t) + b(t)\end{aligned}$$

- $\bar{c}(t)$: **the capability of an asset that embodies the best technology at time** t
- $c(t)$: **the capability of the owner's asset at time** t
- $a(t)$: **the state of deterioration due to wear and tear**
- $b(t)$: **the state of obsolescence due to technical advance**

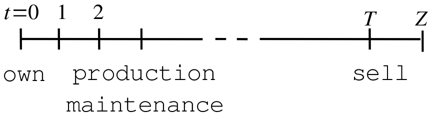
The Model: Receipt

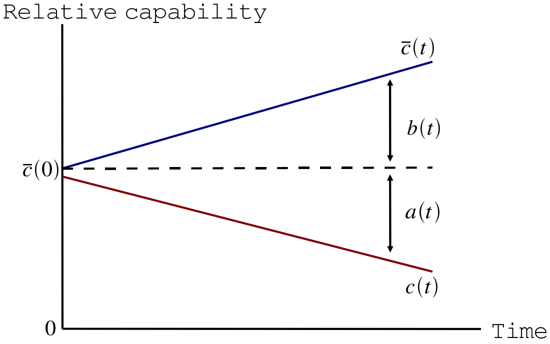
- Production revenue at time t: $R(a(t) + b(t))$
  - Decreasing: $R'(a(t) + b(t)) < 0$
  - More than proportionally: $R''(a(t) + b(t)) < 0$
- Resale price (Salvage value) at time t: $S(a(t) + b(t))$
  - Decreasing: $R'(a(t) + b(t)) < 0$
  - More than proportionally: $R''(a(t) + b(t)) < 0$
- These receipts are larger, the more relatively capable is the asset at the moment

The Model: Maintenance

- Maintenance expenditure at time t: $m(t) \geq 0$
- Maintenance reduces physical wear and tear but has no effect on obsolescence
- Specifically,
  - $\dot{a}(t) = \alpha a(t) - z(m(t))$ and $a(t) \geq a_0$ with $\alpha > 0$
  - $\dot{b}(t) = \beta > 0$
- $z(m(t))$: maintenance production function
  - Increasing: $z'(m(t)) > 0$
  - Concave: $z''(m(t)) < 0$
  - Vanishes: $z(0) = 0$

The Model: Some Figures





The Model: Problem

- **Owner's discounted profits:**
  $$J = \int_0^T e^{-rt}\left[R(a+b) - m\right]\mathrm{d}t + e^{-rT}S(a_T + b(T))$$
- **Problem: choose T, $m(t)$ and a_T to maximize J subject to the inequality state constraint**
- **An optimal policy: the solution $\{T^*, m^*, a_T^*\}$**
- **Current-value Hamiltonian (with costate function $\mu(t)$):**
  $$H = H(a, m, \mu) = R(a+b) - m + \mu\left[\alpha a - z(m)\right]$$
- **Maximum Principle**

Optimal Policy: Sale Date

Proposition (Necessity)

Suppose that $\{T^*, m^*, a_T^*\}$ *exists. Then, necessarily,*

(i) *At an optimal sale date* T^*,

$$R(a_T^* + b) - m^* \geq$$
$$rS(a_T^* + b) - S'(a_T^* + b)(\alpha a_T^* - z(m^*) + \beta)$$

with equality when $T^* < Z$.

- **LHS is the marginal benefit from postponing the sale**
- **RHS is the marginal cost of doing so**

Optimal Policy: Maintenance

Proposition (Necessity, continued)

Suppose that $\{T^*, m^*, a_T^*\}$ *exists. Then, necessarily,*

(ii) An optimal maintenance policy m^* *satisfies*

$$\begin{cases} 1 = -\mu z'(m^*), & t \in I \text{ and } 1 < -\mu z'(0) \\ m^* = 0, & t \in I \text{ and } 1 \geq -\mu z'(0) \\ m^* = z^{-1}(\alpha a_0), & t \in B. \end{cases}$$

Here μ *satisfies the differential equation*

$$\dot{\mu} = \begin{cases} (r - \alpha)\mu - R'(a^* + b), & t \in I \\ 0, & t \in B \end{cases}$$

with the terminal condition $\mu = S'(a_T^* + b)$ *at* $t = T^*$.

Optimal Policy: Interpretation

- RHS is the marginal benefit from an additional dollar expenditure on maintenance
  - μ: the marginal value of the deterioration level a^* at time t
  - So, the maximum forgone profit from a unit increase in a^* at time t
- LHS is the marginal cost of doing so
- $\dot{\mu}$: the rate of change in the marginal value of the deterioration level a^* at time t
- $r - \alpha$: the effective discount rate

Optimal Policy: Sufficiency

Proposition (Sufficiency)

Given T^*, suppose that $\{m^*, a_T^*\}$ is a policy satisfying the above Proposition. Then, $\{m^*, a_T^*\}$ is optimal.

- **For proof, use the Mangasarian condition.**
- **Therefore, the necessary condition is also sufficient.**

Optimal Policy: Qualitative Properties

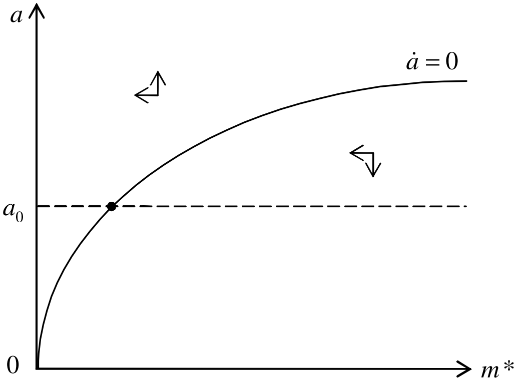
- **Asset types:**
  - **high deterioration type if $r < \alpha$**
  - **low deterioration type if $r > \alpha$**

Proposition (High type)

Let $r < \alpha$. Then, m^* is the highest at the initial date, and steadily and strictly decreases with time in an optimal plan. Moreover, a^* is the lowest at the initial date, and steadily and strictly increases with time at an increasing rate.

- **For proof, use the phase analysis.**

Optimal Policy: Phase Diagram (H)



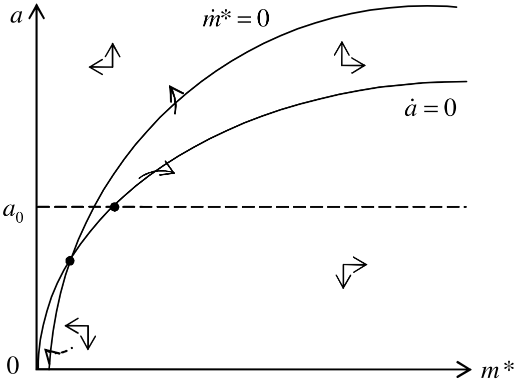
Phase diagram for $r < \alpha$

Optimal Policy: Qualitative Properties

Proposition (Low type)

Let $r > \alpha$. Then, m^* either first increases and then decreases, or evolves monotonically. Moreover, if $\dot{m}^* \leq 0$ at some t' in $(0, T^*)$, then m steadily and strictly decreases with time for all t in (t', T^*).

- **An optimal maintenance expenditure is thus either inversed-U shaped (increase and then decrease) or monotonic.**



phase diagram for $r > \alpha$

- **Note: the m^* null isocline shifts down with time.**

Optimal Policy: More Results

- **Some more results:**

Proposition (Comparative dynamics)

An increase in β does not raise the maintenance investment for all t in $(0, T^*)$ in an optimal plan.

- **Maximized net discounted production profit:**

$$V = V(\alpha, \beta) \equiv \int_0^{T^*} e^{-rt} \left[R(a^* + b) - m^* \right] \mathrm{d}t$$

Proposition (Envelope result)

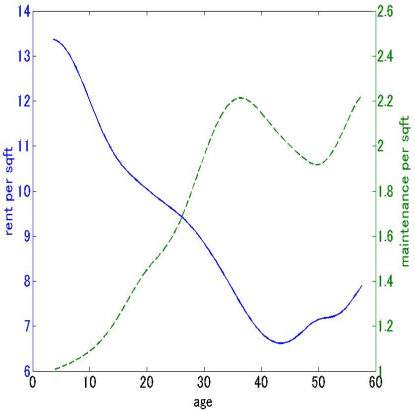
$V_\alpha(\alpha, \beta) < 0$, $V_\beta(\alpha, \beta) < 0$ *and* $V_{\beta\beta}(\alpha, \beta) > 0$.

Estimation: Data

- U.S. office building data (from BOMA International)
  - Corrected by Gort, Greenwood, Rupert (1999)
- Dataset consists of two panels:
  - One covers 200 office buildings from 1989 to 1997
  - The other covers 800 office buildings from 1993 to 1997
  - Include the info. on age, size, rent and several expenses

	mean	std. dev.	min	max
size (sq. ft.)	$254,670$	$304,530$	$10,656.6$	$2,860,100$
maint./sq. ft.	1.5936	0.97363	0	6.3168
rent/sq. ft.	10.352	5.316	0.06557	43.432
age	26.875	22.614	2	144

Estimation: Kernel Estimate Data

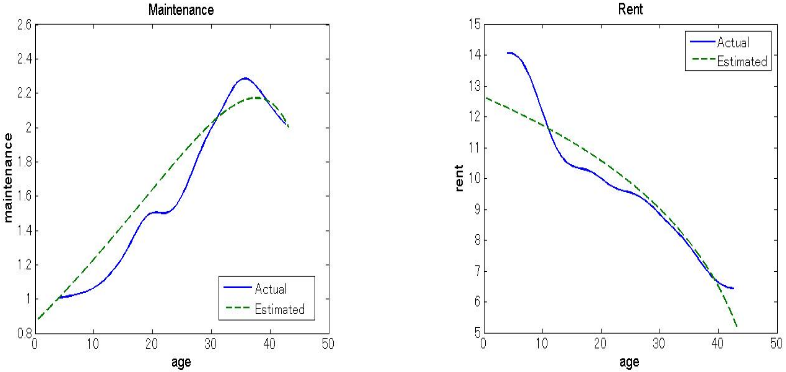


Kernel estimates of rent and maintenance expenditure (with a Gaussian kernel and a MISE-minimizing bandwidth)

Estimation: Parameterization

- **Parameterization**
  - **Maintenance:** $z(m) = \zeta \ln(m+1)$
  - **Revenue:** $R(a+b) = \rho_0 + \rho_1 \ln(\rho_2 - a - b)$
  - **Resale:** $S(a+b) = \sigma_0 + \sigma_1 R(a+b)$
- **Parameters:** r, a_0, α, β, ρ_0, ρ_1, ρ_2, σ_0, σ_1 **and** ζ
- **Procedure:**
  - **Fix a sale date** T^*
  - **Given parameter values, Proposition (Necessity) together with a guess on** $m^*(0)$ **implies time series of** a^*, b, $R(a^*+b)$, m^* **and** μ.
  - **A set of the values is chosen so that the model's prediction fit closely to the data.**

Estimation: Result



- **Kolmogorov-Smirnov test**
- **Null hypothesis: two datasets (actual and estimated) are from the same distribution**
- **Not rejected at the 1% significance level**

Estimation: Result (Table)

r	a_0	α	β	ρ_0
0.08	1.073	0.045	0.115	4.5

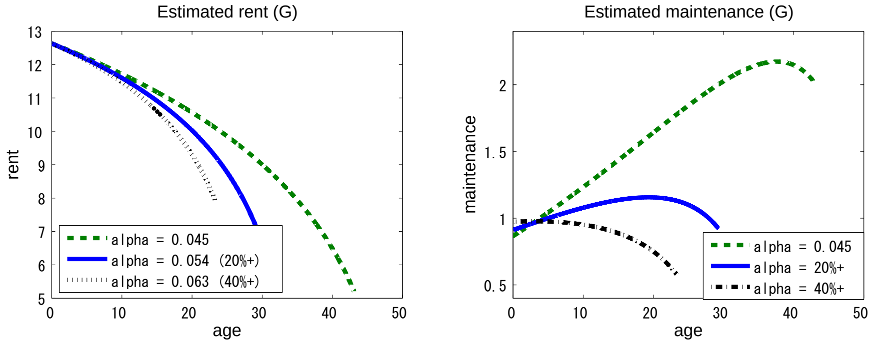
ρ_1	ρ_2	σ_0	σ_1	ζ
4.25	7.85	−149.85	16.71	0.05

Parameter estimates

	KS-statistics	p-value
rent	0.1625	0.22014
maintenance	0.1625	0.22014

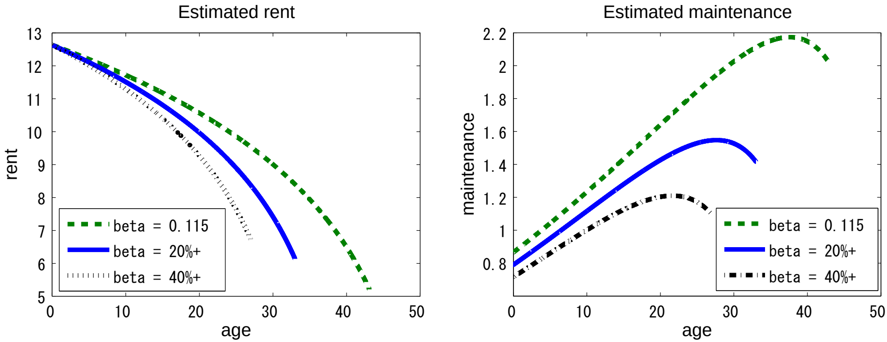
KS test (# observations = 90)

Counterfactual Simulation: α



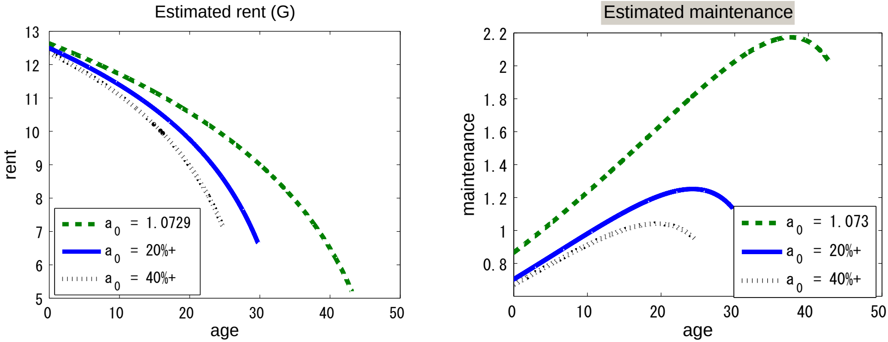
Optimal rent and maintenance expenditure when α increases by 20% and 40%

Counterfactual Simulation: β



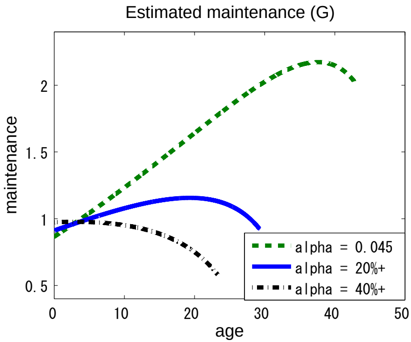
Optimal rent and maintenance expenditure when β increases by 20% and 40%

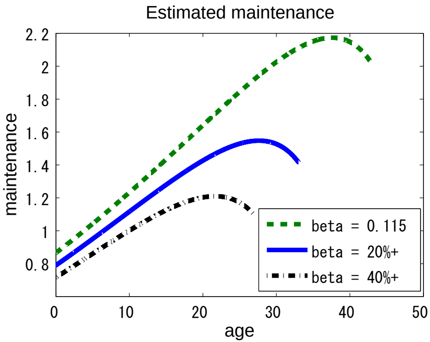
Counterfactual Simulation: a_0



Optimal rent and maintenance expenditure when a_0 increases by 20% and 40%

Counterfactual Simulation: Maintenance

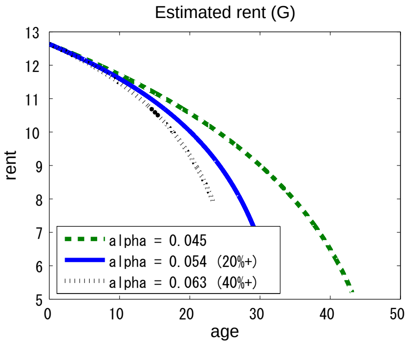




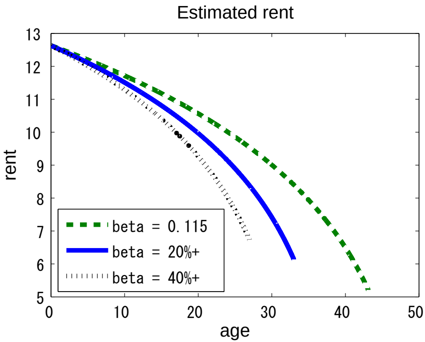
optimal maintenance when α increases by 20% and 40%

optimal maintenance when β increases by 20% and 40%

Counterfactual Simulation: Rent



optimal maintenance when α increases by 20% and 40%



optimal maintenance when β increases by 20% and 40%

Summary

- How to maintain a capital asset that is subject to wear and tear and obsolescence was examined
- An optimal maintenance pattern interestingly varies with asset types
- Deterioration and obsolescence could have different effects on an optimal maintenance pattern