Mixed Duopoly, Privatization and Subsidization in an Endogenous Timing Framework

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Abstract

This is the first paper to consider the endogenous timing in mixed duopoly with subsidization. Pal (1998) shows that the private leadership is always an equilibrium outcome in mixed duopoly without any subsidy. We find that private leadership disappears from equilibrium and Cournot and public leadership are likely to be equilibrium outcomes once the government provides both private and public firms with production subsidy. Furthermore, we find that privatization under the optimal subsidy never enhances social welfare. Especially, when firms have identical technologies, the 'irrelevance result' à la White (1996) — the first-best allocation can be attained by the same subsidy before and after privatization — holds even though production timings of firms are endogenized. Finally, we examine privatization with lobbying activities and show that such privatization leads to deterioration of social welfare.

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1 Introduction

This paper demonstrates how subsidization affects firms' behaviours in a mixed market or mixed duopoly in which public firms compete against private firms.[1] In particular, we focus on the importance of these firms' order of moves. Despite the large body of theoretical literature analyzing mixed oligopoly, the existing works have not conducted minute analyses on how this order of moves changes the effects of subsidization on firms' behaviours, profits and welfare. The purpose of this paper is to fill this gap by introducing subsidization into a mixed oligopoly model and by shedding light on how both public and private firms' order of moves influences their payoffs for various levels of subsidies.

Many studies suggest the importance of endogenous timing in mixed duopoly and oligopoly without any subsidy. Using the observable delay game formulated by Hamilton and Slutsky (1990), Pal (1998) shows that Stackelberg competition with private leadership and public leadership is equilibrium outcomes in mixed duopoly with constant marginal costs. Tomaru and Kiyono (2005) also show it in the setting where both public and private firms have increasing marginal costs. Moreover, Matsumura (2003a) find that in a different endogenous timing game, the private leadership can always be an equilibrium outcome while public leadership is never an equilibrium outcome.[2]

Their results indicate that private leadership is plausible in mixed duopoly. However, in the real world, Stackelberg competition with public leadership best describes many industries with pubilc firms. Industries such as telecommunications, electricity, natural gas, airline, and increasingly the postal sector, are dominated by former public monopolies with a first mover advantage. These industries more closely resemble Stackelberg competition with public leadership than that with private leadership. One of the aims of this paper is to resolve this inconsistency between the theory and the reality. As one way to do this, this paper introduces subsidization policy. We find that if the government subsidizes public and private firms and the level of subsidy is not so low, then public leadership becomes an equilibrium outcome and private leadership never.

In the broad literature, a strand analyses mixed oligopolies in the context of government subsidies designed to promote an increase in outputs of private firms. White (1996) shows that the government can realize the first-best allocation by utilizing the subsidization policy in Cournot mixed oligopoly. Surprisingly, he also shows that the first-best allocation is achievable by the same subsidy as in mixed oligopoly even after the privatization of a public firm. Starting with White (1996), a series of "irrelevance results" has been generated. Poyago-Theotoky (2001) demonstrates that the optimal subsidy is

[1]See DeFraja and Delbono (1990) and Nett (1993) for general reviews of mixed oligopoly models.

[2]For other papers on endogenous timing in mixed oligopoly, see Lu (2006) and Bácena-Ruiz (2007).

identical and that profits, outputs, and welfare are also identical irrespective of whether (i) all the public and private firms simultaneously move or (ii) the public firm acts as a Stackelberg leader or (iii) all firms are privatized and maximize profits. Myles (2002) proves this series of results in the setting of more general cost and demand functions.[3]

The conclusion from these results is that privatization is fruitless in terms of social welfare as long as the subsidization policy is available for the government. However, this conclusion relies critically on the assumption of the given timing of moves after privatization. Fjell and Heywood (2004) show that when the public leader is privatized and becomes the private leader, the optimal subsidy and welfare are reduced. Their result suggests the need to examine what move structures are likely to arise in mixed and private oligopoly when we consider privatization along with subsidization policy. To conduct this examination, we consider a stage of firms' selecting the production timing right after the stage of decision of the level of subsidies by the government and then compare the results from mixed and private duopoly with endogenous timing and subsidy. We find that public leadership and Cournot are equilibrium outcomes in mixed duopoly and that Cournot is an equilibrium outcome in private duopoly. Along with the results of Poyago-Theotoky (2001) and Myles (2001), our results imply that the irrelevance results hold when endogenising the production timing.

The findings of the literature mentioned above and ours, however, are dependent on the fact that the government has a discretion over the subsidy. It might lose its discretion if interest groups lobby and the political process is highly complicated. In this case, the government cannot set the optimal subsidy. Many papers on lobbying activities and campaign contribution show that the production subsidies and export subsidies are likely to be excessive. Then, we focus on the welfare and profits for given higher subsidy levels than the optimal subsidy and analyze the effects of privatization. The result of this analysis is that under such subsidies privatization decreases both profits of the private firm and welfare.

The remainder of this paper is organized as follows. Section 2 presents our model for comparing three types of competition, namely, Cournot competition and Stackelber competition with public leadership and Stackelberg competition with private leadership. In addition, it explains how the subsidy level influences welfare and both private and public firms' profits, also investigating the rankings of welfare and profits in the three types of competition. Section 3 discusses what the optimal subsidy is when the production timing is endogenized. Section 4 investigates the effect of privatization, and Section 5

[3]For other studies on the irrelevance results, see Tomaru (2006) and Kato and Tomaru (2007). Tomaru (2006) examines robustness of the irrelevance results from the view of partial privatization formulated by Matsumura (1998). Kato and tomaru (2007) show that the irrelevance results hold when private firms have other objectives than profits.

explores privatization with lobbying activities. Finally, Section 6 concludes this paper.

2 The model

We analyze mixed duopoly with public firm 0 and private firm 1 producing a single homogeneous good. The private firm maximizes its own profits. On the other hand, the public firm is owned by the welfare-maximizing government, so firm 0 maximizes the welfare. The output of firm i is q_i $(i = 0, 1)$, such that $Q = q_0 + q_1$ represents the total output. Let $P(Q)$ be the inverse demand function; further, each firm has the technology, represented by the cost function $C_i(q_i)$ $(i = 0, 1)$. Throughout this paper, we assume the following:

Assumption 1. *For any $Q \geq 0$, the inverse demand function $P(Q)$ is twice-continuously differentiable, where $P'(Q) < 0$ and $P''(Q) \leq 0$.*

Assumption 2. *For any $q_i \geq 0$, firm i's cost function $C_i(q_i)$ is twice-continuously differentiable, where $C_i'(q_i) > 0$ and $C_i''(q_i) > 0$.*[4]

Social welfare $W(q_0, q_1)$ and each firm's profit $\Pi_i(q_0, q_1, s)$, $(i = 0, 1)$ are given by

$$\begin{aligned} W(q_0, q_1) &:= \int_0^Q P(z)dz - C_0(q_0) - C_1(q_1), \\ \Pi_i(q_0, q_1, s) &:= P(Q)q_i - C_i(q_i) + sq_i, \end{aligned} \tag{1}$$

respectively, where s is the production subsidy. When s is negative, firms faces production taxes. Note that both firms' profits rely on subsidies while social welfare is not directly affected by the subsidies. This is because the subsidies for the firms are just lump sum transfers.

To complete the aims of our paper —analysing endogenous timing in mixed duopoly with subsidy — we need to explore how both firms' payoffs are influenced by subsidies under fixed move strucures: Cournot competition and Stackelberg competition with public and private leadership. For this purpose, we start by deriving both the private and public firms' reaction functions. The first-order conditions of

[4] If both public and private firms have constant marginal costs, the public firm's cost must be higher than the private firm's to preclude public monopoly. We consider the optimal subsidy in the later sections. Then, it is absolutely obvious that the private monopoly yields the first best outcome and that time structure (either Cournot, Stackeleberg or endogenous timing) does not matter. Thus, we assume increasing marginal costs to avoid such an obvious outcome. For further discussion on an importance of increasing marginal costs in mixed oligopoly, see Matsumura and Kanda (2005).

public firms 0 and 1 are given as

$$\frac{\partial W}{\partial q_0} = P(Q) - C_0'(q_0) = 0, \tag{2}$$

$$\frac{\partial \Pi_1}{\partial q_1} = P(Q) + P'(Q)q_1 - C_1'(q_1) + s = 0. \tag{3}$$

The second-order conditions for both firms' maximization problems are satisfied by virtue of Assumptions 1 and 2. These equations yield firm i's reaction function R_i, which satisfies

$$\frac{\partial R_0}{\partial q_1} = -\frac{P'(Q)}{P'(Q) - C_0''(q_0)} \in (-1, 0), \quad \frac{\partial R_1}{\partial q_0} = -\frac{P'(Q) + P''(Q)q_1}{2P'(Q) + P''(Q)q_1 - C_1''(q_1)} \in (-1, 0),$$

$$\frac{\partial R_1}{\partial s} = -\frac{1}{2P'(Q) + P''(Q)q_1 - C_1''(q_1)} > 0. \tag{4}$$

Hence, if a Cournot equilibrium exists, then it is globally stable[5] and is thus uniquely determined.[6]

2.1 Three types of move structures

First, we derive Cournot equilibrium under mixed duopoly. Let the superscript 'mC' denote Cournot equilibrium under mixed duopoly. The equilibrium outputs in Cournot competition are characterized by the first-order conditions (2) and (3). Then, we define them as $q_i^{mC}(s)$ $(i = 0, 1)$ and $Q^{mC}(s) = q_0^{mC}(s) + q_1^{mC}(s)$. For analysis, we examine the comparative statics under Cournot competition. Simple calculation yields

$$\Delta \cdot q_0^{mC\prime}(s) = R_0'(q_1) \cdot \frac{\partial R_1}{\partial s} < 0, \;\; \Delta \cdot q_1^{mC\prime}(s) = \frac{\partial R_1}{\partial s} > 0, \;\; \Delta \cdot Q^{mC\prime}(s) = \frac{\partial R_1}{\partial s}\left\{1 + R_0'(q_1)\right\},$$

where $\Delta = 1 - R_0'(q_1) \cdot (\partial R_1/\partial q_0) > 0$. Production subsidies increase the output of private firm 1 as well as total outputs while they decrease the output of public firm 0.

Second, we consider Stackelberg competition with public and private leadership. Since a Stackelberg leader chooses its output antispating the output of the follower, the public firm with leadership maximizes $\widehat{W}(q_0, s) := W(q_0, R_1(q_0, s))$ while the private firm with leadership maximizes $\widehat{\Pi}_1(q_1, s) := \Pi_1(R_0(q_1, s), q_1, s)$. We assume that these objective functions are concave, which yields the following first-order conditions;

$$\frac{\partial \widehat{W}}{\partial q_0} = P(q_0 + R_1(q_0, s)) - C_0'(q_0) + \left[P(q_0 + R_1(q_0, s)) - C_1'(R_1(q_0, s))\right] \cdot \frac{\partial R_1}{\partial q_0} = 0,$$

$$\frac{\partial \widehat{\Pi}_1}{\partial q_1} = P(R_0(q_1) + q_1) - C_1'(q_1) + \left[1 + R_0'(q_1)\right] P'(R_0(q_1) + q_1)q_1 + s = 0. \tag{5}$$

[5] This assumption is the standard Cournot adjustment process in duopoly. Under this process, it is a sufficient condition for the stability of the equilibrium that the absolute value of the slope of each firm's reaction function is less than 1.

[6] The existence of unique equilibrium is assured when each firm's marginal cost at zero output is lower than the price set at either private or public monopoly equilibrium by the other firm.

The equilibrium outputs in public and private leadership are derived from these equations and the reaction functions of the followers. Let the superscripts 'mL' and 'mF' denote public and private leadership, respectively. We define the equilibrium outputs in leadership structure 'mj' as $q_i^{mj}(s)$ $(j = L, F,\ i = 0, 1)$. Equilibrium total output, in turn, is given as $Q^{mj}(s) = q_0^{mj}(s) + q_1^{mj}(s)$. In addition, the payoffs of public firm 0 and private firm 1 are respectively as follows: $W^{mj}(s) = W(q_0^{mj}(s), q_1^{mj}(s))$ and $\Pi_1^{mj}(s) = \Pi_1(q_0^{mj}(s), q_1^{mj}(s))$.

To ensure that all the above equilibrium outputs q_i^{mj} $(i = 0, 1$ and $j = C, L, F)$ are positive, we should restrict the range of levels of subsidy. Define set S as follows: $S = \{s \mid q_i^{mj}(s) > 0,\ i = 0, 1\ and\ j = C, L, F\}$. Hereafter, we concentrate on the analysis of subsidized mixed duopoly for $s \in S$. Futher, we make an assumption on welfare functions, W^{mC}, W^{mL}, and W^{mF}, for analysis in the later sections. The assumption ensures to make welfare maximization problem with respect to subsidy level s sensible.

Assumption 3. *Three welfare functions, W^{mC}, W^{mL}, and W^{mF}, are concave in $s \in S$.*

2.2 Comparison among the Cournot equilibrium and two Stackelberg equilibria

Some existing works analyse the effect of subsidy on welfare in mixed duopoly. White (1996) shows that the government can attain the Pareto-efficient allocation by utilizing the optimal subsidy in Cournot competition. Poyago-Theotoky (2001) and Myles (2002) have shown that, even in public leadership, the government can also attain the alloaction by the same level of subsidy as in Cournot. This is called the "irrelevance result". We find that these results can be derived from our general setting with cost heterogeneity and further find that the Pareto-efficient allocation is attainable in private leadership but the optimal subsidy is different from that of White (1996).

Proposition 1. *Pareto-efficient allocation is achievable in all the three games, Cournot, public leadership, and private leadership, by the optimal subsidies. Moreover, the optimal subsidies in Cournot, s^C, and public leadership, s^L, are the same (i.e., $s^C = s^L := s^*$) but that in private leadership, s^F, is lower than s^*.*

Proof: See Appendix B.

In Japan, Japan Post was a major public firm which provided postal and delivery services until it was privatized in 2007. This firm had the small market shares in delivery service industry, whereas other private firms, Yamato Delivery and Nippon Express, kept in dominant positions. As in the delivery industry of Japan, some industries might more closely resemble Stackelberg competition with

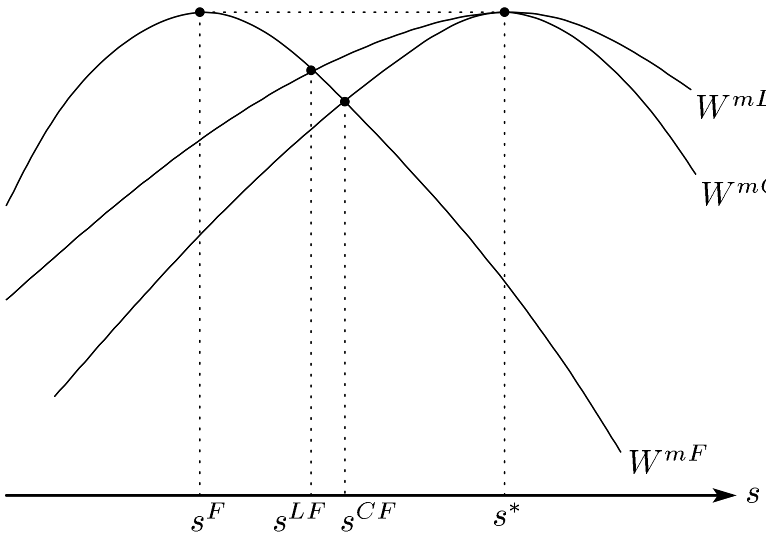

Figure 1: Welfare curves of three types of competition

private leadership than the other competition. Proposition 1 shows idiosyncracy of such competition and industries.

The result of Proposition 1, new to the literature, shows that private leadership is not irrelevant with Cournot and public leadership. The result seems surprising but the intuition is clear. In the case of private leadership, the private firm has an incentive to expand its production for any given level of subsidy in order to get the higher market share. In addition, the public firm tends to produce excessively because it takes into account not only its own profit but also consumer surplus. Thus, the lower subsidy suffices to reach the high level of outputs in Pareto-optimal allocation.

The diffrence of optimal subsidies in given move structures provides an importance of the level of subsidy in relationships of welfare in these structures. To understand the relationships, compare them for any level of subsidy. Figure 1 illustrates them. In this figure, the welfare in each game is drawn as a hump-shaped curve. As stated in Proposition 1, the maximal of the curves in Cournot and public leadership is reached at s^* while that of the curve in private leadership is reached at s^F which is smaller than s^*. All the three welfare curves is increasing in s when $s \leq s^F$ and decreasing in s when $s > s^*$. On the other hand, when $s^F < s \leq s^*$, welfare curves in Cournot and public leadership, W^{mC} and W^{mL}, are increasing whereas that in private leadership, W^{mF}, is decreasing. In this range of subsidy, relationships among welfare of three games are switched.[7] These results are summarized as Lemma 1

[7]Although not drawn in Figure 1, there is some possibility that curve W^{mF} intersects with curve W^{mL} in a certain level

Lemma 1. *Suppose that* s^{jF} *is the subsidy level such that* $W^{mj}(s)$ *equals to* $W^{mC}(s)$ *(*$j = C, L$*). Then we have*

$$\begin{array}{lll} (a) & W^{mL}(s) \geq W^{mC}(s) > W^{mF}(s) & \textit{if } s > s^{CF}, \\ (b) & W^{mL}(s) > W^{mF}(s) \geq W^{mC}(s) & \textit{if } s^{CF} \geq s > s^{LF}, \\ (c) & W^{mF}(s) \geq W^{mL}(s) > W^{mC}(s) & \textit{if } s^{LF} \geq s > s^{F}, \\ (d) & W^{mF}(s) > W^{mC}(s),\ W^{mL}(s) > W^{mC}(s) & \textit{otherwise}. \end{array}$$

Compared with the ranking of welfare, that of the private firm's profits is relatively simple, which is given in the following lemma;

Lemma 2. *Depending on the subsidy level, the following results are obtained regarding both firms' profits.*

$$\begin{array}{lll} (a) & \Pi_1^{mL}(s) \leq \Pi_1^{mC}(s) < \Pi_1^{mF}(s), & \textit{if } s \geq s^*, \\ (b) & \Pi_1^{mC}(s) < \Pi_1^{mL}(s),\ \Pi_1^{mC}(s) < \Pi_1^{mF}(s), & \textit{if } s < s^*, \end{array}$$

Proof: See Appendix.

Since it is obvious that the profits in private leadership is not smaller than that in Cournot, we only explain the intuition behind the relationship between the profits in public leadership and Cournot. Suppose that the subsidy is relatively low ($s < s^*$). In this case, the private firm produces less whereas the public firm produces more in Cournot, which means that the public firm is inefficient due to its increasing marginal cost. Thus, inefficient public firm as a Stackelberg leader can improve social welfare by transfer its production to the efficient private firm. Since this transfer increases the market share of the private firm, $\Pi_1^{mC}(s) < \Pi_1^{mL}(s)$ for $s < s^*$. On the other hand, if the level of subsidy is relatively high ($s \geq s^*$), the private firm produces excessively and, thus, is inefficient. Then, the public leader has an incentive to substitute its output for the output of the private firm, which leads to a decrease in the market share of the private firm and, thus, $\Pi_1^{mC}(s) \geq \Pi_1^{mL}(s)$.

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of subsidy $s < s^F$. However, this possibility never influences the later discussions. In addition, we can show that curve W^{mF} lies above curve W^{mC} for any $s < s^F$ and the reverse is true for any $s > s^*$ as long as $s \in S$.

3 Endogenous timing game

As discussed in the introduction, some studies investigate endogenous timing in mixed duopoly without any subsidy and then show that private leadership is likely to be an equilibrium outcome. In this section, we attempt to examine how their result could be altered if the government provides the public and private firms with production subsidy. Following Pal (1998), we apply the observable delay game of Hamilton and Slutsky (1990), in which firms simultaneously choose the production timing, and thereafter, produce their output at their production timing.

Our game considered in this section proceeds as follows. At stage 1, the government sets a unit production subsidy for firms. At stage 2, the firms simultaneously announce the period in which they will produce their output and are committed to this choice. Let $t_i \in \{1, 2\}$ be the time period chosen by firm i ($i = 0, 1$) at stage 2. Finally, at stage 3, each firm chooses the output level q_i at the period decided at stage 2. More precisely, if both the firms announce the same production period at stage 2, Cournot competition emerges at stage 3. Otherwise, when each firm selects a different period, Stackelberg competition appears in stage 3. We solve the subgame perfect equilibrium in this game by using backward induction.

Now, we proceed to stage 2, because stage 3 was described in section 2. At stage 2 public and private firms determine their production timings for any given level of subsidy. Pal (1998) and Tomaru and Kiyono (2005) show that the private leadership is always an equilibrium outcome of observable game in mixed duopoly without subsidy policy. Matsumura (2003a) also show that in a different endogenous timing game, the private leadership can always be an equilibrium outcome while public leadership is never an equilibrium outcome. However, we find that these results completely change once the government subsidize firms.

Proposition 2. *The following equilibria hold at stage 2:*

$$\begin{aligned}
&(a)\ (t_0, t_1) = (1, 1), && \textit{if } s > s^*,\\
&(b)\ (t_0, t_1) = (1, 1), (1, 2), && \textit{if } s = s^*,\\
&(c)\ (t_0, t_1) = (1, 2), && \textit{if } s^{CF} < s < s^*,\\
&(d)\ (t_0, t_1) = (1, 2), (2, 1), && \textit{otherwise}.
\end{aligned}$$

Proof: See Appendix B.

Surprisingly, Proposition 2 states that contrary to Pal, Tomaru and Kiyono, and Matsumura, the private leadership never appears as an equilibrium outcome of stage 2 when the level of subsidy is not low.

Let us turn to explanation to the intution behind Proposition 2 by using this lemma. In case (a) of Proposition 2, a large amount of subsidy promotes the excess production by the private firm. To mitigate total production costs due to this excess production, the public firm wants to reallocate production from the private firm to itself by acting agressively if this action is commitable. This is the same situation as that of private duopoly. Thus, both firms select period 1 in equilibrium. In case (b), that is $s = s^*$, we know that for this subsidy Cournot and public leadership is irrelevant in the sense that the first-best allocation prevails and Cournot and public leadership are indifferent for both firms. Thus $(t_0, t_1) = (1, 2)$ is added to equilibrium outcomes. In case (c), the subsidy is in midle range in which the private firm with a leader advantage produces more than in the Pareto-effcient allocation, but that without the advantage produces less. In order to avoid such overproduction by the private firm, the public firm tries to procuce in advance. As a result, only $(t_0, t_1) = (1, 2)$ becomes an equilibrium outcome. Finally, we explain case (d) in which the subsidy is too low and the private firm does not produce so much. This implies that a transfer of production by the overproducing public firm to the underproducing private firm decreases total costs and increases social welfare due to increasing marginal costs. Hence, the public firm acts so as to realize either public leadership or private leadership.

Now, we explore the analysis of stage 1. In this stage, the government sets the subsidy to maximize social welfare. According to the analysis in the previous section, it seems that the government should set the subsidy s to s^* or s^F. However, this is not obvious. Proposition 2 implies that the realized competition as the equilibrium at stage 2 for one subsidy can differ from that for another subsidy. Thus, even though the government sets the subsidy s to s^* or s^F, it is not certain whether the Pareto-optimal allocation can be achieved.

Then, let us consider what the social welfare function faced by the government at stage 1 is. Note that two types of competition appear in the equilibrium of stage 2 in (b) and (d) of Lemma 2. In these cases, the welfare function is indeterminate because the government does not know which of the two types of competition are actually a priori. To preclude this indetermination, we assume that the government has the expected welfare function. Let $\mu \in (0, 1)$ be the probability that induces Cournot competition and $1 - \mu$ be the probability that causes Stackelberg competition with public leadership in (b). In addition, define $\lambda \in (0, 1)$ as the probability of Stackelberg competition with public leadership, and $1 - \lambda$ as the probability of Stackelberg competition with private leadership in (d). Thus, the social welfare $\widetilde{W}^m(s)$ that the government encounters is as follows:

$$\widetilde{W}^m(s) = \begin{cases} W^{mC}(s), & \text{if } s > s^*, \\ W^{mL}(s), & \text{if } s^{CF} < s \leq s^*, \\ \lambda W^{mL}(s) + (1-\lambda) W^{mF}(s), & \text{otherwise,} \end{cases} \tag{6}$$

where, if $s = s^*$, the social welfare becomes $\mu W^{mC}(s^*) + (1-\mu) W^{mL}(s^*) = W^{mL}(s^*)$. The government maximizes this welfare function with respect to s. If the government sets s^F, then the Pareto-efficient allocation is not attained, because $\lambda \neq 0$. Meanwhile, subsidy s^* maximizes welfare function (6) and yields the Pareto-efficient allocation.

Proposition 3. *Suppose that $s^* \in S$. The subgame perfect equilibrium under mixed duopoly is characterized as follows:*

$$(q_0, q_1, s) = (q_0^{mC}(s^*), q_1^{mC}(s^*), s^*) = (q_0^{mL}(s^*), q_1^{mL}(s^*), s^*).$$

Proposition 3 states that when the government optimally chooses the subsidy, only Cournot and/or private leadership are the equliribium outcome of endogenous timing and private leadership is not. In the real world, there are many situations where Cournot and Stackelberg with public leadership are suitable. Industries such as telecommunications, electricity, and postal sector, are dominated by former public monopolies with a first mover advantage.[8] In addition, the result of Proposition 3 strengthens an importance in the irrelevance result of Poyago-Theotoky (2001) and Myles (2002), in the sense that Proposition 3 shows that Cournot and public leadership are likely to arise in mixed duopoly with subsidization and private leadership in which the irrelevance result does not hold is not likely.

4 Privatization

White (1996) discusses the other irrelevance result than that of Poyago-Theotoky (2001) and Myles (2002). He shows that the government is able to realize the Pareto-efficient allocation in Cournot mixed and private oligopoly by setting the same optimal subsidy. In this section, we examine whether or not this irrelevance hold even in our endogenous timing model.

For this purpose, we first derive the equilibrium of the endogenous timing model in private duopoly. Because firm 0 maximizes its own profits (1) after privatization, the first-order condition for firm 0's

[8]For detail examples of such industries, see Fjell and Heywood (2002).

profit maximization in Cournot competition gives the reaction function of firm 0, $R_0^p(q_1, s)$. This reaction function satisfies

$$\frac{\partial \Pi_0}{\partial q_0} = P(R_0^p(q_1, s) + q_1) + P'(R_0^p(q_1, s) + q_1)R_0^p(q_1, s) - C'(R_0^p(q_1, s)) + s = 0.$$

Firm 1 also maximizes its profits, and thus, its reaction function still remains $R_1(q_0, s)$. As in section 2, we define firms' equilibrium outputs in Cournot competition in private duopoly as follows: $q_i^{pC}(s)$ and $Q^{pC}(s) = q_0^{pC}(s) + q_1^{pC}(s)$. Further, equilibrium outputs in Stackelberg competition with firm 0's leadership (pL) and with firm 1's leadership (pF) are given as $q_i^{pj}(s)$ and $Q^{pj}(s) = q_0^{pj}(s) + q_1^{pj}(s)$ ($i = 0, 1$, $j = L, F$). Then, we define firm i's profits as $\Pi_i^{pj}(s) := \Pi_i(q_0^{pj}(s), q_1^{pj}(s), s)$ ($i = 0, 1$, $j = C, L, F$). As is well known, the following result is derived in private duopoly.

Lemma 3. *For all subsidies, each profit function of privatized firm 0 and private firm 1 in the private duopoly satisfies the following relationships:*

$$\Pi_0^{pF}(s) < \Pi_0^{pC}(s) < \Pi_0^{pL}(s), \quad \Pi_1^{pL}(s) < \Pi_1^{pC}(s) < \Pi_1^{pF}(s).$$

We now examine the decision of the production timing at stage 2, that is, each firm announces the production period at stage 3. Lemma 3 implies that each firm has the incentive to be the leader. Thus, each firm always chooses the period $t_i = 1$ ($i = 0, 1$) in this stage, such that for any subsidy, $(t_0, t_1) = (1, 1)$ is realized as the equilibrium in this observable delay game. Therefore, Cournot competition occurs at $t_i = 1$ ($i = 0, 1$) in stage 3.

In stage 1, the government sets the subsidy level to maximize social welfare. Then, it recognizes that Cournot competition appears as the equilibrium, so that its objective function becomes

$$\widetilde{W}^p(s) = W(q_0^{pC}(s), q_1^{pC}(s)) = \int_0^{Q^{pC}(s)} P(z)dz - C_0\big(q_0^{pC}(s)\big) - C_1\big(q_1^{pC}(s)\big).$$

The first-order condition for $\widetilde{W}^p(s)$ leads to the following optimal subsidy s^{**}:

$$s^{**} = \underset{\{s\}}{\arg\max}\ \widetilde{W}^p(s). \tag{7}$$

Thus, the subgame perfect equilibrium in the private duopoly after the privatization of firm 0 is characterized in the following proposition.

Proposition 4. *Suppose that $s^{**} \in S$. Under privatization, the following subgame perfect equilibrium is realized:*

$$(q_0, q_1, s) = (q_0^{pC}(s^{**}), q_1^{pC}(s^{**}), s^{**}).$$

We now turn to the comparison between the subgame perfect equilibria derived under mixed and private duopolies. In mixed duopoly, Cournot and public leadership appears in equilibrium. As shown in Proposition 1, in these market structures, one control variable of uniform production subsidy does well for the Pareto-efficient allocation. On the other hand, due to asymmetry of cost functions, uniform subsidy does not always yields the Pareto-efficient allocation in private Cournot duopoly. In this case, the irrelevance result à la White does not hold. Without any heterogeneity of cost functions, this irrelevance is recovered, since Cournot competition in both mixed and private doupoly follows in endogenous timing, which is the same situation as that of White (1996).

Proposition 5. *Suppose that public and private firms face the same cost functions. Then, even if we consider each firm's endogenous production timing, when the government utilizes output subsidization, whether this situation is that of mixed duopoly or private duopoly, identical is the optimal subsidy that gives the first-best allocation.*

Fjell and Heywood (2004) demonstrate that if the privatized firm is still a Stackelberg leader, then the optimal subsidy of private oligopoly is different from that of mixed oligopoly, and moreover, privatization reduces social welfare. This suggests that after privatization, the first-best allocation may require a subsidy other than that in mixed oligopoly when a change in the market and competition structures accompanying privatization is taken into account. However, Proposition 5 states that the results of White (1996) hold even though both the private and public firms choose their own production timings.

5 Subsidization policy and privatization with lobbying

Although the above discussion on optimal subsidy and privatization may attract our interest, we should bear in mind that it presumes that the omniscient government has a free discretion over the determination of the level of subsidy. Past literature on subsidized mixed oligopoly assumes that the government has perfect controlability over setting subsidy and thus can set the optimal subsidy. Yet, this may not be the case when lobbying by interest groups and a highly complicated political process are considered. Many papers on lobby activities and campaign contribution have shown that production subsidies and export subsidies are likely to be excessive. This section attempts to examine the effect of privatization for given higher level of subsidy than optimal one.

Unfortunately, in our present setting, what we can say is limited due to its generality of demand and cost functions. To make our discussion clear, we specify these functions. The inverse demand is assumed to be linear and is given by $P = a - Q$. Following DeFraja and Delbono (1989) and ohter existing works, we also assume that the firms face the symmetric quadratic cost functions, $C_i(q_i) = \frac{1}{2}kq_i^2$. The simple calculation yields $s^* = a/(k+2)$. As stated in the previous section, symmetry of cost functions equalizes the optimal subsidy in mixed Cournot duopoly s^* with that in private Cournot duopoly s^{**} and this level of subsidy leads to the Pareto-optimal allocation in these two types of duopoly, which implies that when the government can set the subsidy $s^* = s^{**}$, privatization does not make any change in welfare and the profit of private firm.

Suppose that the government is forced to set the level of subsidy to $s > s^* = s^{**}$ by the lobbying activities of the owners of the private firm. For this level of subsidy, before and after privatization of the public firm, Cournot competition is the only equilibrium outcome of our endogenous timing game. Based on this, the difference between welfare of private and mixed duopoly is

$$\widetilde{W}^p(s) - \widehat{W}^m(s) = \widetilde{W}^p(s) - W^{mC}(s) = -\frac{(k^3 + 3k^2 + k + 1)\left\{a - (k+2)s\right\}^2}{2(k+3)^2(k^2+3k+1)^2} < 0.$$

This is because the excessive subsidy stimulates not only the exsiting private firm but also the privatized firm, which results in a large amount of total production costs. Similarily the difference of private firm's profits is

$$\Pi_1^{pC}(s) - \Pi_1^{mC}(s) = -\frac{(k+2)\left\{(k+2)s - a\right\}\left\{(2k^2+7k+4)s + (2k^2+6k+1)a\right\}}{(k+3)^2(k^2+3k+1)^2} < 0.$$

Thus, privatization with lobbying activities decreases social welfare as well as the profit of the private firm. This decrease in profits gives owners of the private firm incentives to oppose to and to hamper privatization.

Figure 2 illustrates the relationship between welfare before and after privatization. In this figure the thick curve represnts welfare in mixed duopoly, $\widetilde{W}^m$, and the thick broken curve represents welfare in private duopoly. This figure demonstrates that privatization decreases social welfare for all the range of subsidy.[9] The above results are summarized as Proposition 6

Proposition 6. *Suppose that lobbying activities result in excessive subsidy ($s > s^*$). Then, privatization decreases not only the profit of private firm but also social welfare. Moreover, it decreases welfare even though subsidy is not excessive ($s < s^*$).*

We should notice that our results in this proposition rely on symmetry and specificity in functions. In fact, we can not confirm whether or not privatization deteriorates social welfare for $s > s^*$ in general

[9] In this figure, the range $(\underline{s}, \bar{s})$ is set S. $\underline{s}$ is the subsidy such that the private firm is not active in Cournot and private leadership. $\bar{s}$ is such that the public firm is not active in private leadership.

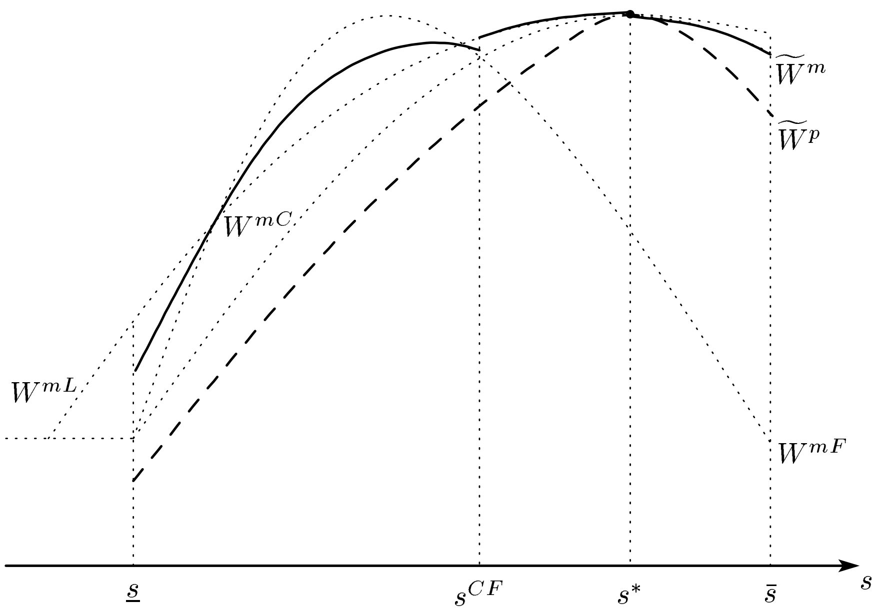

Figure 2: Comparison between mixed and private duopoly

models. One of the reasons is indeterminancy of whether s^* is larger than s^{**}. Suppose that s^* is smaller than s^{**}. In this case, welfare in mixed duopoly is decreasing in s for $s > s^*$ and that in private duopoly is increasing in s for $s^* < s^{**}$. Then, we cannot exclude the case where the maximal of welfare in private duopoly is larger than that in mixed duopoly for $s = s^{**}$. Nevertheless, we can easily show that the profit of the existing private firm always decreases for the relevant range of subsidy even in our general model. This implies that privatization in subsidized mixed duopoly with lobbying activities is likely to be opposed.

6 Concluding remarks

In this paper, we investigate the endogenous timing in mixed duopoly with subsidization by using the observable delay game by Hamilton and Slutsky (1990). First, we find that for not so much low level of subsidy, Stackelberg competition with public leadership and Cournot are likely to appear as equilibrium outcomes and that with private leadership does not become an equilibrium outcome. This is contrary to the results of Pal (1998), Tomaru and Kiyono (2005), and Matsumura (2003a). Second, we show that the government can achieve the Pareto-efficient allocation in Stackelberg with private leadership by providing firms with the lower level of subsidy than the optimal one in Cournot mixed duopoly. This result implies that if private leadership is in subgame perfect equilibrium of our endogenous timing game with subsidy, there is some possiblity that irrelevance result à la White (1996) does not hold. However, we show that public leadership and Cournot are in equilibrium of mixed duopoly and that Cournot is in equilibrium of private duopoly. Along with the results of Poyago-Theotoky (2001) and Myles (2002), these findings indicate that the irrelevance results à la White (1996) hold even when we consider the endogenous timing. Finally, we examine the effect of privatization on profits of the private firm and social welfare for the higher level of subsidy than the optimal subsidy. It is shown that such privatization always decreases both of them.

We make some remarks on our model and findings. This paper assumes that public and private firms compete in quantity. As Bárcena-Ruiz (2007) presents the endogenous timing model where these firms compete in price, we can easily extend our model to price competition. In the linear demand model of differentiated goods, simple calculation shows that welfare curves W^{mL} and W^{mC} are on the left of curve W^{mF}, which alters the outcomes of Proposition 2; (a) $(t_0, t_1) = (1, 2), (2, 1)$ for $s < s^*$, (b) $(t_0, t_1) = (1, 2)$ for $s = s^*$, (c) $(t_0, t_1) = (1, 1), (1, 2)$ for $s^* < s \leq s^{CF}$, and (d) $(t_0, t_1) = (1, 1)$ for $s > s^{CF}$. Further, we assume that the number of the private firm is one. In Pal (1998) which investigates mixed oligopoly without any subsidy, the result depends crucially on the number of private firms. He shows that public and private leadership are in equilibrium of mixed oligopoly with a small number of private firms but that only the private leadership is in equilibrium with a large number of private firms. In the model of linear demand and symmetric quadratic cost, we can show that one private firm always has an incentive to deviate from Cournot and public leadership under the optimal subsidy level. It indicates that there is no symmetric equilibrium on the private firms.

Finally, we discuss the possibility of extention. we adopted the observable delay game by Hamilton and Slutsky (1990). There may be circumstances under which this game is inadequate to examine endogenizing the production timings. Saloner (1987) and Matsumura (2003a) use the two-period model to analyze the manner in which each firm decides how much output to produce in each period. It is of interest that we investigate how formulations other than that of Hamilton and Slutsky (1990), like Saloner (1987) and Matsumura (2003a), change the results. In addition, we consider only full privatization. In reality, many privatized firms are owned by private and public sector. It might also be interesting to examine how our results such as Proposition 2 are altered when we apply the approach of Matsumura (1998) that models such partial privatization. Considering these problems remains an issue for future research.

Appendix

Proof of Proposition 1

First, we prove that the Pareto-efficient allocation can be realized in all three games. For this, we show that there exist subsidies such that both firm's marginal costs are tantamount to price in Cournot competition and two types of Stackelberg competition. Let us consider the case where the government sets the production subsidy to $s^C = -q_1^{mC}(s^C)P'(Q^{mC}(s^C))$ in Cournot game. Under this level of subsidy, the first order condition of private firm is given by

$$\begin{aligned}\frac{\partial \Pi_1}{\partial q_1} &= P(Q^{mC}(s^C)) + P'(Q^{mC}(s^C))q_1^{mC}(s^C) - C_1'(q_1^{mC}(s^C)) + s^C q_1^{mC}(s^C), \\ &= P(Q^{mC}(s^C)) - C_1'(q_1^{mC}(s^C)).\end{aligned}$$

Thus, along with the fact that the public firm is welfare-maximizer, the Pareto-optimal allocation can be attained in Cournot game.

In the case of Stackelberg competition with public leadership, similarily apply subsidy $s^L = -q_1^{mL}(s^L)P'(Q^{mL}(s^L))$.

$$\begin{aligned}0 = \frac{\partial \widehat{W}}{\partial q_0} &= P(Q^{mL}(s^L)) - C'(q_0^{mL}(s^L)) + \left\{P(Q^{mL}(s^L)) - C'(q_1^{mL}(s^L))\right\} \cdot \frac{\partial R_1}{\partial q_0}, \\ &= P(Q^{mL}(s^L)) - C'(q_0^{mL}(s^L)).\end{aligned}$$

In the case of Stackelberg competition with private leadership, suppose that the government selects $s^F = -P'(Q^{mF}(s^F))q_1^{mF}(s^F)[1 + R_0'(q_1^{mF}(s^F))]$. Then both the public and private firms' first-order conditions (2) and (5) are given as

$$P(Q^{mF}(s^F)) - C_0'(q_0^{mF}(s^F)) = P(Q^{mF}(s^F)) - C_1'(q_1^{mF}(s^F)) = 0.$$

Hence, in any of two Stackelberg games, the Pareto-optimal allocation is acchieved.

Next, we prove that $s^F < s^C = s^L$. For convenience, we define the output level q_i^* as

$$P(q_0^* + q_1^*) = C_i'(q_i^*), \qquad i = 0, 1. \tag{8}$$

From the definition of s^C and s^L, we obtain

$$s^C = -q_1^{mC}(s^C)P'(Q^{mC}(s^C)) = -q_1^* P'(q_0^* + q_1^*) = -q_1^{mL}(s^L)P'(Q^{mL}(s^L)) = s^L.$$

In addition, $s^F = -P'(q^* + q^*)q^*\,[1 + R_0^{m\prime}(q^*)]$. Since $R_0'(\cdot) < 0$, we have $s^F < s^C = s^L$. ■

Proof of Lemma 2

To prove (a) and (b), we first show that $q_0^{mL}(s) \leq q_0^{mC}(s)$ if $s \leq s^*$ and $q_0^{mL}(s) > q_0^{mC}(s)$ if $s > s^*$. Define $f(s) := s + P(Q^{mC})q_1^{mC}(s)$. Notice that $f(s^*) = 0$ and the differential of this function f is positive. In fact,

$$\begin{aligned}
f'(s) &= 1 + \left[P'(Q^{mC}(s)) + P''(Q^{mC}(s))q_1^{mC}(s)\right] q_1^{mC\prime}(s) + P''(Q^{mC}(s))q_1^{mC}(s) \cdot q_1^{mC\prime}(s), \\
&= 1 + \frac{\frac{\partial R_1}{\partial s} \cdot \left[P'(Q^{mC}(s)) + P''(Q^{mC}(s))q_1^{mC}(s)\right]}{1 - R_0'(q_1^{mC}(s)) \cdot \frac{\partial R_1}{\partial q_0}} + P''(Q^{mC}(s))q_1^{mC}(s) \cdot q_1^{mC\prime}(s), \\
&= 1 + \frac{\frac{\partial R_1}{\partial q_0}}{1 - R_0'(q_1^{mC}(s)) \cdot \frac{\partial R_1}{\partial q_0}} + P''(Q^{mC}(s))q_1^{mC}(s) \cdot q_1^{mC\prime}(s), \\
&= \frac{1 + (1 - R_0'(q_1^{mC}(s))) \cdot \frac{\partial R_1}{\partial q_0}}{1 - R_0'(q_1^{mC}(s)) \cdot \frac{\partial R_1}{\partial q_0}} + P''(Q^{mC}(s))q_1^{mC}(s) \cdot q_1^{mC\prime}(s), \\
&> 0.
\end{aligned}$$

Thus, by the private firm's first-order condition (3), we obtain the following fact;

$$s \gtreqless s^* \quad \Longleftrightarrow \quad P(Q^{mC}(s)) - C'(q_1^{mC}(s)) \lesseqgtr 0.$$

Further, evaluating $\partial\widetilde{W}/\partial q_0$ at $q_0^{mC}(s)$, we find

$$\begin{aligned}
\left.\frac{\partial\widehat{W}}{\partial q_0}\right|_{q_0 = q_0^{mC}(s)} &= P(Q^{mC}(s)) - C'(q_0^{mC}(s)) + \left[P(Q^{mC}(s)) - C'(q_1^{mC}(s))\right] \cdot \frac{\partial R_1}{\partial q_0}, \\
&= \left[P(Q^{mC}(s)) - C'(q_1^{mC}(s))\right] \cdot \frac{\partial R_1}{\partial q_0},
\end{aligned}$$

Thus, the second-order condition of the public firm as a leader gives

$$s \gtreqless s^* \quad \Longleftrightarrow \quad q_0^{mC}(s) \lesseqgtr q_0^{mL}(s).$$

We now procede to proof of Lemma 2 (a) and (b). Since private firm 1 as a Stackelberg leader can choose its output to prevent its profit from becoming lower than Π_1^{mC}, and $\widehat{\Pi}_1$ is strictly concave, we obtain $\Pi_1^{mC}(s) < \Pi_0^{mF}(s)$ for any s. In order to prove the relationship between $\Pi_1^{mC}(s)$ and $\Pi_1^{mL}(s)$, we define $\bar{\Pi}_1(q_0, s) := \Pi_1(q_0, R_1(q_0, s), s)$. We should notice that $\bar{\Pi}_1(q_0^{mC}(s), s) = \Pi_1^{mC}(s)$ and $\bar{\Pi}_1(q_0^{mL}(s), s) = \Pi_1^{mL}(s)$. Then, from the definition of R_1,

$$\frac{\partial\bar{\Pi}_1}{\partial q_0} = \frac{\partial\Pi_1}{\partial q_0} + \frac{\partial\Pi_1}{\partial q_1} \cdot \frac{\partial R_1}{\partial q_0} = \frac{\partial\Pi_1}{\partial q_0} + 0 \cdot \frac{\partial R_1}{\partial q_0} = \frac{\partial\Pi_1}{\partial q_0} < 0.$$

Since $q_0^{mL}(s) \leq q_0^{mC}(s)$ if $s \leq s^*$ and $q_0^{mL}(s) > q_0^{mC}(s)$ if $s > s^*$, we get $\Pi_1^{mL}(s) \leq \Pi_1^{mC}(s)$ if $s \leq s^*$ and $\Pi_1^{mL}(s) > \Pi_1^{mC}(s)$ if $s > s^*$. ■

Proof of Proposition 2

Consider the following four cases: (a) $s > s^*$, (b) $s = s^*$, (c) $s^{CF} < s < s^*$ and (d) $s < s^{CF}$.

(a) $s > s^*$

In this case, we know that $W^{mL}(s) > W^{mC}(s) > W^{mF}(s)$ and $\Pi_1^{mF}(s) > \Pi_1^{mC}(s) > \Pi_1^{mL}(s)$. Thus, an act of production at period 1, i.e. $t_i = 1$ $(i = 0, 1)$, is a dominant strategy for both the firms. For $s > s^*$, the equilibrium is $(t_0, t_1) = (1, 1)$.

(b) $s = s^*$

In this case, we find that $W^{mL}(s) = W^{mC}(s) > W^{mF}(s)$ and $\Pi_1^{mF}(s) > \Pi_1^{mC}(s) = \Pi_1^{mL}(s)$. The public firm's best responses are $t_0 = 1$ for $t_1 = 1$ and $t_0 = 1$ and $t_0 = 2$ for $t_1 = 2$. On the other hand, those of the private firm are $t_1 = 1$ and $t_1 = 2$ for $t_0 = 1$ and $t_1 = 1$ for $t_0 = 2$. Thus, the equilibrium is $(t_0, t_1) = (1, 1),\ (1, 2)$.

(c) $s^{CF} < s < s^*$

In this case, social welfare and the private firm's profits satisfy $W^{mL}(s) > W^{mC}(s) > W^{mF}(s)$, $\Pi_1^{mF}(s) > \Pi_1^{mC}(s)$ and $\Pi_1^{mL}(s) > \Pi_1^{mC}(s)$. The public firm's best responses are $t_0 = 1$ for $t_1 = 1$ and $t_0 = 1$ for $t_1 = 2$, and those of the private firm are $t_1 = 2$ for $t_0 = 1$ and $t_1 = 1$ for $t_0 = 2$. Thus, the equilibrium is $(t_0, t_1) = (1, 2)$.

(d) $s < s^{CF}$

In this case, we find that $W^{mF}(s) > W^{mC}(s)$, $W^{mL}(s) > W^{mC}(s)$, $\Pi_1^{mF}(s) > \Pi_1^{mL}(s)$ and $\Pi_1^{mL}(s) > \Pi_1^{mC}(s)$. The public firm's best responses are $t_0 = 2$ for $t_1 = 1$ and $t_0 = 1$ for $t_1 = 2$, and those of the private firm are $t_1 = 2$ for $t_0 = 1$ and $t_1 = 1$ for $t_0 = 2$. Hence, the equilibrium is $(t_0, t_1) = (1, 2),\ (2, 1)$. ■

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