Innovative Interaction in Mixed Market *

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Abstract

We investigate an interaction of innovative effort in a mixed duopoly industry. Committing to induce the manager of public firm less effort can improve the expected social welfare when decisions on innovative effort are ordinarily strategic substitute between public and private firms. We show that employing a manager with less wealth can serve for the commitment since it makes the manager’s limited liability constraints be binding more tightly and inducing higher effort more expensive. In such circumstances, privatization is always harmful, when potential buyers of the public firm have sufficient wealth (or easily increase his capital) or when the government can easily alter the buyer, since it destroys the commitment device. **JEL Classification:** D43; H42; L13; L32; L33

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1 Introduction

In recent years, studies of mixed markets have become popular to a degree. Competitions between public sector and private firms are common in most countries,

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especially in European countries, Japan, and BRICs. In mixed oligopoly industries, decisions of public sector can affect those of private firms. Then the public sector’s decision influences the social welfare directly and indirectly.

Most literatures analyzing mixed oligopoly focus on the competitions in goods markets (Merrill and Schneider (1966), Harris and Wiens (1980), Beato and Mas-Colell (1984), Cremer et al. (1989), and so on). Moreover, a number of literatures analyze the impact of privatization of public firm in mixed oligopoly industry (De Fraja and Delbono (1989), Fershtman (1990), Matsumura (1998), Matsumura and Matsushima (2003), and so on). ¹ On the contrary, Poyago-Theotoky (1998), Nishimori and Ogawa (2002), and Ishibashi and Matsumura (2005) analyze innovative investments. But these literatures do not analyze any agency problems.

There are several studies of agency problems in public firms. De Fraja (1993) shows that public firms achieve high productive efficiency, if the public owner can offer a complete contract, in a model with moral hazard and adverse selection. Schmidt (1996) analyzes the effect of privatization of public firms within an incomplete contracting approach. He shows that privatization enhances the productive efficiency although it harms the efficiency of allocation, by contraries. Hart et al. (1997) and Corneo and Rob (2003) analyze agency problems within multi-task models. Hart et al. (1997) present that incentive contracts can enhance an incentive of cost-reducing investment but harm the quality of the service with property rights approach. Corneo and Rob (2003), on the contrary, present that public owners offer less intensive incentive-contracts when the agents get a private benefit from one task (cooperative task) which is not observable. ²

¹ See, for example, Nett (1993) for a survey of mixed oligopoly literatures.
² See Shleifer (1998) to survey on studies about problems derived from public or private ownership.
Several studies deal with interactions between contracts and competitions in oligopoly (duopoly) markets. Fershtman and Judd (1987) is an early literature concerning markets and contracts. They present a model using contracts as a tool of strategic commitment in a quantitative competition. Barros (1995) introduces this structure into a model of mixed duopoly market and shows the difference of incentive schemes between public and private firms. These literatures above consider the ex-post allocations only. Schmidt (1997) is a literature considering both ex-post allocation and ex-ante investment. He shows that an increase in competition affects the incentive of innovative investment by two ways. One is a ‘threat-of-liquidation effect’, which implies that an increase in competition increases the effort level because it leads to a reduction of profit and liquidation of the business when the manager fails in the innovation. The other is ‘value-of-a-cost-reduction effect’, which implies that an increase in competition reduces marginal benefit of innovation. Although he analyzes a relation between market structures and incentives of innovative investment, he does not consider an interaction of investments among firms and the existence of public firms.

We introduce an agency problem in public firm into a mixed duopoly model. The result shows that committing to induce the manager of public firm less effort can improve the expected social welfare. Contracting an incentive contract which induces the manager less aggressive is not sustainable as a commitment device since government has an incentive to recontract with him. We show that employing a

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3 A recent literature similar to this literature is Nishimori and Ogawa (2005). They extend Barros (1995) to a two period model and show that public firm prefers a short-term contract while private firm prefers a long-term contract.

4 For a survey on this class of literatures, see Chapter 13 in Bolton and Dewatripont (2004) for example.
manager with less wealth can serve for the commitment since it makes the manager’s limited liability constraints be binding more tightly and inducing higher effort more expensive. Privatization is always harmful, when potential buyers of the public firm have sufficient wealth (or easily increase his capital) or when the government can easily alter the buyer, since it destroys the commitment device.

The rest of this article is organized as follows: In section 2, we formulate the basic model. Section 3 presents the benchmark outcome without agency problem, the effect of agency problem (main result), and the effect of privatization. Finally, we present concluding remarks in Section 4.

2 Basic Model

2.1 Players

Consider an industry with two firms producing a homogeneous good. One (firm 0) is owned by a government (G) and the other (firm 1) is owned and managed by a private entrepreneur (P).\(^5\) G is unable or unwilling to manage their firms by herself and must employ one manager. We describe the manager employed for firm 0 as B.

The objective for P under a certain technical environment is the profit generated from providing the good, \(\pi_1(q)\), subtracted by the disutility of effort engaged in an innovative investment. That for B is the wage \(w\) subtracted by the disutility of the

\(^5\) We impose this assumption that P is the owner-manager of firm 1 in order to focus on an agency problem in firm 0. Even when P must employ a manager for her firm, she is willing to employ a manager who raises no agency problem.
effort. G’s objective is generally written as

\[ Y(q) = \pi_0(q) - w + \alpha CS(q) + \beta(\pi_1(q) - e_1) + \zeta(w - e_0), \]

where \( \alpha, \beta, \) and \( \zeta \) are constants in \([0, 1]\), \( CS(\cdot) \) denotes the consumer surplus, and \( e_i (i = 0, 1) \) is the disutility of the effort.\(^6\) When every coefficient is equal to one, the government wants to maximize the social welfare

\[ W(q, e) = CS(q) + \sum_{i=0, 1} \pi_i(q) - \sum_{i=0, 1} e_i. \] (1)

\( q \) denotes the quantity vector \((q_0, q_1)\), which affects the consumer surplus \( CS \) and the profit of both firms, \( \pi_0 \) and \( \pi_1 \). On the contrary, \( e \) denotes the effort vector, which does not affect the values in a certain environment but does affect the distribution of the environment as explained later. Suppose that both \( \alpha \) and \( \beta \) equal zero. We impose this assumption in order to focus on the effect of the agency problem in the public firm (firm 0). Then G’s objective is rewritten as

\[ Y(q) = \pi_0(q) - w + CS(q) + \pi_1(q) - e_1 + \zeta(w - e_0). \] (2)

Suppose all agents, \( G, P, \) and \( B, \) are risk neutral. Moreover, we suppose that the \( P \) has sufficient wealth but the managers have only insufficient wealth. This assumption implies that \( G \) confronts limited liability constraints in contracting with \( B \).

We can characterize \( B \) with \( \hat{\omega} \), where \( \hat{\omega} \) denotes the initial wealth of the manager. We suppose the reservation utility of managers are \( \hat{\omega} \). The initial wealth can relax the limited liability constraints directly. We endogenously analyze \( G \)’s decision on employing a manager \((B)\) later.

\(^6\) We explain the role of this effort in the next subsection.
2.2 Technology

There are two production instruments, firm 0 and firm 1, as mentioned in the previous subsection. Each firm has a technology producing a homogenous good. The productivity of the good depends on a technological level, which has two possible levels, *good* and *bad*. Supposing, for example, that the marginal cost under good technology is lower than that under bad technology for every quantity.\(^7\)

The realization of the technological levels depends on the effort level \(e_i (i = 0, 1)\) that the manager of each firm engages before production.\(^8\) Suppose that each manager can get the good technology with probability \(v(e)\) and the effort space is \([0, \tilde{e}]\). Assuming that both the innovation technology \(v(e)\) and the effort space \([0, \tilde{e}]\) are common for every manager is just for simplicity. We can consider the effort level as the level of an innovative investment. The more the manager invests the more likely he enjoys the good technology. The effort level \(e\) is measured in units of disutility caused to him. We impose some assumptions on the function \(v(e)\) as follows:

**Assumption 1** For all \(e \in [0, \tilde{e}]\), (i) \(v_e(e) \geq 0\) and \(v_{ee}(e) < 0\), (ii) \(v(0) = 0\) and \(v(\tilde{e}) = 1\), and (iii) \(v_e(0) = +\infty\) and \(v_e(\tilde{e}) = 0\).

Assumption 1 (i) implies that this investment is innovative, that is, increases the probability of drawing the good technology, but marginal productivity of this investment is decreasing. Assumption 1 (ii) is imposed since \(v(e)\) is probability. As-

\(^7\) The possible technological levels of public firms and those of private firms may be identical. In extreme cases, the good technology of a public firm can be inferior to the bad technology of private firms. We allow such extreme cases in this technological setting as long as satisfying several assumptions imposed later in the next section.

\(^8\) We call \(P\) the manager of firm 1 for explanatory use.
sumption 1 (iii) assures an interior solution with respect to the investment level.

Depending on the technological levels of both firms, the technological environment in the market competition is distinguished into four environments, $gg, gb, bg, \text{ and } bb$, where $gb$ represents, for example, that firm 0 has got the good technology but firm 1 has got the bad technology. We use the subscription $s \in \{gg, gb, bg, bb\}$ when we describe some outcomes depending on the environments.

2.3 Timing and Information Structure

Now, we present the timing of actions and informational structure explicitly. There are three time periods, 0, 1, and 2. In the period 0, both $G$ employ a manager, $B$, for her firm. In this period, every technology described above and the $B$’s characteristic $\hat{w}$ are common knowledge. At the beginning of period 1, both managers decide their effort levels. The effort levels are private information for each manager. The technological levels of both firms are realized, in the stochastic manner, at the end of this period. In the period 2, both managers run their firms and produce the good. At this time, all the agents, $G$, $P$, and $B$, observe the technological levels realized at date 1. At the end of this period, a transfer written in the contract is fulfilled.\(^9\)

\(^9\) We explain the terms in the contract and the timing of contracting in the next section.
3 Equilibrium Outcomes and Implications

3.1 Benchmark — Case without Agency Problem

We introduce a benchmark solution without any agency problems, where $G$ directly manage her firm, that is, decide her effort level at the beginning of period 1 and her quantities at the time period 2. First, we analyze the decision of $G$ and $P$ with respect to the quantities. Since $G$ and $P$ know the technological environments in which they live at the beginning of period 2, they decide the quantities to maximize their objectives in each environment. We define the equilibrium quantity vector as $q_s = (q_{0s}, q_{1s})$ in each environment ($s \in \{gg, gb, bg, bb\}$). Although $G$ and $P$ produce at the time period 2 simultaneously, if $G$ can commit the quantity in each environment before the period, $G$ can improve the social welfare in each environment since $G$ is a welfare-maximizer with respect to the quantity-setting.\(^{10}\) Then $q_s$ represents the equilibrium outcome in Stackelberg-competition if such commitment is sustainable and does that in Cournot-competition if else. We are not interested in which types of competitions are taken place as long as the assumptions we impose later are satisfied.

We describe the equilibrium value of components of the objective functions, in each environment, as $CS_s = CS(q_s^e)$, $\pi_{0s}^e = \pi_0(q_s^e | s)$, and $\pi_{1s}^e = \pi_1(q_s^e | s)$. The gross benefit for $G$ in each environment is given by

$$Z_s^e = \pi_{0s}^e + CS_s^e + \pi_{1s}^e$$

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\(^{10}\) See De Fraja and Delbono (1989).
and the social welfare in each environment is given by

\[ W_i^e = C \cdot S_i^e + \pi_0^e + \pi_{14}^e - e_0 - e_1 \]  \hspace{1cm} (4)

\[ = Z_i^e - e_0 - e_1. \]  \hspace{1cm} (5)

The maximization problems of \( G \) and \( P \) without agency problem are given as follows:

- **\( G \)'s problem without agency problem:**

  \[ \text{Given } e_1 \quad \max_{e_0} E[Z^e] - e_0 - \beta e_1, \]

- **\( P \)'s problem without agency problem**

  \[ \text{Given } e_0 \quad \max_{e_1} E[\pi^e_1] - e_1, \]

where \( E[X] \) represents the expected value of \( X \) and then we have

\[ E[X] = v(e_0)[v(e_1)X_{gg} + (1 - v(e_1))X_{gb}] + (1 - v(e_0))[v(e_1)X_{bg} + (1 - v(e_1))X_{bb}]. \]

The first order conditions on both \( G \)'s and \( P \)'s problems are

\[ v'(e_0) \left[ v(e_1) \left( Z_{gg}^e - Z_{bg}^e \right) + (1 - v(e_1)) \left( Z_{bg}^e - Z_{bb}^e \right) \right] = 1 \]  \hspace{1cm} (6)

and

\[ v'(e_1) \left[ v(e_0) \left( \pi_{1gg}^e - \pi_{1bg}^e \right) + (1 - v(e_0)) \left( \pi_{1bg}^e - \pi_{1bb}^e \right) \right] = 1, \]  \hspace{1cm} (7)

respectively. The second order conditions are always satisfied following from Assumption 1. On the contrary, welfare-maximizing problem is

\[ \max_{e_0, e_1} E[W] \]
and the first order conditions on this problem are given by

\[ v'(e_0) \left[ v(e_1) \left( Z_{gg}^c - Z_{bg}^c \right) + (1 - v(e_1)) \left( Z_{gb}^c - Z_{bb}^c \right) \right] = 1, \quad (8) \]

\[ v'(e_1) \left[ v(e_0) \left( Z_{gg}^c - Z_{gb}^c \right) + (1 - v(e_0)) \left( Z_{bg}^c - Z_{bb}^c \right) \right] = 1. \quad (9) \]

Now, we impose several assumptions for further analyses as follows:

**Assumption 2** Internal effects of the innovation: For any \( \alpha \) and \( \beta \),

(i) \( Z_{bg}^c - Z_{bb}^c > Z_{gg}^c - Z_{bg}^c > 0 \) and \( Z_{bg}^c - Z_{bb}^c > Z_{gg}^c - Z_{gb}^c > 0 \)

(ii) \( \pi_{1bg}^e - \pi_{1bb}^e > \pi_{1gg}^e - \pi_{1gb}^e > 0 \).

Assumption 2 (i) implies that, although the innovation of one firm always increases the social welfare, the degree of increase is larger when the other firm fails in the innovation. These conditions guarantee the second order condition on the welfare-maximizing problem. This assumption is plausible since the innovation of one firm leads to not only an increase in the total supply but also production substitution when the other firm fails in the innovation. Assumption 2 (ii) implies that, although the innovation of firm 1 always increases the profit of the firm, the degree of increase is larger when the firm 0 fails in the innovation. These conditions guarantee that the decision on the effort level is strategic substitute. Assumption 2 guarantees the existence of response functions reduced from (6) and (7), \( e_0 = R_0^c(e_1) \) and \( e_1 = R_1^c(e_0) \) respectively, and functions reduced from (8) and (9), \( e_0 = R_{0}^{\ast}(e_1) \) and \( e_1 = R_{1}^{\ast}(e_0) \) respectively. Note that \( R_0^c(e_1) \) and \( R_{0}^{\ast}(e_1) \) are identical. We define the welfare-maximizing effort vector as \( e^{\ast} = (e_0^{\ast}, e_1^{\ast}) \) satisfying \( e_0^{\ast} = R_{0}^{\ast}(e_1^{\ast}) \) and \( e_1^{\ast} = R_{1}^{\ast}(e_0^{\ast}) \).

**Assumption 3** There exists a unique and stable equilibrium \( e^c = (e_0^c, e_1^c) \) satisfying \( e_0^c = R_0^c(e_1^c) \) and \( e_1^c = R_1^c(e_0^c) \).
Following from Assumption 2, the derivatives of response function are always negative, that is, \( \frac{dR_i^c(e_j)}{de_j} < 0 \) for all \( e_j \) for \( i \neq j \). Moreover, the investment levels, \( e_i \), are less sensitive to the other’s levels, \( e_j \), that is, \( \frac{dR_i^c}{de_j} > -1 \), for sufficiently positive value of the other’s level, \( e_j \), following from Assumption 1. In addition, we have both \( 0 < R_i^c(1) < R_i^c(0) < \overline{e} \) for \( i = 0, 1 \). Therefore this assumption is plausible.\textsuperscript{11}

**Assumption 4**  
*External effects of the innovation:* For any \( \alpha \) and \( \beta \),  
\[
CS_{\alpha \alpha}^c + \pi_{0 \alpha \alpha}^c > CS_{\beta \alpha}^c + \pi_{0 \beta \alpha}^c \quad \text{and} \quad CS_{\alpha \beta}^c + \pi_{0 \alpha \beta}^c > CS_{\beta \beta}^c + \pi_{0 \beta \beta}^c.
\]

As assumed in Assumption 2, the innovation of one firm increases the payoff for the owner of the firm. However, it also varies the value of components in both \( G \)’s and \( P \)’s objective functions. Assumption 4 implies that, although the production substitution following from the innovation in firm 1 probably reduces the profit of firm 0, it increases the payoff for \( G \) since it increases the consumer surplus. Ordinarily, the innovation in one firm increases the firm’s quantity, decreases the other’s quantity, increases the total supply, and lowers the market price, which increase both the firm’s profit and the consumer surplus but decrease the other’s profit. Assumption 2 requires that the effect on the consumer surplus of one firm’s innovation outweighs that on the other’s profit.

**Lemma 1**  
*With respect to the response function of firm 1, we have* \( R_i^c(e_0) < R_i^{**}(e_0) \) *for all* \( e_0 \).

**PROOF.** See Appendix A.

The implication of this lemma is simple. When \( P \) maximizes her own objective, she decides her effort level without taking the effect on the other components in

\textsuperscript{11} See also Figure 1.
Fig. 1. Response curves and effort vectors: The response curve $e_1 = R^e_i(e_0)$ locates on the lower area of the curve $e_1 = R^{**}_i(e_0)$ and the equilibrium effort vector without agency problem, $e^e$, locates on the lower-right area of the welfare-maximizing effort vector, $e^{**}$.

the social welfare into account. Since we assume that the external effect is positive, that is, $CS + \pi_0$ increases when $P$ succeeds in the innovation, then the objective-maximizing effort level is less than the welfare-maximizing effort level for any level of $G$’s effort.

The relation between the functions $R^e_i(e_j)$ and $R^{**}_i(e_j)$ and that between the effort vectors $e^e$ and $e^{**}$ are described in Fig. 1. Lemma 1 implies that the response curve $e_1 = R^e_i(e_0)$ locates on the lower area of the curve $e_1 = R^{**}_i(e_0)$. Following from Assumption 3, the intersection which represents the equilibrium effort vector without agency problem, $e^e$, locates on the lower-right area of that representing the welfare-maximizing effort vector, $e^{**}$.
Define $EW(e_0)$ as follows:

$$EW(e_0) = v(e_0)[v(R^c_0)W_{gg}+(1-v(R^c_1))W_{gb}]+(1-v(e_0))[v(R^c_1)W_{bg}+(1-v(R^c_0))W_{bb}].$$

This function describes the expected social welfare as a function of $G$’s effort when $P$ best responses to it.

**Lemma 2** $\frac{d}{de_0}EW(e_0) < 0$.

**Proof.** See Appendix A.

The rationale of this lemma is as follows: When $G$ directly manages firm 0, she is a welfare-maximizer. Moreover, the decisions on effort level are strategic substitute in our model. Under such circumstances, making $G$’s strategy less aggressive can improve the social welfare as a result. \(^{12}\) Lemma 2 merely implies that $G$’s effort level which maximizes $EW(e_0)$ is less than $e^*_0$. However, we impose the following assumption for explanatory simplicity.

**Assumption 5** There exists a unique $e_0$ which maximizes $EW(e_0)$.

Fig. 2 describes iso-welfare curves on $e_0$-$e_1$ plane. The slope of the tangent line at $e^c$ is zero as is shown in the proof of Lemma 2. On the contrary, the slope of

\(^{12}\)The same effect is considered in the prior studies. De Fraja and Delbono (1989) and Matsumura (1998), for example, use this effect in the quantity-setting competition and show that privatization in mixed oligopoly and partial privatization in mixed duopoly, respectively, can improve the social welfare. Ishibashi and Matsumura (2005) use the effect in the R&D competition and show that committing less investment with imposing a budget constraint improves the social welfare. In our model, as analyzed in the next subsection, we use an agency cost as a commitment device.
Fig. 2. Iso-welfare curves: Since $e^{**}$ represents the single peak of the expected social welfare, the expected social welfare is improved as $e_0$ ($e_1$) goes close to $e_1^{**}$ ($e_1^{**}$). There exists a effort level which maximizes $EW(e_0)$ denoted by $\hat{e}_0$.

the response curve $e_1 = R_1^e(e_0)$ is negative. Then the iso-welfare curve which is through the point $e^c$ ordinarily has another intersection with the curve $e_1 = R_1^c(e_0)$, which locates on the upper left area of $e^c$. Since $e^{**}$ represents the single peak of the expected social welfare, the expected social welfare is improved as $e_0$ ($e_1$) goes close to $e_0^{**}$ ($e_1^{**}$). Then there exists a (range of) effort level which maximizes $EW(e_0)$. Following from Assumption 5, we define the unique value as $\hat{e}_0$. We also define that $R_1^c(\hat{e}_0) = \hat{e}_1$ and $\hat{e} = (\hat{e}_0, \hat{e}_1)$.

3.2 Agency Problem

Now we consider the contract between $G$ and $B$. $G$ employs $B$ characterized with $(\hat{w}, \hat{w})$ and offers a contract ($\{\hat{w}, \hat{w}{\}}_s$, $\{q_{0s}\}_s$). Note that, since the realized envi-
Environment is observable at the beginning of period 2, such environment-contingent contract above is optimal. For the time being, we treat the characteristic of both the managers as given and analyze the contracting problems.

The optimum term concerning the quantities is those of Stackelberg-leader if the contract is useful as a commitment device, and is those under Cournot-competition if the contract is not useful to commit her quantities. For simplicity, we consider that the contracting the quantities does not affect the sustainability of commitment, that is, $G$ decides to enforce $\{q^c_{0s}\}$ in each environment and the equilibrium quantity vector is $q^c_s$ in each environment.

Suppose $G$ can decide the term $\{w_s\}$ before $P$ decides her effort level and disclose the term to $P$. Then $G$ is willing to offer a wage structure which induce $B$ to engage the effort level $\hat{e}_0$, which maximizes the expected social welfare given the $P$’s response, at first. However, $G$ has an incentive to rewrite the contract. If $P$ has straightforwardly responded to the level to which $G$ induces $B$’s effort, $G$ is willing to increase $B$’s effort level. In addition, since the effort levels are not observable, the actual effort level itself cannot affect the other’s decision. Then, if recontracting is feasible, committing the level $\hat{e}_0$ is not sustainable and the decision of the wage structure and $P$’s effort level are, in effect, occurred simultaneously. Therefore, the maximization problems of $G$ and $P$ can be rewritten as follows;

- $G$’s problem: Given $e_1$

\[
\max_{e_0, \{w_s\}} E[Y] \\
\text{s.t. } E[w] - e_0 \geq w, \\
\frac{\partial}{\partial e_0} E[w] = 1, \\
w_s + \hat{w} \geq 0 \quad \forall s \in \{gg, gb, bg, bb\}. 
\]
• $P$’s problem: The same one defined in the previous subsection.

In the $G$’s problem, the first constraint represents the participation constraint, the second does the incentive compatible constraint, and the last constraints do the limited liability constraints. Now we solve the $G$’s problem. Lagrangean of the problem is given by

$$L = E[Y^c] + \lambda(e_1) [E[w] - e_0 - w] + \gamma(e_1) \left[ \frac{\partial}{\partial e_0} E[w] - 1 \right], \quad (10)$$

where $\lambda(e_1)$ and $\gamma(e_1)$ denote Lagrange multipliers on the participation constraint and the incentive compatible constraint, respectively, and are nonnegative. Note that those multipliers depend on $e_1$. The first order conditions derived from the partial derivatives of $L$ can be reduced as follows:

$$v'(e_0) \left[ v(e_1) \left( Z_{gg}^c - Z_{bg}^c \right) + (1 - v(e_1)) \left( Z_{gb}^c - Z_{bb}^c \right) \right] = 1 - \gamma(e_1) \frac{v''(e_0)}{v'(e_0)}, \quad (11)$$

$$1 - \zeta - \lambda(e_1)v(e_0) - \gamma(e_1)v'(e_0) \geq 0, \quad (12)$$

$$1 - \zeta - \lambda(e_1)(1 - v(e_0)) + \gamma(e_1)v'(e_0) \geq 0, \quad (13)$$

$$\lambda(e_1) [E[w] - e_0 - w] = 0, \quad (14)$$

$$\frac{\partial}{\partial e_0} E[w] = 1. \quad (15)$$

**Lemma 3** When $\zeta$ is equal to one, then we have $\lambda(e_1) = \gamma(e_1) = 0$ for all $e_1$.

**PROOF.** See Appendix A.

The rationale of this lemma is simple. When $\zeta$ is equal to one, the wage structure $\{w_s\}$ never affects $G$’s objective since $G$ gives equal weight on both the payoff for the public firm and that for the manager of the firm. In such circumstances, $G$ has no incentive to reduce her agent’s wage levels. Then $G$ can induce her manager any incentive by offering a contract with high level of wages in case of succeeding...
in the innovation, which never tightens the limited liability constraints. Following from this lemma, we can shortly conclude the following proposition.

**Proposition 1** When $\zeta$ is equal to one, then the equilibrium effort vector is $e^e$.

This Lemma 3 implies that, if $\zeta$ is equal to one, the response function of $G$ is identical to that without agency problem, $R_0^c(e_1) = R_0^{*c}(e_1)$. It follows that the equilibrium outcome is identical. This proposition implies that, when $G$ is most benevolent, that is, $\alpha = \beta = \zeta = 1$, $G$ cannot achieve the welfare improving effort vector which maximizes the expected social welfare given the response of $P$, $\hat{e}$. Now, we analyze the problem with less benevolence, $\zeta < 1$.

**Lemma 4** When the limited liability constraints are not binding, we have $\lambda(e_1) = 1 - \zeta$ and $\gamma(e_1) = 0$ for all $e_1$.

**PROOF.** See Appendix A.

Lemma 4 implies that, if the limited liability constraints are not binding, the response function of $G$ is identical to that without agency problem, $R_0^c(e_1) = R_0^{*c}(e_1)$ and, as a result, the equilibrium outcome is identical to that without agency problem.

Now we consider the condition for which the limited liability constraints are binding. Reducing $w_{bg}$ and $w_{bb}$ and increasing $w_{gb}$ and $w_{gg}$ heighten the incentive of effort. Then distortion derived from limited liabilities takes place if and only if the incentive is insufficient for such a wage structure that satisfies both $E[w] - e_0 - w = 0$.
and \( w_{bg} = w_{bb} = -\dot{w} \), that is,

\[
v(e_1) (w_{gg} + \dot{w}) + (1 - v(e_1)) (w_{gb} + \dot{w}) < v(e_1) (Z^c_{gg} - Z^c_{bg}) + (1 - v(e_1)) (Z^c_{gb} - Z^c_{bb}),
\]

\[
v(e_0) [v(e_1) w_{gg} + (1 - v(e_1)) w_{gb}] - (1 - v(e_1)) \dot{w} = e_0 + \dot{w}.
\]

These conditions can be reduced as

\[
E[Z^c] - \left[ v(e_1) Z^c_{bg} + (1 - v(e_1)) Z^c_{bb} \right] > e_0 + \dot{w} + \dot{w}
\]

(16)

for any \( e_1 \). This condition is more likely to be held as \( e_1 \) becomes smaller since the left-hand side of (16) is decreasing in \( e_1 \).

When the two limited liability constraints are binding, that is, \( w_{bg} = w_{bb} = -\dot{w} \), we have

\[
\lambda(e_1) \left[ \frac{v(e_0)}{v'(e_0)} - e_0 - \dot{w} - \dot{w} \right] = 0
\]

(17)

following from (14) and (15). This equation implies that, when \( \lambda(e_1) \) is strictly positive, \( e_0 \) is a constant \( e_0' \) such that satisfies \( \frac{v(e_0)}{v'(e_0)} - e_0 - \dot{w} - \dot{w} = 0 \). On the contrary, when \( \lambda(e_1) \) is equal to zero, we have \( \gamma(e_1) = \frac{(1 - \zeta) v(e_0)}{v'(e_0)} \) since (12) holds with equality.\(^{13}\) Substituting this into (11), we have

\[
v'(e_0) \left[ v(e_1) \left( Z^c_{gg} - Z^c_{bg} \right) + (1 - v(e_1)) \left( Z^c_{gb} - Z^c_{bb} \right) \right] = 1 - \frac{(1 - \zeta) v(e_0) v''(e_0)}{[v'(e_0)]^2}.
\]

(18)

Suppose the existence of a function \( R^l_0(e_1) \) derived from (18). Then the response function is described as follows:

\[
R_0(e_1) = \begin{cases} 
R^c_0(e_1) & \text{if the limited liability constraints are not binding,} \\
p_0' & \text{if } \lambda(e_1) > 0, \\
R^l_0(e_1) & \text{if } \lambda(e_1) = 0.
\end{cases}
\]

(19)

\(^{13}\) This is because the limited liability constraints on \( w_{gg} \) and \( w_{gb} \) are not binding. If else, \( G \) cannot induce \( B \) any effort level.
Fig. 3. Transformation of response curves:

Note that an increase in $e_1$ decreases $\gamma(e_1)$, when $e_0 = e_0'$ following from (11), and then increases $\lambda(e_1)$. Then the response function can be drawn as Fig. 3. For sufficiently high level of $e_1$, the limited liability constraints are not binding in some cases. On the contrary, for sufficiently low level of $e_1$, the limited liability constraints and the incentive compatible constraint are most tightly binding. For an intermediate level of $e_1$, $e_0$ is a constant $e_0'$.

**Lemma 5** $e_0'$ is increasing in $\hat{w}$.

**PROOF.** See Appendix A.

The implication of this lemma is that, when $G$ can commit the employment of $B$ decided in the period 0, she can transform her response function less aggressive by choosing a manage with less wealth. We consider that the commitment of employment is more sustainable than the commitment of contract since the employment is
protected by some kind of laws. Therefore we have one of the main result of our study.

**Proposition 2** If \( R_0' (\hat{e}_1) \leq \hat{e}_0 \) holds, then \( G \) can achieve \( \hat{e} \) by employing a manager with an appropriate characteristic.

### 3.3 Effect of Privatization

We now explain an effect of privatization in mixed duopoly industry. When \( G \) privatizes the public firm (firm 0), \( G \) is unable to observe the environment at the beginning of period 2 since she is a mere outsider. However, she can observe the quantities the privatized firm has generated since production is a public information. Sappington and Stiglitz (1987) present that \( G \) can internalize her objective into the buyer of public firm by a quantity-contingent auction and achieve the optimal outcome.\(^{14}\) In our mixed duopoly model, \( G \) can achieve the ‘objective-maximizing’ response function. Contracts are not sustainable as a commitment device when recontracting is feasible as mentioned in the previous subsection. Then another commitment device is required to achieve the welfare-maximizing effort vector \( \hat{e} \). However, using the limited liability as the device, similar to the previous subsection, is suspected following from the reasons below: (i) \( G \) can easily alter the buyer, (ii) potential buyers have sufficient wealth (or capital), (iii) potential buyer can easily increase their capital. When some of the conditions are plausible, \( G \)’s commitment to induce her agent (buyer) less aggressive is not sustainable and then

\(^{14}\) SchmidtCBP1996 analyzes problems of privatization derived from an incompleteness of contract in monopoly industry. He shows that privatization causes distortions in productive allocation although it enhances innovative incentive when long-term contracts are not feasible.
the equilibrium effort vector is $e^c$. Therefore, privatization enhances the innovative effort of privatized firm, which harms $P$’s incentive and the expected social welfare as a result.

4 Conclusion

We investigate an interaction of innovative effort in a mixed duopoly industry. Decisions on innovative effort are ordinarily strategic substitute between public and private firms. If any agency problems do not exist, the government induces her agent more effort, as a result, even when the government is welfare-maximizer. That is committing to induce the manager of public firm less effort can improve the expected social welfare. Contracting an incentive contract which induces the manager less aggressive is not sustainable as a commitment device since government has an incentive to recontract with him. We show that employing a manager with less wealth can serve for the commitment since it makes the manager’s limited liability constraints be binding more tightly and inducing higher effort more expensive. In such circumstances, privatization is always harmful, when potential buyers of the public firm have sufficient wealth (or easily increase his capital) or when the government can easily alter the buyer, since it destroys the commitment device.
Appendix A

A.1 Proof of Lemma 1

Following from (7) and (9), we have

\[ v'(R_1^c(e_0)) \left[ v(e_0) \left( \pi_{1gg}^c - \pi_{1gb}^c \right) + (1 - v(e_0)) \left( \pi_{1bg}^c - \pi_{1bb}^c \right) \right] = 1, \]

\[ v'(R_1^{**}(e_0)) \left[ v(e_0) \left( W_{gg}^c - W_{gb}^c \right) + (1 - v(e_0)) \left( W_{bg}^c - W_{bb}^c \right) \right] = 1. \]

Since we have both \( W_{gg}^c - W_{gb}^c > \pi_{1gg}^c - \pi_{1gb}^c \) and \( W_{bg}^c - W_{bb}^c > \pi_{1bg}^c - \pi_{1bb}^c \) following from Assumption 4 (i), then we have \( v'(R_1^c(e_0)) > v'(R_1^{**}(e_0)) \), which is reduced as \( R_1^c(e_0) < R_1^{**}(e_0) \). □

A.2 Proof of Lemma 2

The expected social welfare on \( P \)'s response curve is rewritten as

\[
EW(e_0) = v(e_0) \left[ v(R_1^c(e_0))Z_{gg} + (1 - v(R_1^c(e_0)))Z_{gb} \right] + (1 - v(e_0)) \left[ v(R_1^c(e_0))Z_{bg} + (1 - v(R_1^c(e_0)))Z_{bb} \right] - e_0 - R_1^c(e_0).
\]

The derivative of this function is

\[
v'(e_0) \left[ v(R_1^c(e_0)) \left( Z_{gg}^c - Z_{gb}^c \right) + (1 - v(R_1^c(e_0))) \left( Z_{bg}^c - Z_{bb}^c \right) \right] - 1
+ \frac{dR_1^c(e_0)}{de_0} \left[ v'(e_1) \left( Z_{gg}^c - Z_{gb}^c \right) + (1 - v(e_0)) \left( Z_{bg}^c - Z_{bb}^c \right) \right] - 1.
\]

Then we have

\[
\frac{dEW(e_0)}{de_0} = \frac{dR_1^c(e_0)}{de_0} \left[ v'(e_1) \left( Z_{gg}^c - Z_{gb}^c \right) + (1 - v(e_0)) \left( Z_{bg}^c - Z_{bb}^c \right) \right] - 1.
\]

(A.1)
Note that we have
\[
v'(R^*_1(e_0^c)) \left[ v(e_0^c) \left( Z_{g}^c - Z_{g b}^c \right) + (1 - v(e_0^c)) \left( Z_{b g}^c - Z_{b b}^c \right) \right] = 1.
\]

Since \( \frac{dR_1^c(e_0^c)}{de_0} \) and \( e_1^c = R_1^c(e_0^c) < R^*_1(e_0^c) \) following from Assumption 2 and Lemma 1, then we have \( v'(R^*_1(e_0^c)) < 0. \)

\[ \square \]

### A.3 Proof of Lemma 3

Substituting \( \zeta = 1 \) into (12) and 13, we have
\[
\begin{align*}
-\lambda(e_1)v(e_0) - \gamma(e_1)v'(e_0) & \geq 0, \quad (A.2) \\
-\lambda(e_1)(1 - v(e_0)) + \gamma(e_1)v'(e_0) & \geq 0. \quad (A.3)
\end{align*}
\]

Note that both \( \lambda(e_1) \) and \( \gamma(e_1) \) are not less than zero for all \( e_1 \). In addition, both \( v(e_0) \) and \( v'(e_0) \) are not less than zero following from Assumption 1. Therefore, (A.2) must hold with equality, that is, \( \lambda(e_1)v(e_0) + \gamma(e_1)v'(e_0) = 0. \) Substituting this into (A.3), we have \( -\lambda(e_1) \geq 0 \), which implies that \( \lambda(e_1) \) must be equal to zero for all \( e_1 \). Note that we have \( e_0 < \bar{e} \) following from Assumption 1 and (11), which implies \( e_0 \leq R^*_0(e_1) \) for all \( e_1 \). Therefore, \( \lambda(e_1)v(e_0) + \gamma(e_1)v'(e_0) = 0 \) and \( \lambda(e_1) = 0 \) implies \( \gamma(e_1) = 0 \) for all \( e_1 \). \[ \square \]

### A.4 Proof of Lemma 4

When the limited liability constraints are not binding, both (12) and (13) hold with equality. It follows that \( \lambda(e_1) = 1 - \zeta \) and \( \gamma(e_1) = 0 \) hold. \[ \square \]
When \( \frac{v(e_0')}{v'(e_0')} - e_0' - \hat{w} - \bar{w} = 0 \) holds, we have
\[
- \frac{de_0'}{d\hat{w}} \frac{v'(e_0')v''(e_0')}{[v'(e_0')]^2} - 1 = 0.
\]
Since \( \frac{v'(e_0')v''(e_0')}{[v'(e_0')]^2} \) is negative following from Assumption 1, we have \( \frac{de_0'}{d\hat{w}} < 0. \) □

References


