Strategic Outsourcing and Quality Choice: Is a Vertical Integration Model Sustainable?

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Abstract

This paper uses two versions of a vertical product differentiation model with price competition to investigate how strategic behaviour between rivalrous firms will influence which organizational production mode each firm adopts, vertical integration or outsourcing. We show that not only a symmetric configuration, where both high- and low-quality firms outsource, but an asymmetric configuration, where the high-quality firm produces in-house while the low-quality firm outsources, is accepted as a subgame perfect equilibrium outcome. Furthermore, the implications of these results are explored with a discussion about a recent business slump in Japanese electronics enterprises in the flat panel television industry. (99 words)

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Proposed Running Head: Strategic Outsourcing and Quality Choice

1 Introduction

We first take the Japanese digital consumer electronics industry, in particular the flat panel television (TV) market, for example, and make a comparison between the status quo and the situation prevailing in the 2000s in the industry. Fujimoto (2012, p. 5) says that many of Japan's digital electronics products have lost their competitive advantages since the 1990s, when their architectures quickly became modular; a case in point is flat panel TV manufacturing. Consider the Japanese digital consumer electronics industry in the 2000s. The question at issue in the industry of those days was which business model had a strategic advantage in enhancing an electronics company's corporate value, profitability and global competitiveness, "vertical integration" or "horizontal specialization" (see Ohki, 2008, on "suihei bungyou" in Japanese). This question is directly related to a firm's 'make-or-buy' decision in industrial organization.

A value chain for a liquid crystal display television (LCD TV) is composed of the following activities; concept, development and design, production of LCD panels, assembly of LCD modules for TVs, production of system large-scale integrated circuits (LSIs) for LCDs, final assembly of TV sets and brand. Outsourcing is essentially a division of labour. Thus, the degree of "horizontal specialization" depends on which and how many activities an electronics company outsources to the outside. *The Nikkei* (November 16, 2009) says that Panasonic and Sharp were vertically integrated companies while Hitachi, Toshiba and Sony were aiming for "horizontal specialization".[1] Take Toshiba for example. In those days this company specialized in the production of system LSIs for LCDs and procured LCD modules for TVs from the outside (see *The Nikkei*, October 19, 2009).

Vizio is a producer of consumer electronics in the USA. It specializes in activities such as concept and design, and sales and after-sales service. Since Vizio is a fabless LCD TV company, it does not have its own production facility. Amtran, a Taiwan-based company, specializes in the tasks of procuring components from Taiwan and South Korea, assembly of LCD TV sets in China, quality assurance and so on. It supplies finished LCD TV sets to Vizio.

Outsourcing will offer a move from fixed costs to variable costs, thereby changing the ratio of fixed to variable costs and leading to a firm's cost restructuring. It may be said that Vizio's degree of 'horizontal specialization' is higher than that of Toshiba. Since Vizio

[1] *The Nikkei* is a Japanese newspaper focusing on the Japanese economy.

is a fabless company, it doesn't have to make any irreversible investment in a production facility for supplying inputs in-house and any investment in R&D for the design and development of new products. It will follow from this fact that Vizio's ratio of fixed to variable costs is quite lower than that of Toshiba. In contrast, Panasonic and Sharp aiming for vertical integration have so far made massive investments in state-of-the-art facilities that carry out integrated production of large LCD TVs from manufacture of LCD panels to final assembly of TV sets. Moreover, because they considered that R&D served as the basis for value-added activities and the development of new products with advanced technology, they attached great importance to investment in R&D, too. Vertical integration, therefore, implies that Panasonic and Sharp's ratios of fixed to variable costs were much higher than those of Vizio and Toshiba.

In the 2000s, what was consumers' image for Japanese consumer electrical appliances? Japanese consumer electronics companies had a reputation for high quality and innovation in both the Japanese and global flat panel TV markets. Sharp thought that the quality of the plant correlates directly with the quality of the LCD TVs produced there. Since 2004, this company has invested heavily in the LCD panel manufacturing plants. One is the Kameyama Plant, and the other the Sakai plant. The latter is still the only 10th generation LCD panel manufacturing plant in the world. Furthermore, this company aims at promoting manufacturing innovations such as dramatically reducing cost and improving manufacturing processes. As is generally known, Sharp began producing the world's first high performance LCD panels incorporating IGZO oxide semiconductors in March 2012.[2]

As to the present situation of Sharp and Panasonic, *The Wall Street Journal* (February 4, 2013) says, "Sharp Corp. and Panasonic Corp. improved their performance in the most recent quarter from big losses in the previous three months, though the Japanese electronics manufacturers warned they still face a difficult situation." It should be added that Panasonic will trim its money-losing TV business by ceasing production of Plasma TVs at its plant in Amagasaki by 2014, at the earliest (see *The Nikkei*, March 18 and 19, 2013).

As mentioned above, in the 2000s the flat panel TV manufacturers faced the problem of choosing between outsourcing and in-house production. There are two 'extremes' of organizational production mode in flat panel TV markets: one is an organizational production mode with exclusive in-house production; the other is that with exclusive outsourcing (see Shy and Stenbacka, 2005, p. 1174). We may say that Panasonic and Sharp were representa-

[2] http://www.sharp-world.com/corporate/news/120413.html

tives of the former and Vizio was typical of the latter. Also, there is partial outsourcing in between the two extremes. Hitachi, Toshiba and Sony seemed to belong to this category. This fact means that in those days in real world flat panel TV markets electronics manufacturers aiming for vertical integration (in-house production) coexisted with electronics producers outsourcing the production of all or part of the inputs they needed. Nowadays, however, it seems that the flat panel TV industry is the one where large enterprises make use of production processes which technically lend themselves to outsourcing and where the outsourced activities can support independent subcontractors.[3]

Grossman and Helpman (2002) and Shy and Stenbacka (2003) handle firms' choice of organizational production mode. The former present an equilibrium model of industrial structure in which the organization of firms is endogenous. They demonstrate that, except in a knife-edge case, there are no equilibria in which an industry is populated by both vertically integrated and specialized firms. The latter make use of the Hotelling duopoly model in a differentiated industry context in order to analyze oligopolistic firms' choice of whether to outsource the production of the input good or whether to self-produce it. They show that asymmetric production modes, where one firm outsources while the other produces in-house, are ruled out as subgame perfect equilibrium outcomes.

Nickerson and Bergh (1999) investigate rivalrous firms' asset specificity and organizational mode choices in Cournot competition, and demonstrate that strategic interactions lead rivals to make not only symmetric choices but also asymmetric choices from which intra-industry organizational heterogeneity follows. In contrast to Shy and Stenbacka (2003), Buehler and Haucap (2006) find that in addition to symmetric equilibria, there may be asymmetric equilibria where one firm buys the input from an existing input market, whereas the other firm produces the input internally. The difference stems from the fact that they consider a non-specific input good, whereas Shy and Stenbacka focus on a specific input good. This paper focuses on the firms' choice of organizational production mode in a vertically differentiated duopoly model. We show that in this model there is not only a symmetric configuration, where both firms outsource, but an asymmetric configuration, where one firm produces in-house while the other outsources.

The existing literature on vertical product differentiation has devoted little attention to the choice of an organizational production mode in an oligopolistic environment in which

[3]Shy and Stenbacka (2003, p. 220) view the mobile phone, computer and aircraft industries as a sample of such an industry as mentioned in the text. See also *The Nikkei*, November 2, 2012 and March 18, 2013.

strategic considerations are of primary importance. A great deal of attention has been paid to the issues of a comparison of equilibrium qualities in price and quantity competition (see Motta, 1993; Amacher et al., 2005), the characterization of quality choice under full or partial market coverage (see Choi and Shin, 1992; Wauthy, 1996), the persistence of the high-quality advantage (see Lehmann-Grube, 1997; Aoki and Prusa, 1997) and the implications of a 'strategic-trade policy' for quality choice (see Zhou et al., 2002).

The purpose of this paper is to investigate how firms' choice of two types of organizational production mode, vertical integration and outsourcing, will influence their quality choice in a vertically differentiated duopoly. We present a simple game-theoretic model which is concerned with outsourcing and quality choice, where the organizational production mode can be treated as a strategic instrument affecting quality and organizational production mode choices by rivalrous firms.

There are a number of related earlier contributions on aspects of outsourcing different from those we focus on. Arya et al. (2008a) demonstrate that standard conclusions regarding the effects of Bertrand and Cournot competition (e.g., Singh and Vives, 1984) can be altered when the production of inputs is outsourced to retail rivals. Baake et al. (1999) consider a duopoly model to examine what they call "cross-supplies" within an industry. Focusing on the "endogenous Stackelberg effect" pointed out by Baak et al., Chen et al. (2011) find that it is typically not the case that a firm will outsource supplies to its rivals. Arya et al. (2008b) show that a rival's reliance on a supplier may prompt a firm to outsource to the same supplier rather than produce inputs internally even when outsourcing is more costly than internal production. Van Long (2005) considers the outsourcing decision of a firm facing a foreign rival that could benefit from technology spillovers associated with the training of workers by the outsourcing firm. In Spiegel (1993) horizontal subcontracting is driven by the assumption that the upstream cost functions are strictly convex. Chen (2005) demonstrates that downstream competitors may strategically choose not to purchase from a vertically integrated firm, unless the latter's price for the intermediate good is sufficiently lower than those of alternative suppliers. In contrast, Chen (2001) reaches the result that vertical integration occurs in equilibrium if and only if one of the upstream producers is more efficient than the others. Chen et al. (2004) explore the strategic incentives of international outsourcing and its potential collusive effects associated with trade liberalization.

The remainder of this paper is arranged as follows. In Section 2 we describe the model and its assumptions. In Section 3 we use a three-stage game model of duopoly. First, each firm chooses the organizational production mode, and then, quality. Finally, both firms compete in prices. In Section 4 the application of the model is illustrated with a discussion about a recent business slump in Japanese electronics enterprises in the flat panel TV industry. Section 5 concludes.

2 The Model

There are two firms in the industry. Each firm produces a vertically differentiated good of quality s_i and sells it at price p_i, where $i = \mathrm{H}, \mathrm{L}$ and $s_\mathrm{H} > s_\mathrm{L} > 0$. Motta (1993, p. 113) states that two different assumptions are made about the nature of costs. One is that there are fixed costs of quality improvement, while variable costs do not change with quality. We assume that this cost function is quadratic in quality with the form below:

$$F(s_i) = \frac{1}{2}ks_i^2, \tag{1}$$

where $k > 0$. This may be thought of as a case in which firms should engage in R&D activity to improve quality. The other takes place when the main burden of quality improvement falls on more skilled labour or more expensive raw materials and inputs. This cost function does not include fixed costs and is given by:

$$c(s_i) = \frac{1}{2}vs_i^2, \tag{2}$$

where $v > 0$. Since the total cost of firm i is linear in quantity q_i, its marginal cost is constant.

We consider a duopoly in which two firms play a three-stage game. The two firms simultaneously determine organizational production mode in the first stage of the game. We assume that two organizational production modes are available for the firms. To avoid unnecessary complications, our model focuses on polar organizational production modes, vertical integration (in-house production) and outsourcing. Vertical integration means a fully integrated manufacturing style of carrying out every activity from production of key components to final assembly of finished goods, while outsourcing is defined to mean the style of choosing to outsource the production of products to the outside and also to sell them under a seller's brand in a finished goods market.[4] In the second stage, each firm

[4]As mentioned above, there is partial outsourcing in between the two polar organizational modes. For partial outsourcing see Shy and Stenbacka (2005).

chooses a quality level of its product. In the third stage, given their own cost structures and quality levels, both firms compete in prices.

Under vertical integration, a firm makes a large investment in R&D activities to develop the advanced manufacturing technology and its related technologies, including quality improvement, and thereby a newly developed technology yields a new product of high quality. Thus, the technological development will prompt the firm to make a massive investment in a production facility for supplying components in-house that are needed to manufacture the new products. Also, such a large-scale investment will result in a dramatic rise in the ratio of fixed to variable costs. In this case, the main burden of quality improvement falls on R&D activities and R&D-related investments, while variable costs do not change with quality. This enables us to take constant unit costs of production to be zero. Therefore, the firm pursuing the strategy of vertical integration faces a cost function represented by (1). The firm's profits are written as:

$$\Pi_{i\mathrm{K}} = p_i q_i - F(s_i), \tag{3}$$

where the subscript K means that firm i adopts the strategy of vertical integration, and q_i denotes demand for the firm.

A firm adopting the strategy of outsourcing does not have to make any investments in R&D activities and a production facility. If it aims for an improvement in product quality, it will have to ask a subcontractor to improve the quality of key components. This request will lead the subcontractor to procure the key components of higher quality from the outside, otherwise it may improve their quality at its own plant. Thus the firm's request will lead to a rise in a price which the firm pays to the subcontractor for a finished product. In this case, because the firm's fixed costs are negligible as compared to variable costs, its cost function is given by (2). This firm's profits are then:

$$\Pi_{i\mathrm{V}} = (p_i - c(s_i))q_i, \tag{4}$$

where the subscript V means that firm i chooses the strategy of outsourcing.

There is a continuum of consumers uniformly distributed over the interval $[a, b]$ with unit density, $b - a = 1$, where $b > a \geq 0$. Each consumer, indexed by $\theta \in [a, b]$, purchases at most one unit of a differentiated good and maximizes the following utility function (see

Tirole, 1988, pp. 96–97, pp. 296–298):

$$U = \begin{cases} \theta s_i - p_i & \text{if he buys one unit of the good with quality } s_i \text{ at price } p_i, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

In (5), θ represents consumers' taste parameter and consumers with a higher θ will be willing to pay more for a higher quality good. Since θ can be interpreted as the inverse of the marginal rate of substitution between income and quality, wealthier consumers have a lower marginal utility of income and therefore a higher θ.

Let $\hat{\theta}$ denote the marginal willingness to pay for quality defined for the consumer who is indifferent between buying the high-quality good at price p_{H} or the low-quality good at price p_{L}, i.e., $\hat{\theta} = (p_{\mathrm{H}} - p_{\mathrm{L}})/(s_{\mathrm{H}} - s_{\mathrm{L}})$. The consumer with index $\tilde{\theta}$ for which $\tilde{\theta}s_{\mathrm{L}} - p_{\mathrm{L}} = 0$ will be indifferent between buying the low-quality good and buying nothing at all, so $\tilde{\theta} = p_{\mathrm{L}}/s_{\mathrm{L}}$. We assume that a market is not covered. This assumption requires $a < p_{\mathrm{L}}/s_{\mathrm{L}}$. Moreover, demands for the high-quality and the low-quality firm are given by, respectively:

$$q_{\mathrm{H}} = b - \hat{\theta}, \tag{6}$$

$$q_{\mathrm{L}} = \hat{\theta} - \tilde{\theta}. \tag{7}$$

Let $\gamma = b/a$ and $\mu = s_{\mathrm{H}}/s_{\mathrm{L}}$ denote the degree of population heterogeneity (a, b) and the degree of product differentiation $(s_{\mathrm{L}}, s_{\mathrm{H}})$, respectively. By definition we have $\mu > 1$. The Nash equilibrium in price depends on these degrees.

First, each firm chooses the organizational production mode, then quality, and finally its price. The third stage equilibrium is a Nash equilibrium in price, taking each firm's choices of organizational production mode and quality as given by the preceding stages. Using this third stage solution, we can write the objective function of each firm as a function of the pair of quality levels chosen in the preceding stage.

In Table 1 $\pi_{i,j}$ is the payoff to firm i from the third stage of the game, given that both firms are in a state represented by the subscript j, which means a state where each firm chooses between the two alternative modes and thereby determines the shape of its cost function given in (1) or (2). $j = 1$ stands for the state in which both firms adopt vertical integration referred to as (K, K). $j = 2$ denotes the state where the high-quality firm chooses outsourcing while the low-quality firm chooses vertical integration referred to as (V, K). $j = 3$ means the state where the high-quality firm chooses vertical integration while the

Table 1: Vertical Integration vs. Outsourcing

		Low-Quality Firm	
	Strategies	Vertical Integration (K)	Outsourcing (V)
High-Quality Firm	Vertical Integration (K)	$\pi_{\mathrm{H},1},\ \pi_{\mathrm{L},1}$	$\pi_{\mathrm{H},3},\ \pi_{\mathrm{L},3}$
	Outsourcing (V)	$\pi_{\mathrm{H},2},\ \pi_{\mathrm{L},2}$	$\pi_{\mathrm{H},4},\ \pi_{\mathrm{L},4}$

low-quality firm chooses outsourcing referred to as (K, V). $j = 4$ stands for the state in which both firms adopt outsourcing referred to as (V, V). We solve for a Nash equilibrium in that game. The solution concept is that of a subgame perfect equilibrium.

3 Choices of Organizational Production Mode and Quality

In this section, in the first stage firms choose the organizational production mode, and in the second stage, quality, and they compete à la Bertrand in the marketing stage of the game. We first solve for Nash equilibria in the third stage. The solutions to this stage are then substituted into the payoff functions to produce $\pi_{i,j}$s in Table 1. When $j = 1$, letting $i = \mathrm{H}$, then $i = \mathrm{L}$ in (3) produces the firms' profits corresponding to (K, K), i.e., $(\Pi_{\mathrm{HK}}, \Pi_{\mathrm{LK}})$. Similarly, when $j = 2$, we have the firms' profits corresponding to (V, K), i.e., $(\Pi_{\mathrm{HV}}, \Pi_{\mathrm{LK}})$. When $j = 3$, the firms' profits corresponding to (K, V) are $(\Pi_{\mathrm{HK}}, \Pi_{\mathrm{LV}})$. When $j = 4$, the firms' profits corresponding to (V, V) are $(\Pi_{\mathrm{HV}}, \Pi_{\mathrm{LV}})$.

3.1 Both firms choose vertical integration: (K, K)

First, differentiating Π_{HK} and Π_{LK} with respect to p_{H} and p_{L}, respectively, we have two first-order conditions from which a two-equation simultaneous system in unknowns p_{H} and p_{L} follows. Solving this system yields each firm's price that can be thought of as a function of qualities. Then, substituting these prices into each firm's profits and partially differentiating its profits with respect to its quality yields a first-order condition for each firm. Solving the two-equation system composed of the two first-order conditions leads to the determination of qualities.

We therefore have (see Motta, 1993 and Amacher et al., 2005):

$$s_{\mathrm{H},1} = 0.253311b^2/k; \quad s_{\mathrm{L},1} = 0.0482383b^2/k; \quad s_{\mathrm{H},1} - s_{\mathrm{L},1} = 0.205072b^2/k, \tag{8}$$

$$p_{\mathrm{H},1} = 0.107662b^3/k; \quad p_{\mathrm{L},1} = 0.0102511b^3/k; \quad p_{\mathrm{H},1}/p_{\mathrm{L},1} = 10.502468, \tag{9}$$

$$q_{\mathrm{H},1} = 0.524994b; \quad q_{\mathrm{L},1} = 0.262497b; \quad q_{\mathrm{H},1} + q_{\mathrm{L},1} = 0.787491b, \tag{10}$$

$$\pi_{\mathrm{H},1} = 0.0244386b^4/k; \quad \pi_{\mathrm{L},1} = 0.00152741b^4/k; \quad \pi_{\mathrm{H},1} + \pi_{\mathrm{L},1} = 0.0259660b^4/k, \tag{11}$$

$$\tilde{\theta}_1 = 0.212509b; \quad \hat{\theta}_1 = 0.475006b, \tag{12}$$

$$\gamma_1 > 4.705677; \quad \mu_1 = 5.251234, \tag{13}$$

where $\tilde{\theta}_1$, $\hat{\theta}_1$ and μ_1 stand for values of $\tilde{\theta}$, $\hat{\theta}$ and μ in the state of $j = 1$, respectively.[5]

3.2 The high-quality firm chooses outsourcing while the low-quality firm adopts vertical integration: (V, K)

This subsection is concerned with the case in which the high-quality firm chooses outsourcing while the low-quality firm adopts vertical integration. Let $\beta \equiv k/v$. k and v are interpreted as efficiency parameters related to vertical integration and outsourcing, respectively. For example, higher values of k mean that vertical integration is a less efficient strategy for a firm. Thus, β is referred to as the efficiency ratio. β is small when the efficiency of vertical integration compared to that of outsourcing is high. Conversely, β is large when the efficiency of outsourcing compared to that of vertical integration is high.

The two first-order conditions for both firms fixing quality levels can be reduced to (see the Appendix for the derivation):

$$\beta = \frac{b\mu^3(4 - 11\mu + 8\mu^2)(-20 + 81\mu - 84\mu^2 + 32\mu^3)}{4(-1+\mu)(-1+4\mu)(2 - 3\mu + 4\mu^2)(-4 + 23\mu - 46\mu^2 + 24\mu^3)}. \tag{14}$$

If β were fixed at a certain value, we could determine a value of μ. Let $g(\mu)$ denote the right-hand side of this equation. This function is at first decreasing and then increasing in μ. However, there is a one-to-one correspondence between β and μ through $\beta = g(\mu)$ on the interval $[2.080460, +\infty)$. In this case, μ that can be thought of as a function of β and b is increasing in β.

[5]Since $b - \tilde{\theta}_1 < 1$ has to hold true, we have $b < 1.269856$, in which case the degree of population heterogeneity represented by γ_1 is greater than 4.705677. In addition, if $\mu > 0.25$ and $\mu > 1.75$, then the second-order conditions for the high-quality and the low-quality firm are negative, respectively.

For the moment we describe the following results in terms of μ:

$$s_{\mathrm{H},2} = \frac{4b(-1+\mu)(2-3\mu+4\mu^2)}{v(-4+23\mu-46\mu^2+24\mu^3)}; \; s_{\mathrm{L},2} = \frac{4b(-1+\mu)(2-3\mu+4\mu^2)}{v\mu(-4+23\mu-46\mu^2+24\mu^3)}, \tag{15}$$

$$p_{\mathrm{H},2} = \frac{8b^2(-1+\mu)^2(2-3\mu+4\mu^2)(4-11\mu+8\mu^2)}{v(-4+23\mu-46\mu^2+24\mu^3)^2};$$

$$p_{\mathrm{L},2} = \frac{4b^2(-1+\mu)^2(2-3\mu+4\mu^2)(4-11\mu+8\mu^2)}{v\mu(-4+23\mu-46\mu^2+24\mu^3)^2}, \tag{16}$$

$$q_{\mathrm{H},2} = \frac{4b\mu(1-4\mu+2\mu^2)}{-4+23\mu-46\mu^2+24\mu^3}; \qquad q_{\mathrm{L},2} = \frac{b\mu(4-11\mu+8\mu^2)}{-4+23\mu-46\mu^2+24\mu^3};$$

$$q_{\mathrm{H},2} + q_{\mathrm{L},2} = \frac{b\mu(8-27\mu+16\mu^2)}{-4+23\mu-46\mu^2+24\mu^3}, \tag{17}$$

$$\pi_{\mathrm{H},2} = \frac{64b^3(-1+\mu)^2\mu(1-4\mu+2\mu^2)^2(2-3\mu+4\mu^2)}{v(-4+23\mu-46\mu^2+24\mu^3)^3};$$

$$\pi_{\mathrm{L},2} = \frac{2b^3(-1+\mu)(2-3\mu+4\mu^2)^2(4-15\mu+8\mu^2)(4-11\mu+8\mu^2)}{v(-1+4\mu)(-4+23\mu-46\mu^2+24\mu^3)^3}, \tag{18}$$

$$\tilde{\theta}_2 = \frac{b(-1+\mu)(4-11\mu+8\mu^2)}{-4+23\mu-46\mu^2+24\mu^3}; \; \hat{\theta}_2 = \frac{b(-1+2\mu)(4-11\mu+8\mu^2)}{-4+23\mu-46\mu^2+24\mu^3}. \tag{19}$$

3.3 The high-quality firm chooses vertical integration while the low-quality firm adopts outsourcing: (K, V)

This subsection is concerned with the case in which the high-quality firm chooses vertical integration while the low-quality firm adopts outsourcing. The two first-order conditions for both firms leading to the determination of quality levels can be reduced to (see the Appendix for the derivation):

$$\beta = \frac{b(4-15\mu+12\mu^2)(8-42\mu+99\mu^2-104\mu^3+48\mu^4)}{2(-1+\mu)\mu(-7+4\mu)(-1+4\mu)(-2+19\mu-38\mu^2+24\mu^3)}. \tag{20}$$

Let $f(\mu)$ stand for the right-hand side of this equation. There is a one-to-one correspondence between β and μ through $\beta = f(\mu)$. In this case, μ can be thought of as a function of β and b, and it is decreasing in β. For the moment we describe the following results in terms of μ:

$$s_{\mathrm{H},3} = \frac{2b(-1+\mu)\mu^2(-7+4\mu)}{v(-2+19\mu-38\mu^2+24\mu^3)}; \; s_{\mathrm{L},3} = \frac{2b(-1+\mu)\mu(-7+4\mu)}{v(-2+19\mu-38\mu^2+24\mu^3)}, \tag{21}$$

$$p_{\mathrm{H},3} = \frac{2b^2(-1+\mu)^2\mu^2(-7+4\mu)(4-15\mu+12\mu^2)}{v(-2+19\mu-38\mu^2+24\mu^3)^2};$$

$$p_{\mathrm{L},3} = \frac{2b^2(-1+\mu)^2\mu(-7+4\mu)(2-11\mu+8\mu^2)}{v(-2+19\mu-38\mu^2+24\mu^3)^2}, \tag{22}$$

$$q_{\mathrm{H},3} = \frac{b\mu(4-15\mu+12\mu^2)}{-2+19\mu-38\mu^2+24\mu^3}; \qquad q_{\mathrm{L},3} = \frac{2b\mu(1-2\mu+2\mu^2)}{-2+19\mu-38\mu^2+24\mu^3};$$

$$q_{H,3}+q_{L,3}=\frac{b\mu(6-19\mu+16\mu^2)}{-2+19\mu-38\mu^2+24\mu^3}, \tag{23}$$

$$\pi_{H,3}=\frac{b^3(7-4\mu)^2(-1+\mu)\mu^4(4-15\mu+12\mu^2)(4-13\mu+12\mu^2)}{v(-1+4\mu)(-2+19\mu-38\mu^2+24\mu^3)^3};$$

$$\pi_{L,3}=\frac{8b^3(-1+\mu)^2\mu^2(-7+4\mu)(1-2\mu+2\mu^2)^2}{v(-2+19\mu-38\mu^2+24\mu^3)^3}, \tag{24}$$

$$\tilde{\theta}_3=\frac{b(-1+\mu)(2-11\mu+8\mu^2)}{-2+19\mu-38\mu^2+24\mu^3};\quad \hat{\theta}_3=\frac{b(-2+15\mu-23\mu^2+12\mu^3)}{-2+19\mu-38\mu^2+24\mu^3}. \tag{25}$$

3.4 Both firms choose outsourcing: (V, V)

In this case we obtain (see Motta, 1993 and Amacher et al., 2005)[6] :

$$s_{H,4}=0.819521b/v;\qquad s_{L,4}=0.398722b/v;\qquad s_{H,4}-s_{L,4}=0.420798b/v, \tag{26}$$

$$p_{H,4}=0.453313b^2/v;\qquad p_{L,4}=0.150020b^2/v;\qquad p_{H,4}/p_{L,4}=3.021676, \tag{27}$$

$$q_{H,4}=0.279245b;\qquad q_{L,4}=0.344503b;\qquad q_{H,4}+q_{L,4}=0.623747b, \tag{28}$$

$$\pi_{H,4}=0.0328129b^3/v;\qquad \pi_{L,4}=0.0242980b^3/v;\qquad \pi_{H,4}+\pi_{L,4}=0.0571108b^3/v, \tag{29}$$

$$\tilde{\theta}_4=0.376253b;\qquad \hat{\theta}_4=0.720755b, \tag{30}$$

$$\gamma_4>2.657789;\qquad \mu_4=2.055367. \tag{31}$$

3.5 Characterization of the Equilibria

In the first stage each firm chooses an organizational production mode, and in the second stage, its quality level. In this game, there are four possible outcomes as illustrated in Table 1: (K, K), (V, K), (K, V) and (V, V). We can use the following four lemmas to find out which pair(s) will be a Nash equilibrium (see the Appendix for proofs):

Lemma 1 *If* $0.411542b\le\beta\le0.546074b(=\beta^{*}_{12})$*, then we have* $\pi_{H,1}\ge\pi_{H,2}$*. Conversely, if* $0.546074b<\beta$*, then we obtain* $\pi_{H,1}<\pi_{H,2}$*.*

Lemma 2 *If* $\beta\le0.217161b(=\beta^{**}_{13})$*, then we have* $\pi_{L,1}\ge\pi_{L,3}$*. Conversely, if* $0.217161b<\beta$*, then* $\pi_{L,1}<\pi_{L,3}$*.*

Lemma 3 *If* $\beta\in(0.411542b,\ 0.416891b)$ *or* $[0.411542b,\ 0.413810b]$*, then we have* $\pi_{L,2}\ge\pi_{L,4}$*. Conversely, if* $\beta>0.416891b$*, then* $\pi_{L,2}<\pi_{L,4}$*.*

[6] Since $b-\tilde{\theta}_4<1$ has to hold true, $b<1.603213$, in which case $\gamma_4>2.657789$. In addition, if $\mu>1$ and $\mu\in(1.75,\ 3.611555)$, then the second-order conditions for the high-quality and the low-quality firm are negative, respectively.

Lemma 4 *If $\beta \leq 0.581924b (= \beta^{**}_{34})$, then we have $\pi_{\mathrm{H},3} \geq \pi_{\mathrm{H},4}$. Conversely, if $0.581924b < \beta$, then $\pi_{\mathrm{H},3} < \pi_{\mathrm{H},4}$.*

We therefore establish (see the Appendix for the proof):

Proposition 1 *Under price competition there are two subgame perfect equilibria of the game.*
(i) *If $0.217161b \leq \beta \leq 0.581924b$, then* (K, V) *is a subgame perfect equilibrium of the game.*
(ii) *If $0.581924b \leq \beta$, then* (V, V) *is a subgame perfect equilibrium of the game.*

This proposition shows that there are two subgame perfect equilibrium outcomes for the game. The outcome that we can obtain varies with the efficiency ratio β. < Insert Figure 1. > The first part of the proposition states that the high-quality firm chooses vertical integration while the low-quality firm chooses to outsource when the efficiency of in-house production compared to that of outsourcing is high, i.e., when β is small. This implies that low values of β will lead to the firms' asymmetric choices meaning intra-industry heterogeneity.

The last part of the proposition shows that both firms choose to outsource when the efficiency of outsourcing compared to that of in-house production is high, i.e., when β is large. This outcome means intra-industry homogeneity. It should be noted that the other configurations, (K, K) and (V, K), are ruled out as subgame perfect equilibrium outcomes. Vertical integration is a dominated strategy for the low-quality firm.[7]

Let us make a comparison between the two subgame perfect equilibrium outcomes, (K, V) and (V, V), on the basis of Table 2. We cannot use (20) to determine an equilibrium value of the degree of product differentiation μ in (K, V). Choosing $\mu = 2.866840$ as a benchmark produces equilibrium values in the Table.[8] < Insert Table 2. >

First it should be noted that a value of v in (K, V) differs from the one in (V, V), because a change in $\beta (\equiv k/v)$ influences which organizational production mode each firm chooses. Focus on the ratios such as the degree of product differentiation, the ratio of a price of a high-quality firm to that of a low-quality firm, a market share and the ratio of the high-quality firm's profits to total profits in the industry. The degree of product differentiation in (K, V) is higher than that in (V, V). Moreover, the ratio of the price of the high-quality

[7] We have the same kind of proposition under quantity competition, too. See Miyamoto (2011) for the proof.

[8] The ratio of $\pi_{\mathrm{H},3}$ to $\pi_{\mathrm{H},4}$ leads to $30.47584414(7-4\mu)^2(-1+\mu)\mu^4(4-15\mu+12\mu^2)(4-13\mu+12\mu^2)/(-1+4\mu)(-2+19\mu-38\mu^2+24\mu^3)^3$. Note that $\pi_{\mathrm{H},3} = \pi_{\mathrm{H},4}$ holds true at $\mu = 2.866840$. If $\mu > 2.866840$, then $\pi_{\mathrm{H},3} > \pi_{\mathrm{H},4}$, while if $\mu < 2.866840$, then $\pi_{\mathrm{H},3} < \pi_{\mathrm{H},4}$. We can easily verify that the ratio $\pi_{\mathrm{H},3}/\pi_{\mathrm{H},4}$ is increasing in μ in the interval $(1.75, +\infty)$.

firm to that of the low-quality firm is much higher in (K, V) than in (V, V). The high-quality firm's market share is higher in (K, V) than in (V, V). The ratio of the high-quality firm's profits to the industry profits is greater in (K, V) than in (V, V).

The high-quality firm's profits in (K, V) are increasing in μ in the interval $(1.75, +\infty)$, whereas its profits in (V, V) do not vary with μ in the same interval. Both the high-quality firm's price in (K, V) and the ratio of this price to the low-quality firm's price in (K, V) are also increasing in μ in the interval $(1.75, +\infty)$. Moreover, the high-quality firm's market share in (K, V) is increasing in μ in the same interval as mentioned above. If $\mu > 2.866840$, then $\pi_{H,3} > \pi_{H,4}$, from which it follows that a subgame perfect equilibrium outcome in (K, V) is brought about at a greater value of μ than that which we use to find equilibrium values in (K, V) in Table 2.

We therefore have (see the Appendix for the proof):

Proposition 2 *Relaxing price competition through product differentiation takes place in* (K, V)*, whereas both firms in* (V, V) *face much fiercer price competition.*

This proposition demonstrates that relaxing price competition through product differentiation occurs when the efficiency of in-house production compared to that of outsourcing is high, i.e., when β is small. In this situation the high-quality firm enjoys much larger profits than the low-quality firm. If the efficiency of outsourcing compared to that of in-house production is high, i.e., β is large, then both high- and low-quality firms choose to outsource and face much fiercer price competition. This leads to the result that the high-quality firm earns the lower profits in (V, V) than in (K, V).[9]

4 Discussion

In this section, the implications of Propositions 1 and 2 for the use of a vertical integration model are explored with a discussion about a recent business slump in Japanese electronics

[9] Table 2 shows that the degree of product differentiation in (K, V) is larger than that in (V, V), i.e., $\mu_3 = 2.866840 > \mu_4 = 2.055367$. However, the high-quality firm's price in (K, V) is lower than that in (V, V), i.e, $p_{H,3} = 0.163338b^2/v < 0.453313b^2/v = p_{H,4}$, from which it does not follow that as k gets larger compared to v, the high-quality firm's price falls agaist the price level in (K, V) and reaches that in (V, V). However, we have $\mu_1 = 5.251234$ in (K, K) and $\mu = 4.610514$ at which $p_{H,3} = p_{H,4}$ holds true. Thus, there is a real possibility that the degree of product differentiation in (K, V) will be larger than $\mu = 4.610514$, in which case we can obtain the result that as k gets larger relative to v, the high-qualiity firm's price falls agaist the price level in (K, V) and reaches that in (V, V).

enterprises in the flat panel TV industry.

As mentioned in the Introduction, many of Japan's digital electronics products have lost their competitive advantages since the 1990s. As to such a big change in the Japanese digital consumer electronics industry, Fujimoto (2012, p. 6) says, "post-war Japan, where many manufacturing sites with a rich endowment of coordinative capability emerged for historic reasons (e.g., economy of scarcity), tended to enjoy design-based comparative advantages in integral (i.e., coordination-intensive) products rather than modular (coordination-saving) ones. This logic may persuasively explain why Japan's auto industry (relatively integral) sustains its competitive advantage, while its digital consumer appliances (relatively modular) quickly lost their competitiveness as of the early 21st century."

'Integral' and 'modular' do not correspond to 'vertical integration' and 'outsourcing', respectively, because the former are closely connected with a given product's architectural attributes.[10] The latter mean organizational production modes directly related to a firm's 'make-or-buy' decision.

Suppose that a product's architecture is of a modular type, more specifically, an open-modular type. The question which needs to be addressed is whether modular architecture leads to a firm's choice of outsourcing as an organizational production mode. In this architecture "mix and match" of component designs is technically and commercially feasible not only within a firm but also across firms (see Fujimoto, 2007, p. 86). There is a one-to-one correspondence between the product's functional elements and its structural elements. This implies that an organizational production mode that fits in well with this architecture is the one in which the firm makes use of components supplied by separate profit-pursuing producers. Also, these suppliers are characterized by specialization. Thus, Fujimoto (2007, p. 98) says, "in case of an open-modular architecture product, a firm with a high level of strategic capability tends to enjoy higher level of profit because its ability to create profitable business plans by combining existing product-process elements tends to bring about higher profits more straightforwardly." Examples in point are Vizio referred to in the Introduction and Apple supplying the iPhone. A distinguishing feature that both companies have in common is that they do not have their own plants and facilities for the

[10]Fujimoto (2007, p. 82) states that architecture means a basic design approach to link a system's functional elements to its structural elements, and/or to cut and connect a system's structural elements (components or modules) of the system. In the present context 'system' means 'product'.

production of their products. It seems that the firms supplying products with modular architecture have a strong tendency toward outsourcing as an organizational production mode.

In the case of closed-integral products, a lack of integrative manufacturing capability may become a bottleneck that could hamper a firm making a profit from potentially effective business plans (see Fujimoto, 2007, p. 98). One might say that products with a more integral architecture require a high level of coordination efforts that are characterized by a teamwork of multi-skilled workers. Thus, it is likely that firms producing those products have a strong inducement to choose vertical integration as an organizational production mode.

These remarks lead us to consider the implications of Propositions 1 and 2 for a major change in the Japanese flat panel TV industry in recent years. There are two subgame perfect equilibria of the game in Proposition 1, i.e., (K, V) and (V, V). The former corresponds to the situation where electronics manufacturers choosing vertical integration coexisted with those choosing to outsource in the Japanese flat panel TV market in the 2000s, whereas the latter to the situation where leading electronics producers tend to outsource key components or even production of all TV sets these days. According to Fujimoto (2012), what caused a recent business slump in Japanese electronics enterprises in the flat panel TV industry is a rapid change in the product-process architecture expressed by the fact that digital electronics products' architectures quickly became modular.

Since μ can be viewed as a decreasing function of β, the modularization implies that v rapidly declined relative to k after the bankruptcy of Lehman Brothers in September 2008. The big change in β led to the transition from intra-industry organizational heterogeneity, (K, V), to intra-industry organizational homogeneity, (V, V). In (V, V) we have a smaller degree of product differentiation and fiercer price competition than in (K, V). This example is found in recent real-world flat panel TV markets. According to *The Nikkei* (July 15, 2013), Panasonic decided to outsource the production of LCD panels it needs to assemble its TV sets in China, to reduce costs. Furthermore, *The Nikkei* (June 26, 2013) says that Sharp changed its management strategy from vertical integration to outsourcing. Also, Sharp has decided to provide high definition TFT-LCD panels and module technology to CEC Panda in China. Both companies have agreed to operate a new plant, and Sharp preserves the right to purchase the LCD panels that will be produced at the plant.[11]

[11] http://www.sharp-world.com/corporate/ir/topics/pdf/130627.pdf

It seems that outsourcing that Sharp has chosen quite differs from that chosen by Panasonic from the point of view of a management strategy. There are two noticeable features that distinguish Sharp from the other Japanese electronics manufacturers; one is that Sharp has the only 10th generation LCD panel manufacturing plant in the world in Sakai; the other is that Sharp and Semiconductor Energy Laboratory have jointly developed a new IGZO technology with high crystallinity.[12] One of the aims that Sharp set out to achieve a recovery from a difficult situation is to provide LCD panels using the new IGZO technology to electronics manufacturers. The question may arise as to whether this aim will lead Sharp to be on the road to recovery.

Consider the situation where a high-quality firm in (K, V) is driven to decide between a shift from (K, V) to (V, V) and the road to a producer supplying a key component for two firms in (V, V). Note that the degree of product differentiation in (V, V) is given by $\mu = 2.055367$, at which $\pi_{H,3,} < \pi_{H,4}$ holds true and we have $\beta = 2.155655b$ through $\beta = f(\mu)$ that the high-quality firm faces. If it chooses the road to the supplier, it is assumed that the upstream firm is a monopoly. This assumption reflects Sharp's main feature of technology.

Profits that a high-quality firm makes in (V, V) are $\pi_{H,4} = 0.0328129b^3/v$. In order to make a comparison between the two cases, it is necessary to find the amount of profits that the road to a supplier will lead the high-quality firm to make. We apply Nash's bargaining solution to the latter case.[13] In (V, V) the low-quality firm's profits are $\pi_{L,4} = 0.0242980b^3/v$ and then total profits are $\pi_4 = \pi_{H,4} + \pi_{L,4} = 0.0571108b^3/v$. Moreover, let q_4 represent total output and we obtain $q_4 = q_{H,4} + q_{L,4} = 0.623747b$. We assume that when the firms in (V, V) want to enter a market, they incur a small amount of fixed entry cost F_e. Let π_{34} stand for profits that the road to the supplier brings for the high-quality firm and we obtain $\pi_{34} = vq_4 - (\frac{1}{2}ks^2_{H,4} + \frac{1}{2}ks^2_{L,4})$ where $s_{H,4} = 0.819521b/v$ and $s_{L,4} = 0.398722b/v$.

So far β has been referred to as the efficiency ratio. In this framework, however, k and v can be viewed as a price of unit quality corresponding to vertical integration and outsourcing, respectively. Making use of the expression $k/v = 2.155655b$ enables the parameters k and v to be determined in the bargaining game. The two downstream producers choosing outsourcing and the upstream supplier providing a key component for them bargain over

[12] http://www.sharp-world.com/corporate/news/120601.html

[13] See de Fontenay and Gans (2008), for example, for an approach to bargaining between upstream and downstream firms.

the price of unit quality v. Their payoffs are given by, respectively:

$$U_u(v) = \pi_{34} = vq_4 - (\frac{1}{2}ks^2_{\mathrm{H},4} + \frac{1}{2}ks^2_{\mathrm{L},4}),$$
$$U_d(v) = \pi_4 - 2F_e = \frac{0.0571108b^3}{v} - 2F_e.$$

In addition, the subscripts u and d stand for the upstream supplier and the downstream producers, respectively. Nash's bargaining solution is the one to the following maximization problem (see Okada, 1996):

$$\max_v \{vq_4 - (\frac{1}{2}ks^2_{\mathrm{H},4} + \frac{1}{2}ks^2_{\mathrm{L},4})\}\{\frac{0.0571108b^3}{v} - 2F_e\}, \tag{32}$$

where $k = 2.155655bv$.

We therefore obtain (see the Appendix for the proof):

Claim 1 *If the following inequality holds true:*

$$-0.00890568b^4F_e + 15.675436F_e^3 < 0,$$

then there exists some v at which we obtain the first-order condition for the maximization problem (32). A sufficient condition for the inequality above to hold true is that $F_e < 0.0238355b^2$. The second-order condition for maximization is always satisfied.

Moreover, if the entry cost is set equal to $F_e = 0.01b^2$, then we have:

$$\bar{U}_u = 0.607524b;\ \bar{U}_d = 0.0120809b;\ \bar{U}_u\bar{U}_d = 0.00733940b$$
$$\bar{k} = 3.837531b^2;\ \bar{v} = 1.780216b.$$

This claim states that the payoff of the upstream producer is larger than that of the downstream producers for a small amount of entry cost. Thus, it suggests that a management strategy that Sharp is pursuing may lead it to take the opportunity of reducing an enormous amount of fixed costs and thereby to be on the road to recovery.

5 Conclusion

In this paper we have used a vertically differentiated duopoly model to analyze how firms' choice of two types of organizational production mode, vertical integration and outsourcing, will influence their quality choice, pricing and profits. Among other things, we have defined conditions under which the asymmetric configuration, where the high-quality firm chooses vertical integration while the low-quality firm chooses outsourcing, is accepted as a subgame perfect equilibrium outcome. Moreover, we have shown that not only the other asymmetric configuration but also the symmetric configuration where both firms choose in-house production is ruled out as the subgame perfect equilibrium outcome. The other symmetric configuration where the two firms choose outsourcing emerges as the subgame perfect equilibrium outcome.

Shy and Stenbacka (2003) provide an example exemplifying (V, V) as a subgame perfect equilibrium outcome. They suggest that it is a common business practice for competing product market firms to outsource production to a joint input producer in order to exploit economies of scale. Its good example is given by the competing mobile phone producers Ericsson and Nokia that outsource production to take place in Elcoteq's (a joint subcontractor) production facilities. It is a fact that in the mobile phone industry the unit price is much lower than in the flat panel TV industry. This may imply that each final goods producer does not have a tendency to make a heavy investment in state-of-the-art production facilities and R&D, but he has an incentive to outsource production to a joint subcontractor in order to enable the whole industry to fully utilize economies of scale.

Which of the two subgame perfect equilibrium outcomes takes place depends on what value the efficiency ratio β takes on. (K, V) occurs when β is small, i.e., when the efficiency of in-house production relative to that of outsourcing is high. (V, V) takes place when β is large, i.e., when the efficiency of outsourcing compared to that of in-house production is high.

As already mentioned in our study, in real world flat panel TV markets in the 2000s electronics manufacturers aiming for vertical integration coexisted with electronics producers outsourcing the production of all or part of the inputs they need. In Japan it is Sharp and Panasonic that made massive investments in state-of-the-art facilities that carry out integrated production of large LCD TVs from manufacture of LCD panels to final assembly of TV sets in the early 2000s. This offers an example of the asymmetric equilibrium configuration (K, V). Nowadays leading electronics producers tend to outsource key components or even production of all TV sets and moreover, an enterprise chooses to be a supplier that provides high definition TFT-LCD panels to the other producers. In real world flat panel TV markets the electronics manufacturers face much fiercer price competition than in the earlier times and are now suffering from a fall in the price of LCD TV sets. Furthermore, it is well known that they made a heavy investment in R&D for an improvement in the quality of products and the development of new products. This investment would cause k to be much larger than before, thereby leading to an increase in

β. This situation corresponds to the symmetric equilibrium configuration (V, V).

As a high-quality firm invests more heavily in state-of-the-art production facilities and R&D, returns to investment are decreasing, thereby causing k to increase compared to v and thus leading to an increase in β. This remark suggests that the high-quality firm would find it hard to sustain the asymmetric equilibrium configuration (K, V) over a long period of time. However, it should be noted that what product the remark can be applied to depends on how quickly and how easily its architecture will be modular (see Baldwin and Clark, 1997).

Appendix

Derivation of (14). In the third stage, firms choose prices given the organizational production modes and quality levels. From the first-order conditions, $\partial\Pi_{\mathrm{HV}}/\partial p_{\mathrm{H}} = 0$ and $\partial\Pi_{\mathrm{LK}}/\partial p_{\mathrm{L}} = 0$, we can solve for each firm's price as a function of qualities:

$$p_{\mathrm{H}} = \frac{2s_{\mathrm{H}}[c(s_{\mathrm{H}}) + b(s_{\mathrm{H}} - s_{\mathrm{L}})]}{4s_{\mathrm{H}} - s_{\mathrm{L}}}; \quad p_{\mathrm{L}} = \frac{s_{\mathrm{L}}[c(s_{\mathrm{H}}) + b(s_{\mathrm{H}} - s_{\mathrm{L}})]}{4s_{\mathrm{H}} - s_{\mathrm{L}}}, \tag{33}$$

where $c(s_{\mathrm{H}}) = \frac{1}{2}vs_{\mathrm{H}}^2$.

Substituting these prices into Π_{HV} and Π_{LK} yields:

$$\Pi_{\mathrm{HV}} = \frac{s_{\mathrm{H}}^2[4b(s_{\mathrm{H}} - s_{\mathrm{L}}) + vs_{\mathrm{H}}(-2s_{\mathrm{H}} + s_{\mathrm{L}})]^2}{4(s_{\mathrm{H}} - s_{\mathrm{L}})(4s_{\mathrm{H}} - s_{\mathrm{L}})^2}, \tag{34}$$

$$\Pi_{\mathrm{LK}} = \frac{s_{\mathrm{L}}[v^2s_{\mathrm{H}}^5 + 4bvs_{\mathrm{H}}^3(s_{\mathrm{H}} - s_{\mathrm{L}}) + 4b^2s_{\mathrm{H}}(s_{\mathrm{H}} - s_{\mathrm{L}})^2 - 2ks_{\mathrm{L}}(s_{\mathrm{H}} - s_{\mathrm{L}})(-4s_{\mathrm{H}} + s_{\mathrm{L}})^2]}{4(s_{\mathrm{H}} - s_{\mathrm{L}})(4s_{\mathrm{H}} - s_{\mathrm{L}})^2}. \tag{35}$$

Differentiating (34) and (35) with respect to each firm's quality, given the other firm's quality, gives the first-order conditions:

$$\begin{aligned}
&s_{\mathrm{H}}[16b^2(s_{\mathrm{H}} - s_{\mathrm{L}})^2(4s_{\mathrm{H}}^2 - 3s_{\mathrm{H}}s_{\mathrm{L}} + 2s_{\mathrm{L}}^2) - 8bvs_{\mathrm{H}}(s_{\mathrm{H}} - s_{\mathrm{L}})^2(16s_{\mathrm{H}}^2 - 12s_{\mathrm{H}}s_{\mathrm{L}} + 3s_{\mathrm{L}}^2) \\
&\quad + v^2s_{\mathrm{H}}^2(48s_{\mathrm{H}}^4 - 116s_{\mathrm{H}}^3s_{\mathrm{L}} + 92s_{\mathrm{H}}^2s_{\mathrm{L}}^2 - 31s_{\mathrm{H}}s_{\mathrm{L}}^3 + 4s_{\mathrm{L}}^4)]/[4(s_{\mathrm{H}} - s_{\mathrm{L}})^2(4s_{\mathrm{H}} - s_{\mathrm{L}})^3] = 0, \quad (36) \\
&[4b^2s_{\mathrm{H}}^2(4s_{\mathrm{H}} - 7s_{\mathrm{L}})(s_{\mathrm{H}} - s_{\mathrm{L}})^2 - 4ks_{\mathrm{L}}(s_{\mathrm{H}} - s_{\mathrm{L}})^2(4s_{\mathrm{H}} - s_{\mathrm{L}})^3 + 4bvs_{\mathrm{H}}^3(s_{\mathrm{H}} - s_{\mathrm{L}})^2(4s_{\mathrm{H}} + s_{\mathrm{L}}) \\
&\quad + v^2s_{\mathrm{H}}^5(4s_{\mathrm{H}}^2 + s_{\mathrm{H}}s_{\mathrm{L}} - 2s_{\mathrm{L}}^2)]/[4(s_{\mathrm{H}} - s_{\mathrm{L}}^2)(4s_{\mathrm{H}} - s_{\mathrm{L}})^3] = 0. \quad (37)
\end{aligned}$$

Define $s_{\mathrm{L}} \equiv x$. By definition we have $s_{\mathrm{H}} = \mu x$. Substituting these expressions into (36), we can solve for x as a function of μ:

$$x_2^* = \frac{4b(-1+\mu)}{v\mu(-1+2\mu)}; \quad x_2^{**} = \frac{4b(-1+\mu)(2-3\mu+4\mu^2)}{v\mu(-4+23\mu-46\mu^2+24\mu^3)}. \tag{38}$$

Evaluating the second-order condition for the high-quality firm at x_2^*, we obtain:

$$\frac{2bv\mu(1-4\mu+2\mu^2)^2}{(-1+2\mu)(1-5\mu+4\mu^2)} > 0 \quad \text{for} \quad \mu > 0.5,$$

while evaluating the second-order condition for the high-quality firm at x_2^{**} yields:

$$-\frac{2bv\mu(1-4\mu+2\mu^2)(-8+60\mu-123\mu^2+128\mu^3-78\mu^4+24\mu^5)}{(-1+\mu)^2(-1+4\mu)(2-3\mu+4\mu^2)(-4+23\mu-46\mu^2+24\mu^3)} < 0 \quad \text{for} \quad \mu > 1.707107.$$

Thus, x_2^{**} in (38) is accepted as a 'solution'.

Substituting x_2^{**} into (37) and arranging terms leads to:

$$\frac{k}{v} = \frac{b\mu^3(4-11\mu+8\mu^2)(-20+81\mu-84\mu^2+32\mu^3)}{4(-1+\mu)(-1+4\mu)(2-3\mu+4\mu^2)(-4+23\mu-46\mu^2+24\mu^3)}. \tag{39}$$

Letting $\beta \equiv k/v$ and denoting the right-hand side of (39) by $g(\mu)$ leads to $\beta = g(\mu)$.

Derivation of (21). Solving the first-order conditions, $\partial\Pi_{\mathrm{HK}}/\partial p_{\mathrm{H}} = 0$ and $\partial\Pi_{\mathrm{LV}}/\partial p_{\mathrm{L}} = 0$, yields:

$$p_{\mathrm{H}} = \frac{s_{\mathrm{H}}[c(s_{\mathrm{L}}) + 2b(s_{\mathrm{H}} - s_{\mathrm{L}})]}{4s_{\mathrm{H}} - s_{\mathrm{L}}}; \quad p_{\mathrm{L}} = \frac{2s_{\mathrm{H}}c(s_{\mathrm{L}}) + bs_{\mathrm{L}}(s_{\mathrm{H}} - s_{\mathrm{L}})}{4s_{\mathrm{H}} - s_{\mathrm{L}}}, \tag{40}$$

where $c(s_{\mathrm{L}}) = \frac{1}{2}vs_{\mathrm{L}}^2$.

Substituting these prices into Π_{HK} and Π_{LV} produces:

$$\Pi_{\mathrm{HK}} = \frac{s_{\mathrm{H}}^2[16b^2(s_{\mathrm{H}} - s_{\mathrm{L}})^2 + 8bvs_{\mathrm{L}}^2(s_{\mathrm{H}} - s_{\mathrm{L}}) + v^2s_{\mathrm{L}}^4 - 2k(s_{\mathrm{H}} - s_{\mathrm{L}})(-4s_{\mathrm{H}} + s_{\mathrm{L}})^2]}{4(s_{\mathrm{H}} - s_{\mathrm{L}})(4s_{\mathrm{H}} - s_{\mathrm{L}})^2}, \tag{41}$$

$$\Pi_{\mathrm{LV}} = \frac{s_{\mathrm{H}}s_{\mathrm{L}}[2b(s_{\mathrm{H}} - s_{\mathrm{L}}) + vs_{\mathrm{L}}(-2s_{\mathrm{H}} + s_{\mathrm{L}})^2]}{4(s_{\mathrm{H}} - s_{\mathrm{L}})(4s_{\mathrm{H}} - s_{\mathrm{L}})^2}. \tag{42}$$

Differentiating (41) and (42) with respect to each firm's quality, given the other firm's quality, gives the first-order conditions:

$$\begin{aligned}
&-s_{\mathrm{H}}[4k(s_{\mathrm{H}} - s_{\mathrm{L}})^2(4s_{\mathrm{H}} - s_{\mathrm{L}})^3 + 16bvs_{\mathrm{L}}^3(s_{\mathrm{H}} - s_{\mathrm{L}})^2 + v^2s_{\mathrm{L}}^4(4s_{\mathrm{H}}^2 + s_{\mathrm{H}}s_{\mathrm{L}} - 2s_{\mathrm{L}}^2) \\
&-16b^2(s_{\mathrm{H}} - s_{\mathrm{L}})^2(4s_{\mathrm{H}}^2 - 3s_{\mathrm{H}}s_{\mathrm{L}} + 2s_{\mathrm{L}}^2)]/[4(s_{\mathrm{H}} - s_{\mathrm{L}})^2(4s_{\mathrm{H}} - s_{\mathrm{L}})^3] = 0, \qquad (43)\\
&s_{\mathrm{H}}[4b^2s_{\mathrm{H}}(4s_{\mathrm{H}} - 7s_{\mathrm{L}})(s_{\mathrm{H}} - s_{\mathrm{L}})^2 - 4bvs_{\mathrm{L}}(s_{\mathrm{H}} - s_{\mathrm{L}})^2(16s_{\mathrm{H}}^2 - 12s_{\mathrm{H}}s_{\mathrm{L}} + s_{\mathrm{L}}^2) \\
&+v^2s_{\mathrm{L}}^2(48s_{\mathrm{H}}^4 - 100s_{\mathrm{H}}^3s_{\mathrm{L}} + 76s_{\mathrm{H}}^2s_{\mathrm{L}}^2 - 23s_{\mathrm{H}}s_{\mathrm{L}}^3 + 2s_{\mathrm{L}}^4)]/[4(s_{\mathrm{H}} - s_{\mathrm{L}})^2(4s_{\mathrm{H}} - s_{\mathrm{L}})^3] = 0. \qquad (44)
\end{aligned}$$

Substituting $s_{\mathrm{L}} = x$ and $s_{\mathrm{H}} = \mu x$ into (44), we can solve for x as a function of μ:

$$x_3^* = \frac{2b(-1+\mu)}{v(-1+2\mu)}; \quad x_3^{**} = \frac{2b\mu(-1+\mu)(-7+4\mu)}{v(-2+19\mu-38\mu^2+24\mu^3)}. \tag{45}$$

Evaluating the second-order condition for the low-quality firm at x_3^*, we have:

$$\frac{bv\mu(1-2\mu+2\mu^2)^2}{(-1+2\mu)(1-5\mu+4\mu^2)} > 0 \quad \text{for} \quad \mu > 0.5,$$

while evaluating the second-order condition for the low-quality firm at x_3^{**} yields:

$$-\frac{bv\mu(1-2\mu+2\mu^2)(28-87\mu+102\mu^2-70\mu^3+24\mu^4)}{(-1+\mu)^2(-7+4\mu)(-1+4\mu)(-2+19\mu-38\mu^2+24\mu^3)} < 0 \quad \text{for} \quad \mu > 1.75.$$

Thus, x_3^{**} in (45) is accepted as a 'solution'.

Substituting x_3^{**} into (43) and arranging terms gives:

$$\frac{k}{v} = \frac{b(4-15\mu+12\mu^2)(8-42\mu+99\mu^2-104\mu^3+48\mu^4)}{2(-1+\mu)\mu(-7+4\mu)(-1+4\mu)(-2+19\mu-38\mu^3+24\mu^4)}. \tag{46}$$

Letting $f(\mu)$ denote the right-hand side of (46) leads to $\beta = f(\mu)$.

Proof of Lemma 1. Consider first whether $\pi_{\mathrm{H},1} \geq \pi_{\mathrm{H},2}$. $\pi_{\mathrm{H},1}$ given by the first expression in (11) includes parameters b and k, while $\pi_{\mathrm{H},2}$ has parameters b and v and the degree of product differentiation μ. When the high-quality firm makes a comparison between $\pi_{\mathrm{H},1}$ in (K, K) and $\pi_{\mathrm{H},2}$ in (V, K), it faces $\beta = g(\mu)$ represented by (14). This equation relates k and v to μ. Using (14) enables us to express $\pi_{\mathrm{H},1}$ in terms of v rather than k.

Substituting $k = vg(\mu)$ into the first expression in (11) yields:

$$\pi_{\mathrm{H},1} = \frac{0.0977544b^3(-1+\mu)(-1+4\mu)(2-3\mu+4\mu^2)(-4+23\mu-46\mu^2+24\mu^3)}{v\mu^3(4-11\mu+8\mu^2)(-20+81\mu-84\mu^2+32\mu^3)}. \tag{47}$$

Let $R_{12}(\mu) \equiv \pi_{\mathrm{H},1}/\pi_{\mathrm{H},2}$. This ratio is given below:

$$R_{12}(\mu) = \frac{0.00152741(-1+4\mu)(-4+23\mu-46\mu^2+24\mu^3)^4}{\mu^4(-1+\mu)(4-11\mu+8\mu^2)(-20+81\mu-84\mu^2+32\mu^3)(1-4\mu+2\mu^2)^2}. \tag{48}$$

$\mu = 1.707107$ is an asymptotic line of $R_{12}(\mu)$. Because $\lim_{\mu\to 1.707107+} R_{12}(\mu) = +\infty$, $\lim_{\mu\to\infty} R_{12}(\mu) = 0$, $\lim_{\mu\to 1.707107+} dR_{12}(\mu)/d\mu = -\infty$ and $\lim_{\mu\to\infty} dR_{12}(\mu)/d\mu = 0$, $R_{12}(\mu)$ is positive and strictly decreasing in μ on (1.707107, $+\infty$).[14] This implies that there exists a value of μ in the interval $(1.707107, +\infty)$ at which $R_{12}(\mu) = 1$, i.e., $\pi_{\mathrm{H},1} = \pi_{\mathrm{H},2}$. This value is $\mu^*_{12} = 3.287677$. Correspondingly, making use of $\beta = g(\mu)$ yields $\beta^*_{12} = 0.546074b$ at which $\pi^*_{\mathrm{H},2} = 0.0447532b^3/v$ is equal to $\pi_{\mathrm{H},1} = 0.0244386b^4/k$. Thus, $\pi_{\mathrm{H},1} \geq \pi_{\mathrm{H},2}$ for $\mu \leq \mu^*_{12}$, from which it follows that $\beta \leq \beta^*_{12}$. It should be noted that $g(\mu)$ attains a minimum $0.411542b$ at $\mu = 2.080460$. The above condition for $\pi_{\mathrm{H},1} \geq \pi_{\mathrm{H},2}$ is changed to $0.411542b \leq \beta \leq 0.546074b$. Conversely, if $0.546074b < \beta$, then $\pi_{\mathrm{H},1} < \pi_{\mathrm{H},2}$.

However, when μ is in the interval (1.585120, 2.255379), $\pi_{\mathrm{L},2} > \pi_{\mathrm{H},2}$. Find out whether $\pi_{\mathrm{H},1} > \pi_{\mathrm{L},2}$ for $\mu \in (1.585120, 2.255379)$, on which $\pi_{\mathrm{L},2}$ attains a maximum $0.0306901b^3/v$ at $\mu = 1.784800$. Because the second-order condition for the low-quality firm in (V, K) requires $\mu > 1.946960$, the maximum value of $\pi_{\mathrm{L},2}$ occurs at $\mu = 1.946960$ in [1.946960, 2.255379). It is $0.0286504b^3/v$. A value of β corresponding to $\mu = 1.946960$ is $0.416891b$ through $\beta = g(\mu)$, from which it follows that $\pi_{\mathrm{L},2} = 0.0119441b^4/k < 0.0244386b^4/k = \pi_{\mathrm{H},1}$ for $\mu \in [1.946960, 2.255379)$.[15] □

Proof of Lemma 2. Compare $\pi_{\mathrm{L},1}$ and $\pi_{\mathrm{L},3}$. Since the second expression in (11) includes parameters b and k, we use (20) relating k and v to μ. The low-quality firm's payoff $\pi_{\mathrm{L},1}$ can be rewritten as:

$$\pi_{\mathrm{L},1} = \frac{0.00305482b^3(-1+\mu)\mu(-7+4\mu)(-1+4\mu)(-2+19\mu-38\mu^2+24\mu^3)}{v(4-15\mu+12\mu^2)(8-42\mu+99\mu^2-104\mu^3+48\mu^4)}. \tag{49}$$

[14] It should be noted that $\pi_{\mathrm{H},2}$ in (18) is positive and the second-order condition for the high-quality firm is negative for $\mu \in (1.707107, +\infty)$.

[15] This question is closely related to the persistence of the high-quality advantage referred to by Lehmann-Grube (1997). In this case also we verify the persistence of the high-quality advantage.

Let $R_{13}(\mu) \equiv \pi_{L,1}/\pi_{L,3}$. We use $\pi_{L,3}$ given by the second expression in (24) to obtain:

$$R_{13}(\mu) = \frac{0.000381853(-1+4\mu)(-2+19\mu-38\mu^2+24\mu^3)^4}{(4-15\mu+12\mu^2)(8-42\mu+99\mu^2-104\mu^3+48\mu^4)(-1+\mu)\mu(1-2\mu+2\mu^2)^2}. \tag{50}$$

We have $\lim_{\mu\to 1+} R_{13}(\mu) = +\infty$, $\lim_{\mu\to\infty} R_{13}(\mu) = +\infty$, $\lim_{\mu\to 1+} dR_{13}(\mu)/d\mu = -\infty$ and $\lim_{\mu\to\infty} dR_{13}(\mu)/d\mu = 0.219947$. $R_{13}(\mu)$ attains a minimum 0.136995 at $\mu = 1.200335$ where $d^2R_{13}(\mu)/d\mu^2 = 2.702819 > 0$. $R_{13}(\mu)$, then, is strictly increasing in $\mu \in (1.200335, +\infty)$. Therefore, there is a value of μ at which $R_{13}(\mu) = 1$, i.e., $\pi_{L,1} = \pi_{L,3}$. Its value is $\mu_{13}^{**} = 4.862582$. Making use of $\beta = f(\mu)$ leads to the result that we have $\beta_{13}^{**} = 0.217161b$ corresponding to μ_{13}^{**} and correspondingly $\pi_{L,3}^{**} = 0.00703356b^3/v = 0.00152741b^4/k = \pi_{L,1}$. Because $\pi_{L,3}$ is positive and the second-order condition for the low-quality firm is negative for $\mu \in (1.75, +\infty)$, $\pi_{L,1} \geq \pi_{L,3}$ for $\mu \in [4.862582, \infty)$. Using $\beta = f(\mu)$ yields $\pi_{L,1} \geq \pi_{L,3}$ for $\beta \leq 0.217161b$. Conversely, if $0.217161b < \beta$, then $\pi_{L,1} < \pi_{L,3}$. □

Proof of Lemma 3. $\pi_{L,2}$ is given by the second expression in (18) and $\pi_{L,4}$ by that in (29). Let $R_{24}(\mu)$ denote the ratio of the former to the latter:

$$R_{24}(\mu) \equiv \frac{82.311433(-1+\mu)(2-3\mu+4\mu^2)^2(4-15\mu+8\mu^2)(4-11\mu+8\mu^2)}{(-1+4\mu)(-4+23\mu-46\mu^2+24\mu^3)^3}. \tag{51}$$

Since $\mu = 1.261890$ is an asymptotic line of $R_{24}(\mu)$, $\lim_{\mu\to 1.261890+} R_{24}(\mu) = -\infty$. In addition, $\lim_{\mu\to\infty} R_{24}(\mu) = 0$. $R_{24}(\mu)$ attains a maximum 1.263072 at $\mu = 1.784800$ where $d^2R_{24}(\mu)/d\mu^2 = -11.773962 < 0$. $\pi_{L,2} > 0$ for $\mu > 1.553054$ and the second-order condition for the low-quality firm is negative for $\mu > 1.946960$. Because $d^2R_{24}(\mu)/d\mu^2 < 0$ on $(1.784800, +\infty)$, $R_{24}(\mu)$ is strictly decreasing in $\mu \in (1.946960, +\infty)$. Letting $R_{24}(\mu)$ be set equal to one, we have $\mu_{24}^{*} = 2.182609$ and correspondingly $\beta_{24}^{*} = 0.413810b$ through $\beta = g(\mu)$ where $\pi_{L,2} = \pi_{L,4}$. In this lemma the intervals of β, $(0.411542b, 0.416891b)$ and $[0.411542b, 0.413810b]$, correspond to those of μ, $(1.946960, 2.080460)$ and $[2.080460, 2.182609]$, respectively. Because $g(\mu)$ is at first decreasing and then increasing in $\mu \in (1.946960, +\infty)$, two values of μ correspond to a given value of β. For example, $\mu = 1.946960$ and 2.242880 correspond to $\beta = 0.416891b$. Since $\pi_{L,2}$ is decreasing in μ over $[1.946960, +\infty)$, the low-quality firm will choose the lower one from those two values corresponding to a given value of β. Therefore, if $\beta \in (0.411542b, 0.416891b)$ or $[0.411542b, 0.413810b]$, then $\pi_{L,2} \geq \pi_{L,4}$. Conversely, if $\beta > 0.416891b$, then $\pi_{L,2} < \pi_{L,4}$. □

Proof of Lemma 4. Compare $\pi_{H,3}$ and $\pi_{H,4}$. Let $R_{34}(\mu) \equiv \pi_{H,3}/\pi_{H,4}$:

$$R_{34}(\mu) = \frac{30.475844(7-4\mu)^2(-1+\mu)\mu^4(4-15\mu+12\mu^2)(4-13\mu+12\mu^2)}{(-1+4\mu)(-2+19\mu-38\mu^2+24\mu^3)^3}. \tag{52}$$

$R_{34}(\mu) = 0$ and $dR_{34}(\mu)/d\mu = 0$ at $\mu = 1.75$. In addition, $d^2R_{34}(\mu)/d\mu^2 = 3.625273 > 0$ at $\mu = 1.75$. Furthermore, $\lim_{\mu\to\infty} R_{34}(\mu) = +\infty$ and $\lim_{\mu\to\infty} dR_{34}(\mu)/d\mu = 1.269827$. Since $R_{34}(\mu)$ is strictly increasing in $\mu \in [1.75, +\infty)$, there exists a value of μ at which $R_{34}(\mu) = 1$, i.e., $\pi_{\mathrm{H},3} = \pi_{\mathrm{H},4}$. This value is $\mu_{34}^{**} = 2.866840$ and correspondingly $\beta_{34}^{**} = 0.581924b$ through $\beta = f(\mu)$, at which both firms' profits are $0.0328129b^3/v$. Therefore, if $\beta \leq 0.581924b$, then $\pi_{\mathrm{H},3} \geq \pi_{\mathrm{H},4}$. Conversely, if $0.581924b < \beta$, then $\pi_{\mathrm{H},3} < \pi_{\mathrm{H},4}$.[16] □

Proof of Proposition 1. From Table 1 we see that (K, K) is a Nash equilibrium when

$$\pi_{\mathrm{H},1} \geq \pi_{\mathrm{H},2} \quad \text{and} \quad \pi_{\mathrm{L},1} \geq \pi_{\mathrm{L},3}. \tag{53}$$

Lemmas 1 and 2 say that a sufficient condition to have $\pi_{\mathrm{H},1} \geq \pi_{\mathrm{H},2}$ is $0.411542b \leq \beta \leq 0.546074b$ while the condition for $\pi_{\mathrm{L},1} \geq \pi_{\mathrm{L},3}$ is $\beta \leq 0.217161b$. Then, these conditions are incompatible and thus (K, K) is not a Nash equilibrium.

Conditions for (V, K) to be a Nash equilibrium are:

$$\pi_{\mathrm{H},2} \geq \pi_{\mathrm{H},1} \quad \text{and} \quad \pi_{\mathrm{L},2} \geq \pi_{\mathrm{L},4}. \tag{54}$$

It follows from Lemma 1 that if $\beta \geq 0.546074b$, then $\pi_{\mathrm{H},2} \geq \pi_{\mathrm{H},1}$. Lemma 3 says that a sufficient condition for $\pi_{\mathrm{L},2} \geq \pi_{\mathrm{L},4}$ is $\beta \in (0.411542b, 0.416891b)$ or $[0.411542b, 0.413810b]$. These conditions are incompatible with each other. Thus, (V, K) is not a Nash equilibrium.

Similarly, if the following conditions are satisfied:

$$\pi_{\mathrm{H},3} \geq \pi_{\mathrm{H},4} \quad \text{and} \quad \pi_{\mathrm{L},3} \geq \pi_{\mathrm{L},1}, \tag{55}$$

then (K, V) is a Nash equilibrium. Lemma 2 says that a condition for $\pi_{\mathrm{L},1} \leq \pi_{\mathrm{L},3}$ is $\beta \geq 0.217161b$. Lemma 4 gives a sufficient condition for $\pi_{\mathrm{H},3} \geq \pi_{\mathrm{H},4}$, i.e., $\beta \leq 0.581924b (= \beta_{34}^{**})$. Both lemmas therefore lead to the result that if $0.217161b \leq \beta \leq 0.581524b$, then (K, V) is a Nash equilibrium.

Turn to conditions under which (V, V) is a Nash equilibrium:

$$\pi_{\mathrm{H},4} \geq \pi_{\mathrm{H},3} \quad \text{and} \quad \pi_{\mathrm{L},4} \geq \pi_{\mathrm{L},2}. \tag{56}$$

It follows from Lemma 4 that if $\beta \geq 0.581924b$, then $\pi_{\mathrm{H},3} \leq \pi_{\mathrm{H},4}$. Lemma 3 says that if $\beta \geq 0.416891b$, then $\pi_{\mathrm{L},2} \leq \pi_{\mathrm{L},4}$. Thus, if $\beta \geq 0.581924b$, then $\pi_{\mathrm{L},2} \leq \pi_{\mathrm{L},4}$ while $\pi_{\mathrm{H},3} \leq \pi_{\mathrm{H},4}$. This means that if $\beta \geq 0.581924b$, then (V, V) is a Nash equilibrium. □

[16] In (K, V), if $\mu > 1.75$, then $\pi_{\mathrm{H},3} > 0$ and $\pi_{\mathrm{L},3} > 0$. Moreover, the second-order condition for each firm is negative for $\mu > 1.75$.

Proof of Proposition 2. Focus on the case of (K, V). We prove that in this case, the high-quality firm's profits and price, the ratio of this firm's price to the low-quality firm's price and the firm's market share are increasing in μ in the interval $(1.75, +\infty)$.

The high-quality firm's profits are given by the first expression in (24). Differentiating this firm's profits with respect to μ produces:

$$\begin{aligned}\frac{d\pi_{\mathrm{H},3}}{d\mu} =& \frac{1}{v(-1+4\mu)^2(24\mu^3-38\mu^2+19\mu-2)^4}[b^3(-7+4\mu)\mu^3(55296\mu^{10}-281088\mu^9 \\ &+758016\mu^8-1417920\mu^7+1866392\mu^6-1679226\mu^5+1014288\mu^4-403627\mu^3 \\ &+101436\mu^2-14544\mu+896)].\end{aligned} \tag{57}$$

Making use of this expression, we find the greatest real value of μ, $\mu_{max} = 1.75$, that satisfies $d\pi_{\mathrm{H},3}/d\mu = 0$. Moreover, we obtain the second derivative of the firm's profits with respect to μ:

$$\begin{aligned}\frac{d^2\pi_{\mathrm{H},3}}{d\mu^2} =& \frac{-1}{v(-1+4\mu)^3(24\mu^3-38\mu^2+19\mu-2)^5}[6b^3\mu^2(8257536\mu^{13}-81199104\mu^{12} \\ &+305922048\mu^{11}-617951232\mu^{10}+759937536\mu^9-596970752\mu^8+293807232\mu^7 \\ &-74280736\mu^6-6439760\mu^5+13207221\mu^4-5290012\mu^3+1124476\mu^2 \\ &-129024\mu+6272)].\end{aligned} \tag{58}$$

Evaluating the second derivative at $\mu = 1.75$ yields $d^2\pi_{\mathrm{H},3}/d\mu^2 = 0.118956b^3/v > 0$. This means that the high-quality firm's profits are increasing in μ in the interval $(1.75, +\infty)$.

Next, we prove that the high-quality firm's price and the ratio of this price to the low-quality firm's price are increasing in μ in the interval $(1.75, +\infty)$. The high-quality firm's price is given by the first expression in (22). Differentiating this expression with respect to μ, we have:

$$\begin{aligned}\frac{dp_{\mathrm{H},3}}{d\mu} =& \frac{1}{v(24\mu^3-38\mu^2+19\mu-2)^3}[2b^2(-1+\mu)\mu(1152\mu^7-4320\mu^6 \\ &+7512\mu^5-8894\mu^4+7291\mu^3-3597\mu^2+950\mu-112)].\end{aligned} \tag{59}$$

The second derivative of the price is written as:

$$\begin{aligned}\frac{d^2p_{\mathrm{H},3}}{d\mu^2} =& \frac{-1}{v(24\mu^3-38\mu^2+19\mu-2)^4}[4b^2(10368\mu^9-95520\mu^8+283488\mu^7 \\ &-410400\mu^6+335177\mu^5-164026\mu^4+47912\mu^3-7088\mu^2+4\mu+112)].\end{aligned} \tag{60}$$

Evaluating the second derivative at $\mu = 1.384529$ that is the greatest value of μ satisfying $dp_{\mathrm{H},3}/d\mu = 0$, we obtain $d^2p_{\mathrm{H},3}/d\mu^2 = 0.442584b^2/v > 0$, from which it follows that the high-quality firm's price is increasing in μ in the interval $(1.75, +\infty)$.

The ratio of the high-quality firm's price to the low-quality firm's price is given by:

$$\frac{p_{\mathrm{H},3}}{p_{\mathrm{L},3}} = \frac{\mu(12\mu^2 - 15\mu + 4)}{8\mu^2 - 11\mu + 2}. \tag{61}$$

Following similar procedures produces $d^2(p_{\mathrm{H},3}/p_{\mathrm{L},3})/d\mu^2 = 5.740911 > 0$ at $\mu = 1.685841$, i.e., the greatest value of μ that satisfies $d(p_{\mathrm{H},3}/p_{\mathrm{L},3})/d\mu = 0$. The ratio of the high-quality firm's price to the low-quality firm's price, then, is increasing in μ in the interval $(1.75, +\infty)$.

The first and the third expressions in (23) lead to:

$$\frac{q_{\mathrm{H},3}}{q_3} = \frac{12\mu^2 - 15\mu + 4}{16\mu^2 - 19\mu + 6}, \tag{62}$$

where $q_3 = q_{\mathrm{H},3} + q_{\mathrm{L},3}$. Following similar procedures yields $d^2(q_{\mathrm{H},3}/q_3)/d\mu^2 = 234.225950 > 0$ at $\mu = 0.602629$, i.e., the greatest value of μ that satisfies $d(q_{\mathrm{H},3}/q_3)/d\mu = 0$. Thus, the high-quality-firm's market share is increasing in μ in the interval $(1.75, +\infty)$. □

Proof of Claim 1. The degree of product differentiation in (V, V) is $\mu_4 = 2.055367$. Since the high-quality firm in (K, V) is supposed to change its production mode and to produce a key component yielding μ_4 for the two firms in (V, V), it faces the expression $\beta(\equiv k/v) = 2.1556655b$ derived from substituting μ_4 into (14). Note that μ_4 leads to $\pi_{\mathrm{H},4} > \pi_{\mathrm{H},3}$.

In (V, V) we have $\pi_{\mathrm{H},4} = 0.0328129b^3/v$ and $\pi_{\mathrm{L},4} = 0.0242980b^3/v$, and so total profits are $\pi_4 = \pi_{\mathrm{H},4} + \pi_{\mathrm{L},4} = 0.0571108b^3/v$. Let $U_d(v) = \pi_4 - 2F_e$ denote the payoff of two firms in (V, V) and this payoff can be written as:

$$U_d(v) = \frac{0.0571108b^3}{v} - 2F_e. \tag{63}$$

They jointly bargain with an upstream supplier over the price of quality v. Let us assume that the supplier is characterized by cutting-edge technologies. Thus, it is a monopoly in the upstream market. Let U_u stand for the payoff of the supplier and we have:

$$\begin{aligned} U_u(v) &= vq_4 - \{\frac{1}{2}k(0.819521b/v)^2 + \frac{1}{2}k(0.398722b/v)^2\} \\ &= 0.623747vb - \frac{0.895237b^3}{v}, \end{aligned} \tag{64}$$

where $q_4 = 0.623747b$ and $k = 2.155655bv$.

Now we can write the bargaining game as: $\max_v U_u(v)U_d(v)$. Let us find Nash's bargaining solution to this problem. Manipulating (63) produces:

$$v = \frac{0.0571108b^3}{U_d + 2F_e}. \tag{65}$$

Substituting (65) into (64) leads to:

$$U_u = \frac{0.0356227b^4}{U_d + 2F_e} - 15.675436U_d - 31.350871F_e. \tag{66}$$

This substitution changes the bargaining game $\max_v U_u(v)U_d(v)$ to:

$$\max U_u U_d \text{ subject to } U_u = \frac{0.0356227b^4}{U_d + 2F_e} - 15.675436U_d - 31.350871F_e. \tag{67}$$

Since this constraint is almost linear in payoff space, we can solve for the $\bar{U}_d$ that maximizes the Nash product.

The first-order condition for a maximum with respect to U_d is that:

$$\begin{aligned} \frac{-8}{(U_d + 2F_e)^2}\{&3.918859U_d^3 + 19.594294U_d^2F_e + 31.350871U_dF_e^2 \\ &- 0.00890568b^4F_e + 15.675436F_e^3\} = 0, \end{aligned} \tag{68}$$

and the second-order condition is that:

$$\begin{aligned} \frac{-1.6}{(U_d + 2F_e)^3}\{&19.594294U_d^3 + 117.565766U_d^2F_e + 235.131532U_dF_e^2 \\ &+ 0.0890568b^4F_e + 156.754355F_e^3\} < 0. \end{aligned} \tag{69}$$

If $-0.00890568b^4F_e + 15.675436F_e^3 < 0$, then there exists some v that the first-order condition for maximization holds true. A sufficient condition for the inequality above to hold true is that $F_e < 0.0238355b^2$. The second-order condition is always satisfied.

Let us suppose that $F_e = 0.01b^2 < 0.0238355b^2$. This value of F_e results in:

$$\bar{U}_u = 0.607524b; \;\; \bar{U}_d = 0.0120809b; \;\; \bar{U}_u\bar{U}_d = 0.00733940b \tag{70}$$

$$\bar{k} = 3.837531b^2; \;\; \bar{v} = 1.780216b. \tag{71}$$

□

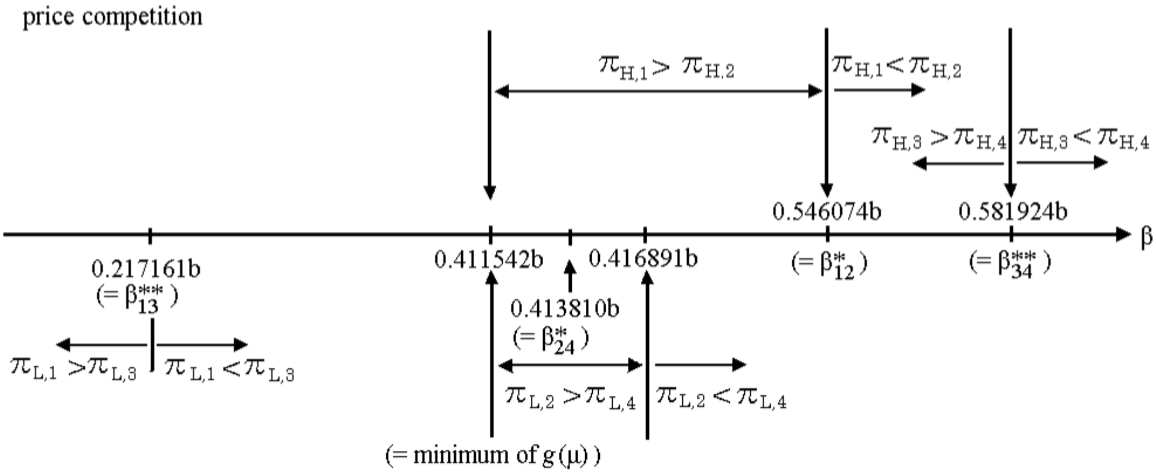
Table 2: Equilibrium Values in (K, V) and (V, V)[17]

(K, V)		
High-quality Firm	Low-quality Firm	
$s_{\mathrm{H},3} = 0.448520b/v$	$s_{\mathrm{L},3} = 0.156451b/v$	$s_{\mathrm{H},3} - s_{\mathrm{L},3} = 0.292069b/v$
$p_{\mathrm{H},3} = 0.163338b^2/v$	$p_{\mathrm{L},3} = 0.0346066b^2/v$	$p_{\mathrm{H},3}/p_{\mathrm{L},3} = 4.719838$
$q_{\mathrm{H},3} = 0.559244b$	$q_{\mathrm{L},3} = 0.219558b$	$q_3 = q_{\mathrm{H},3} + q_{\mathrm{L},3} = 0.778082b$
		$q_{\mathrm{H},3}/q_3 = 0.718082$
$\pi_{\mathrm{H},3} = 0.0328129b^3/v$	$\pi_{\mathrm{L},3} = 0.00491111b^3/v$	$\pi_3 = \pi_{\mathrm{H},3} + \pi_{\mathrm{L},3} = 0.0377240b^3/v$
		$\pi_{\mathrm{H},3}/\pi_3 = 0.869815$
		$CS_3 = 0.0931191b^3/v$
		$SS_3 = 0.130843b^3/v$
		$\tilde{\theta}_3 = 0.221198b$
		$\hat{\theta}_3 = 0.440756b$
		$\gamma_3 > 4.520835$
		$\mu_3 = 2.866840$
		$k = 0.581924bv$

(V, V)		
High-quality Firm	Low-quality Firm	
$s_{\mathrm{H},4} = 0.819521b/v$	$s_{\mathrm{L},4} = 0.398722b/v$	$s_{\mathrm{H},4} - s_{\mathrm{L},4} = 0.420798b/v$
$p_{\mathrm{H},4} = 0.453313b^2/v$	$p_{\mathrm{L},4} = 0.150020b^2/v$	$p_{\mathrm{H},4}/p_{\mathrm{L},4} = 3.021676$
$q_{\mathrm{H},4} = 0.279245b$	$q_{\mathrm{L},4} = 0.344503b$	$q_4 = q_{\mathrm{H},4} + q_{\mathrm{L},4} = 0.623747b$
		$q_{\mathrm{H},4}/q_4 = 0.447689$
$\pi_{\mathrm{H},4} = 0.0328129b^3/v$	$\pi_{\mathrm{L},4} = 0.0242980b^3/v$	$\pi_4 = \pi_{\mathrm{H},4} + \pi_{\mathrm{L},4} = 0.0571108b^3/v$
		$\pi_{\mathrm{H},4}/\pi_4 = 0.574547$
		$CS_4 = 0.0939701b^3/v$
		$SS_4 = 0.151081b^3/v$
		$\tilde{\theta}_4 = 0.376253b$
		$\hat{\theta}_4 = 0.720755b$
		$\gamma_4 > 2.657789$
		$\mu_4 = 2.055367$

[17] In Table 2, let CS_j and SS_j denote consumer surplus and social surplus, respectively. Equilibrium values in (K, V) are evaluated at $\mu = 2.866840$ at which $\pi_{\mathrm{H},3} = \pi_{\mathrm{H},4}$ holds true. Substituting μ into (20) yields $k = 0.581924bv$.

Figure 1: β and Payoffs: price competition



Note: The superscripts * and ** refer to the functions $g(\mu)$ and $f(\mu)$, respectively. The subscripts of β, 12, 13, 24, and 34, refer to two states, respectively. For example, 12 refers to the states, 1 and 2, simultaneously.

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