Game Theoretic Analysis of Positive and Negative Campaign for Policy*

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Abstract

In this paper, we construct and analyze competitive election model which two candidates choose the degree of policy, positive and negative campaigning for policy in order to maximize their own probability of winning an election. Particularly, we analyze relationship between voter's awareness of policy effect and voter's welfare and consider whether we should regulate negative campaigning by using voter's welfare. We obtain three interesting result. First, symmetric equilibrium policy is more extreme than voters' welfare maximization policy. In this paper, voters' awareness for policy effect is imperfect. Therefore, voters' welfare maximization policy is not realized under symmetric equilibrium. Second, if voters' awareness of policy effects is high, then voters' welfare which is obtained by policy is high. In Japan Election, the youth does not have interest of election because he assumes youth voice does not reach candidates' policy very much. However, in this model we consider all of voters who include young ages should realize candidates' policy and manifest if they want to get good welfare. Finally, regulation of negative campaign is not necessarily because voters' welfare in no regulating negative campaign for policy is more than in regulating. Past literature consider bad aspect of negative campaign and in Japanese Election candidates can not use negative campaign on Internet and Election broadcast. However, we take an example which negative campaign for policy should not be regulated.

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1. Introduction

In this paper, we represent and analyze a competitive election model which two candidates choose degree of policy, positive and negative campaigning for policy in order to maximize their own probability of winning an election. Past literatures focus on relationship between candidates' ability and degree of campaign resources. However, these literatures neglect voters' behavior. We consider that relationship between candidates and voters is important for the same or more than relationship between candidates. Therefore, we analyze relationship between voter's awareness of policy effect and voter's welfare and we consider whether we should regulate negative campaigning by using voter's welfare.

Candidates and companies use advertisement or campaign in order to differentiate from competitor. In order to give theirselves an advantage there are two campaign and advertisement types, which are positive and negative. Positive campaign is defined as expression of their own good aspect. Negative campaign is defined as expression of competitor's bad aspect. Negative campaign is often used by political campaign rather than companies' advertisements. In particular, in American president election negative campaign is often used. For example, Young (1987) discusses amount to use negative campaign in the 1980s is greater than in twenty years ago. So, after the 1990s, empirical and experimental researches for negative campaign in American election increase. In Japan, even though regime of Liberal Democratic Party continued in 50 years, there exist negative campaigns. It is obvious by Curini (2011)'s empirical research which shows if parties' ideology is close, then amount of negative campaign for candidates' valence increases by using Japan and Italia data. Recent example is Japanese general election in 2009. In this election, Democratic Party of Japan used negative campaign for Liberal Democratic Party and then Democratic Party of Japan won. In Politics field, negative campaign infiltrates. So, we consider it is worth to studying negative campaign.

As described above, in previous literatures of negative campaign, empirical researches are a lot, however theoretical research is rare.[1] Particularly, first theoretical studies of negative campaign are Skaperdas and Grofman (1995) and Harrington and Hess (1996). Skaperdas and Grofman (1995) defines positive campaigning as increasing independent voters who vote for theirselves and negative campaigning as decreasing competitor's supporters, and then show if candidate's supporters are much more than competitor's, he does not have incentive to use negative campaign. In other word, a candidate who is much advantageous to competitor uses positive campaign only under Nash equilibrium. Harrington and Hess (1996) defines positive campaign as advertising their own ideology is moderate and negative campaign as advertising that opponent's ideology is extreme, and then shows if a candidate's personality is advantageous to competitor's, a candidate uses positive campaign more than competitor. So, Skaperdasand Grofman (1995)'s outcome is similar to Harrington and Hess (1996)'s. In other word, these models show a candidate who is advantageous to competitor uses positive campaign more than opponent. These outcomes are consistent with Druckman, Kifer and Perkin (2009) which shows challenger uses negative campaign more than incumbent by empirically method. However, these analyses are limited, because these two theoretic studies assume an effect of positive and negative campaign. In other word, a campaign effect is not limited, for example, like Chakrabarti (2007) negative campaigning is defined as advertising competitor's personality is bad. So, we consider positive campaign is defined as representing their own good aspect and negative campaign is defined as representing competitor's bad aspect. For example, theoretic studies of campaign by using the above definition of positive and negative campaign are Mattes and Redlawsk (2015) and Schipper and Woo (2014). Mattes and Redlawsk (2015) constructs and analyzes a competitive election model which voter has belief of candidates' type (which is high or low ability for political issue) and voter updates his belief by candidates'

[1] Lau and Rovner (2009) is great survey about negative campaign's theory and empirical research.

campaigning. However, in this model there exist multi equilibria, so this model cannot show proper use of positive and negative campaign. Schipper and Woo (2014) analyzes a microtargeting election. Schipper and Woo (2014) shows an election's outcome where voters know all issues of election is equal to where voters do not know even though voters' rationality is bounded in candidates using negative campaign. So, Schipper and Woo (2014) shows affirmative effect of negative campaigning. However, this model cannot show proper use of positive and negative campaign.

Past theoretical literatures of negative campaign are not sufficient for the following two reasons. First, past theoretical models of negative campaign assume effect of campaign, for example negative campaign affects competitor's ideology. However, campaign does not only affect ideology of policy or candidates but also candidates' policy effect which is announced by their manifest. Therefore, this paper does not assume campaign effects, but defines positive campaign as representing good aspect of candidates' own policy and negative campaign as representing bad aspect of competitor's policy. And this paper constructs and analyzes competitive election model which two candidates choose the degree of policy, positive and negative campaigning for policy in order to maximize their own probability of winning an election. Second, past theoretical models only analyze the relationship between candidates' ability and the degree of campaign resources, in other word they neglect the aspect of votes' behavior. Indeed in real elections, there is a difference between candidates' ability. However, we consider a voters' behavior, aspect and relationship between voters and candidates is as important as or more than relationship between candidates' ability in elections. Thus, in this paper we analyze the relationship between voters' behavior and candidates' behavior.

In this paper we show the following three outcomes. First, symmetric equilibrium policy is more extreme than voters' welfare maximization policy. In this paper, voters' awareness for policy effect is imperfect. Therefore, voters' welfare maximization policy is not realized under symmetric equilibrium. Second, if voters' awareness of policy effects is high, then voters' welfare which is obtained by policy is high. Finally, regulation of negative campaign is not necessarily because voters' welfare in no regulating negative campaign for policy is more than in regulating. Previous empirical literatures of negative campaign show bad effect of negative campaign. For example, Ansolabehere, Iyengar and Simon (1999) shows that negative campaign decreases voter turnout by using empirical method. Geer and Vavreck (2014) show that if a candidate uses negative campaign, then voters recognize his ideology is extreme by using experimental method. Therefore, we can guess negative campaign should be regulated because negative campaign causes bad influence. However, our outcome is the opposite of our guess that negative campaign should be regulated. Thus, we show negative campaign do not only have bad effect.

The remainder of the paper is organized as follows. In the next section, we construct two-stage game which two candidates maximize their probability of winning election. In Section 3, We analyze candidates' behavior and relationship between voters' awareness and the degree of policy under symmetric equilibrium. In Section 4, we analyze whether we should regulate negative campaigning by using voters' welfare. This paper closes in Section 5 with brief remarks on further studies concerning negative campaign.

2. The Model

In this section, we construct a two-stage game which two candidates who have the same ability maximize their probability of winning an election. In the first stage, candidates simultaneously choose their policy $x_i \in [0, \infty)$, which is their manifest and represents degree of innovation from the status quo. Innovation of politics has both good and bad aspect. Good aspect represents how much it brings a positive effect on economy or for voters. Bad aspect represents how much it brings a negative effect on economy or for voters, or how extreme ideology of policy is. For example of this policy, we present Policy of free trade. (More specifically say TPP.) Policy of free trade eliminates the tariff. Therefore, cheap products (whose quality is almost the same as domestic products) are imported from foreign. Thus, consumers' welfare improves. This character is good aspect of Policy of free trade. However, if consumers buy cheap products made by foreign, then domestic products is not consumed. Therefore, domestic industry may decline. This character is bad aspect of Policy of free trade. In summary of the above, political policy includes positive and negative aspect. In this model, to characterize this political policy's property, we define the positive aspect of policy x_i as $f(x_i)$ and the negative aspect of policy as $g(x_i)$. Now, we assume function f and g satisfy the following condition.

Assumption 1

i. Function $f: [0, \infty) \to (0, \infty)$ and $g: [0, \infty) \to (0, \infty)$ are C^2-function and satisfy $f' > 0, f'' \leq 0, g' > 0, g'' \geq 0$.

ii. $f(0) = g(0)$.

iii. $\lim_{x \to 0} f'(x) > \lim_{x \to 0} g'(x)$, $\lim_{x \to \infty} f'(x) < \lim_{x \to \infty} g'(x)$.

Assumption 1's i represents the more innovation of policy x_i increases, the more good and bad aspect increase and marginal effect of good (bad) aspect decreases (increases). In other word, extreme policy gives a bad influence for voter. Assumption 1's ii represents if candidate chooses status quo, then good aspect of the status quo is equal to bad aspect. In other word, if candidate chooses the status quo, then voter cannot receive welfare by policy. Assumption 1's iii means a little innovation is better than no innovation, however extreme innovation causes more bad aspect of policy than good aspect of policy.

In the second stage, candidates have one resource (or time). They distribute one resource to positive campaigning p_i and negative campaigning n_i . In other word, they divide one resource which satisfies $p_i + n_i = 1$. In this model, positive (negative) campaigning is defined as that candidates convey $P(p_i)$ ($N(n_i)$) about the good (bad) aspect of his own (opponent's) policy to median voter. Now, we assume function P and N satisfy the following condition.

Assumption 2

i. Function $P: [0,1] \rightarrow [0,1]$ and $N: [0,1] \rightarrow [0,1]$ are C^2-function and satisfy $P' > 0, P'' \leq 0, N' > 0, N'' \leq 0$.

ii. $P(0) = N(0) = 0, P(1) = N(1) = 1$.

Assumption 2's i means that if a candidate increases the resource of campaigning, then the effect of campaigning increases but the marginal effect of campaigning decreases. Assumption 2's ii means if candidates choose no resources for campaigning, then the effect of campaigning does not exist and if candidates allocate positive (negative) campaigning to all resources, then voter recognizes perfectly his good (bad) aspect of policy.

Next, we consider the probability of winning an election. Alesina and Spear (1988) and Harrington (1991) construct the probability of winning election by using median voter's utility. However, their researches consider campaigning affects only ideology of policy. In this paper, we consider a good and bad aspect which concludes ideology of policy. Therefore, we construct the probability of winning an election by using voters' utility which voters get when a candidate i carry an election. In this paper, we assume that set of voters is $[0, 1]$ and each voter knows a part of good and bad aspect of policy from the first and understands a unknown part of a good and bad aspect by using candidates' campaigning. In other word, each voter knows $\alpha \in (0,1)$ about good aspect and $\beta \in (0,1)$ about bad aspect from the first. Therefore, in this model, each voter does not understand perfectly that their welfare is $f(x) - g(x)$. Then, to summarize the above, we define voters' utility when he chooses candidate i as the following equation (1).

$$u(i) = \left(\alpha + (1-\alpha)P(p_i)\right) f(x_i) - \left(\beta + (1-\beta)N(n_{-i})\right)g(x_i) \tag{1}$$

Next, we construct voters' strategy and probability of winning an election by using equation (1). Each voter k has $z_{k,ij}$ which is the degree of aversion of a candidate i compared to a candidate j. In this model, for each $k = 1,2$, $z_{k,ij}$ follows identically and independently distribution whose median is θ_{ij}, and satisfies $z_{k,ij} = -z_{k,ji}$. We assume that each candidate does not know aversion distribution's median θ_{ij}, however each candidate knows the distribution which aversion distribution's median follows. In this model, aversion distribution's median θ_{ij} follows the symmetric distribution whose mean is 0. In other word, each candidate does not know true aversion's value of median voter, who is defined as the voter whose aversion is median value, but each candidate knows the distribution of aversion's median. Next, we introduce voters' strategies. We assume that each voter k votes for candidate i if $u(i) - u(j) > z_{k,ij}$ and votes for candidate j if $u(i) - u(j) < z_{k,ij}$. (If $u(i) - u(j) = z_{k,ij}$, then each voter votes for i with probability 0.5.) If we assume the above voters' strategy, candidate wins an election when he gets vote of median voter. Therefore, we consider the probability of candidate i winning an election is equal to the probability of the median voter voting for candidate i. Thus, we construct the probability of candidate i winning an election for the following equation (2) by using the difference of voters' utility when he votes for each candidate and median voter's aversion.

$$\begin{aligned} EP_i &= EP\big(u(i) - u(-i)\big) \\ &= EP\Big(\big\{\big(\alpha + (1-\alpha)P(p_i)\big) f(x_i) - \big(\beta + (1-\beta)N(n_{-i})\big)g(x_i)\big\} \qquad (2) \\ &\quad - \big\{\big(\alpha + (1-\alpha)P(p_{-i})\big) f(x_{-i}) - \big(\beta + (1-\beta)N(n_i)\big)g(x_{-i})\big\}\Big) \end{aligned}$$

Now, we assume function EP satisfies the following conditions since the above discussion. (For the following assumption, we write y as $u(i) - u(j)$.)

Assumption 3

i. $\forall\, y \in (-\infty, \infty)$, $EP(y) \in (0,1)$.

ii. EP is C^1–function and $\forall\, y \in (-\infty, \infty)$, $EP'(y) > 0$.

iii. $EP(y) = 1 - EP(-y)$.

iv. $\lim_{y\to-\infty} EP(y) = 0$, $\lim_{y\to\infty} EP(y) = 1$.

Assumption 3's ii means if voters' utility in his voting for a candidate i increases or voters' utility in his voting for opponent decreases, then probability of a candidate i's winning an election increases. Assumption 3's iii means the sum of candidate i's probability and opponent's probability must be equal to one.

In this paper, each candidate chooses policy x_i and resource for positive campaigning p_i in order to maximize his probability which is defined by equation (2). In following Section 3, we show symmetric equilibrium of this game by using backward induction and examine property of candidate's behavior under symmetric equilibrium.

3. Symmetric Equilibrium

In this section, we consider the situation which satisfies following Assumption 4.

Assumption 4

i. $\alpha = \beta$.

ii. $P(p_i) = p_i^t \, , N(n_i) = n_i^t \; \big(t \in (0,1)\big)$.

iii. $f(x)^{t/1-t} f'(x)$ is non-increasing function.

Assumption 4's i means that voters know the good and bad aspect of policy by the same. This is because in this model, we construct the probability function of candidate's winning an election by using median voter. Therefore, we focus on median voter, and actually, it is rare that medium voter knows one of aspects well like independent voters. Thus, we consider this case. Assumption 4's ii means we assume specific campaigning function P and N satisfying Assumption 2. Assumption 4's iii means marginal effect of policy's good aspect is always stronger than or equal to policy effect of good aspect. In other word, extreme innovation does not really provide good effect. Then, each candidate maximizes the following probability function of winning an election.

$$\max_{x_i,p_i} EP(\{(\alpha+(1-\alpha)p_i^t)f(x_i)-(\alpha+(1-\alpha)(1-p_{-i})^t)g(x_i)\}$$

$$-\{(\alpha+(1-\alpha)p_{-i}^t)f(x_{-i})-(\alpha+(1-\alpha)(1-p_i)^t)g(x_{-i})\})$$

Next, we analyze this game by using backward induction. In the second stage, each candidate simultaneously chooses resource of positive campaigning p_i. So, we differentiate the above probability function in p_i. Then we get the following condition (3).[2]

$$p_i=\frac{f(x_i)^{1/1-t}}{f(x_i)^{1/1-t}+g(x_{-i})^{1/1-t}} \tag{3}$$

Next, we input (3) for the probability function and we analyze first stage game. In the first stage, each candidate simultaneously chooses policy x_i . So, we differentiate probability function in x_i for which we input equation (3). Then, we get the following condition (4).[3]

$$EP'\times\left\{\alpha[f'(x_i)-g'(x_i)]+(1-\alpha)\left[\frac{f(x_i)^{\frac{t}{1-t}}f'(x_i)}{\left(f(x_i)^{\frac{1}{1-t}}+g(x_{-i})^{\frac{1}{1-t}}\right)^t}-\frac{g(x_i)^{\frac{t}{1-t}}g'(x_i)}{\left(f(x_{-i})^{\frac{1}{1-t}}+g(x_i)^{\frac{1}{1-t}}\right)^t}\right]\right\}=0 \tag{4}$$

Policy x_i which satisfies the equation (4) is optimal policy for candidate i, and resource of positive campaigning p_i which satisfies equation (3) and (4) is optimal resource of positive campaigning for candidate i. Because of Assumption 3 which is that EP' is always positive, therefore we neglect EP' in equation (4) for the following discussion. Next, we consider symmetric case where we input $x_i=x_{-i}=x$ for equation (4). Then we get the following condition of symmetric equilibrium $x_1=x_2=x^*$.

$$\alpha\{f'(x^*)-g'(x^*)\}+\frac{(1-\alpha)\left\{f(x^*)^{\frac{t}{1-t}}f'(x^*)-g(x^*)^{\frac{t}{1-t}}g'(x^*)\right\}}{\left(f(x^*)^{\frac{1}{1-t}}+g(x^*)^{\frac{1}{1-t}}\right)^t}=0 \tag{5}$$

Now, we analyze symmetric equilibrium. If Assumption 1~3 hold, we can prove existence of interior solution. Before this proposition shows, we show lemma in order to prove existence of interior

[2] Probability function EP is increasing function (in other word, EP' is always positive) because of Assumption 3. Therefore, we neglect EP' in first order condition.

[3] Because of Assumption 1, 2 and 4, the section 2 of equation (4) is decreasing in x_i . Therefore, there exists a reaction function.

solution.

Lemma 1

Suppose x^1, x^2 and x^3 satisfy $f'(x^1) = g'(x^1)$, $f(x^2)^{t/1-t}f'(x^2) = g(x^2)^{t/1-t}g'(x^2)$, $f(x^3) = g(x^3)$. If Assumption1 holds, then $x^1 < x^2 < x^3$.

Proof

Because of Assumption 1 and 4's iii, $f(x)^{t/1-t}f'(x) - g(x)^{t/1-t}g'(x)$ is non-increasing function. And Because of Assumption 1, $f(x)^{t/1-t}f'(x) - g(x)^{t/1-t}g'(x)$ is positive if $0 < x \leq x^1$, and $f(x)^{t/1-t}f^{'}(x) - g(x)^{t/1-t}g^{'}(x)$ is negative if $x \geq x^3$. Therefore, because of Mean Value Theorem there exists x^2 which satisfies $x^1 < x^2 < x^3$.■

Next, we show existence of the symmetric equilibrium by using Lemma1.

Proposition 1

If Assumption 1, 3 and 4 hold, there exists the symmetric equilibrium policy $x_1 = x_2 = x^*$ and symmetric equilibrium policy x^* is greater than voters' welfare maximization policy x^1.

Proof

In order to get symmetric equilibrium policy x^*, we differentiate left hand side of equation (5) (which we calls $B(x)$) in x. Then, we get the following equation.

$$B'(x) = \alpha(f'' - g'') + (1-\alpha)\frac{K'(x)L(x) - K(x)L'(x)}{L(x)^2}$$

$$where\ K(x) = f(x)^{t/1-t}f'(x) - g(x)^{t/1-t}g'(x)\,, L(x) = \left(f(x)^{\frac{1}{1-t}} + g(x)^{\frac{1}{1-t}}\right)^t$$

Because of Assumption 1, 3's iii and Lemma 1, $K^{'}(x) < 0, L(x) > 0, L^{'}(x) > 0$ for all x and because of Lemma 1, $K(x) > 0$ if $0 \leq x < x^2$. Therefore, $B^{'}(x) < 0$ when $0 \leq x \leq x^2$. Thus,

$B(x)$ is decreasing function when $0 \leq x \leq x^2$. Equation $B(x)$ satisfies $B(x) > 0$ if $0 \leq x \leq x^1$ and $B(x) < 0$ if $x \geq x^2$. Therefore, there exists x^* which satisfies $x^1 < x^* < x^2$.■

By using Proposition 1, we can show upper bound of symmetric equilibrium policy.

Lemma 2

If Assumption 1, 3 and 4 hold, then symmetric equilibrium policy x^* is smaller than policy x^2 which is realized when voters do not recognize all of policy effect.

Because of Proposition 1 and Lemma 2, we discovered where symmetric equilibrium policy exists. And we turn out symmetric equilibrium policy x^* is more extreme than the policy x^1 which maximizes voters' welfare. In other word, candidates have no incentive to realize voters' welfare maximizing policy x^1 when voters' awareness is imperfect. In our model, voters' awareness for policy effects is not completely. So, each candidate wants to choose a little extreme policy and increase resource of positive campaigning. For example, we consider the case where candidates choose policy x^1 which is voters' welfare maximization policy. Then, amount of positive campaign resource is more than amount of negative campaign resource because of equation (3) and Assumption 1. Thus, candidate's marginal benefit which is represented by equation (5) is positive because marginal benefit from campaign which is represented by section 2 of equation (5) is positive. Therefore, each candidate have incentive to choose more extreme policy than voters' welfare maximization policy x^1 .

Next, we consider relationship between voter's awareness α and symmetric equilibrium policy x^*. By equation (5), if voters do not understand the effect of policy (in other word, α is close to 0) , then symmetric equilibrium policy x^* is close to extreme policy x^2, and if voters understand the effect of policy (in other word, α is close to 1) , then symmetric equilibrium policy x^* is close to the policy of the maximizing voters' welfare x^1. So, we can guess if voters know the effect of policy more, then voters' welfare increases. So, we show this guess by the following proposition.

Proposition 2

Under Assumption 1, 3 and 4, if voters' awareness α increases, then symmetric equilibrium policy x^* decreases and symmetric equilibrium resource of positive (negative) campaigning increases (decreases).

Proof

Firstly, we show relationship between α and x^*. In order to prove this relationship, we apply implicit function theorem with respect to α for equation (5). Then, we get the following outcome.

$$\frac{dx^*}{d\alpha} = \left\{ -f'(x^*) + g'(x^*) + \frac{\left\{ f(x^*)^{\frac{t}{1-t}} f'(x^*) - g(x^*)^{\frac{t}{1-t}} g'(x^*) \right\}}{\left(f(x^*)^{\frac{1}{1-t}} + g(x^*)^{\frac{1}{1-t}} \right)^t} \right\} \bigg/ B'(x^*) \qquad (6)$$

Equation (6)'s numerator of right hand side is positive and denominator is negative because of Proposition 1 and Lemma 2. Therefore, equation (6) is negative.

Next, we show relationship between α and symmetric equilibrium resource of positive and negative campaigning. In order to prove this relationship, we differentiate equation (3) in α for which we input symmetric equilibrium policy x^* (which we call $p(x^*)$). Then, we get the following equation (8).

$$\frac{dp(x^*)}{d\alpha} = \frac{(1-t)^{-1} \{ f(x^*) g(x^*) \}^{\frac{1}{1-t}}}{\left(f(x^*)^{\frac{1}{1-t}} + g(x^*)^{\frac{1}{1-t}} \right)^2} \left(\frac{f'(x^*)}{f(x^*)} - \frac{g'(x^*)}{g(x^*)} \right) \times \frac{dx^*}{d\alpha} \qquad (7)$$

Section 1 of equation (7)'s right hand side is positive and section 3 is negative because of equation (6). Thus, we consider section 2 of equation (7)'s right hand side. By using Lemma1, 2 and Proposition 1, $x^1 < x^* < x^2 < x^3$ holds. And because of Assumption 1, $f(x^*) > g(x^*)$ and $f'(x^*) < g'(x^*)$ hold. Thus, section 2 is negative. Therefore equation (7) is positive. And in this model, symmetric equilibrium resource of negative campaigning $n(x^*)$ satisfies $n(x^*) = 1 - p(x^*)$. So, $dn(x^*)/d\alpha = -dp(x^*)/d\alpha < 0$.■

By using Proposition 2, we show value of symmetric equilibrium policy when voters' awareness is asymptotically close to 1, in other word voters know almost all of policy effect.

Corollary 1

Under Assumption 1, 3 and 4, if voters' awareness α is asymptotically close to 1, then symmetric equilibrium policy x^* is asymptotically equal to voters' welfare maximization policy x^1 .

Proof

In order to show this Collorary, we input $\alpha = 1$ for equation (5). Then, we get symmetric equilibrium policy x^* is equal to voters' welfare maximization policy x^1 .■

Because of Proposition 2, if voters' awareness increases, then resource of positive campaigning and voters' welfare increase and symmetric equilibrium policy decreases. Therefore, if voters know the effect of policy more, then the policy which voters like more realizes. In this model, if voters' awareness α increases, then voters can understand policy effect by not very using candidates' campaign. Thus, candidates chooses close of voters' welfare maximization policy. Next we consider intuition of relationship between voters' awareness and campaign resource. In this model, because of equation (3), amount of campaign resource is determined by comparative assessment between positive and negative effect of policy. Thus, if degree of policy innovation decreases, in other word policy is not extreme, then bad effect of policy decreases rapidly and good effect of policy decreases gently. Therefore, candidates' incentive to use negative campaign decreases.

In following Section 4, we consider whether we regulate negative campaigning by using voters' welfare when voters' awareness is $\alpha = 0.5$ and Campaign function's power is $t = 0.5$.

4. Regulation versus No Regulation For Negative Campaign When Voter's Awareness is Half

In Section 3, we discussed candidates' behavior when they can use negative campaigning. However, negative campaign does not only express opponent's bad aspect of policy. For example, Ansolabehere, Iyengar and Simon (1999) show that negative campaign decreases voter turnout by using empirical method. Geer and Vavreck (2014) show that if a candidate uses negative campaign, then voters recognize his ideology is extreme by using experimental method. Therefore, we can guess negative campaign should be regulated because negative campaign causes bad influence. So, in this section, in order to verify this guess we compare voters' welfare in regulating negative campaigning with welfare in no regulating.

In this section, we consider the situation which satisfies following Assumption 5.

Assumption 5

i. $\alpha = \beta = 0.5$.

ii. $P(p_i) = p_i^{0.5}$, $N(n_i) = n_i^{0.5}$.

iii. $f(x) = ax^{\gamma}$, $g(x) = cx^{\delta}$ where $a, c > 0$, $0 < \gamma \leq 0.5$ and $\delta \geq 1$

Assumption 5's iii means we assume specific policy effect function which satisfies Assumption 1 and Assumption 4's iii. Then we get the following symmetric equilibrium policy x^* because of equation (5).

$$x^* = \left(\frac{(\delta+\gamma)^2 - \sqrt{(\delta+\gamma)^4 - 16\delta^2\gamma^2}}{4\delta\gamma}\right)^{\frac{1}{\delta-\gamma}} \left(\frac{a}{c}\right)^{\frac{1}{\delta-\gamma}} \tag{8}$$

Next, we analyze candidates' behavior when they cannot use negative campaigning. In this case, each candidate chooses the resource of positive campaigning only. Therefore, we present next

Proposition 3 which means that how much each candidate chooses resource of positive campaigning and symmetric equilibrium policy in regulating negative campaigning.

Proposition 3

Under Assumption 3 and 5, if candidates cannot use negative campaigning, then each candidate chooses symmetric equilibrium resource of positive campaigning in regulating negative campaigning $p_i = 1$ and symmetric equilibrium policy in regulating negative campaigning satisfies $x' = (2\gamma/\delta)^{1/\delta-\gamma}(a/c)^{1/\delta-\gamma}$ under subgame perfect equilibrium.[4]

Proof

Fixed any policy $x_i', x_{-i}' \in (0.\infty)$ which each candidate chooses in first stage. Then we check that candidate i has incentive to deviate $p_i' \in [0,1)$ when other candidate chooses $p_{-i}' \in [0,1]$. Therefore, in order to check this we differentiate equation (2) in p_i for which we input $n_i = n_{-i} = 0$. Then we get the following equation (9).

$$\frac{dEP_i}{dp_i} = EP' \times (1-\alpha)P'(p_i')f(x_i') \tag{9}$$

Because of Assumption 3, EP' is always positive. Because of Assumption 5, $(1-\alpha)P'(p_i)f(x_i')$ is always positive. Therefore, equation (9) is always positive. Therefore, candidate i has incentive to deviate from $p_i' \in [0,1)$. Thus, next we check if each candidate chooses $p_i' = 1$, then he does not have incentive to deviate. In order to check this, we input $p_i' = 1$ for equation (9). Then, because this equation (9) is always positive, candidate i wants to deviate. But, he cannot choose $p_i > 1$. Thus, under subgame perfect equilibrium each candidate chooses $p_i = 1$.

Next, we consider symmetric equilibrium policy in regulating negative campaigning. In order to

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[4] Under Assumption 1, 3 and 4, this Proposition 3 also holds. (Assumption 1, 3 and 4 is weaker assumption than Assumption 3 and 5.)

get symmetric equilibrium policy, we differentiate equation (2) in x_i for which we input $n_i = n_{-i} = 0$ and $p_i = p_{-i} = 1$. Then, we can get symmetric equilibrium policy in regulating negative campaigning $x' = (2\gamma/\delta)^{1/\delta-\gamma}(a/c)^{1/\delta-\gamma}$.■

Proposition 3 means if candidates' negative campaign is regulated, then they use full of resources for positive campaign in order to increase their own probability of winning an election. And because of Proposition 3, we showed symmetric equilibrium policy in regulating negative campaign x'. Next, we compare voters' welfare in regulating negative campaigning with welfare in no regulating.

Proposition 4

If Assumption 3 and 5 hold, then voters' welfare in no regulating negative campaigning is greater than in regulating negative campaigning.

Proof

Firstly, we can get the following policy of maximizing voters' welfare x^1 by using first order condition of voters' welfare $f(x) - g(x)$.

$$x^1 = \left(\frac{\gamma}{\delta}\right)^{\frac{1}{\delta-\gamma}} \left(\frac{a}{c}\right)^{\frac{1}{\delta-\gamma}} \tag{10}$$

Because of Assumption 5, Proposition 1 and 3, $x^1 < x^*, x'$. And if $x \geq x^1$, then voter's welfare function $f(x) - g(x)$ is decreasing function. So, in order to prove this proposition, we compare x^* with x' . Then, we consider the following inequality.

$$\frac{(\delta+\gamma)^2 - \sqrt{(\delta+\gamma)^4 - 16\delta^2\gamma^2}}{4\delta\gamma} - \frac{2\gamma}{\delta} \leq 0 \tag{11}$$

We transform inequality (11) as following.

$$(\delta+\gamma)^2 - 8\gamma^2 \leq \sqrt{(\delta+\gamma)^4 - 16\gamma^2\delta^2} \tag{12}$$

If $\gamma/\delta \geq \left(2\sqrt{2}-1\right)^{-1}$, we obtain inequality (12) because left hand side of inequality (13) is negative. So, we consider the case where $0 < \gamma/\delta < \left(2\sqrt{2}-1\right)^{-1}$. In this case, both left and right hand side of inequality (12) are positive. Therefore, we consider the case where we raise both sides of (12) to the second power. Then if the following condition holds, the second power of inequality (12) holds.

$$\frac{\gamma}{\delta} \leq \frac{2}{3} \tag{13}$$

Because $0 < \gamma/\delta < \left(2\sqrt{2} - 1\right)^{-1}$ satisfies condition (13), therefore inequality (11) always holds.■

By Proposition 4, voters' welfare in no regulating negative campaigning is better than in regulating. Thus, we consider negative campaign for policy should not be regulated. Next, we consider intuition of Proposition 4. If candidates cannot use negative campaigning, because of Proposition 3, candidates allocate all resource for positive campaigning. Then, candidates advertise strongly their good aspect of policy on campaigning. Therefore, they choose more extreme policy. However, if negative campaigning is not regulated, each candidate monitors each other's bad aspect of policy. Thus, it is difficult for candidate to use extreme policy. So, voters' welfare in no regulating negative campaigning is better than in regulating.

For example Ansolabehere, Iyngar & Simon (1999), bad effect of negative campaign is focused. In Japan, candidates can not use negative campaign on Internet advertisement and election broadcast because of Japanese Public Offices Election Act paragraph 7, Article 142 and paragraph 2, Article 150. However, we consider negative campaign for policy should not be regulated.

5. Concluding Remark

Most of previous literature analyzed the case where campaign affects ideology of policy or effect of campaign is specialized. So, we construct and analyze the model which candidates use campaign for policy effect. First outcome of this paper is that the more voters' awareness increases, the more voters' welfare increases. Actually, in Japanese election young voters have little interest in politics and policy effect. However, we show little interest in politics leads to extreme policy and then voters' welfare decreases. So, we consider voters should increase their awareness of politics and policy effect in order to improve their welfare. Second outcome of this paper is that if voters' awareness of policy effect is half, negative campaign should be regulated because voters' welfare in no regulating negative campaigning is greater than in regulating negative campaigning. Most of past literature focuses on bad aspect of negative campaign. However, we showed negative campaigning for policy improves voters' welfare, so negative campaign does not only cause bad influence.

In this paper, we analyzed symmetric equilibrium only. In other word, we focused on candidates who have the same ability. However in real election, there exists the case where a candidate's ability is much greater than other's ability. So, in future literature we expect that the general case of this election model will be analyzed.

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