Relative Performance and Stability of Collusive Behavior

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Abstract

We formulate a model where firms care about relative profits as well as their own profits. We investigate whether these objectives facilitate collusive behavior. We find a monotone relationship between weight of relative performance in objectives and the stability of collusive behavior.

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1 Introduction

We investigate the relationship between the payoff function of firms and the stability of collusive behavior. Issues related to cartel stability have been intensively discussed for many years. One concern is determining under what conditions the firms can easily collude. If these conditions become clear, anti-monopoly departments can more effectively monitor anti-competitive behavior. In this paper, we investigate the relationship between the severity of competition among firms and their ability to collude, by introducing relative performance in the objective functions of the firms.

We consider a duopoly model where firms maximize relative profits rather than their own profits. Firm $i$’s payoff is $\pi_i - \alpha \pi_j$, where $\pi_i$ is its own profits, $\pi_j$ is the rival’s profits, and $\alpha \in (-1, 1)$. We investigate the relationship between $\alpha$ and the stability of collusion of the firms.

We consider a model where duopolists choose their outputs simultaneously. In the model, the equilibrium outcome converges to the monopoly one when $\alpha$ converges to $-1$, and it converges to the perfectly competitive one (Walrasian) when $\alpha$ is close to 1. A larger $\alpha$ accelerates competition and is welfare improving. More severe competition, however, may increase the incentive for collusion; thus, an increase in $\alpha$ can be anti-competitive rather than pro-competitive.\(^1\) In this paper, we take a close look at this problem.

We believe that there are rationales of objective functions based on relative performance. First, evaluations of management activities are often based on their relative performances as well as the absolute performances. Outperforming managers often obtain good positions in their management job markets. Those facts rationalize considering positive $\alpha$ in our model. Second, many laboratory (experimental) works pointed out spiteful behavior as well as reciprocal behavior or altruistic behavior. Both behaviors are closely related to the objective functions based on relative performance (positive and negative $\alpha$).\(^2\) Third, relative performance, especially the positive $\alpha$ case, is quite important from the viewpoint of evolutionary stability.\(^3\) Fourth, if we adopt the approach taken by Fershtman and Judd (1987) and replace sales with (minus) rival’s

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\(^1\) For example, Gupta and Venkatu (2002) show that an increase in the distance between two firms reduces the stability of collusion in their delivered pricing linear-city model, whereas a minimal distance between firms yields most severe competition in the competitive phase. It implies that a severe competitive situation can yield anti-competitive collusion.

\(^2\) See, among others, Brandts et al. (2004), Cason et al. (2002), and Coats and Neilson (2005).

\(^3\) See Alchian (1950) and Vega-Redondo (1997). Vega-Redondo (1997) also shows that Cournot competition with relative performance objectives yield the Bertrand outcome, even in duopoly, and this outcome is evolutionary stable. The relative performance approach is also important in the political science. Obviously, the party cares about the number of votes obtained, not in absolute terms but in relative terms.
profit in their model, the firms in fact adopt a positive $\alpha$. We believe that our formulation has sufficient importance for investigation on policy matters.\footnote{The payoff function, which is based on relative wealth status, has been intensively discussed in the macroeconomics context, too. See, among others, Corneo and Jeanne (1997, 1999) and Futagami and Shibata (1998).}

## 2 The Model

We formulate a symmetric duopoly model. Firms produce perfectly substitutable commodities for which the market demand function is given by $p = a - Y$ (price as a function of quantity), where $Y$ is the total output of duopolists. Let $y_i$ denote the output of firm $i$. We assume that marginal cost is constant and is normalized to be zero. Each firm $i$ chooses $y_i$ independently. The payoff of firm $i$ ($i = 1, 2$) is given by $U_i = \pi_i - \alpha \pi_j$ ($i \neq j$), where $\pi_i$ is the profit of firm $i$ and $\alpha \in (0, 1)$. $\alpha$ indicates the important of relative performance for firm $i$’s management.

Firms engage in an infinitely repeated game. Let $\delta$ denote the discount factor between periods. We examine the effect of $\alpha$ on the sustainability of the collusion. Along the punishment path, the firms are assumed to use the grim trigger strategy of Friedman (1971).\footnote{This punishment strategy is not optimal (See Ahreu (1988)). We use the grim trigger strategy for simplicity and tractability. We believe that this is a very realistic punishment strategy because of its simplicity. Many works adopt this strategy when analyzing stability of agreements. See, among others, Deneckere (1983), Chang (1991), Hückner (1994, 1995), Lambertini et al. (1998), Maggi (1999), and Gupta and Venkatu (2002).}

First, we discuss joint-payoff maximization. The joint payoff is $(1 - \alpha)(\pi_1 + \pi_2)$ and it is maximized when $y_1 = y_2 = a/4$. The resulting profit of each firm is $a^2/4$ (half of the monopoly profit), and the resulting payoff is:

$$U^C_i = \frac{(1 - \alpha)a^2}{8}. \quad (1)$$

Second, we discuss the deviation from the tacit collusion. Given the cooperative output of the rival, firm 2, firm 1 maximizes its payoff $U_1$. The first order condition is:

$$a - 2y_1 - (1 - \alpha)y_2 = 0. \quad (2)$$

From this we obtain the following reaction function:

$$y_1 = \frac{a - (1 - \alpha)y_2}{2}. \quad (3)$$

Substituting $y_2 = a/4$, we have:

$$y^D_1 = \frac{a(3 + \alpha)}{8}. \quad (4)$$
where superscript ‘D’ denotes the outcome when a firm deviates from the collusion. The resulting payoff is:

\[ U_1^D = \frac{a^2(3 - \alpha)^2}{64}. \]  \hspace{1cm} (5)

Third, we discuss the competitive situation. Each firm independently chooses its output so as to maximize its own payoff. From the reaction function above (equation (3)), we have that the Cournot-Nash equilibrium is:

\[ y_1^E = y_2^E = \frac{a}{3 - \alpha}, \]  \hspace{1cm} (6)

where superscript ‘E’ denotes the equilibrium outcome in the competitive phase. The resulting profit and payoff are:

\[ \pi_1^E = \pi_2^E = \frac{a^2(1 - \alpha)}{(3 - \alpha)^2}, \hspace{0.5cm} U_1^E = U_2^E = \frac{a^2(1 - \alpha)^2}{(3 - \alpha)^2}. \]  \hspace{1cm} (7)

### 3 Results

Given the collusive behavior of firm 2, firm 1 can increase its one-shot profit by deviating from the cartel. Its payoff is \( U_1^D \). This deviation induces the competition thereafter. Firm 1’s payoff at the competitive phase is \( U_1^E \). If firm 1 does not deviate from the collusion, its current payoff is \( U_1^C \). If firm 1 has no incentive for deviation now, it will have no incentive in future, too. Thus, the tacit collusion is sustainable if and only if:

\[ \frac{U_1^C}{1 - \delta} \geq U_1^D + \frac{\delta U_1^E}{1 - \delta}. \]

Let \( \delta^\ast \) be the \( \delta \) satisfying the above equation with equality. The tacit collusion is sustainable if and only if \( \delta \geq \delta^\ast \). We have

\[ \delta^\ast = \frac{U_1^D - U_1^C}{U_1^D - U_1^E} = \frac{(3 - \alpha)^2}{17 - 14\alpha + \alpha^2}. \]  \hspace{1cm} (8)

Following the tradition of this field, we measure the stability of collusion by this minimum discount factor \( \delta^\ast \). We have that an increase in \( \alpha \) causes more instability in collusive behavior.

**Proposition** \( \delta^\ast \) is increasing in \( \alpha \).

**Proof** From (8) we have:

\[ \frac{\partial \delta^\ast}{\partial \alpha} = \frac{8(1 + \alpha)(3 - \alpha)}{(17 - 14\alpha + \alpha^2)^2}, \]

It is positive for \( \alpha \in (-1, 1) \). Q.E.D.

On the one hand, an increase in \( \alpha \) accelerates competition and increases the effectiveness of punishment. This stabilizes the collusion. On the other hand, an increase in \( \alpha \) increases the incentive for spiteful behavior...
and thus increases the incentive for deviation. This destabilizes the collusion. Our proposition states that the latter effect always dominates the former effect. Thus, a larger $\alpha$ implies a more competitive market both from the static and dynamic viewpoints.

4 Concluding Remarks

In this paper, we adopt the relative performance approach and investigate the stability of collusive behavior. We think that this approach is important in more general contexts. We can interpret $\alpha$ as a parameter indicating severity of competition. $\alpha = 0$ indicates the standard Cournot case, $\alpha = 1$ indicates the perfectly competitive case (Bertrand case), and $\alpha = -1$ indicates the monopoly case. Thus, a larger $\alpha$ indicates a more competitive market. The relative performance approach enables us to treat competitiveness as a continuous variable, and this approach contains three models, Cournot, Bertrand, and monopoly, as special cases.\(^6\)

We can also interpret that $\alpha$ indicates the degree of reciprocal attitude or altruism. An increase in $\alpha$ indicates a less reciprocal attitude. We believe that this approach is applicable to the broad area of social science.

References


\(^6\) The conjectural variation approach is another approach which contains three models as special cases. However, the conjectural variation model assumes that firm 1’s output affects that of firm 2 and vice versa. Needless to say, this assumption is inconsistent if we respect conjectural the variation model as a static model. The relative performance approach does not have this defect, and this is an important advantage of this approach.


