Essays on Competition, Regulation, and Privatization Policies

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Main theme of my dissertation:

government intervention towards market power.

Contents:

1. Chapter 2: Competition policy in digital economy
2. Chapter 3: Regulation policy in natural monopoly
3. Chapter 4: Privatization policy in mixed oligopoly (joint with Toshihiro Matsumura)
1 Chapter 2: Horizontal Mergers in the Presence of Network Externalities

2 Chapter 3: Monopoly Regulation in the Presence of Consumer Demand-Reduction

3 Chapter 4: Dynamic Privatization Policy
Concentration in the tech industry:

- Google, Apple, Facebook, Amazon, Microsoft, etc.
- Characterized by network externalities.
  → Exhibits “winner-takes-all” feature.
- Poses challenges to competition authorities.
- One typical area: Merger Control
  - Numerous mergers and acquisitions by Big Tech.
Some casual discussion on merger in tech industry:

- Static impacts of network externalities:
  - Infringing minor firms (−)
  - Direct gain from demand-side scale economies (+)

Which effect dominates under what condition?
This study tries to offer some theoretical guidance on the impacts of network effects:

- Adopt an aggregative-games approach to multiproduct-firm oligopoly (Nocke and Shutz, 2018a, 2018b) and extend it to incorporate network externalities

- Characterize the “scrutiny" of CS-oriented merger policy.
Overview of the Results

Scrutiny of merger policy in a static environment:

- **decreases** with network externalities when merging parties are small or industry is symmetric.
- **increases** with network externalities when merging parties are dominant.
Network externalities:  
(Katz and Shapiro 1984, 1985; Farrell and Saloner, 1986; Cabral, 2011)

My paper  
(1) providing analytical framework  
(2) showing non-monotone relation between CS-effects of mergers and firm sizes  
(3) Implications on killer acquisitions and platform mergers

Merger analysis  
Environment:

- A mass of consumers in one-sided market.
  - Consumer \( z \in [0, 1] \) purchases one product from a set \( \mathcal{N} \).

- There is a set of firms \( \mathcal{F} \).
  - Firm \( f \) produces a set \( \mathcal{N}_f \) of products.

- Consumers derive firm-level network externalities from a purchase.
Framework

Consumers

Messenger

WhatsApp

Instagram

Other networks

Facebook
Model of Consumer Demand

- Logit-type demand model (for presentation).
- Indirect utility from a purchase of product \( i \in N_f \)

\[
\frac{a_i - p_i}{\lambda} + \alpha \log n_f + \varepsilon_{iz},
\]

- \((a_i - p_i)/\lambda\): stand-alone indirect subutility;
- \(p_i\): unit price;
- \(\alpha \in (0, 1)\): direct network externalities;
- \(n_f\): network share of firm \( f \).
- \(\varepsilon_{iz} \sim \text{TIEV}\).

- No outside option and single-homing.
Model of Consumer Demand

Network size $n_f$ is determined by rational expectation equilibrium:

- Given network sizes, share $s_i$ of each product $i \in \mathcal{N}_f$ is given by

$$s_i = \frac{\exp\left(\frac{a_i-p_i}{\lambda}\right)(n_f)^\alpha}{\sum_{f' \in \mathcal{F}} \sum_{j \in \mathcal{N}_{f'}} \exp\left(\frac{a_j-p_j}{\lambda}\right)(n_{f'})^\alpha}.$$

- The network share $n_f$ is the sum of the share of products:

$$n_f = \sum_{i \in \mathcal{N}_f} s_i.$$
Model of Consumer Demand

- Firm-level and industry-level aggregators:
  \[ H_f = \sum_{i \in \mathcal{N}_f} \exp \left( \frac{a_i - p_i}{\lambda} \right), \quad H = \sum_{f' \in \mathcal{F}} \left( H_{f'} \right)^{1-\alpha} \]

- Network share in rational expectation equilibrium is given by
  \[ n_f = \frac{H_f^{1-\alpha}}{H} \]

- Finally, the demand for product \( i \in \mathcal{N}_f \) under discrete-continuous choice is given by
  \[ \hat{D}_i (p_i, H_f, H) = n_f \times \frac{\exp \left( \frac{a_i - p_i}{\lambda} \right)}{H_f} = \frac{H_f^{\alpha}}{H} \exp \left( \frac{a_i - p_i}{\lambda} \right) \]
Firm Pricing

- Each product $i \in \mathcal{N}$ has a constant marginal cost $c_i > 0$ of production.

- Firm $f$’s profit is

$$\Pi_f = \sum_{i \in \mathcal{N}_f} \hat{D}_i (p_i, H_f, H) (p_i - c_i)$$

- Pricing game: firms simultaneously choose their price profiles.
• Common markup property: there exists $\mu_f$ such that firm’s FOC yields

$$p_i - c_i = \lambda \mu_f$$

for all $i \in \mathcal{N}_f$.

• Type-aggregation property: $\mu_f$ can be written as

$$\mu_f = m \left( \frac{\gamma(T_f)}{H} \right),$$

where

• $T_f = \sum_{i \in \mathcal{N}_f} \exp\{(a_i - c_i)/\lambda\}$: “type” of firm $f$.
• $\gamma(x) = x^{1-\alpha}$.
Equilibrium

- Network share can also be written as:
  \[ n_f = N \left( \frac{\gamma(T_f)}{H} \right) . \]

- \( N(\cdot) \) concave.
- Equilibrium condition for the aggregator \( H \):
  \[ \sum_{f \in \mathcal{F}} N \left( \frac{\gamma(T_f)}{H} \right) = 1. \]

- \( H^* \): equilibrium aggregator, a function of \((T_f)_{f \in \mathcal{F}}\).
  - increasing in each element.

- Equilibrium consumer surplus: \( CS = (1 - \alpha) \log H^* . \)
Modelling a Merger

Merger between firms $f$ and $g$:

- Firms $f$ and $g$ with types $T_f$ and $T_g$ are transformed into firm $M$ with

$$T_M = T_f + T_g + \Delta,$$

- $\Delta$ is the technological synergy generated by the merger.
Consumer-surplus effects of merger:

- Consumer surplus is increasing in $H$.
  $\Rightarrow$ merger is CS-improving iff $H$ is increased.

- Simple condition for CS-improving merger: Merger is CS-improving if and only if $\Delta \geq \hat{\Delta}$, where

$$N \left( \frac{\gamma(T_f + T_g + \hat{\Delta})}{H^*} \right)$$

$$= N \left( \frac{\gamma(T_f)}{H^*} \right) + N \left( \frac{\gamma(T_g)}{H^*} \right)$$

where $H^*$ is pre-merger equilibrium aggregator.

- $\hat{\Delta}$: CS-neutral technological synergy, interpreted as a "scrutiny of merger policy".
Results:

1. In the presence of network effects, merger can be CS-improving without technological synergies ($\Delta < 0$) when
   - merging parties are small ($T_f \approx 0$), or
   - industry is symmetric ($T_{f'} = T$ for all $f'$)

2. $\Delta$ decreases with $\alpha$ as long as merging parties are small.

3. $\Delta$ increases with $\alpha$ when merging parties are large relative to the industry.
Results: Numerical illustration

Numerical example:

- 12 firms, including 10 firms with $T_f = 5$, one firm with $T_f = 20$, and one firm with $T_f = 25$.

Figure: Weak firms ($T_f = T_g = 5$).
Results

Figure: Strong firms \((T_f = 25, T_g = 20)\).
Summary

Main findings:

- Implications of network externalities on merger policy depend on firm sizes relative to markets

Other exercises:

- Killer acquisitions:
- Similar analysis in two-sided markets:
  - single-homing
  - ad-sponsored media models.

Future directions:

- Applying the framework to problems other than mergers.
- Merger and innovation incentive in general (cf. Motta and Tarantino, 2017)
1 Chapter 2: Horizontal Mergers in the Presence of Network Externalities

2 Chapter 3: Monopoly Regulation in the Presence of Consumer Demand-Reduction

3 Chapter 4: Dynamic Privatization Policy
Motivation

Consumers often reduce the demand for some goods by

- Establishing rooftop solar generation (electricity),
- Purchasing electricity-efficient consumer electronics (electricity).
- Living in the electrified house (gas).
- Purchase bicycle or car (public transportation).
Motivation

Death spiral:
An example of the effects of demand-reducing investment on the rate-setting in the regulated sector.

How should regulation policy react to this?
What I do:

- Let consumers engage in demand-reducing investments.
- Then investigate how monopoly regulation is affected by the presence of demand-reducing investments.

Findings:

- Consumer demand reduction is excessive – regulator should mitigate it.
- Asymmetric cost information lowers prices for efficient firms.
Model of monopoly regulation à la Laffont and Tirole (1993), where a continuum of consumers can engage in demand-reducing investments:

- **Players**: a regulator, a monopolist, and unit mass of consumers.

Consumers derive the utility $S(q, x) = pq$,

- $q \in \mathbb{R}_+$: the amount of purchase
- $x \in \mathbb{R}_+$: the level of demand-reducing investment.
Assumptions:

1. $S$ is concave,
2. $S_q > 0, S_{qq} < 0, S_{qx} < 0, S_{xx} < 0$.
3. $S_x(q, 0) > 0$ for any $q$,
4. for any $q$, there exists $\bar{x}_q$ such that $S_x(q, \bar{x}_q) = 0$.

- These assumptions guarantee the existence of a demand function $D(p, x)$ derived from the FOC:
  \[ S_q(D(p, x), x) - p = 0 \] (1)
- $D_x(p, x) < 0$. 

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Model

The monopolist:

- Constant marginal cost of production $\beta \in [\beta_L, \beta_H]$.
- $\beta \sim F$ is privately known to the monopolist.
- With price $p$ and sales $q$, the monopolist yields the profit
  \[(p - \beta)q - K + s,\]
  - $K$: fixed cost of production
  - $s$: subsidy from regulator.
- Monopolist operates only if profit $\geq 0$
The regulator:

- The regulator can offer a menu \((p(\beta), s(\beta))\) of pairs of price and subsidy.
- Financing subsidy is socially costly and incurs \(\lambda > 0\) excess burden.
- Regulator’s objective = aggregate welfare is given by

\[
W = CS + PS - \text{social cost of subsidy.}
\]

\[
= S(D(p, x), x) - \beta D(p, x) - K - \lambda s \quad (2)
\]
Timing:

1. The regulator offers a menu \((p(\beta), s(\beta))\) of contracts.

2. Consumers decide whether to engage in demand-reducing investments. At the same time, monopolist observes \(\beta\) and choose the contract \((p(\beta'), s(\beta'))\) that maximizes his profit.

3. Given the price \(p(\beta')\) consumers choose the amount of purchase.
I proceed to the analysis in step-by-step manner:

1. complete information with exogenous investments,

2. complete information with endogenous investments, and

3. asymmetric information with endogenous investments.
Benchmark: Exogenous Investments

What happens if the regulation policy ignores consumers’ investments?

- As a benchmark, consider another timeline where:
  1. consumers first choose the investment decision, and
  2. regulator chooses her policy.

- Also assume that the cost parameter $\beta$ is observed by the regulator.
• Consumers choose $x$ according to the FOC

$$E_\beta[S_x(D(p(\beta), x), x)] = 0.$$  \hspace{1cm} (3)

• In this setting, a standard derivation yields

$$s(\beta) = K - (p(\beta) - \beta)D(p(\beta), x)$$

and

$$\frac{p(\beta) - \beta}{p(\beta)} = \frac{\lambda}{1 + \lambda \eta(p(\beta), x)},$$  \hspace{1cm} (4)

where

$$\eta(p, x) := -\frac{D_p(p, x)p}{D(p, x)} > 0$$  \hspace{1cm} (5)

is the price elasticity of demand.
Exogenous Investments

- This is the standard Lerner formula obtained in the models of monopoly regulation with a cost of public funds.

- At the price schedule given by the formula above, is \( x \) optimal, excess, or insufficient?

**Proposition**

Under the complete information, if the regulator sets the policy taking the consumers’ investments as given, the amount of the investments is too high in terms of social welfare.
Intuition:

- Aggregate welfare includes the cost of public funds.
- Regulator wants to guarantee some profit of monopolist to reduce subsidy.
- Consumers ignore the effect on monopolist’s profit, and thus increases the amount of costly subsidy.
- As a result, consumers’ investments are excessive.

Implication:

- The optimal regulation should be designed so as to limit consumer investments.
Next, consider the original order, while maintaining the complete information assumption.

The formula for $s(\beta)$ is the same as the benchmark case.

The expected-welfare maximization problem now includes the constraint

$$\mathbb{E}_\beta[S_x(D(p(\beta), x), x)] = 0.$$
Setting up Lagrangian and solving for FOC yields the condition for the complete-information policy:

\[
\frac{p(\beta) - \beta}{p(\beta)} = \frac{\lambda}{1 + \lambda \eta(p(\beta), x)} \frac{1}{\eta(p(\beta), x)}
\]

\(\frac{dx}{dp(\beta)} \mathbb{E}_\beta [(p(\beta) - \beta) D_x].\)

\(\text{Standard inverse elasticity}\)

\(\text{Investment reduction term (−)}\)

\(p(\beta)\) is lower than that with exogenous investments.
Now consider the original game of optimal regulation with asymmetric information.

- Regulation policy must be incentive compatible:
  \[
  \beta = \arg \max_{\beta'} (p(\beta') - \beta) D(p(\beta'), x) - K + s(\beta')
  \]

- Then, the FOC is altered as follows.
  \[
  p(\beta) - \beta - \frac{\lambda}{1+\lambda} \frac{F(\beta)}{f(\beta)} = \frac{\lambda}{1 + \lambda \eta(p(\beta), x)} \]

  Standard inverse elasticity

  \[
  + \frac{dx}{dp(\beta)} \mathbb{E}_\beta \left[ \left( p(\beta) - \beta - \frac{\lambda}{1 + \lambda f(\beta)} \right) D_x \right].
  \]  

  Investment reduction term
How the consumer investment and asymmetric information interact?

- Asymmetric information generally increases the regulated prices since it increases the *virtual* marginal cost.
- This in turn would increase the consumer investments.
- This may generate a downward pressure on the pricing decision.
Comparison

• Let $p^*(\beta), m^* := \mathbb{E}_\beta[p^*(\beta)]$, and $x^*$ be the price schedule, average price, and the threshold consumer under complete-information optimal regulation.

• Let $p^{**}(\beta), m^{**} := \mathbb{E}_\beta[p^{**}(\beta)]$, and $x^{**}$ be those under asymmetric-information optimal regulation.

Linear-quadratic utility:

• $S(q, x) = aq - \frac{b}{2}(\theta x + q)^2 + Ax - \frac{B}{2}x^2$.

• Then, $D(p, x) = (a - p)/b - \theta x$,

• $x^* = (A + \theta m^* - a\theta)/B$, and

• $x^{**} = (A + \theta m^{**} - a\theta)/B$.

This specification has a feature that

• Higher average price leads to more consumer investments.
Comparison of $p^i$, $m^i$, and $x^i$ for $i \in \{\ast, \ast\ast\}$.

**Proposition**

The average price is higher under the asymmetric information, and thus the threshold consumer’s type is greater under the asymmetric information. That is,

$$m^{\ast\ast} > m^{\ast} \quad \text{and} \quad x^{\ast\ast} > x^{\ast}.$$
Asymmetric information generates an upward pricing pressure to the regulator to reduce rents.

This simply increases the average prices.

Anticipating this price increase, the more consumers engage in demand-reducing investments.
Higher level of consumer investments has two implications for pricing:

1. Lower marginal gain from investment implies the greater gain from deterring the investment.
2. Lower level of demand basically implies the more elastic demand and thus lowers the optimal price.

It is possible that this effect dominate for some type of monopolists.
For the most efficient monopolists, the regulated price under asymmetric information is lower than that under complete information. That is,

\[ p^{**}(\beta_L) < p^*(\beta_L). \]
There is *no distortion at the top*: 
\[ F(\beta_L)/f(\beta_L) = 0. \]

Upward-pricing pressure is absent for efficient monopolists.

Thus, the only the downward-pricing pressure from consumer investments prevails.

As a result, the prices for efficient monopolist decreases through the introduction of asymmetric information.
• The presence of asymmetric information exacerbates the excess investment by consumers.

• Then, the presence of asymmetric information require even more price decrease for efficient types of monopolists to tackle with excess investments.

• asymmetric information as amplifier of the consumer investment problem and the lower prices as a solution.
Summary

1. Consumer investment is too much in its natural form.

2. Regulator need to limit investments.

3. Asymmetric information exacerbates this problem and leads to even lower prices than the first-best policy.

Future direction:

- Regulation in two-part tariffs.
Chapter 2: Horizontal Mergers in the Presence of Network Externalities

Chapter 3: Monopoly Regulation in the Presence of Consumer Demand-Reduction

Chapter 4: Dynamic Privatization Policy
Mixed market = a market where (semi)public and private firms coexist.

Examples:
- banking (DBJ)
- telecommunication (NTT),
- automobiles (Renault),
- tobacco (JT), etc.
Some countries privatize the state-owned enterprises (e.g., UK), and others do not much (e.g., China).

Privatization as a form of changes in government control over public firm is one issue of mixed markets.

Gradual privatization:

- Most existing studies assume that privatization is one-shot event.

- In the process of privatization, governments sometimes sells its share gradually over time.
Introduction: Gradual Privatization

- Example 1: NTT (Japan): state-owned monopolist until 1985; its government’s share is continuously sold from 1986; government still holds one-third of share.

- Example 2: Renault (French): French government increased its share from 15% to 19.4% in 2015.

- Other examples: JT, JR, Japan Post, Postal Bank, Kampo, etc.
Introduction: Gradual Privatization

- Need to analyze such dynamics of privatization policy.

- What is the cause of gradual privatization?
  - Changes in environments,
  - moderating impacts on financial markets,
  - Revenue motives of governments.
Introduction: Why Shadow Cost?

Some primary purpose of privatization:

1. Achieve higher allocation/production efficiency
2. Promote a development of financial market
3. Collect government revenues

→ Introduce shadow cost of public funds as a revenue motive.
Privatization is a stock-selling process of the public firm.

Its stock price reflects the enterprise value (=present value of profits) of the public firm.

Future actions may affect current stock price

However, since the stock is already sold, this effect is ignored in future (time inconsistency).

This time inconsistency generate a number of dynamics in privatization policy.
Two-period \((t = 1, 2)\) model

Players:
- firm 0 (public firm),
- firm 1, . . . , n (private firms), and
- government.

In each period, the government first sells 
\(\alpha_t - \alpha_{t-1}\) share of the public firm, and then public/private firms compete in quantities.

Government first holds all share of the public firm, i.e., \(\alpha_0 = 0\).
Model: Objective Functions

- Government maximizes welfare $W_t$.  
- Private firms maximize its own profit $\pi_{i,t}$.  
- Public firm maximizes a convex combination of welfare and profit $\alpha_t \pi_{0,t} + (1 - \alpha_t)W_t$.  

... each is measured by present value.
Model: Notations

- \( \alpha_t \): degree of privatization in period \( t \).
- \( R_t \): revenue from the stock-selling of the public firm.
- \( D_t \): dividend of the government from the public firm.
- \( \lambda \): shadow cost of public funding. We assume \( \lambda \leq 1 \).
- \( \theta \): share of foreign investors in private firms.
- \( \delta \): common discount factor.
We assume that financial market is perfect, i.e., investors pays the enterprise-value of the public firm.

In period 1, enterprise value of firm 0 is $\pi_{0,1} + \delta\pi_{0,2}$.

In period 2, enterprise value of firm 0 is $\pi_{0,2}$.

$R_t = (\alpha_t - \alpha_{t-1}) \times $ enterprise value of firm 0
Model: Profit and Welfare

\[ W_t = CS_t + \pi_{0,t} + (1 - \theta) \sum_{i=1}^{n} \pi_{i,t} + \lambda(R_t + D_t) \]

- \( \pi_{0,t} = p(Q_t)q_{0,t} - c_0(q_{0,t}) \)
- \( \pi_{i,t} = p(Q_t)q_{i,t} - c(q_{i,t}) \)
- \( R_1 = \alpha_1(\pi_{0,1} + \delta \pi_{0,2}) \)
- \( R_2 = (\alpha_2 - \alpha_1)\pi_{0,2} \)
- \( D_t = (1 - \alpha_t)\pi_{0,t} \)
Model: Shadow Cost of Public Funds

- One unit of government revenue has $(1 + \lambda)$ units of values in terms of welfare.

- $\lambda > 0 \rightarrow$ higher stock price leads to higher welfare gain through privatization revenue and dividend revenue $\rightarrow$ government has a strong incentive to raise the stock price.
In each period $t$, the government chooses $\alpha_t$.

Then firms face Cournot competition.

In each period, the present value of government revenue is:

- $R_1 + D_1 + \delta(R_2 + D_2) = \pi_{0,1} + \delta\pi_{0.2}$

- $R_2 + D_2 = (1 - \alpha_1)\pi_{0,2}$
Time Inconsistency

• In period 1, one unit increase in the profit of firm 0 increases welfare by $(1 + \lambda)$ unit.

• In period 2, one unit increase in the profit of firm 0 increases welfare by $(1 + \lambda) - \alpha_1 \lambda$ unit.
→ a distorted incentive in choice of $\alpha_2$

• In period 2, the government has a stronger incentive to improve CS or PS at the cost of the profit of firm 0 unless $\alpha_1 = 0$.

• To mitigate this distortion, the government chooses $\alpha_1$ smaller than the optimal one, $\alpha^{**}$. 
Cournot Equilibrium

- Because there is no intertemporal effect in output, public firm’s output is the same in periods 1 and 2 as long as $\alpha_1$ and $\alpha_2$ is the same.

- $q_0(\alpha)$: public firm’s output
- $q(\alpha)$: private firms’ output
- $Q(\alpha)$: total output.

### Lemma 1

$q_0(\alpha)$ and $Q(\alpha)$ are decreasing in $\alpha$, and $q(\alpha)$ is increasing in $\alpha$. 
Suppose that the government can choose both \( \alpha_1 \) and \( \alpha_2 \) in period 1.

Let \( \alpha_t^{**} \) be this commitment optimal degree of privatization.

**Lemma 2**

1. \( \alpha_1^{**} = \alpha_2^{**} \)

2. \( \alpha_1^{**} = \alpha_2^{**} =: \alpha^{**} = 0 \) if and only if

\[
\theta(Q(0) - q_0(0)) + (1 - \theta)q(0) - \lambda q_0(0) \leq 0.
\]

3. \( \alpha^{**} < 1 \) if \( \theta = 1 \) or \( c_0(\cdot) = c(\cdot) \).
Let $\alpha_2(\alpha_1)$ be the second-period optimal degree of privatization given $\alpha_1$.

**Lemma 3**

\[ \alpha_2(0) = \alpha^{**}. \]

- If $\alpha_1 = 0$, $\alpha_2$ is optimally chosen since there is no source of distortion.
• Let $\alpha_1^*$ and $\alpha_2^* := \alpha_2(\alpha_1^*)$ be the equilibrium degrees of privatization.

### Proposition 1

1. $\alpha_1^* \leq \alpha^{**}$.
2. $\alpha_1^* = 0$ if and only if $\alpha^{**} = 0$.
3. $\alpha_1^* = 1$ if and only if $\alpha^{**} = 1$. 
1. To mitigate the distortion in later stage, the government choose the lower degree of privatization than optimal one in earlier stage.

2. One-shot full nationalization emerges $\iff$ full nationalization is optimal.

3. One-shot full privatization emerges $\iff$ full privatization is optimal.
Intuition behind Proposition 1 (ii):

- If $\alpha^{**} = 0$ is optimal, choosing $\alpha_1 = 0$ is optimal since $\alpha^*_1 \leq \alpha^{**}$.

- In addition, because $\alpha_1 = 0$, there is no distortion in period 2, and $\alpha^*_2 = \alpha^{**} = 0$ is realized.
Intuition behind Proposition 1 (iii):

- Suppose that $\alpha^{**} = 1$ holds.

- Since the government cares less about public firm’s profit, $\alpha_2^* \geq \alpha^{**}$ if the further privatization decreases the profit.

- At $\alpha^{**} = 1$, partial nationalization increases the profit (Fershtman and Judd), and thus $\alpha_2^* = 1$.

- Since $\alpha_2^* = 1$, there is no distortion in period 2, and thus the government optimally chooses $\alpha_1^* = \alpha^{**} = 1$. 
Results

- In period 2, the government chooses the degree of privatization which achieves lower profit of public firm in terms of welfare.

- Thus, whether the second-period degree of privatization $\alpha_2^*$ is too high or too low depends on whether a further privatization from the optimal degree of privatization reduces public firm’s profit.

- Higher $\theta \rightarrow$ lower $\alpha^{**}$.

- Higher $n \rightarrow$ higher $\alpha^{**}$.

- Higher $\alpha^{**} \rightarrow$ more likely that a further privatization reduces the profit.
Proposition 2

Suppose that \( p(Q) = a - Q \) and \( c_0(q) = c(q) = q^2 / 2 \).

1. \( \alpha_2^* > \alpha^{**} \) if and only if

\[
\theta < \theta(n) := \frac{n^2 - 8}{3n(n + 4)},
\]

and \( \theta(n) \) is increasing in \( n \).

2. \( \alpha_1^* = \alpha_2^* = \alpha^{**} = 0 \) if and only if

\[
g(n, \lambda, \theta) := (n - 1)\theta(2 + \lambda) + 2(1 - \lambda^2) - n\theta^2 \leq 0.
\]

3. \( g(n, \lambda, \theta) \leq 0 \) only if \( n < 2 \), and \( g(n, \lambda, \theta) \) is decreasing in both \( \lambda \) and \( \theta \) for \( n < 2 \).
Results

\[ \lambda = \frac{7}{8} \]
Since $\alpha_1^* \leq \alpha^{**}$, $\alpha_2^* > \alpha_1^*$ if $\alpha_2^* > \alpha^{**}$.

**Lemma 5**
Under the linear demand and quadratic cost specified in Proposition 2, $\alpha_2^* > \alpha_1^*$ if $\theta < \theta(n)$.

**Proposition 3**
Even if $\alpha^{**} < 1$, $\alpha_2^*$ can be one.

- Even if full privatization is not optimal, the government may fully privatize later.
Conclusion

1. Early stage privatization distorts the later stage privatization.
   → commitment not to adjust privatization policy over time improves welfare.

2. Gradual privatization appears under reasonable conditions.

3. If full privatization or full nationalization is optimal, the government implements its policy.

4. Government may fully privatize the public firm at the latest stage even if it is not optimal.
Thank you!