Retailer's product line choice with manufacturer's multi-channel marketing*

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Abstract

This paper studies how a retailer decides the length of wholesale products from an upstream manufacturer. We consider a vertically related market with two product varieties in which each retailer determines the number of product varieties and makes an order to the manufacturer. The manufacturer may open its online store by which it encroaches upon the resale market. We use a monopoly retailer case to show that anticipating the online store's encroachment, a retailer may be willing to shorten its product line, although it can choose a full-length one. In a duopoly retailers case, we show that retailers may make their product lines completely overlapped, partially overlapped, or non-overlapped. Moreover, the total surplus may decrease as the number of varieties in the resale market increases.

Keywords: channels of distribution; encroachment; buyer power; product line choice

JEL Classification Numbers: L14, L22, L41

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1 Introduction

In a traditional manner, a manufacturer is often thought to decide its preferred product line to be distributed through retailers. However, recent markets pay more attention to cases in which dominant retailers possess strong channel power that the manufacturer has to adjust production line in favor of the retailers' orders (Kadiyali et al., 2000). Losing power in the traditional wholesale channel, more upstream manufacturers begin to consider taking the benefit of Internet commercial and adopting online channels through which they gain additional profits. Such emergence of manufacturer's multi-channel marketing brings the traditional retailers new challenges that they face competition from the potential online stores (Tannenbaum, 1995; Dixon and Quinn, 2004), which is sometimes referred to as "franchise encroachment" (Arya et al., 2007; Emerson, 2010). Benefiting from drop-shipping (Randall et al., 2006), the manufacturer's online stores can simply decide which product variety to put on the Internet whenever necessary. However, due to inconveniences in inventory and limits in display spaces in traditional resale channels, the retailer must prudently decide which variety to order from the manufacturer.

Our objective is to identify how an incumbent retailer exercises its buyer power of specifying product line when facing potential entry of upstream manufacturer's online store, and how the consequences affect the social welfare. Although the online store brings the manufacturer additional channel profits, it causes *intrabrand competition* against the incumbent retailer, harming the profit from the wholesale channel. Realizing this channel trade-off, the manufacturer strategically chooses its online store's product line. To illustrate, suppose that the manufacturer carries two varieties. When there is only one variety in the wholesale channel, it is possible that the manufacturer sells only a different variety in the online channel (a partial encroachment). In this way, because the retailer and the online store are differentiated in product line, the direct *intrabrand competition* is alleviated. However, when there are already both varieties in the wholesale channel, no matter which variety the online store sells there always exists overlapping part. Then, the *intrabrand competition* is always direct and intense, which cannot be alleviated by a partial encroachment. Without any benefit from a partial encroachment, the manufacture never sells only one variety online. Instead, it chooses either selling both varieties online (a full encroachment) or completely shutting down the online channel. The key to summarize these facts is that the manufacturer uses the online store as a tool to keep a balanced channel distribution of product varieties. It does not expect either variety to be oversupplied.

Anticipating the manufacturer's two encroaching patterns (partial or full), the retailer strategically manages its product line in order to alleviate the negative effect from the online store's encroachment. When online retail cost is low enough, the encroachment is inevitable. Realizing this, the retailer would rather abandon a part of product line so as to induce the manufacture's partial encroachment. Although losing profits from a shrunk range, the resulting partial encroachment is less harmful, which enables the retailer to stay differentiated with the online store, keeping an adequate profitability in a short product line. This result gains wider significance if we consider an oligopoly downstream market where retailers' strategic interactions play an important role. Suppose the simplest duopoly retailer case, when online retail cost is low enough, the retailers tend to make their product lines overlapped in only one variety, as if declaring a relatively higher profitability of the less supplied variety. In this way, the retailers induce the online store's partial encroachment instead of a full one. Alternatively, when online retail cost is high enough, the retailers may tend to make their product line non-overlapped so as to make a balanced variety distribution in the wholesale channel, which stimulates the manufacturer's incentive for shutting down its online channel.

Actually, our results can be easily found in several real-world cases in which physical retailers sell certain products' limited versions which are not sold by manufactures' online stores. For example, a dominant physical retailer in Japan, "Japan Consumers' Cooperate Union (JCCU)," sells several customized models of Panasonic's notebook computers, Casio's electronic dictionaries, and Cannon's laser printers in main universities, targeting college users. But these models are not sold by respective manufacturer's online stores. Fashion magazines sold in physical stores are always bundled with CDs, small samples, or supplemental materials. However, online subscribers are often not given these additional bonuses.

Moreover, our study provides implications in explaining why some retail stores choose overlapping product lines, while some others choose to be differentiated.

For the manufacture's side, we show that committing not to run an online store can sometimes be beneficial. Although sacrificing channel profits from the online store, a manufacturer can benefit from stimulating the retailers' incentive for enlarging product lines, which enhances channel efficiency when physical retailers are more adapted to resale activities. Finally, we show that even though a full encroachment results in more varieties in the resale market than a partial one does, it may result in a worse social welfare. This phenomenon occurs when the physical retailers are much more adapted to resale activities than the online store and different varieties are close substitutes. Although a full encroachment is more pro-competitive than a partial one, it reallocates more share to the inefficient online channel, which causes considerable social loss and may even be a dominant effect.

Our study has managerial implications for both retailers and manufacturers' strategies. First, although we often observe retailers competing with each other in product diversity so as to attract more consumers with different variety preferences, our results imply that the head-to-head competition in enlarging product lines can sometimes be inefficient even when the product variety expansion does not require additional costs. When facing a weak online stores, it may be more important for retailers to better coordinate with each other so that the sale of different varieties can be distributed in balance. Second, even though the online retail cost is relatively low, it is an important issue for a manufacturer to assure its upstream retailers that they are safe in keeping a full-length product line and that their territories will not be encroached upon. For the social planners, our research shows an opposite side of the manufacture's online retailing. When varieties are too close substitutes, it is important for social planners to well organize the product variety distribution so that different varieties can be specialized in different channels and the overlapping problems do not occur.

Now, we discuss the theoretical literature. To the best of my knowledge, our research is the first attempt discussing retailers' product line choices while considering manufacturer's direct marketing. However, there are several papers with results and methodologies that are closely related to ours. Dukes et al. (2009) consider a similar setting in which two retailers decide their respective product line from a multi-product manufacturer. The longer the product line is, the higher assortment cost is incurred. It is shown that one of the retailers may unilaterally cut its product line so as to induce the rival retailer to carry the full-length line with higher assortment costs. Although similar results that retailers do not choose full-length product line are derived, the intuitions behind are different–in the current work, product line expansion does not require additional costs. Moner-Colonques et al. (2011) consider a case with two single-product manufacturers and two retailers. Retailers' product line expansion means a multi-sourcing manner. The authors provide us with a theoretical explanation why some retailers choose overlapping product lines. Inderst and Shaffer (2007) considers retailers' incentives for "single-sourcing" purchasing strategy (cutting product line) and their cross-border mergers under cases of different bargaining powers. Gabrielsen and Sørgard (1999) allow monopoly retailer to decide whether producers should have exclusive dealership (or not) under a linear contract manner. Mills (2015) considers a similar setting but under a nonlinear contract setting, while contract terms are decided by negotiations between the monopoly retailer and either or both suppliers. Gabrielsen and Sørgard (2007) consider retailers' incentives for carrying private labels in a setting of vertically differentiated products. Although all these works focus on buyer power in product line choices, the upstream manufacturer's incentive of direct marketing is not considered. Our work complements the literature by taking into account the incentive that a manufacturer strategically recaptures the channel power.[1]

Arya et al. (2007) may be among the earliest attempts theoretically discussing the manufacturer's encroaching behaviour. A manufacturer encroaches upon a retailer's territory if its retail cost is low enough. The encroachment may even benefit the retailer because the manufacturer resets a lower wholesale price to maintain the retailer's demand to an adequate level. This benefit exists only if the retailer decides its quantity before the manufacturer does

[1]For parallel researches, Inderst and Shaffer (2010) discusses how a supplier uses different contract forms (based on the share it receives of a retailer's total purchases, or on how much a retailer purchases of its products) when retailers have out-sourcing options. Milliou and Sandonís (2015) studies the relationship between multi-product manufacturers' mergers and their incentive for enlarging product line.

so that the retailer's output reaches the Stackelberg leader's level.[2] On the basis of the main structure of Arya et al. (2007), we consider manufacture's encroachment problem in which a retailer decides its preferred product line, but assume quantities to be simultaneously decided so as to remove the Stackelberg leader's advantage and to focus on the effect of product line choices. Mizuno (2012) considers a case in which two manufacturers distribute their products to n retailers by competing for a wholesale market, while simultaneously deciding whether to encroach upon the resale market (or not). Li et al. (2015) considers a model with n vertical supply chains and analyzes each manufacturer's incentive to encroach.[3] By considering retailers' channel power in product line choices, we add several new insights into this literature.

The remaining paper is organized as follows. Section 2 introduces a basic model with one manufacturer owned online store and one retailer, from which we see how the retailer employs different variety orders to affect the manufacturer owned online store's encroachment. Section 3 extends the basic model to a duopoly retailers case. The retailers' variety orders in equilibrium and some related propositions will be derived. Section 4 concludes the paper.

2 Monopoly Case

We start with a simplest monopoly retailer case and demonstrate a basic result–the retailer's product line choice affects the online store's encroachment.

[2] Liao (2014) extends this model by considering a signalling game in which the manufacturer has private information on its own retail cost, and shows that in a separating equilibrium of retail competition, the manufacturer signals its inefficiency in retail behaviour by setting a lower wholesale price than that under complete information.

[3] Some other parallel literature discusses the manufacturer's direct selling channel in the Salop (1979) setting. In Balasubramanian (1998), the consumers buy products from the conventional retailers incurring location-dependent transportation cost, and are charged a fixed freight per unit that is irrelevant of location, if they buy from a direct seller. Shulman (2014) considers Balasubramanian's setting in a vertically related market. The direct seller can either buy wholesale products from the authorized retailers who are supplied by the manufacturer, which is referred to as a "gray-market," or buy wholesale products directly from the manufacturer.

2.1 The Basic Model

Let's consider a standard vertically related market consisting of one manufacture M, and one retailer R. The manufacturer produces imperfect substitutable products X and Y, and the retailer orders either or both varieties from the manufacturer. The products are distributed to the retailer who sells them to consumers through a wholesale channel. Additionally, the manufacturer can also open its online store and encroach upon the retailer's territory by directly supplying either or both product varieties to consumers. We call the competition between the retailer and the online store the *intrabrand competition*. The retailer's retail cost is normalized to zero, and the online store's when the encroachment occurs is $c > 0$.[4] The manufacturer's production cost and the fixed cost of introducing both varieties are normalized to zero for simplicity. We assume that a representative consumer's utility to be quadratic with the form

$$u(Q_X, Q_Y) = a(Q_X + Q_Y) - \frac{1}{2}(Q_X^2 + 2\gamma Q_X Q_Y + Q_Y^2),$$

where Q_n is the aggregate quantity of variety n with $n = X$ or Y, the parameter $\gamma \in (0, 1)$ denotes the substitutability between two varieties. The products of the same variety are perfect substitutes whether they are sold by the retailer or the online store.[5] Then, the consumer demand is represented by a linear downward sloping inverse demand function $p_n = a - Q_n - \gamma Q_{-n}$, where n and $-n$ are different varieties.

Let B denote both varieties, and N denote none of the varieties. N cannot be the retailer's strategy because it gains profits only when it orders wholesale products and sells them to consumers, but this is not the case for the manufacturer because it still gains profits from the wholesale channel even if it does not open an online store. The game proceeds as follows: In period 1, the retailer orders variety X or both (B).[6] In period 2, the manufacturer decides

[4]This assumption theoretically follows the setting of Arya et al. (2007). In real world cases, online stores usually need to bear higher operating costs than physical stores. Unlike the physical stores, because consumers cannot physically inspect a product before ordering, the online stores have to undertake the risk of returns and redress. Besides, they have to endeavor to make consumers learn about the products as much as they can when browsing the Internet (Lieber and Syverson, 2010).

[5]This assumption is made for analytical simplicity. The main propositions still hold if we allow for the imperfect substitutability for the same variety sold by the retailer or the online store.

[6]By symmetry, because it is the same that the retailer chooses either X or Y, we can consider either case

whether (or not) to open the online store (N if not to open), or which variety(ies) to sell through the online store: X, Y, or both (B). Then, the manufacturer sets the corresponding wholesale price(s) w_n. In period 3, the wholesale products are delivered to the retailer, and the retailer simultaneously competes with the online store in quantity if the encroachment occurs; otherwise the retailer monopolizes the resale market.[7]

The timing assumption of the retailer's early variety ordering are observed in many industries, especially those in which retailers need to regard product line design as a long-term issue and forecast potential challenges, such as an online store owned by the manufacturer, long before the market forms (e.g. the apparel industry) (Iyer and Bergen, 1997). One explanation is that it is costly for a traditional retailer to take the consequences of an inappropriate variety order.[8] However, with advantages in inventory, display and product line adjustment, an online store often regards product line design as a short-term issue. Notice that our results still hold if the retailer and the manufacture simultaneously decide their respective product line.

2.2 Equilibrium Product Variety

Define $K \subseteq \{X, Y\} \setminus \emptyset$, and $L \subseteq \{X, Y\}$. Denote the retailer's variety choice, $r \in \{X, B\}$, and the manufacturer's, $m \in \{N, X, Y, B\}$. Any pair the retailer and the manufacturer's variety choices, rm, defines a product line system. There may be seven different product line systems in period 3: $rm \in \{XN, XX, XY, XB, BN, BY, BB\}$.[9] Let q_{nR}^{rm} and q_{nM}^{rm} be the retailer and the online store's selling quantities of variety n, where $n = X$ or Y. In period 3, the retailer chooses q_{nR}^{rm} to maximize its profit which is given by:

$$\pi_R^{rm} = \sum_{n \in K} [p_n(Q_n, Q_{-n}) - w_n^{rm}] q_{nR}^{rm}; \tag{1}$$

instead of both.

[7]For simplicity, we assume that the manufacturer cannot reject the retailer's order. If we allow for the possibility of rejection, it becomes the case in which the manufacturer appoints the variety(ies) to the retailer. If the retailer is appointed with both varieties, it has alternatives to sell one variety or both. We find the results still hold whether the manufacturer has rejective option or not.

[8]For example, the retailers are less likely to order those "very-low-demand" varieties which cannot cover the fixed cost of storing them. However, it is not the case for the online stores because the demand comes from a larger geographic market (Lieber and Syverson, 2010).

[9]By symmetry, BX is equal to BY.

If the manufacturer does not open the online store, its profit is only from the wholesale channel, which is given by:

$$\pi_M^{rm} = \sum_{n \in K} q_{nR}^{rm} w_n^{rm}; \tag{2}$$

otherwise, it competes with the retailer in the resale market and chooses q_{nM}^{rm} ($m \neq N$) to maximize its profit which is given by:

$$\pi_M^{rm} = \sum_{n' \in L} [p_{n'}(Q_{n'}, Q_{-n'}) - c] q_{n'M}^{rm} + \sum_{n \in K} q_{nR}^{rm} w_n^{rm}. \tag{3}$$

Solving the profit maximization problems in period 3, we obtain the equilibrium quantities, if the online store encroaches, $q_{nR}^{rm}(w_n^{rm}, w_{-n}^{rm})$ and $q_{n'M}^{rm}(w_n^{rm}, w_{-n}^{rm})$ (or $q_{nR}^{rm}(w_n^{rm})$ and $q_{n'M}^{rm}(w_n^{rm})$).

In period 2, w_n and w_{-n} are decided by the manufacturer, anticipating the equilibrium outcomes in period 3.When the manufacturer does not open the online store (XN and BN), it solves its maximization problem by

$$\max_{w_n^{rm}, w_{-n}^{rm}} \sum_{n \in K} q_{nR}^{rm}(w_n^{rm}, w_{-n}^{rm}) w_n^{rm}; \tag{4}$$

When the online store encroaches ($rm = XX$, XY, XB, BY, or BB), the manufacturer solves its maximization problem by

$$\max_{w_n^{rm}, w_{-n}^{rm}} \sum_{n' \in L} [p_{n'}(Q_{n'}(w_n^{rm}, w_{-n}^{rm}), Q_{-n'}(w_n^{rm}, w_{-n}^{rm})) - c] q_{n'M}^{rm}(w_n^{rm}, w_{-n}^{rm}) + \sum_{n \in K} q_{nR}^{rm}(w_n^{rm}, w_{-n}^{rm}) w_n^{rm}. \tag{5}$$

The manufacturer decides whether to open the online store and the product line for the online store based on the resulting profits. The equilibrium wholesale price, w_n and w_{-n}, are denoted in Table 1, where "I.S." denotes the conditions for interior solutions. For cases in which the manufacturer encroaches, c must be low enough. Moreover, for the BY case, c mustn't be too low ($c/a > 5\gamma/(8 + 8\gamma)$), otherwise w_X^{rm} becomes so high that the retailer gives up supplying X. Thus, the BY case with $q_{YR}^{BY} = 0$ becomes the XY case.

It is straightforward to find that given $r = X$ or B, the wholesale prices decrease with more product varieties sold online, which is summarized in the next proposition:

Proposition 1 *Given the retailer's variety order (X or B), the wholesale prices decrease with more product varieties sold online.*

r	X				B		
m	N	X	Y	B	N	Y	B
w_X^{rm}	$\frac{a}{2}$	$\frac{5a-c}{10}$	$\frac{(8-4\gamma^2+\gamma^3)a-\gamma^3 c}{2(8-3\gamma^2)}$	$\frac{5a-c}{10}$	$\frac{a}{2}$	$\frac{5a-c}{10}$	$\frac{5a-c}{10}$
w_Y^{rm}	\	\	\	\	$\frac{a}{2}$	$\frac{5a-\gamma c}{10}$	$\frac{5a-c}{10}$
$I.S.$	$a>0$	$\frac{c}{a} \leq \frac{5}{7}$	$\frac{c}{a} \leq \frac{8-2\gamma-\gamma^2}{8-\gamma^2}$	$\frac{c}{a} \leq \frac{5}{7+2\gamma}$	$a>0$	$\frac{c}{a} \in (\frac{5\gamma}{8+8\gamma}, \frac{5}{7}]$	$\frac{c}{a} \leq \frac{5}{7}$

Table 1: Equilibrium wholesale prices

Proposition 1 follows straightforward from Arya et al. (2007)'s main result that a manufacturer may strategically reduce the wholesale price when it encroaches upon the retailer's territory. The manufacturer gains profits from both the wholesale channel and the online channel. In the current research, the increasing number of product varieties sold online intensifies the *intrabrand competition*, which causes a decrease in the manufacturer's profits in the wholesale channel. The manufacturer thus lowers the wholesale price to shift some business back to the retailers so as to keep the wholesale demand at an adequate level. The wholesale price conversely reflects how intensively the presence of online store causes *intrabrand competition*. The lower wholesale price is charged, the more badly the wholesale channel is affected, the stronger incentive by which the manufacturer attempts to retrieve the retailer's demands.

For simplicity, we call the manufacturer's encroachment with only one variety, a partial encroachment; that with both varieties, a full encroachment. Taking the conditions for interior solutions into consideration and making sure that unilateral deviations do not take place in each case, we find the equilibrium outcomes in period 2 as follows:

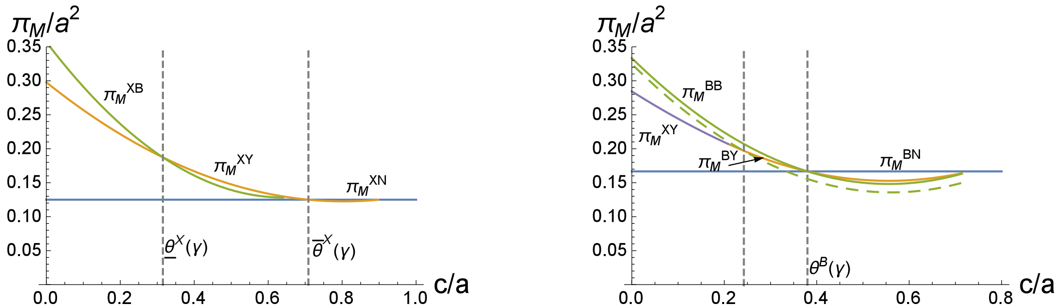
Lemma 1 *(1) When the retailer orders variety X in period 1, there exist $\underline{\theta}^X(\gamma)$ and $\overline{\theta}^X(\gamma)$, with $\overline{\theta}^X(\gamma) > \underline{\theta}^X(\gamma)$, so that the online store*

(i) fully encroaches if $c/a \leq \underline{\theta}^X(\gamma)$,

(ii) partially encroaches with variety Y if $\underline{\theta}^X(\gamma) < c/a \leq \overline{\theta}^X(\gamma)$,

(iii) is shut down if $c/a > \overline{\theta}^X(\gamma)$;

(2) When the retailer orders both varieties in period 1, there exists $\theta^B(\gamma)$, so that the online store



(a) The retailer orders X (b) The retailer orders both varieties

Figure 1: The online store's variety choice in period 2 when $\gamma = 0.5$

(i) fully encroaches if $c/a \leq \theta^B(\gamma)$*,*

(ii) is shut down in period 2 if $c/a > \theta^B(\gamma)$*,*

where $\underline{\theta}^X(\gamma) < \theta^B(\gamma) < \overline{\theta}^X(\gamma)$*.*

Here, we don't take a *weakly dominated strategy* as an equilibrium candidate. Figure 1 depicts the threshold values in Lemma 1. Please see calculations in Appendix 5.2.

Notice that given $r = X$ or B, the manufacturer's profit curve becomes steeper when it sells more varieties online. This is because the increasing online retail cost incurs a higher loss when the online store has a wider product range.

In (a) of Figure 1, given that the retailer orders only variety X, as the online retail cost increases, the online store first stops selling variety X. The benefits of partial encroachment with a different variety rather than a same one online come from two angles. First, the manufacturer obtains higher shares in the wholesale channels when its online store indirectly compete with the retailer. Second, a higher wholesale price ($w_X^{XY} > w_X^{XX}$) refelcts a less intensive *intrabrand competition*, implying a less profit loss in the wholesale channel.

In (b) of Figure 1, π_M^{BB}, π_M^{BY} and π_M^{BN} intersects at $\theta^B(\gamma)$, implying that when different varieties are evenly distributed in the wholesale channel ($r = B$), selling only one variety online is a *weakly dominated strategy* for the manufacturer. This fact follows from the assumption of symmetric online retail costs for both varieties. When the retailer orders both varieties and the manufacturer starts the online channel and sells only variety Y, it lowers the wholesale prices for both X an Y from $w_X^{BN} = w_Y^{BN}$ to w_X^{BY} and w_Y^{BY} respectively. w_Y^{BY} is lower than w_X^{BY}

because the online store directly invades the retailer's share in Y but indirectly in X. Then, the partial encroachment causes intensive *intrabrand competition* in both varieties. However, if the online store additionally sells X, it only further decreases the wholesale price of X ($w_Y^{BB} = w_Y^{BY}$, $w_X^{BB} < w_X^{BY}$), implying that the additional sale of X by the online store does not further intensify the *intrabrand competition* in Y. The additional sale of X by the online store reduces its sale of Y through the *cannibalization effect*, which prevents retailer's sale of Y from decreasing sharply. Then, the manufacturer does not need to further reduce the wholesale price of Y. We see that a full encroachment does not seriously cause more profit loss in the wholesale channel than a partial one does. If selling one variety online is profitable for the manufacturer than selling nothing, it always further sells the other variety. Then, selling one variety is a *weakly dominated strategy* when $r = B$. If we assume that there exists a positive real number τ such that selling X incurs more cost ($c + \tau$) than selling Y, π_M^{BB} in (b) of Figure 1 shifts left downwards so that the range in which the online store sells only the more efficient variety Y becomes wider. All results here still hold if the cost difference τ is small enough so that the range of selling only Y online is neglectable. We assume the symmetric online retail costs just for analytical simplicity.

The case that $r = X$ can be seen as an unbalanced order, because only one variety is distributed to the retailer; the case that $r = B$ can be seen as a balanced order, because both varieties are distributed. The manufacturer tends to keep balance of the variety distribution-it may sell only the variety that is less distributed to the retailer (when $r = X$), but never sells only one variety when both varieties are already evenly distributed (when $r = B$).

Lemma 1 shows how the retailer's product line choice affects the manufacturer's incentive for opening the online store. When the online retail cost is relatively low, the manufacturer considers whether (or not) it fully encroaches. The threshold values $\underline{\theta}^X(\gamma) < \theta^B(\gamma)$ show that the manufacturer has a stronger incentive to do so when $r = B$ than when $r = X$. Reducing the number of varieties sold online affects the manufacturer's profits form two angles: First, it alleviates the *intrabrand competition* and results in higher wholesale prices, from which the manufacturer's profit in the wholesale channel increases. Second, because of a narrower product line, the manufacturer's profit in the online channel decreases. We first see the wholesale channel. When the online store sells both varieties, $w_X^{XB} = w_X^{BB} = w_Y^{BB}$. But when the online store decreases its varieties, the manufacturer sets higher wholesale prices (w_X^{BN} and w_Y^{BN}) when $r = B$ than that (w_X^{XY}) when $r = X$. This is because when $r = B$, once the manufacturer reduces its varieties, it directly shuts down the online channel so that the *intrabrand competition* is fully removed. Thus, the manufacturer's gain in the wholesale channel is lager when $r = B$ than when $r = X$. However, the manufacturer's loss in the online channel is more severe when $r = B$ than when $r = X$, because it loses profits from both varieties in the former case. Because now the online retail cost is low so that the loss dominates the gain, the manufacturer is more likely to keep full encroachment when $r = B$ than when $r = X$.

Remark 1 $\underline{\theta}^X(\gamma) < \theta^B(\gamma)$.

When the online retail cost is relatively high, the manufacturer considers whether (or not) it shuts down the online store. The threshold values $\theta^B(\gamma) < \overline{\theta}^X(\gamma)$ show that the manufacturer has a stronger incentive to do so when $r = B$ than when $r = X$. Opening an online store has two effects on the manufacturer's profits: First, it incurs the *intrabrand competition*, from which the manufacturer's profit in the wholesale channel decreases. Second, it brings the manufacturer additional profits in the online channel. We fist see the wholesale channel. When the online store sells both varieties, $w_X^{XN} = w_X^{BN} = w_Y^{BN}$. But when the manufacturer opens its online store, it sets lower wholesale prices (w_X^{BB} and w_Y^{BB}) when $r = B$ than that (w_X^{XY}) when $r = X$. This is because when $r = B$, the online store directly competes with the (incumbent) retailer in both varieties, but when $r = X$, the online store can sell a different variety so that the decreases in the retailer's share is not that large. Thus, the manufacturer's loss in the wholesale channel is lager when $r = B$ than when $r = X$. However, the manufacturer gains more profits in the online channel when $r = B$ than when $r = X$, because it gains profits from both varieties in the former case. Because now the online retail cost is high so that the loss dominates the gain, the manufacturer is more likely to shut down its online store when $r = B$ than when $r = X$.

Remark 2 $\theta^B(\gamma) < \overline{\theta}^X(\gamma)$.

2.3 The Retailer's Product Line Choice in Equilibrium

In period 1, the retailer orders either variety X or both varieties, anticipating the manufacturer's reaction in period 2. The following proposition shows the equilibrium variety outcome.

Proposition 2 *In the monopoly retailer case, the equilibrium variety outcome is*

(i) the retailer orders both varieties in period 1 and the online fully encroaches in period 2 if $c/a \leq \underline{\theta}^X(\gamma)$ *(hereafter the BB variety outcome);*

(ii) the retailer orders variety X in period 1 and the online store partially encroaches with variety Y in period 2 if $\underline{\theta}^X(\gamma) < c/a \leq \theta^B(\gamma)$ *(hereafter the XY variety outcome);*

(iii) the retailer orders both varieties in period 1 and the online store is shut down in period 2 if $c/a > \theta^B(\gamma)$ *(hereafter the BN variety outcome).*

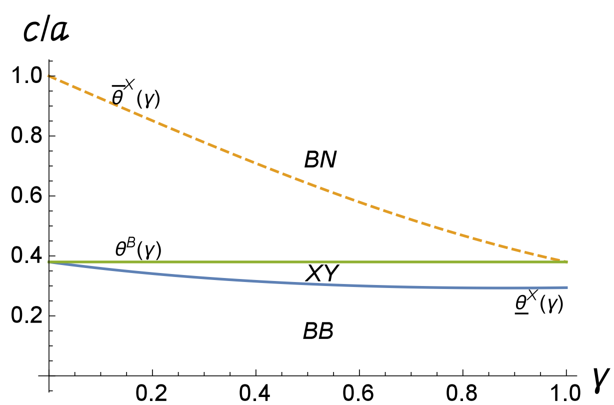


Figure 2: The variety choices *rm* in equilibrium

It is important for the retailer to consider two main issues in the ordering process: First, ordering as more varieties as possible so as to obtain a wider product line; Second, weakening the negative effect of the online store's encroachment. When the retail cost is too low ($c/a \leq \underline{\theta}^X(\gamma)$), whether the retailer orders one variety or both cannot change the outcome that the online store fully encroaches. Therefore, the retailer chooses a full-length product line. But when the online retail cost is not that low ($\underline{\theta}^X(\gamma) < c/a \leq \theta^B(\gamma)$), the retailer can only order X

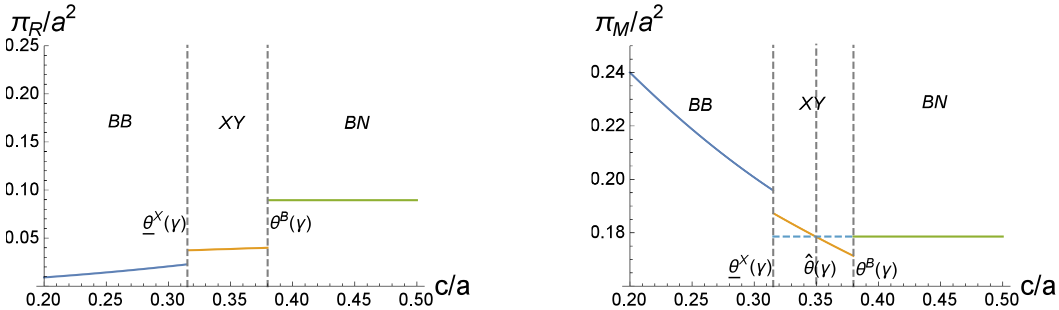


Figure 3: The retailer and the manufacturer's profit function when $\gamma = 0.3$

so as to stimulate the manufacturer's incentive of encroaching with the less supplied variety Y. However, if it orders both varieties, the manufacturer follows with a full encroachment. Because now the online store is a relatively efficient one, alleviating the negative effect of the encroachment is prior to enlarging product line. Therefore, although losing profits from one variety, the retailer still benefits from making a mitigant market condition by inducing less varieties sold by the online store. When the retail cost is very high ($c/a \geq \theta^B(\gamma)$), the retailer a full-length product line so as to induce the manufacturer to shut down the online store. Then, the retailer fully removes the negative effect of the encroachment and obtains a wider product line as well.

Concerning how the change of online retail cost affects the retailer and the manufacturer's profits (see Figure 3), we find that as the online retail cost increases, the retailer's profit always has an increasing tendency. This is because an increasing c makes the retailer more competitive and gradually removes the manufacturer's encroaching incentive. However, the manufacturer's profit does not always show a decreasing tendency, which is summarized in the following proposition.

Proposition 3 *As the online retail cost increases, the manufacturer's profit drops at* $\underline{\theta}^X(\gamma)$*, but upward jumps at* $\theta^B(\gamma)$*.*

When c just surpasses $\underline{\theta}^X(\gamma)$, the retailer and the online store begins to compete in different varieties, which alleviates the *intrabrand competition*. However, because both variety Y in the wholesale channel and variety X in the online channel are suddenly removed, the manufacturer bears profit losses from both channels, which overweighs the positive effect from alleviating the *intrabrand competition*. When c just surpasses $\theta^B(\gamma)$, the profit from supplying variety Y through the online channel is suddenly replaced by that through the wholesale channel. This replacement benefits the manufacturer from the following two aspects: First, because the *intrabrand competition* is fully removed, the manufacturer obtains a higher profit by specializing the wholesale channel. Second, the channel efficiency is improved because Y is supplied by the efficient retailer instead of the costly online store.

Proposition 3 has further implications concerning the manufacturer's encroaching decision. Suppose that there is a pre-determinate period, period 0. The upward jump feature implies that when $\hat{\theta}(\gamma) < c/a \leq \theta^B(\gamma)$, it is better for the manufacturer to commit to the retailer in period 0 that it will not open its online store, where $\hat{\theta}(\gamma)$ is the threshold satisfying $\pi_M^{XY} = \pi_M^{BN}$. Being guaranteed that the resale market will not be encroached upon, the retailer chooses to carry both varieties. Without such a commitment, at $\theta^B(\gamma)$, the encroachment makes the retailer reduce one variety and enforces the manufacturer to carry this variety in an inefficient way, which implies a "lose-lose" consequence. The committing manner enables the manufacturer to insure that both varieties are supplied efficiently. In Arya et al. (2007), without considering the possibility that the retailer specifies its product line, because the manufacturer never encroaches if doing so causes profit decrease, such "lose-lose" consequence never occurs.

Next we see how the consumer surplus (CS) and the total surplus (TS) are affected by the retailer and the online store's variety choices. Because the social loss only comes from the online retail cost, the total surplus is the representative consumer's gross utility net of the total online retail cost, which is denoted by $TS = U(Q_X, Q_Y) - (q_{XM} + q_{YM})c$. We find that as c increases, the consumer surplus always decreases, but the total surplus upwards jumps at $\theta^B(\gamma)$. This fact is summarized in the following proposition.

Proposition 4 *As c increases, the consumer surplus always shows an increasing tendency, and the total surplus drops at $\underline{\theta}^X(\gamma)$, but upward jumps at $\theta^B(\gamma)$.*

Figure 4 depicts how c affects the consumer and the total surplus.

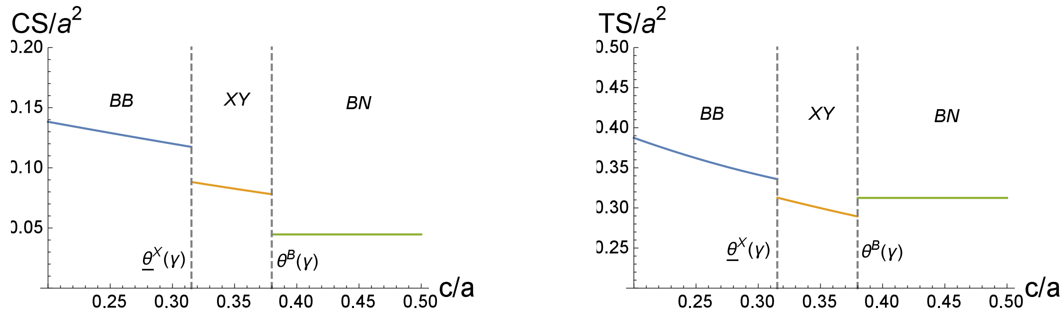


Figure 4: The consumer and the total surplus when $\gamma = 0.4$

The increasing c gradually removes the manufacturer's incentive of online retailing and enhances the (incumbent) retailer's market power who finally monopolizes the resale market. At $\underline{\theta}^X(\gamma)$, both the retailer and the online store reduce one variety and begin to indirectly compete, which results in higher prices. This harms both the consumer and the total surpluses. At $\theta^B(\gamma)$, the online store is shut down and the retailer becomes a monopolist of both varieties. This drives up prices, which further decreases the consumer surplus. However, although the monopolization of the (incumbent) retailer decreases the gross utility $U(Q_X, Q_Y)$, the shutdown of the online channel fully removes the social loss, which results in a higher total surplus.

3 Extension: Duopoly Retailers Case

Our basic model tells us how a monopoly retailer decides its product line when its manufacturer can strategically recapture its channel power by employing an online channel. There raises a question how the product line outcomes changes if there are additionally strategic interactions between retail competitors. Then, it is naturally to consider a simple extension with duopoly retailers. Suppose there are a monopoly manufacturer, M, two retailers, denoted by R_i, with $i = 1$ or 2.

To keep consistent with the benchmark model, we assume that all players are kept active in the market so that N (order nothing) cannot be any retailer's choice. Besides, we normalize both retailers' resale cost for both varieties to zero. Define $K_i \subseteq \{X, Y\} \setminus \emptyset$. Denote each

r_1r_2	m	$w_{X1}^{r_1r_2m}$	$w_{Y1}^{r_1r_2m}$	$w_{X2}^{r_1r_2m}$	$w_{Y2}^{r_1r_2m}$	$I.S.$
XX	N	$\frac{a}{2}$	$\backslash$	$\frac{a}{2}$	$\backslash$	$a>0$
	X	$\frac{3a-c}{6}$	$\backslash$	$\frac{3a-c}{6}$	$\backslash$	$\frac{c}{a}\leq\frac{3}{5}$
	Y	$\frac{(6-4\gamma^2+\gamma^3)a-\gamma^3c}{6(2-\gamma^2)}$	$\backslash$	$\frac{(6-4\gamma^2+\gamma^3)a-\gamma^3c}{6(2-\gamma^2)}$	$\backslash$	$\frac{c}{a}\leq\frac{6-2\gamma-\gamma^2}{6-\gamma^2}$
	B	$\frac{3a-c}{6}$	$\backslash$	$\frac{3a-c}{6}$	$\backslash$	$\frac{c}{a}\leq\frac{3}{5+2\gamma}$
XY	N	$\frac{a}{2}$	$\backslash$	$\backslash$	$\frac{a}{2}$	$a>0$
	Y	$\frac{(10-5\gamma^2+\gamma^3)a-2\gamma c}{4(5-2\gamma^2)}$	$\backslash$	$\backslash$	$\frac{(10-\gamma-3\gamma^2)a-2c}{4(5-2\gamma^2)}$	$\frac{c}{a}\in(\frac{\gamma-\gamma^2}{8-4\gamma^2},\frac{5-\gamma-\gamma^2}{7-2\gamma^2}]$
	B	$\frac{(5+\gamma)a-(1+\gamma)c}{2(5+\gamma)}$	$\backslash$	$\backslash$	$\frac{(5+\gamma)a-(1+\gamma)c}{2(5+\gamma)}$	$\frac{c}{a}\leq\frac{5+\gamma}{7+3\gamma}$
XB	N	$\frac{a}{2}$	$\backslash$	$\frac{a}{2}$	$\frac{a}{2}$	$a>0$
	X	$\frac{3a-c}{6}$	$\backslash$	$\frac{3a-c}{6}$	$\frac{3a-\gamma c}{6}$	$\frac{c}{a}\in(\frac{3\gamma}{4(1+\gamma)},\frac{3}{5}]$
	Y	$\frac{(15-4\gamma^2+\gamma^3)a-(3\gamma+\gamma^3)c}{6(5-\gamma^2)}$	$\backslash$	$\frac{(15-4\gamma^2+\gamma^3)a-(3\gamma+\gamma^3)c}{6(5-\gamma^2)}$	$\frac{(15-\gamma-2\gamma^2)a-(3+\gamma^2)c}{6(5-\gamma^2)}$	$\frac{c}{a}\in(\frac{\gamma}{2(1+\gamma)},\frac{15-2\gamma-\gamma^2}{21-\gamma^2}]$
	B	$\frac{3a-c}{6}$	$\backslash$	$\frac{3a-c}{6}$	$\frac{15a-(3+2\gamma)c}{30}$	$\frac{c}{a}\leq\frac{15}{25+4\gamma}$
BB	N	$\frac{a}{2}$	$\frac{a}{2}$	$\frac{a}{2}$	$\frac{a}{2}$	$a>0$
	Y	$\frac{3a-\gamma c}{6}$	$\frac{3a-c}{6}$	$\frac{3a-\gamma c}{6}$	$\frac{3a-c}{6}$	$\frac{c}{a}\in(\frac{\gamma}{2(1+\gamma)},\frac{3}{5}]$
	B	$\frac{3a-c}{6}$	$\frac{3a-c}{6}$	$\frac{3a-c}{6}$	$\frac{3a-c}{6}$	$\frac{c}{a}\leq\frac{3}{5}$

Table 2: Equilibrium wholesale prices

retailer's variety choice $r_i \in \{X, Y, B\}$, and the manufacturer's, $m \in \{N, X, Y, B\}$. The game proceeds as in the monopoly retailer case except that the retailers decide their respective product line simultaneously and independently in period 1, and the manufacturer discriminates the retailers by offering different wholesale price $w_{ni}^{r_1r_2m}$ in period 2. The retailers' order combination r_1r_2 in period 1 is simplified as follows: $r_1r_2 \in \{XX, XY, XB, BB\}$.[10]

3.1 Equilibrium Product Variety

In period 3, the Cournot competition proceeds based on the product line system, r_1r_2m, which are of fourteen cases: XXN, XXX, XXY, XXB, XYN, XYY, XYB, XBN, XBX, XBY, XBB, BBN, BBY, BBB.[11] The optimization system is the same as in the basic model.

In period 2, the manufacturer decides whether to open its online store and which variety(ies) to sell based on the resulting profits. The equilibrium wholesale price, $w_{ni}^{r_1r_2m}$ and $w_{-ni}^{r_1r_2m}$, are denoted in Table 2, where "I.S." denotes the conditions for interior solutions.

[10]By symmetry, YY is equal to XX; YX is equal to XY; YB, BY and BY is equal to XB.

[11]By symmetry, XYX is equal to XYY, BBX is equal to BBY.

The online store encroaches only if its retail cost is low enough. Moreover, for the interior solutions in the XBY case, the online retail cost must not be too low. When the online store is very efficient in selling variety Y, even if R_2 orders both varieties, it will be charged an unacceptable w_{Y2}^{XBY} so that R_2 does not really sell Y. The XBY case with a corner solution ($q_{YR2}^{XBY} = 0$) becomes the XXY case. Thus, we need the online retail cost to be higher than $\theta^{XB}(\gamma) \equiv \gamma/(2 + 2\gamma)$ in the XBY case. After several calculations, taking the corner solutions into consideration and checking the manufacturer's incentive for deviation, we find the equilibrium outcomes in period 2 as follows:

Lemma 2 *(1) When both retailers order variety X in period 1, there exist $\underline{\theta}^{XX}(\gamma)$ and $\overline{\theta}^{XX}(\gamma)$, with $\overline{\theta}^{XX}(\gamma) > \underline{\theta}^{XX}(\gamma)$, so that the online store*

(i) fully encroaches in period 2 if $c/a \leq \underline{\theta}^{XX}(\gamma)$,

(ii) partially encroaches with variety Y in period 2 if $\underline{\theta}^{XX}(\gamma) < c/a \leq \overline{\theta}^{XX}(\gamma)$,

(iii) is shut down in period 2 if $c/a > \overline{\theta}^{XX}(\gamma)$;

(2) When one retailer orders variety X, and the other one orders both varieties in period 1, there exist $\underline{\theta}_1^{XB}(\gamma)$, $\underline{\theta}_2^{XB}(\gamma)$, $\overline{\theta}_1^{XB}(\gamma)$ and $\overline{\theta}_2^{XB}(\gamma)$, so that the online store

(i) fully encroaches in period 2 when $\gamma \leq 0.690$ if $c/a \leq \underline{\theta}_1^{XB}(\gamma)$, as well as when $\gamma > 0.690$ if $c/a \leq \underline{\theta}_2^{XB}(\gamma)$,

(ii) partially encroaches with variety Y in period 2 when $\gamma \leq 0.690$ if $\underline{\theta}_1^{XB}(\gamma) < c/a \leq \overline{\theta}_1^{XB}(\gamma)$, as well as when $0.69 \leq \gamma \leq 0.897$ if $\theta^{XB}(\gamma) < c/a \leq \overline{\theta}_1^{XB}(\gamma)$.

(iii) is shut down in period 2 when $\gamma \leq 0.897$ if $c/a > \overline{\theta}_1^{XB}(\gamma)$, as well as when $\gamma > 0.897$ if $c/a > \overline{\theta}_2^{XB}(\gamma)$;

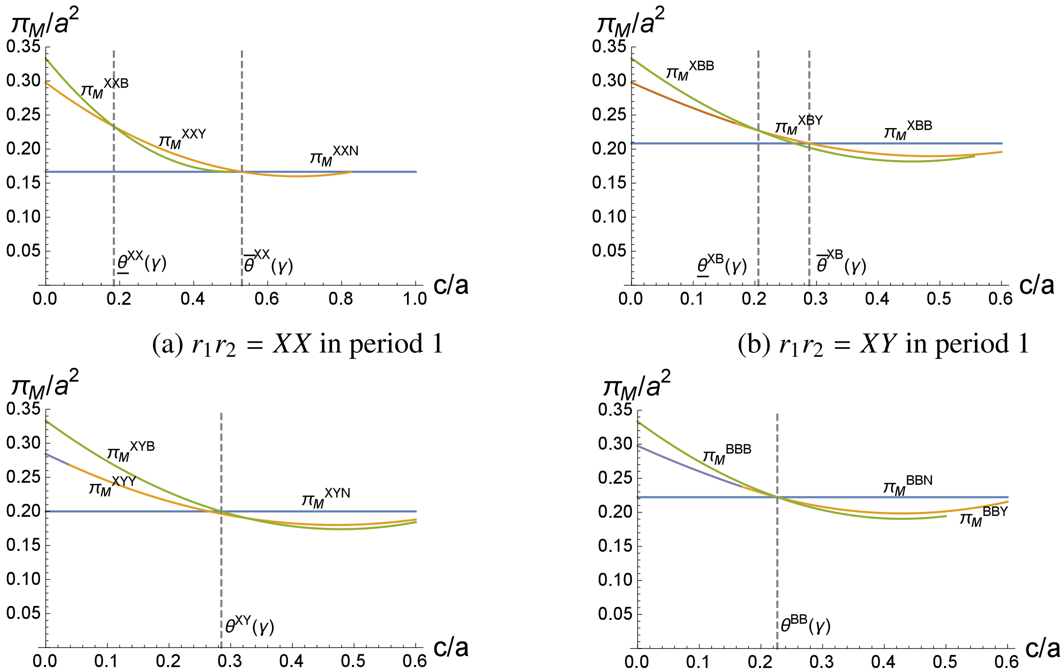
(3) When one retailer orders variety X and the other one orders variety Y in period 1, there exist $\theta^{XY}(\gamma)$, so that the online store

(i) fully encroaches in period 2 if $c/a \leq \theta^{XY}(\gamma)$,

(ii) is shut down in period 2 if $c/a > \theta^{XY}(\gamma)$;

(4) When both retailers order both varieties in period 1, there exist $\theta^{BB}(\gamma)$, so that the online store

(i) fully encroaches in period 2 if $c/a \leq \theta^{BB}(\gamma)$,



(a) $r_1 r_2 = XX$ in period 1

(b) $r_1 r_2 = XY$ in period 1

(c) $r_1 r_2 = XB$ in period 1

(d) $r_1 r_2 = BB$ in period 1

Figure 5: The online store's variety choice in period 2 when $\gamma = 0.5$

(ii) is shut down in period 2 if $c/a > \theta^{BB}(\gamma)$.

Please see calculations in Appendix 5.4. The ranges for equilibrium outcomes in period 2 are summarized in Figure 5. For simplicity in notation, we denote the upper bound and lower bound values of the XB case as $\overline{\theta}^{XB}(\gamma)$ and $\underline{\theta}^{XB}(\gamma)$ respectively.[12]

Following the logic of the monopoly retail case, the case that $r_1 r_2 = XX$ or XB corresponds to that $r = X$ in Section 2 (see (*a*) and (*b*) in Figure 5). These two cases can be seen as the unbalanced orders, because variety X is distributed to both retailers, but Y is distributed to at most one retailer. Knowing that Y will be less supplied by the retailers comparing with X, the online store sells only Y when the c is intermediate ($\underline{\theta}^{XX}(\gamma) < c/a \leq \overline{\theta}^{XX}(\gamma)$ or $\underline{\theta}^{XB}(\gamma) < c/a \leq \overline{\theta}^{XB}(\gamma)$). The threshold values, $\underline{\theta}^{XX}(\gamma) < \underline{\theta}^{XB}(\gamma) < \overline{\theta}^{XB}(\gamma) < \overline{\theta}^{XX}(\gamma)$, imply that a partial encroachment with Y is more likely to happen in the XX case than in the XB case. This is because the distribution unbalance is more serious in the former case. In the

[12] $\underline{\theta}_1^{XB}$ and $\underline{\theta}_2^{XB}(\gamma)$ are connected at $\gamma = 0.690$; $\overline{\theta}_1^{XB}$ and $\overline{\theta}_2^{XB}(\gamma)$ are connected at $\gamma = 0.897$.

XB case, R_2's share of Y makes selling Y online less profitable for the manufacturer than in the XX case.

The case that $r_1 r_2 = XY$ or BB corresponds to that $r = B$ in Section 2 (see (c) and (d) in Figure 5). These two cases can be seen as the balanced orders, because both varieties are evenly distributed to the retailers. Then, in order not to break such balance, the manufacturer chooses either to fully encroaches or completely shut down the online channel. The threshold values, $\theta^{BB}(\gamma) < \theta^{XY}(\gamma)$, imply that a full encroachment is more likely to happen in the XY case than in the BB case. In the XY case, the retailers are differentiated in product lines so that the resale market before the presence of encroachment is less competitive for online retailing, comparing with the BB case where retailers have overlapping product lines so that they compete intensively.

Notice that in the case $r_1 r_2 = XY$, a partial encroachment is strictly dominated by a fully encroachment or a shutdown of the online channel,[13] which differs from the case $r_1 r_2 = BB$ in the duopoly retailers case and $r = B$ in the basic model. Partially encroaching with Y forces the manufacturer to reduce the wholesale prices of X and Y from w_{X1}^{XYN} to w_{X1}^{XYY} and from x_{Y2}^{XYN} to w_{Y2}^{XYY} respectively. If the manufacturer additionally sells X online, there would be in a lower wholesale price of X ($w_{X1}^{XYB} < w_{X1}^{XYY}$), but a higher wholesale price of Y ($w_{Y2}^{XYB} > w_{Y2}^{XYY}$), implying that additionally selling X alleviates the *intrabrand competition* in Y. Because now each variety is ordered by only one retailer, the demand in the wholesale channel is small, which stimulates the manufacturer's strong incentive for online retailing. When the online store sells only Y, R_2's share is severely invaded so that the manufacturer has to greatly lower the wholesale price of Y. However, if the manufacturer additionally sells X online, the *cannibalization effect* diminishes the online sale of Y, thus alleviating the negative impact on R_2's share in Y. Additionally selling X not only helps the manufacturer to enlarge the online store's product line, but also enhances the wholesale channel's profit in Y. If fully encroaching is more profitable than shutting down the online store, a manufacturer never partially encroaches.

[13] π_M^{XYB} and π_M^{XYX} intersects at the point that is larger than $\theta^{XY}(\gamma)$, implying that a partial encroachment with either variety is a *strictly dominated strategy* for the manufacturer.

3.2 The Retailers' Variety Orders at Equilibrium

In period 1, the retailers decide which variety(ies) to order, anticipating the potential online store's encroachment. The equilibrium variety outcome in period 1 is summarized as follows:

Proposition 5 *(i) the retailers order both varieties, and the online store fully encroaches (hereafter the BBB outcome) if* $c/a \leq \underline{\theta}^{XB}(\gamma)$*;*

(ii) the retailers order X, and the online store partially encroaches with Y in period 2 (hereafter the XXY outcome) if $\underline{\theta}^{XX}(\gamma) < c/a \leq \underline{\theta}^{XB}(\gamma)$*, or if* $\underline{\theta}^{XB}(\gamma) < c/a \leq \min\{\overline{\theta}^{XB}(\gamma), \theta^{XB}(\gamma)\}$.

(iii) one retailer orders X and the other retailer orders both varieties, and the online store partially encroaches with Y (hereafter the XBY outcome) if $\max\{\underline{\theta}^{XB}(\gamma), \theta^{XB}(\gamma)\} < c/a \leq \theta^{BB}(\gamma)$*;*

(iv) each retailer orders a different variety respectively, and the online store is shut down (hereafter the XYN outcome) if $\theta^{XY}(\gamma) < c/a \leq \overline{\theta}^{XB}(\gamma)$*;*

(v) the retailers order both varieties, and the online store is shut down (hereafter the BBN outcome) if $c/a > \theta^{BB}(\gamma)$.

Here, because quantities are decided simultaneously, the encroachment always harms the retailers' profits. Although being charged charged lower wholesale prices, the retailers' losses in market share can never be compensated.[14] The main issue for each retailer is to make a balance between enlarging product line and weakening the negative impact of the online store's encroachment.

When the online retail cost is too low ($c/a \leq \underline{\theta}^{XB}(\gamma)$), whether $r_1r_2 = BB$ or XB, the online store always fully encroaches. Then, each retailer chooses a full-length product line (the *BBB* outcome).

As the online retail cost gradually increases, the retailers' unbalanced variety order-s may induce the manufacturer's partial encroachment instead of a full one. This is be-cause the unbalanced orders provide the manufacturer with signal that variety Y would be less supplied. Observing this, the manufacturer sells only Y so that the resulting *in-*

[14] If the online store moves after the retailers, it is possible that the retailers' shares loss caused by the encroachment is compensated by being charged lower wholesale prices (see Arya et al., 2007).

trabrand competition is not that intensive. In the range that $\underline{\theta}^{XX}(\gamma) < c/a \leq \underline{\theta}^{XB}(\gamma)$ or $\underline{\theta}^{XB}(\gamma) < c/a \leq \min\{\overline{\theta}^{XB}(\gamma), \theta^{XB}(\gamma)\}$, only when $r_1 r_2 = XX$ does the online store partially encroaches (the XXY outcome).[15] Although both retailers give up variety Y, both of them benefit from weakening the negative impact of the encroaching online store. In the range that $\max\{\underline{\theta}^{XB}(\gamma), \theta^{XB}(\gamma)\} < c/a \leq \theta^{BB}(\gamma)$, either when $r_1 r_2 = XX$ or XB, the online store's partial encroachment can be induced. In equilibrium, one retailer is willing to give up one variety and let its retail rival have a full-length product line (the XBY outcome). Although the retailer who cuts product line gains less profit than its retail rival does, it would loses more if insisting on choosing a full-length line because the online store would thus fully encroach.

When the online retail cost is relatively high, the retailers' balanced variety orders may induce the manufacturer's incentive for shutting its online store down, because running it means a full encroachment, which seriously causes profit decrease in the wholesale channel. In the range that $\theta^{XY}(\gamma) < c/a \leq \overline{\theta}^{XB}(\gamma)$, the manufacturer partially encroaches when receiving the unbalanced orders, but shuts down the online channel when $r_1 r_2 = XY$ (the XYN outcome). Given that one of the retailers orders one variety, the other one would spontaneously order the different variety. Although both of them give up the full-length product line, their non-overlapping product line choices fully remove the online store's partial encroachment and makes the competition between them indirect and mitigant. In the range that $c/a > \theta^{BB}(\gamma)$, the manufacture shuts down its online store when $r_1 r_2 = BB$ (the BBN outcome). No retailer would unilaterally reduce the number of varieties because this not only gives the retail rival the advantage in diversity but also results in the online store's partial encroachment (when $c/a \leq \overline{\theta}^{XB}(\gamma)$).

Comparing the equilibrium ranges, we find the following three sets of variety outcomes can coexist: the XXY and BBB outcomes, the XXY and BBN outcomes, the XYN and BBN outcomes. This implies that there may exit coordination failure in some ranges, which is summarized in the following corollary:

[15] If $\underline{\theta}^{XX}(\gamma) < c/a \leq \underline{\theta}^{XB}(\gamma)$, when $r_1 r_2 = XY$, XB or BB, the online store sells both varieties. If $\underline{\theta}^{XB}(\gamma) < c/a \leq \min\{\overline{\theta}^{XB}(\gamma), \theta^{XB}(\gamma)\}$, when $r_1 r_2 = XY$ or BB, the online store sells both varieties. But when $r_1 r_2 = XB$, the online store sells only Y. Because when $c/a \leq \theta^{XB}(\gamma)$, R_2 is offered an unacceptable w_{Y2}^{XBY} so that its quantity of Y becomes zero. The $r_1 r_2 m = XBY$ case with $q_{YR2}^{XB} = 0$ becomes the XXY case.

Corollary 1 *The retailers' coordination failure may occur in the following ranges, in which each retailer's profit is higher in the former outcome than the later one:*

(i) If $\underline{\theta}^{XX}(\gamma) < c \leq \underline{\theta}^{XB}(\gamma)$*, the XXY and BBB variety outcomes coexist;*

(ii) If $\theta^{BB}(\gamma) < c \leq \min\{\theta^{XB}(\gamma), \overline{\theta}^{XB}(\gamma)\}$*, the BBN and XXY variety outcomes coexist;*

(iii) If $\theta^{XY}(\gamma) < c \leq \overline{\theta}^{XB}(\gamma)$*, the XYN and BBN variety outcomes coexist.*

Figure 6 depicts the variety outcomes in equilibrium. The shady areas denote the ranges in which coordination failure may exist.

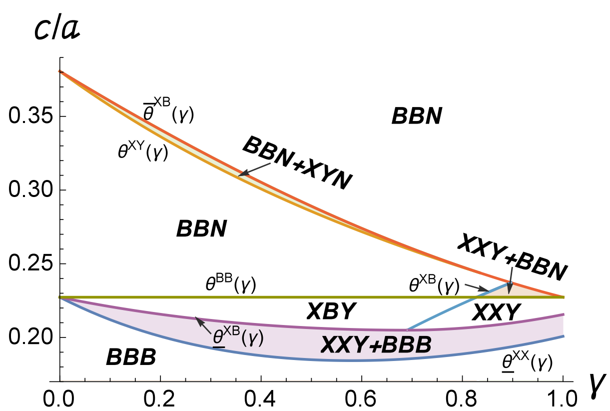


Figure 6: The variety outcomes $r_1 r_2 m$ in equilibrium

In (i) of Corollary 1, although the BBB variety outcome brings each retailer profits from a full-length product line, the resulting online store's full encroachment makes the competition too intensive. In (ii) of Corollary 1, comparing with the BBN variety outcome, the XXY outcome not only removes each retailer' profit from selling Y, but also incurs the online store's partial encroachment. In (iii) of Corollary 1, although both cases that $r_1 r_2 = BB$ and that $r_1 r_2 = XY$ make the manufacturer shut down its online store, the former case is clearly less beneficial than the later one because the competition in the later case is more mitigant. Despite all these facts, neither retailer is willing to deviates from the less profitable variety outcome.

Finally, we see the consumer surplus and the total surplus in the duopoly retailer case. The consumer surplus here shows similar features to that in the monopoly retailer case. It always shows a decreasing tendency as the online retail cost increases within the ranges in which each variety outcome exists. Moreover, it always downward jumps at some threshold points where the number of varieties in the resale market decreases, or where the online store is shut down by the manufacturer. However, the total surplus shows a distortion, which is summarized in the following proposition.

Proposition 6 *When $\gamma > 0.751$, the total surplus upward jumps at $\underline{\theta}^{XX}(\gamma)$, where the variety outcome changes from BBB to XXY.*

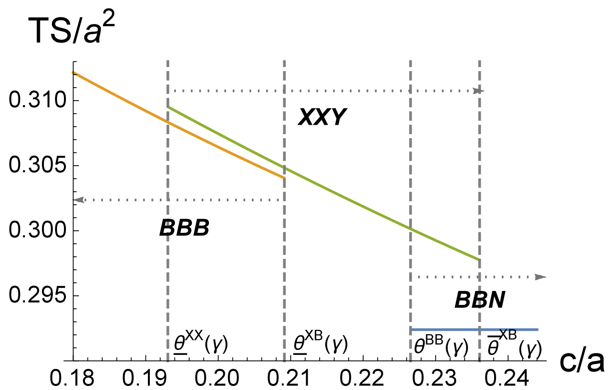


Figure 7: The total surplus when $\gamma = 0.9$

This proposition is counterintuitive at first glance because now less varieties in the resale market may even benefit the whole society (see Figure 7). Recall that the social loss only comes from the online store's sale. We first see the XXY outcome. Assume γ is almost zero so that different varieties are almost heterogeneous. Because the online store almost independently monopolizes the market of Y, the social loss is very large. As γ grows larger, the markets of X and Y become more related. Because the competition between the retailers and the online store intensifies, the online retail cost shifts more business from the online store to the retailers, which decreases the social loss. However, in the BBB outcome, because the retailers and the online store always compete in the same variety, the impact of a growing γ on the online store's share is quite limited. When γ is large enough, the online store's share is less in the XXY outcome than that in the BBB outcome, implying less social loss in the former case. When the variety outcome changes from BBB to XXY, although the decreasing number of varieties in the resale market harms the consumer's gross utility, $U(Q_X, Q_Y)$, the total surplus may still get improved by saving social cost. Notice that in the duopoly retailer case, the shutdown of the online store (when $c/a = \overline{\theta}^{XB}(\gamma)$) does not improve the total surplus as in the monopoly retailer case (Proposition 4). This is because the partial encroachment causes less social loss here than in the monopoly retailer case. Shutting down the online store (removing the partial encroachment) is anti-competitive, which cannot be compensated by the benefit from fully removing the social loss.

4 Concluding Remarks

Our paper discusses a market in which a retailer decides its product line from a manufacturer who may encroach upon the resale market through an online store. We show an important fact that the manufacturer uses the online channel as a tool to keep variety distribution balanced. Specifically, the manufacturer tends to use an online store to sell the variety which is less supplied by the (incumbent) retailers, but not to start the online channel if the variety distribution in the wholesale channel is already balanced. As discussed in the monopoly retailer model, if the online retail cost is moderately low, a retailer chooses a shorter product line instead of a full-length one so as to induce the manufacturer's incentive of selling less supplied varieties through the online store. The retailer benefits from doing so by creating itself a more mitigant market condition.

For analytical simplicity, we consider the case in which there are two varieties and at most two retailers. It is mathematically difficult to consider a more general case because we already have 27 types of subgames to consider under the two variety and two retailer case. The situation becomes more complicated if we generalize either or both of the two parameters to n. An attempt with the continuous product variety can be a possible way to solve this difficulty. However, the results already have strong enough implications to the more general case. Besides the assumption of horizontally differentiated products, it is natural to extend this setting to the one with vertically differentiated products. If consumers show their heterogeneity in evaluating product quality, and the (incumbent) retailer(s) can decide products' quality(ies) from the manufacturer who also decides the quality of products sold online, what would be the product quality outcomes, and how such outcomes affect social welfare.

Moreover, as remarked in the previous part, the result that the online store never sells one variety when receiving the retailers' orders for both varieties follows from the assumption of the symmetric retail cost. This is just for analytical simplicity. Our results still holds if the retail cost gap between two varieties is small enough. How large the retail cost gap can be to make our main results hold can be another interesting issue.

5 Appendix

5.1 Equilibrium Quantities and Profits in Monopoly Retailer Case

We first consider the equilibrium outcomes in period 3. By solving the optimization system of Eq. (1) and (3), the equilibrium quantities denoted by wholesale prices are as follows:

(i) when $rm = XN$:

$$q_{XR}^{XN}(w_X^{XN}) = \frac{a - w_X^{XN}}{2};$$

(ii) when $rm = XX$:

$$q_{XR}^{XX}(w_X^{XX}) = \frac{a + c - 2w_X^{XX}}{3}, \; q_{XM}^{XX}(w_X^{XX}) = \frac{a - 2c + w_X^{XX}}{3};$$

(iii) when $rm = XY$:

$$q_{XR}^{XY}(w_X^{XY}) = \frac{(2-\gamma)a + \gamma c - 2w_X^{XY}}{4-\gamma^2}, \; q_{YM}^{XY}(w_X^{XY}) = \frac{(2-\gamma)a - 2c + \gamma w_X^{XY}}{4-\gamma^2};$$

(iv) when $rm = XB$:

$$q_{XR}^{XB}(w_X^{XB}) = \frac{a + c - 2w_X^{XB}}{3}, \; q_{XM}^{XB}(w_X^{XB}) = \frac{(2-\gamma)a - (4+\gamma)c - (2+2\gamma)\gamma w_X^{XB}}{6(1+\gamma)}, \; q_{YM}^{XB}(w_X^{XB}) = \frac{a-c}{2(1+\gamma)};$$

(v) when $rm = BN$:

$$q_{XR}^{BN}(w_X^{BN}, w_Y^{BN}) = \frac{(1-\gamma)a - w_X^{BN} + \gamma w_Y^{BN}}{2(1-\gamma^2)}, \; q_{YR}^{BN}(w_X^{BN}, w_Y^{BN}) = \frac{(1-\gamma)a + \gamma w_X^{BN} - w_Y^{BN}}{2(1-\gamma^2)};$$

(vi) when $rm = BY$:

$$q_{XR}^{BY}(w_X^{BY}, w_Y^{BY}) = \frac{(2-3\gamma+\gamma^2)a + (2-2\gamma^2)c - (4-\gamma^2)w_X^{BY} + 3\gamma w_Y^{BY}}{6(1-\gamma^2)},$$

$$q_{YR}^{BY}(w_X^{BY}, w_Y^{BY}) = \frac{(1-\gamma)a + \gamma w_X^{BY} - w_Y^{BY}}{2(1-\gamma^2)},\ q_{XM}^{BY}(w_X^{BY}, w_Y^{BY}) = \frac{a - 2c + w_X^{BY}}{3};$$

(vii) when $rm = BB$:

$$q_{XR}^{BB}(w_X^{BB}, w_Y^{BB}) = \frac{(1-\gamma)a + (1-\gamma)c - 2w_X^{BB} + 2\gamma w_Y^{BB}}{3-3\gamma^2},$$

$$q_{YR}^{BB}(w_X^{BB}, w_Y^{BB}) = \frac{(1-\gamma)a + (1-\gamma)c + 2\gamma w_X^{BB} - 2w_Y^{BB}}{3-3\gamma^2},$$

$$q_{XM}^{BB}(w_X^{BB}, w_Y^{BB}) = \frac{(1-\gamma)a - (2-2\gamma)c + w_X^{BB} - \gamma w_Y^{BB}}{3-3\gamma^2},$$

$$q_{YM}^{BB}(w_X^{BB}, w_Y^{BB}) = \frac{(1-\gamma)a - (2-2\gamma)c - \gamma w_X^{BB} + w_Y^{BB}}{3-3\gamma^2}.$$

Substituting the equilibrium outcomes in period 3 back to Eq. (4) or (5) and solving the manufacturer's optimization problem, we derive the equilibrium wholesale prices in period 2 as in Table 1. Substituting the equilibrium wholesale prices, we derive the retailer and manufacturer's equilibrium quantities and profits in period 2 as follows:

(i) when $rm = XN$:

$$q_{XR}^{XN} = \frac{a}{4},\ \pi_R^{XN} = \frac{a^2}{16},\ \pi_M^{XN} = \frac{a^2}{8};$$

(ii) when $rm = XX$:

$$q_{XR}^{XX} = \frac{5a-7c}{10},\ q_{XM}^{XX} = \frac{5a+3c}{10},\ \pi_R^{XX} = \frac{4c^2}{25},\ \pi_M^{XX} = \frac{5a^2 - 10ac + 9c^2}{20};$$

(iii) when $rm = XY$:

$$q_{XR}^{XY} = \frac{2(1-\gamma)a + 2\gamma c}{8-3\gamma^2},\ q_{YM}^{XY} = \frac{(8-2\gamma-\gamma^2)a - (8-\gamma^2)c}{2(8-3\gamma^2)},$$

$$\pi_R^{XY} = \frac{4[(1-\gamma)a + \gamma c]^2}{(8-3\gamma^2)^2},\ \pi_M^{XY} = \frac{(12-8\gamma+\gamma^2)a^2 - (16-8\gamma+2\gamma^2)ac + (8+\gamma^2)c^2}{4(8-3\gamma^2)};$$

(iv) when $rm = XB$:

$$q_{XR}^{XB} = \frac{2c}{5},\ q_{XM}^{XB} = \frac{5a-(7+2\gamma)c}{10(1+\gamma)},\ q_{YM}^{XB} = \frac{a-c}{2(1+\gamma)},$$

$$\pi_R^{XB} = \frac{4c^2}{25},\ \pi_M^{XB} = \frac{5a^2 - 10ac + (7+2\gamma)c^2}{10(1+\gamma^2)};$$

(v) when $rm = BN$:

$$q_{XR}^{BN} = q_{YR}^{BN} = \frac{a}{4(1+\gamma)},\ \pi_R^{BN} = \frac{a^2}{8(1+\gamma)},\ \pi_M^{BN} = \frac{a^2}{4(1+\gamma)};$$

(vi) when $rm = BY$:

$$q_{XR}^{BY} = \frac{-5\gamma a + 8(1+\gamma^2)c}{20(1+\gamma^2)},\ q_{YR}^{BY} = \frac{a}{4(1+\gamma)},\ q_{XM}^{BY} = \frac{5a-7c}{10},$$

$$\pi_R^{BY} = \frac{25(1-\gamma)a^2 + 64(1+\gamma)c^2}{400(1+\gamma)},\ \pi_M^{BY} = \frac{5(3+\gamma)a^2 - 20(1+\gamma)ac + 18(1+\gamma)c^2}{40(1+\gamma)};$$

(vii) when $rm = BB$:

$$q_{XR}^{BB} = q_{YR}^{BB} = \frac{2c}{5(1+\gamma)},\ q_{XM}^{BB} = q_{YM}^{BB} = \frac{5a-7c}{10(1+\gamma)},\ \pi_R^{BB} = \frac{8c^2}{25(1+\gamma)},\ \pi_M^{BB} = \frac{5a^2 - 10ac + 9c^2}{10(1+\gamma)}.$$

Making all quantities positive, we obtain the $I.S.$ in Table 1.

5.2 Proof of Lemma 1

In period 2, we solve for the manufacturer's equilibrium outcome, taking the corner solutions into consideration and checking the incentive for deviation. Simply comparing the threshold values in Table 1 shows that $\frac{5}{7+2\gamma} < \frac{5}{7} < \frac{8-2\gamma-\gamma^2}{8-\gamma^2}$. We show that the online store never encroaches with variety X in this case. If $\frac{5}{7+2\gamma} < \frac{c}{a} \leq \frac{5}{7}$, we find that $\pi_M^{XX} < \pi_M^{XY}$. If $\frac{c}{a} \leq \frac{5}{7+2\gamma}$, we find that $\pi_M^{XX} < max\{\pi_M^{XY}, \pi_M^{XB}\}$. Thus, we have proved that encroaching with variety X is a *strictly dominated strategy* in this case. The manufacturer chooses B if $\frac{c}{a} \leq \frac{5}{7+2\gamma}$ and $\pi_M^{XB} \geq max\{\pi_M^{XN}, \pi_M^{XX}, \pi_M^{XY}\}$, from which we obtain

$$\frac{c}{a} \leq \underline{\theta}^X(\gamma) \equiv \frac{5(8+4\gamma+\gamma^2) - 2\sqrt{40+80\gamma+25\gamma^2-30\gamma^3-15\gamma^4}}{72+64\gamma+17\gamma^2}.$$

The outcome that the manufacturer chooses Y must satisfy the following conditions: if $\frac{5}{7} < \frac{c}{a} \leq \frac{8-2\gamma-\gamma^2}{8-\gamma^2}$ and $\pi_M^{XY} \geq \pi_M^{XN}$, or if $\frac{5}{7+2\gamma} < \frac{c}{a} \leq \frac{5}{7}$ and $\pi_M^{XY} \geq max\{\pi_M^{XN}, \pi_M^{XX}\}$, or if $\frac{c}{a} \leq \frac{5}{7+2\gamma}$ and $\pi_M^{XY} \geq max\{\pi_M^{XN}, \pi_M^{XX}, \pi_M^{XB}\}$, from which we obtain

$$\underline{\theta}^X(\gamma) \leq \frac{c}{a} \leq \overline{\theta}^X(\gamma) \equiv \frac{2(8-4\gamma+\gamma^2) - \gamma\sqrt{16-6\gamma^2}}{2(8+\gamma^2)}.$$

The manufacturer does not open its online store if

$$\frac{c}{a} \geq \overline{\theta}^X(\gamma).$$

We next see the case in which the retailer orders both varieties in period 1. Notice that for $\frac{c}{a} \leq \frac{5\gamma}{8(1+\gamma)}$, q_{XR}^{BY} become zero so that the outcome is the same with that in the YX case. Simply comparing the threshold values in Table 1 shows that $\frac{5\gamma}{8(1+\gamma)} < \frac{5}{7}$. We show that the online store never encroaches with either variety except for a single point. If $\frac{5\gamma}{8(1+\gamma)} < \frac{c}{a} \leq \frac{5}{7}$, we find that $\pi_M^{BY} \geq max\{\pi_M^{BN}, \pi_M^{BB}\}$ for $c/a = \frac{10-\sqrt{10}}{18}$. If $\frac{c}{a} \leq \frac{5\gamma}{8(1+\gamma)}$, we find that $\pi_M^{YX} < max\{\pi_M^{BN}, \pi_M^{BB}\}$. Thus, we have proved that encroaching with either variety is a *weakly dominated strategy* in this case. The manufacturer chooses B if $\frac{5\gamma}{8(1+\gamma)} < \frac{c}{a} < \frac{5}{7}$ and $\pi_M^{BB} \geq max\{\pi_M^{BN}, \pi_M^{BY}\}$, or if $\frac{c}{a} \leq \frac{5\gamma}{8(1+\gamma)}$ and $\pi_M^{BB} \geq max\{\pi_M^{BN}, \pi_M^{YX}\}$, from which we obtain

$$\frac{c}{a} \leq \theta^B(\gamma) \equiv \frac{10 - \sqrt{10}}{18}.$$

The manufacturer does not open its online store if

$$\frac{c}{a} \geq \theta^B(\gamma).$$

□

5.3 Equilibrium Quantities and Profits in Duopoly Retailers Case

Define $q_{nR_i}^{r_1 r_2 m}$ and $q_{nM}^{r_1 r_2 m}$ to be the retailer i and the online store's selling quantity of variety n. The equilibrium outcomes in period 3 are as follows:

(i) when $r_1 r_2 m = XXN$:

$$q_{XR1}^{XXN}(w_{X1}^{XXN}, w_{X2}^{XXN}) = \frac{a - 2w_{X1}^{XXN} + w_{X2}^{XXN}}{3}, \; q_{XR2}^{XXN}(w_{X1}^{XXN}, w_{X2}^{XXN}) = \frac{a + w_{X1}^{XXN} - 2w_{X2}^{XXN}}{3};$$

(ii) when $r_1 r_2 m = XXX$:

$$q_{XR1}^{XXX}(w_{X1}^{XXX}, w_{X2}^{XXX}) = \frac{a + c - 3w_{X1}^{XXX} + w_{X2}^{XXX}}{4}, \; q_{XR2}^{XXX}(w_{X1}^{XXX}, w_{X2}^{XXX}) = \frac{a + c + w_{X1}^{XXX} - 3w_{X2}^{XXX}}{4},$$

$$q_{XM}^{XXX}(w_{X1}^{XXX}, w_{X2}^{XXX}) = \frac{a - 3c + w_{X1}^{XXX} + w_{X2}^{XXX}}{4};$$

(iii) when $r_1 r_2 m = XXY$:

$$q_{XR1}^{XXY}(w_{X1}^{XXY}, w_{X2}^{XXY}) = \frac{(2-\gamma)a + \gamma c - (4-\gamma^2)w_{X1}^{XXY} + (2-\gamma^2)w_{X2}^{XXY}}{2(3-\gamma^2)},$$

$$q_{XR1}^{XXY}(w_{X1}^{XXY}, w_{X2}^{XXY}) = \frac{(2-\gamma)a + \gamma c + (2-\gamma^2)w_{X1}^{XXY} - (4-\gamma^2)w_{X2}^{XXY}}{2(3-\gamma^2)},$$

$$q_{YM}^{XXY}(w_{X1}^{XXY}, w_{X2}^{XXY}) = \frac{(3-2\gamma)a - 3c + \gamma w_{X1}^{XXY} + \gamma w_{X2}^{XXY}}{2(3-\gamma^2)};$$

(iv) when $r_1 r_2 m = XXB$:

$$q_{XR1}^{XXB}(w_{X1}^{XXB}, w_{X2}^{XXB}) = \frac{a + c - 3w_{X1}^{XXB} + w_{X2}^{XXB}}{4}, \ q_{XR2}^{XXB}(w_{X1}^{XXB}, w_{X2}^{XXB}) = \frac{a + c + w_{X1}^{XXB} - 3w_{X2}^{XXB}}{4},$$

$$q_{XM}^{XXB}(w_{X1}^{XXB}, w_{X2}^{XXB}) = \frac{(1-\gamma)a - (3+\gamma)c + (1+\gamma)w_{X1}^{XXB} + (1+\gamma)w_{X2}^{XXB}}{4(1+\gamma)},$$

$$q_{YM}^{XXB}(w_{X1}^{XXB}, w_{X2}^{XXB}) = \frac{a-c}{2(1+\gamma)};$$

(v) when $r_1 r_2 m = XYN$:

$$q_{XR1}^{XYN}(w_{X1}^{XYN}, w_{Y2}^{XYN}) = \frac{(2-\gamma)a - 2w_{X1}^{XYN} + \gamma w_{Y2}^{XYN}}{4-\gamma^2},$$

$$q_{YR2}^{XYN}(w_{X1}^{XYN}, w_{Y2}^{XYN}) = \frac{(2-\gamma)a + \gamma w_{X1}^{XYN} - 2\gamma w_{Y2}^{XYN}}{4-\gamma^2};$$

(vi) when $r_1 r_2 m = XYX$:

$$q_{XR1}^{XYX}(w_{X1}^{XYX}, w_{Y2}^{XYX}) = \frac{(2-\gamma)a + (2-\gamma^2)c - (4-\gamma^2)w_{X1}^{XYX} + \gamma w_{Y2}^{XYX}}{2(3-\gamma^2)},$$

$$q_{YR2}^{XYX}(w_{X1}^{XYX}, w_{Y2}^{XYX}) = \frac{(3-2\gamma)a + \gamma c + \gamma w_{X1}^{XYX} - 3w_{Y2}^{XYX}}{2(3-\gamma^2)},$$

$$q_{XM}^{XYX}(w_{X1}^{XYX}, w_{Y2}^{XYX}) = \frac{(2-\gamma)a - (4-\gamma^2)c + (2-\gamma^2)w_{X1}^{XYX} + \gamma w_{Y2}^{XYX}}{2(3-\gamma^2)};$$

(vii) when $r_1 r_2 m = XYB$:

$$q_{XR1}^{XYB}(w_{X1}^{XYB}, w_{Y2}^{XYB}) = \frac{(3-\gamma)a + (3-\gamma)c - 6w_{X1}^{XYB} + 2\gamma w_{Y2}^{XYB}}{9-\gamma^2},$$

$$q_{YR2}^{XYB}(w_{X1}^{XYB}, w_{Y2}^{XYB}) = \frac{(3-\gamma)a + (3-\gamma)c + 2\gamma w_{X1}^{XYB} - 6w_{Y2}^{XYB}}{9-\gamma^2},$$

$$q_{XM}^{XYB}(w_{X1}^{XYB}, w_{Y2}^{XYB}) = \frac{(3-\gamma)a + (6+\gamma-\gamma^2)c + 3(1+\gamma)w_{X1}^{XYB} - (\gamma+\gamma^2)w_{Y2}^{XYB}}{9+9\gamma-\gamma^2-\gamma^3},$$

$$q_{YM}^{XYB}(w_{X1}^{XYB}, w_{Y2}^{XYB}) = \frac{(3-\gamma)a + (6+\gamma-\gamma^2)c - (\gamma+\gamma^2)w_{X1}^{XYB} + 3(1+\gamma)w_{Y2}^{XYB}}{9+9\gamma-\gamma^2-\gamma^3};$$

(viii) when $r_1 r_2 m = XBN$:

$$q_{XR1}^{XBN}(w_{X1}^{XBN}, w_{X2}^{XBN}, w_{Y2}^{XBN}) = \frac{a - 2w_{X1}^{XBN} + w_{X2}^{XBN}}{3},$$

$$q_{XR2}^{XBN}(w_{X1}^{XBN}, w_{X2}^{XBN}, w_{Y2}^{XBN}) = \frac{(2-3\gamma+\gamma^2)a + 2(1-\gamma^2)w_{X1}^{XBN} - (4-\gamma^2)w_{X2}^{XBN} + 3\gamma w_{Y2}^{XBN}}{6(1-\gamma^2)},$$

$$q_{XR2}^{XBN}(w_{X1}^{XBN}, w_{X2}^{XBN}, w_{Y2}^{XBN}) = \frac{(1-\gamma)a + \gamma w_{X2}^{XBN} - w_{Y2}^{XBN}}{2(1-\gamma^2)};$$

(ix) when $r_1 r_2 m = XBY$:

$$q_{XR1}^{XBY}(w_{X1}^{XBY}, w_{X2}^{XBY}, w_{Y2}^{XBY}) = \frac{a + c - 3w_{X1}^{XBY} + w_{X2}^{XBY}}{4},$$

$$q_{XR2}^{XBY}(w_{X1}^{XBY}, w_{X2}^{XBY}, w_{Y2}^{XBY}) = \frac{(2-2\gamma+\gamma^2)a + (1-\gamma^2)c + (1-\gamma^2)w_{X1}^{XBY} - (3-\gamma^2)w_{X2}^{XBY} + 2\gamma w_{Y2}^{XBY}}{4(1-\gamma^2)},$$

$$q_{YR2}^{XBY}(w_{X1}^{XBY}, w_{X2}^{XBY}, w_{Y2}^{XBY}) = \frac{(1-\gamma)a + \gamma w_{X2}^{XBY} - w_{Y2}^{XBY}}{2(1-\gamma^2)},$$

$$q_{XM}^{XBY}(w_{X1}^{XBY}, w_{X2}^{XBY}, w_{Y2}^{XBY}) = \frac{a - 3c + w_{X2}^{XBY} + w_{Y2}^{XBY}}{4};$$

(x) when $r_1 r_2 m = XBY$:

$$q_{XR1}^{XBY}(w_{X1}^{XBY}, w_{X2}^{XBY}, w_{Y2}^{XBY}) = \frac{(3-\gamma)a + 2\gamma c - 6w_{X1}^{XBY} + 3w_{X2}^{XBY} - \gamma w_{Y2}^{XBY}}{9-\gamma^2},$$

$$q_{XR2}^{XBY}(w_{X1}^{XBY}, w_{X2}^{XBY}, w_{Y2}^{XBY}) = \frac{(3-4\gamma+\gamma^2)a - (1-\gamma^2)c + 3(1-\gamma^2)w_{X1}^{XBY} - 2(3-\gamma^2)w_{X2}^{XBY} + (5\gamma-\gamma^3)w_{Y2}^{XBY}}{9-10\gamma^2+\gamma^4},$$

$$q_{YR2}^{XBY}(w_{X1}^{XBY}, w_{X2}^{XBY}, w_{Y2}^{XBY}) = \frac{(3-4\gamma+\gamma^2)a + 3(1-\gamma^2)c - (\gamma-\gamma^3)w_{X1}^{XBY} + (5\gamma-\gamma^3)w_{X2}^{XBY} - 2(3-\gamma^2)w_{Y2}^{XBY}}{9-10\gamma^2+\gamma^4},$$

$$q_{XM}^{XBY}(w_{X1}^{XBY}, w_{X2}^{XBY}, w_{Y2}^{XBY}) = \frac{a - 3c + w_{X2}^{XBY} + w_{Y2}^{XBY}}{4};$$

(xi) when $r_1 r_2 m = XBB$:

$$q_{XR1}^{XBB}(w_{X1}^{XBB}, w_{X2}^{XBB}, w_{Y2}^{XBB}) = \frac{a + c - 3w_{X1}^{XBB} + w_{X2}^{XBB}}{4},$$

$$q_{XR2}^{XBB}(w_{X1}^{XBB}, w_{X2}^{XBB}, w_{Y2}^{XBB}) = \frac{(3-4\gamma+\gamma^2)a + (3-4\gamma+\gamma^2)c + 3(1-\gamma^2)w_{X1}^{XBB} - (9-\gamma^2)w_{X2}^{XBB} + 8\gamma w_{Y2}^{XBB}}{12(1-\gamma^2)},$$

$$q_{YR2}^{XBB}(w_{X1}^{XBB}, w_{X2}^{XBB}, w_{Y2}^{XBB}) = \frac{(1-\gamma)a + (1-\gamma)c + 2\gamma w_{X2}^{XBB} - 2w_{Y2}^{XBB}}{3(1-\gamma^2)},$$

$$q_{XM}^{XBB}(w_{X1}^{XBB}, w_{X2}^{XBB}, w_{Y2}^{XBB}) = \frac{(3-4\gamma+\gamma^2)a - (9-8\gamma-\gamma^2)c + 3(1-\gamma^2)w_{X1}^{XBB} + (3+\gamma^2)w_{X2}^{XBB} - 4\gamma w_{Y2}^{XBB}}{12(1-\gamma^2)},$$

$$q_{YM}^{XBB}(w_{X1}^{XBB}, w_{X2}^{XBB}, w_{Y2}^{XBB}) = \frac{(1-\gamma)a - 2(1-\gamma)c - \gamma w_{X2}^{XBB} + w_{Y2}^{XBB}}{3(1-\gamma^2)};$$

(xii) when $r_1 r_2 m = BBN$:

$$q_{XR1}^{BBN}(w_{X1}^{BBN}, w_{Y1}^{BBN}, w_{X2}^{BBN}, w_{Y2}^{BBN}) = \frac{(1-\gamma)a - 2w_{X1}^{BBN} + w_{X2}^{BBN} + 2\gamma w_{Y1}^{BBN} - \gamma w_{Y2}^{BBN}}{3(1-\gamma^2)},$$

$$q_{YR1}^{BBN}(w_{X1}^{BBN}, w_{Y1}^{BBN}, w_{X2}^{BBN}, w_{Y2}^{BBN}) = \frac{(1-\gamma)a + 2\gamma w_{X1}^{BBN} - \gamma w_{X2}^{BBN} - 2w_{Y1}^{BBN} + w_{Y2}^{BBN}}{3(1-\gamma^2)},$$

$$q_{XR2}^{BBN}(w_{X1}^{BBN}, w_{Y1}^{BBN}, w_{X2}^{BBN}, w_{Y2}^{BBN}) = \frac{(1-\gamma)a + w_{X1}^{BBN} - 2w_{X2}^{BBN} - \gamma w_{Y1}^{BBN} + 2\gamma w_{Y2}^{BBN}}{3(1-\gamma^2)},$$

$$q_{YR2}^{BBN}(w_{X1}^{BBN}, w_{Y1}^{BBN}, w_{X2}^{BBN}, w_{Y2}^{BBN}) = \frac{(1-\gamma)a - \gamma w_{X1}^{BBN} + 2\gamma w_{X2}^{BBN} + w_{Y1}^{BBN} - 2w_{Y2}^{BBN}}{3(1-\gamma^2)};$$

(xiii) when $r_1 r_2 m = BBY$:

$$q_{XR1}^{BBY}(w_{X1}^{BBY}, w_{Y1}^{BBY}, w_{X2}^{BBY}, w_{Y2}^{BBY}) = \frac{(3-4\gamma+\gamma^2)a + 3(1-\gamma^2)c}{12(1-\gamma^2)}$$

$$-\frac{(9-\gamma^2)w_{X1}^{BBY} - (3+\gamma^2)w_{X2}^{BBY} - 8\gamma w_{Y1}^{BBY} + 4\gamma w_{Y2}^{BBY}}{12(1-\gamma^2)},$$

$$q_{YR1}^{BBY}(w_{X1}^{BBY}, w_{Y1}^{BBY}, w_{X2}^{BBY}, w_{Y2}^{BBY}) = \frac{(1-\gamma)a + 2\gamma w_{X1}^{BBY} - \gamma w_{X2}^{BBY} - 2w_{Y1}^{BBY} + w_{Y2}^{BBY}}{3(1-\gamma^2)},$$

$$q_{XR2}^{BBY}(w_{X1}^{BBY}, w_{Y1}^{BBY}, w_{X2}^{BBY}, w_{Y2}^{BBY}) = \frac{(3-4\gamma+\gamma^2)a + 3(1-\gamma^2)c}{12(1-\gamma^2)}$$

$$+\frac{(3+\gamma^2)w_{X1}^{BBY} - (9-\gamma^2)w_{X2}^{BBY} - 4\gamma w_{Y1}^{BBY} + 8\gamma w_{Y2}^{BBY}}{12(1-\gamma^2)},$$

$$q_{YR2}^{BBY}(w_{X1}^{BBY}, w_{Y1}^{BBY}, w_{X2}^{BBY}, w_{Y2}^{BBY}) = \frac{(1-\gamma)a - \gamma w_{X1}^{BBY} + 2\gamma w_{X2}^{BBY} + w_{Y1}^{BBY} - 2w_{Y2}^{BBY}}{3(1-\gamma^2)},$$

$$q_{XM}^{BBY}(w_{X1}^{BBY}, w_{Y1}^{BBY}, w_{X2}^{BBY}, w_{Y2}^{BBY}) = \frac{a - 3c + w_{X1}^{BBY} + w_{X2}^{BBY}}{4};$$

(xiv) when $r_1 r_2 m = BBB$:

$$q_{XR1}^{BBB}(w_{X1}^{BBB}, w_{Y1}^{BBB}, w_{X2}^{BBB}, w_{Y2}^{BBB}) = \frac{(1-\gamma)a + (1-\gamma)c - 3w_{X1}^{BBB} + w_{X2}^{BBB} + 3\gamma w_{Y1}^{BBB} - \gamma w_{Y2}^{BBB}}{4(1-\gamma^2)},$$

$$q_{YR1}^{BBB}(w_{X1}^{BBB}, w_{Y1}^{BBB}, w_{X2}^{BBB}, w_{Y2}^{BBB}) = \frac{(1-\gamma)a + (1-\gamma)c + 3\gamma w_{X1}^{BBB} - \gamma w_{X2}^{BBB} - 3w_{Y1}^{BBB} + w_{Y2}^{BBB}}{4(1-\gamma^2)},$$

$$q_{XR2}^{BBB}(w_{X1}^{BBB}, w_{Y1}^{BBB}, w_{X2}^{BBB}, w_{Y2}^{BBB}) = \frac{(1-\gamma)a + (1-\gamma)c + w_{X1}^{BBB} - 3w_{X2}^{BBB} - \gamma w_{Y1}^{BBB} + 3\gamma w_{Y2}^{BBB}}{4(1-\gamma^2)},$$

$$q_{YR2}^{BBB}(w_{X1}^{BBB}, w_{Y1}^{BBB}, w_{X2}^{BBB}, w_{Y2}^{BBB}) = \frac{(1-\gamma)a + (1-\gamma)c - \gamma w_{X1}^{BBB} + 3\gamma w_{X2}^{BBB} + w_{Y1}^{BBB} - 3w_{Y2}^{BBB}}{4(1-\gamma^2)},$$

$$q_{XM}^{BBB}(w_{X1}^{BBB}, w_{Y1}^{BBB}, w_{X2}^{BBB}, w_{Y2}^{BBB}) = \frac{(1-\gamma)a - 3(1-\gamma)c + w_{X1}^{BBB} + w_{X2}^{BBB} - \gamma w_{Y1}^{BBB} - \gamma w_{Y2}^{BBB}}{4(1-\gamma^2)},$$

$$q_{YM}^{BBB}(w_{X1}^{BBB}, w_{Y1}^{BBB}, w_{X2}^{BBB}, w_{Y2}^{BBB}) = \frac{(1-\gamma)a - 3(1-\gamma)c - \gamma w_{X1}^{BBB} - \gamma w_{X2}^{BBB} + w_{Y1}^{BBB} + w_{Y2}^{BBB}}{4(1-\gamma^2)}.$$

By substituting the equilibrium outcomes in period 3 and solving the manufacturer's optimization problem, we derive the equilibrium wholesale prices in period 2 as in Table 2. Substituting the equilibrium wholesale prices, we derive the retailer and manufacturer's equilibrium quantities and profits in period 2 as follows:

(i) when $r_1 r_2 m = XXN$:

$$q_{XR1}^{XXN} = q_{XR2}^{XXN} = \frac{a}{6},\ \pi_{R1}^{XXN} = \pi_{R2}^{XXN} = \frac{a^2}{36},\ \pi_M^{XXN} = \frac{a^2}{6};$$

(ii) when $r_1 r_2 m = XXX$:

$$q_{XR1}^{XXX} = q_{XR2}^{XXX} = \frac{c}{3},\ q_{XM}^{XXX} = \frac{3a-c}{6},\ \pi_{R1}^{XXX} = \pi_{R2}^{XXX} = \frac{c^2}{9},\ \pi_M^{XXX} = \frac{3a^2 - 6ac + 7c^2}{12};$$

(iii) when $r_1 r_2 m = XXY$:

$$q_{XR1}^{XXY} = q_{XR2}^{XXY} = \frac{(1-\gamma)a + \gamma c}{3(2-\gamma^2)},\ q_{YM}^{XXY} = \frac{(6-2\gamma-\gamma^2)a - (6-\gamma^2)c}{6(2-\gamma^2)},$$

$$\pi_{R1}^{XXY} = \pi_{R2}^{XXY} = \frac{[(1-\gamma)a + \gamma c]^2}{9(2-\gamma^2)^2},\ \pi_M^{XXY} = \frac{(10-8\gamma+\gamma^2)a^2 - (12-8\gamma+2\gamma^2)ac + (6+\gamma^2)c^2}{12(2-\gamma^2)};$$

(iv) when $r_1 r_2 m = XXB$:

$$q_{XR1}^{XXB} = q_{XR2}^{XXB} = \frac{c}{3},\ q_{XM}^{XXB} = \frac{3a - (5+2\gamma)c}{6(1+\gamma)},\ q_{YM}^{XXB} = \frac{a-c}{2(1+\gamma)},$$

$$\pi_{R1}^{XXB} = \pi_{R2}^{XXB} = \frac{c^2}{9},\ \pi_M^{XXB} = \frac{3a^2 - 6ac + (5+2\gamma)c^2}{6(1+\gamma)};$$

(v) when $r_1 r_2 m = XYN$:

$$q_{XR1}^{XYN} = q_{YR2}^{XYN} = \frac{a}{2(2+\gamma)},\ \pi_{R1}^{XYN} = \pi_{R2}^{XYN} = \frac{a^2}{4(2+\gamma)^2},\ \pi_M^{XYN} = \frac{a^2}{2(2+\gamma)};$$

(vi) when $r_1 r_2 m = XYX$:

$$q_{XR1}^{XYX} = \frac{4(2-\gamma^2)c - (\gamma-\gamma^2)a}{4(5-2\gamma^2)},\ q_{YR2}^{XYX} = \frac{5(1-\gamma)a + 4\gamma c}{4(5-2\gamma^2)},\ q_{XM}^{XYX} = \frac{(5-\gamma-\gamma^2)a - (7-2\gamma^2)c}{2(5-2\gamma^2)},$$

$$\pi_{R1}^{XYX} = \frac{[(\gamma-\gamma^2)a - (8-4\gamma^2)c]^2}{16(5-2\gamma^2)^2},\ \pi_{R2}^{XYX} = \frac{[5(1-\gamma)a + 4\gamma c]^2}{16(5-2\gamma^2)^2},$$

$$\pi_M^{XYX} = \frac{(15-10\gamma+\gamma^2)a - (20-8\gamma)ac + (18-4\gamma^2)c^2}{8(5-2\gamma^2)};$$

(vii) when $r_1 r_2 m = XYB$:

$$q_{XR1}^{XYB} = q_{YR2}^{XYB} = \frac{2c}{5+\gamma},\ q_{XM}^{XYB} = q_{YM}^{XYB} = \frac{(5+\gamma)a - (7+3\gamma)c}{2(5+6\gamma+\gamma^2)},$$

$$\pi_{R1}^{XYB} = \pi_{R2}^{XYB} = \frac{4c^2}{(5+\gamma)^2},\ \pi_M^{XYB} = \frac{(5+\gamma)a^2 - 2(5+\gamma)ac + (9+5\gamma)c^2}{2(5+6\gamma+\gamma^2)};$$

(viii) when $r_1 r_2 m = XBN$:

$$q_{XR1}^{XBN} = \frac{a}{6},\ q_{XR2}^{XBN} = \frac{(2-\gamma)a}{12(1+\gamma)},\ q_{YR2}^{XBN} = \frac{a}{4(1+\gamma)},$$

$$\pi_{R1}^{XBN} = \frac{a^2}{36},\ \pi_{R2}^{XBN} = \frac{(13-5\gamma)a^2}{144(1+\gamma)},\ \pi_M^{XBN} = \frac{(7+\gamma)a^2}{24(1+\gamma)};$$

(ix) when $r_1 r_2 m = XBY$:

$$q_{XR1}^{XBY} = \frac{c}{3},\ q_{XR2}^{XBY} = \frac{4(1+\gamma)c - 3\gamma a}{12(1+\gamma)},\ q_{YR2}^{XBY} = \frac{a}{4(1+\gamma)},\ q_{XM}^{XBY} = \frac{3a - \gamma c}{6},$$

$$\pi_{R1}^{XBY} = \frac{c^2}{9},\ \pi_{R2}^{XBY} = \frac{9(1-\gamma)a^2 + 16(1+\gamma)c^2}{144(1+\gamma)},\ \pi_M^{XBY} = \frac{3(3+\gamma)a^2 - 12(1+\gamma)ac + 14(1+\gamma)c^2}{24(1+\gamma)};$$

(x) when $r_1 r_2 m = XBY$:

$$q_{XR1}^{XBY} = \frac{5(1-\gamma)a + 8\gamma c}{6(5-\gamma^2)},\ q_{XR2}^{XBY} = \frac{(5+\gamma^2)a - 4(\gamma+\gamma^2)c}{6(5+5\gamma-\gamma^2-\gamma^3)}\ q_{YR2}^{XBY} = \frac{(15-2\gamma-\gamma^2)a - (21-\gamma^2)c}{6(5-\gamma^2)},$$

$$q_{XM}^{XBY} = \frac{a - 3c + w_{X2}^{XBY} + w_{Y2}^{XBY}}{4},$$

$$\pi_{R1}^{XBY} = \frac{[5(1-\gamma)a + 8\gamma c]^2}{36(5-\gamma^2)^2},\ \pi_{R2}^{XBY} = \frac{(25-25\gamma+11\gamma^2-11\gamma^3)a^2 - 64(\gamma-\gamma^3)ac + 16(9+9\gamma-5\gamma^2-5\gamma^3)c^2}{36(25+25\gamma-10\gamma^2-10\gamma^3+\gamma^4+\gamma^5)},$$

$$\pi_{M}^{XBY} = \frac{(25+5\gamma-7\gamma^2)a^2 - 2(15+11\gamma-3\gamma^2+\gamma^3)ac + (27+27\gamma+\gamma^2+\gamma^3)c^2}{5+5\gamma-\gamma^2-\gamma^3};$$

(xi) when $r_1r_2m = XBB$:

$$q_{XR1}^{XBB} = \frac{c}{3},\ q_{XR2}^{XBB} = \frac{(5-\gamma)c}{15(1+\gamma)},\ q_{YR2}^{XBB} = \frac{2c}{5(1+\gamma)},\ q_{XM}^{XBB} = \frac{15a-(25+4\gamma)c}{30(1+\gamma)},\ q_{YM}^{XBB} = \frac{5a-7c}{10(1+\gamma)},$$

$$\pi_{R1}^{XBB} = \frac{c^2}{9},\ \pi_{R2}^{XBB} = \frac{(61-11\gamma)c^2}{225(1+\gamma)},\ \pi_{M}^{XBB} = \frac{15a^2-30ac+(31+4\gamma)c^2}{30(1+\gamma)};$$

(xii) when $r_1r_2m = BBN$:

$$q_{XR1}^{BBN} = q_{YR1}^{BBN} = q_{XR2}^{BBN} = q_{YR2}^{BBN} = \frac{a}{6(1+\gamma)},\ \pi_{R1}^{BBN} = \pi_{R2}^{BBN} = \frac{a^2}{18(1+\gamma)},\ \pi_{M}^{BBN} = \frac{a^2}{3(1+\gamma)};$$

(xiii) when $r_1r_2m = BBY$:

$$q_{XR1}^{BBY} = q_{XR2}^{BBY} = \frac{2(1+\gamma)c-\gamma a}{6(1+\gamma)},\ q_{YR1}^{BBY} = q_{YR2}^{BBY} = \frac{a}{6(1+\gamma)},\ q_{XM}^{BBY} = \frac{3a-5c}{6},$$

$$\pi_{R1}^{BBY} = \pi_{R2}^{BBY} = \frac{(1-\gamma)a^2+4(1+\gamma)c^2}{36(1+\gamma)},\pi_{M}^{BBY} = \frac{(5+\gamma)a^2-6(1+\gamma)ac+7(1+\gamma)c^2}{12(1+\gamma)};$$

(xiv) when $r_1r_2m = BBB$:

$$q_{XR1}^{BBB} = q_{XR2}^{BBB} = q_{YR1}^{BBB} = q_{YR2}^{BBB} = \frac{c}{3(1+\gamma)},\ q_{XM}^{BBB} = q_{YM}^{BBB} = \frac{3a-5c}{6(1+\gamma)},$$

$$\pi_{R1}^{BBB} = \pi_{R2}^{BBB} = \frac{2c^2}{9(1+\gamma)},\pi_{M}^{BBB} = \frac{3a^2-6ac+7c^2}{6(1+\gamma)}.$$

Making all quantities positive, we obtain the $I.S.$ in Table 2. We will prove that encroaching with variety X through the online store when receiving the retailer's XX or XB type order, and encroaching with either variety through the online store when receiving the retailer's XY or BB are an dominated strategies later.

5.4 Proof of Lemma 2

In period 2, we solve for the manufacturer's equilibrium outcome, taking the corner solutions into consideration and checking the incentive for deviation.

First we see the case in which the manufacturer receives the XX type order in period 1. Simply comparing the threshold values in Table 2 shows that $\frac{3}{5+2\gamma} < \frac{3}{5} < \frac{6-2\gamma-\gamma^2}{6-\gamma^2}$. We show that the online store never encroaches with variety X in this case. If $\frac{c}{a} \leq \frac{3}{5}$, we find that $\pi_M^{XXX} < \pi_M^{XXB}$. Thus, we have proved that encroaching with variety X is a *strictly dominated strategy* in this case. The manufacturer chooses B if $\frac{c}{a} \leq \frac{3}{5+2\gamma}$ and $\pi_M^{XXB} \geq max\{\pi_M^{XXN}, \pi_M^{XXX}, \pi_M^{XXY}\}$, from which we obtain

$$\frac{c}{a} \leq \underline{\theta}^{XX}(\gamma) \equiv \frac{(6+4\gamma+\gamma^2) - 2\sqrt{2+4\gamma+\gamma^2-2\gamma^3-\gamma^4}}{14+16\gamma+5\gamma^2}.$$

The outcome that the manufacturer chooses Y must satisfy the following conditions: if $\frac{3}{5} < \frac{c}{a} \leq \frac{6-2\gamma-\gamma^2}{6-\gamma^2}$ and $\pi_M^{XXY} \geq \pi_M^{XXN}$, if $\frac{3}{5+2\gamma} < \frac{c}{a} \leq \frac{3}{5}$ and $\pi_M^{XXY} \geq max\{\pi_M^{XXN}, \pi_M^{XXX}\}$, if $\frac{c}{a} \leq \frac{3}{5+2\gamma}$ and $\pi_M^{XXY} \geq max\{\pi_M^{XXN}, \pi_M^{XXX}, \pi_M^{XXB}\}$, from which we obtain

$$\underline{\theta}^{XX}(\gamma) \leq \frac{c}{a} \leq \overline{\theta}^{XX}(\gamma) \equiv \frac{(6-4\gamma+\gamma^2) - \gamma\sqrt{2(2-\gamma^2)}}{6+\gamma^2}.$$

The manufacturer does not open its online store if

$$\frac{c}{a} \geq \overline{\theta}^{XX}(\gamma).$$

Second, we see the case in which the manufacturer receives the XY type order in period 1. Notice that for $\frac{c}{a} \leq \frac{\gamma-\gamma^2}{4(2-\gamma^2)}$, q_{XR1}^{XYX} becomes zero so that the outcome is the same with that in the NYX case. Simply comparing the threshold values in Table 2 shows that $\frac{\gamma-\gamma^2}{4(2-\gamma^2)} < \frac{5+\gamma}{7+3\gamma} < \frac{5-\gamma-\gamma^2}{7-2\gamma^2}$. We show that the online store never encroaches with variety X in this case. If $\frac{5+\gamma}{7+3\gamma} < \frac{c}{a} \leq \frac{5-\gamma-\gamma^2}{7-2\gamma^2}$, we find that $\pi_M^{XYX} < \pi_M^{XYN}$. If $\frac{\gamma-\gamma^2}{4(2-\gamma^2)} < \frac{c}{a} \leq \frac{5+\gamma}{7+3\gamma}$, we find that $\pi_M^{XYX} < max\{\pi_M^{XYN}, \pi_M^{XYB}\}$. If $\frac{c}{a} \leq \frac{\gamma-\gamma^2}{4(2-\gamma^2)}$, we find that $\pi_M^{NYX} < max\{\pi_M^{XYN}, \pi_M^{XYB}\}$. Thus, we have proved that encroaching with variety X is a *strictly dominated strategy* in this case. The outcome that the manufacturer chooses B must satisfy the following conditions: if $\frac{1-\gamma^2}{4(2-\gamma^2)} < \frac{c}{a} < \frac{5+\gamma}{7+3\gamma}$ and $\pi_M^{XYB} \geq max\{\pi_M^{XYN}, \pi_M^{XYX}\}$, or if $\frac{c}{a} \leq \frac{1-\gamma^2}{4(2-\gamma^2)}$ and $\pi_M^{XYB} \geq max\{\pi_M^{XYN}, \pi_M^{NYX}\}$, from which we obtain

$$\frac{c}{a} \leq \theta^{XY}(\gamma) \equiv \frac{(10+7\gamma+\gamma^2) - \sqrt{10+27\gamma+25\gamma^2+9\gamma^3+\gamma^4}}{18+19\gamma+5\gamma^2}.$$

The manufacturer does not open its online store if

$$\frac{c}{a} \geq \theta^{XY}(\gamma).$$

Third, we see the case in which the manufacturer receives the XB type order in period 1. Notice that for $\frac{c}{a} \le \frac{3\gamma}{4(1+\gamma)}$, q_{XR2}^{XBY} becomes zero so that the outcome is the same with that in the XYX case; for $\frac{c}{a} \le \frac{\gamma}{2(1+\gamma)}$, q_{YR2}^{XBY} becomes zero so that the outcome is the same with that in the XXY case; for $\frac{c}{a} \le \frac{\gamma-\gamma^2}{4(2-\gamma)}$, q_{XR1}^{XYX} becomes zero so that the outcome is the same with that in the NYX case. Simply comparing the threshold values in Table 2 shows that $\frac{\gamma-\gamma^2}{4(2-\gamma^2)} < \frac{\gamma}{2(1+\gamma)} < \frac{3\gamma}{4(1+\gamma)} < \frac{15}{25+4\gamma} < \frac{3}{5} < \frac{15-2\gamma-\gamma^2}{21-\gamma^2}$. We show that the online store never encroaches with variety X in this case. If $\frac{15}{25+4\gamma} < \frac{c}{a} \le \frac{3}{5}$ in which the XBB equilibrium outcome does not exist, we find that $\pi_M^{XBY} < max\{\pi_M^{XBN}, \pi_M^{XBY}\}$. If $\frac{3\gamma}{4(1+\gamma)} < \frac{c}{a} \le \frac{15}{25+4\gamma}$, we find that $\pi_M^{XBY} < max\{\pi_M^{XBN}, \pi_M^{XBY}, \pi_M^{XBB}\}$. If $\frac{\gamma}{2(1+\gamma)} < \frac{c}{a} \le \frac{3\gamma}{4(1+\gamma)}$, we find that $\pi_M^{XYX} < max\{\pi_M^{XBN}, \pi_M^{XBY}, \pi_M^{XBB}\}$. If $\frac{\gamma-\gamma^2}{4(2-\gamma^2)} < \frac{c}{a} \le \frac{\gamma}{2(1+\gamma)}$, we find that $\pi_M^{XYX} < max\{\pi_M^{XBN}, \pi_M^{XXY}, \pi_M^{XBB}\}$. If $\frac{c}{a} \le \frac{\gamma-\gamma^2}{4(2-\gamma^2)}$, we find that $\pi_M^{NYX} < max\{\pi_M^{XBN}, \pi_M^{XXY}, \pi_M^{XBB}\}$. Thus, we have proved that encroaching with variety X is a *strictly dominated strategy* in this case. The outcome that the manufacturer chooses B must satisfy the following conditions: if $\frac{3\gamma}{4(1+\gamma)} < \frac{c}{a} \le \frac{15}{25+4\gamma}$ and $\pi_M^{XBB} \ge max\{\pi_M^{XBN}, \pi_M^{XBY}, \pi_M^{XBY}\}$, or if $\frac{\gamma}{2(1+\gamma)} < \frac{c}{a} \le \frac{3\gamma}{4(1+\gamma)}$ and $\pi_M^{XBB} \ge max\{\pi_M^{XBN}, \pi_M^{XYX}, \pi_M^{XBY}\}$, or $\frac{\gamma-\gamma^2}{4(2-\gamma)} < \frac{c}{a} \le \frac{\gamma}{2(1+\gamma)}$ and $\pi_M^{XBB} \ge max\{\pi_M^{XBN}, \pi_M^{XYX}, \pi_M^{XXY}\}$, or $\frac{c}{a} \le \frac{\gamma-\gamma^2}{4(2-\gamma)}$ and $\pi_M^{XBB} \ge max\{\pi_M^{XBN}, \pi_M^{NYX}, \pi_M^{XXY}\}$, from which we obtain

$$\textit{when}\ \gamma \le 0.690,\ \frac{c}{a} \le \underline{\theta}_1^{XB}(\gamma) \equiv \frac{5(15+4\gamma+\gamma^2)-(5+2\gamma)\sqrt{10(5-\gamma^2)}}{175+80\gamma+13\gamma^2},$$

$$\textit{as well as when}\ \gamma > 0.690,\ \frac{c}{a} \le \underline{\theta}_2^{XB}(\gamma) \equiv \frac{5(6+4\gamma+\gamma^2)-2\sqrt{5(-2+20\gamma+5\gamma^2-10\gamma^3-2\gamma^4)}}{94+80\gamma+13\gamma^2}.$$

The outcome that the manufacturer chooses Y must satisfy the following conditions: if $\frac{3}{5} < \frac{c}{a} \le \frac{15-2\gamma-\gamma^2}{21-\gamma^2}$ and $\pi_M^{XBY} \ge \pi_M^{XBN}$, or if $\frac{15}{25+4\gamma} < \frac{c}{a} \le \frac{3}{5}$ and $\pi_M^{XBY} \ge \{\pi_M^{XBN}, \pi_M^{XBY}\}$, or if $\frac{3\gamma}{4(1+\gamma)} < \frac{c}{a} \le \frac{15}{25+4\gamma}$ and $\pi_M^{XBY} \ge \{\pi_M^{XBN}, \pi_M^{XBY}, \pi_M^{XBB}\}$, or if $\frac{\gamma}{2(1+\gamma)} < \frac{c}{a} \le \frac{3\gamma}{4(1+\gamma)}$ and $\pi_M^{XBY} \ge \{\pi_M^{XBN}, \pi_M^{XYX}, \pi_M^{XBB}\}$, from which we obtain

$$\textit{when}\ \gamma \le 0.690,\ \underline{\theta}_1^{XB}(\gamma) \le \frac{c}{a} \le \overline{\theta}_1^{XB}(\gamma) \equiv \frac{2(15-4\gamma+\gamma^2)-(3+\gamma)\sqrt{2(5-\gamma^2)}}{2(27+\gamma^2)},$$

$$\textit{as well as when}\ 0.690 < \gamma \le 0.897,\ \theta^{XB}(\gamma) \equiv \frac{\gamma}{2(1+\gamma)} \le \frac{c}{a} \le \overline{\theta}_1^{XB}(\gamma).$$

The outcome that the manufacturer does not open its online store must satisfy the following conditions: if $\frac{c}{a} > \frac{15-2\gamma-\gamma^2}{21-\gamma^2}$, or if $\frac{3}{5} < \frac{c}{a} \le \frac{15-2\gamma-\gamma^2}{21-\gamma^2}$ and $\pi_M^{XBN} \ge \pi_M^{XBY}$, or $\frac{15}{25+4\gamma} \le \frac{c}{a} < \frac{3}{5}$

and $\pi_M^{XBN} \geq max\{\pi_M^{XBY}, \pi_M^{XBY}\}$, or $\frac{3\gamma}{4(1+\gamma)} < \frac{c}{a} \leq \frac{15}{25+4\gamma}$ and $\pi_M^{XBN} \geq max\{\pi_M^{XBY}, \pi_M^{XBY}, \pi_M^{XBB}\}$, or $\frac{\gamma}{2(1+\gamma)} < \frac{c}{a} \leq \frac{3\gamma}{4(1+\gamma)}$ and $\pi_M^{XBN} \geq max\{\pi_M^{XYX}, \pi_M^{XBY}, \pi_M^{XBB}\}$, or $\frac{\gamma-\gamma^2}{4(2-\gamma^2)} < \frac{c}{a} \leq \frac{\gamma}{2(1+\gamma)}$ and $\pi_M^{XBN} \geq max\{\pi_M^{XYX}, \pi_M^{XXY}, \pi_M^{XBB}\}$, or $\frac{c}{a} \leq \frac{\gamma-\gamma^2}{4(2-\gamma^2)}$ and $\pi_M^{XBN} \geq max\{\pi_M^{NYX}, \pi_M^{XXY}, \pi_M^{XBB}\}$, from which we obtain

$$when\ \gamma \leq 0.897,\ \frac{c}{a} > \overline{\theta}_1^{XB}(\gamma),$$
$$as\ well\ as\ when\ \gamma > 0.897,\ \frac{c}{a} > \overline{\theta}_2^{XB}(\gamma)$$
$$\equiv \frac{(1+\gamma)(12-8\gamma+2\gamma^2) - \sqrt{2(1+\gamma)(36-36\gamma-4\gamma^2+20\gamma^3-7\gamma^4-\gamma^5)}}{2(1+\gamma)(6+\gamma^2)}.$$

Fourth, we see the case in which the manufacturer receives the *BB* type order in period 1. Notice that for $\frac{c}{a} \leq \frac{\gamma}{2(1+\gamma)}$, q_{XR1}^{BBY} and q_{XR2}^{BBY} becomes zero so that the outcome is the same with that in the *YYX* case. Simply comparing the threshold values in Table 2 shows that $\frac{\gamma}{2(1+\gamma)} < \frac{3}{5}$. We show that the online store never encroaches with either variety except for a single point in this case. If $\frac{\gamma}{2(1+\gamma)} < \frac{c}{a} \leq \frac{3}{5}$, we find that $\pi_M^{BBY} \geq max\{\pi_M^{BBN}, \pi_M^{BBB}\}$ when $c/a = \theta^{BB}(\gamma)$. If $\frac{c}{a} \leq \frac{\gamma}{2(1+\gamma)}$, we find that $\pi_M^{YYX} < max\{\pi_M^{BBN}, \pi_M^{BBB}\}$. Thus, we have proved that encroaching with either variety is a *weakly dominated strategy* in this case. The outcome that the manufacturer chooses B must satisfy the following conditions: if $\frac{\gamma}{2(1+\gamma)} < \frac{c}{a} \leq \frac{3}{5}$ and $\pi_M^{BBB} \geq max\{\pi_M^{BBN}, \pi_M^{BBY}\}$, or if $\frac{c}{a} \leq \frac{\gamma}{2(1+\gamma)}$ and $\pi_M^{BBB} \geq max\{\pi_M^{XBN}, \pi_M^{YYX}\}$, from which we obtain

$$\frac{c}{a} \leq \theta^{BB}(\gamma) \equiv \frac{3-\sqrt{2}}{7}.$$

The outcome that the manufacturer does not open its online store must satisfy the following conditions:

$$\frac{c}{a} \geq \theta^{BB}(\gamma).$$

□

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