Emergence of Populism under Ambiguity*

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Abstract

I construct a dynamic elections model in which a representative voter chooses a policymaker among the set of politicians consisting of both biased elites and unbiased non-elites in each period. Using this model, I analyze when populism arises in the sense that an unbiased non-elite, who has only limited knowledge and ability, is chosen as the policymaker instead of a biased elite. I show that an increase in uncertainty in the sense of *ambiguity* (*Knightian uncertainty*) makes populism more likely to arise. This is a contrast to the effect of an increase in *risk* such that more uncertainty makes populism less likely to arise so long as the reward and punishment mechanism to incentivize politicians is limited. The results suggest that an increase in ambiguity rather than risk is a crucial source of populism.

Keywords: Populism; Dynamic elections; Political agency; Ambiguity; Risk

JEL classification codes: D72; H11; D81; C73

1 Introduction

These days, it has been pointed out that populism is emerging in many countries including the Western developed countries.[1] In the past, populism has arisen after a severe change in society (e.g., a significant change in economic circumstances). For example, it is known that after the Great Depression, populism, especially extremism, emerged in several countries. Such a significant change often induces an increase in uncertainty voters face. Thus, an increase in uncertainty may be related to the emergence of populism. The purpose of the present paper is to analyze the relationship

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[1] There is a growing literature on populism in political science. See Gidron and Bonikowski (2013).

between the emergence of populism and an increase in uncertainty about politicians, voters face, by distinguishing *ambiguity* (*Knightian uncertainty*) with *risk*.

To this end, what populism is should be defined, first. Recently, there is one definition that has been widely accepted by many political theorists. According to Mudde, populism is defined as follows:

> I define populism as an ideology that considers society to be ultimately separated into two homogeneous and antagonistic groups, "the pure people" versus "the corrupt elite", and which argues that politics should be an expression of the volonté générale (general will) of the people. (Mudde 2004: 543)

Given this, populism has the following aspect. Populism can arise as protest against politics by elites when ordinary citizens distrust them. Even if the elites have profound knowledge and ability, voters would vote for a non-elite when they think that the elites hurt their interests. This also implies that populism is not necessarily a threat to democracy.[2] The concept of democracy requires that politics reflect ordinary citizens' opinions. Thus, if elites are corrupt and politics by them does not reflect the ordinary citizens' opinions, it is appropriate to elect a populist as the representative in order to maintain the responsiveness of democracy. Therefore, populism can be a corrective to democracy (Mudde and Kaltwasser 2012).[3]

I develop a dynamic elections model that describes this aspect of populism. In the model, there are a representative voter (hereafter the voter), biased elites, and unbiased non-elites. A biased elite has enough ability to find out what is a good policy. However, her/his policy preference is biased from the voter's one. On the other hand, an unbiased non-elite has only limited ability. Instead, her/his policy preference is the same as the voter's one since the both are ordinary citizens. Here, a biased elite's degree of the bias is drawn from a probability distribution, and its value is unobservable to the voter. In other words, the voter faces uncertainty about a biased elite's degree of the bias. The voter elects the representative among biased elites and unbiased non-elites in each period. "Populism emerges" when the voter elects an unbiased non-elite as the representative in spite of her/his limited ability.

The contribution of the present paper is to analyze the effect of uncertainty based on this model.

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[2]To understand this point, see the relationship between populism and direct democracy. "[T]he populist ideology shares the Rousseauian critique of representative government. [...] [P]opulism appeals to Rousseau's republican utopia of self-government, that is the very idea that citizens are able both to make the laws and execute them" (Mudde and Kaltwasser 2013: 503).

[3]This does not mean that populism is always a corrective to democracy. Mudde and Kaltwasser (2012) argue the both cases.

In reality, voters face uncertainty about an elite's degree of the bias. I analyze how such uncertainty affects the emergence of populism. As Knight (1921) points out, we should distinguish *ambiguity* (*Knightian uncertainty*), where even the probability distribution is unknown, from *risk*, where the probability distribution is known. Indeed, Ellsberg (1961)'s paradox shows the importance of this distinction. Thus, I analyze both cases by employing Choquet expected utility with a convex capacity (Schmeidler 1989). In other words, a special case of Maxmin expected utility (Gilboa and Schmeidler 1989) is employed: the voter has a set of priors about a biased elite's degree of the bias and maximizes the payoff evaluated by the prior that gives the voter the minimum payoff.

Then, I show that the effect of an increase in uncertainty differs depending on which type of the uncertainty is involved. When the voter has less confidence about the true distribution (i.e., the degree of ambiguity increases)[4], populism is more likely to arise. In contrast, when the variance of the distribution increases (i.e., the degree of risk increases), populism becomes less likely to arise so long as the reward and punishment mechanism to incentivize politicians is limited.[5] This result is surprising because people hate risk as well as ambiguity, and so both effects seem to be the same.

Why does the effect differ depending on the type of uncertainty? To be sure, there is a common effect that the voter becomes reluctant to vote for a biased elite after an increase in uncertainty because the voter hates uncertainty. However, the interaction between the possibility to replace the incumbent and an increase in uncertainty creates another effect that differs depending on the type of uncertainty. Under dynamic elections, the voter can replace the incumbent by new one if s/he finds out that the incumbent is highly biased. Therefore, the loss due to electing a highly biased elite is limited. This nature of dynamic elections interacts with an increase in uncertainty as follows.

An increase in risk (a mean-preserving spread) means that the probability that an elite is not biased as well as the probability that an elite is highly biased, increase. Here, the loss due to electing a highly biased elite is bounded thanks to the possibility of replacement. Thus, the benefit due to an increase in the probability that an elite is not biased dominates the loss due to an increase in the probability that an elite is highly biased. As a result, the expected payoff when the voter elects a biased elite can increase with the degree of risk. When the degree of risk aversion is not high, this positive effect dominates the effect that the voter hates risk. Therefore, an increase in risk makes populism less likely to arise. To put it differently, the nature of dynamic elections (i.e., the possibility of replacement) makes democracy robust against populism even under high risk.

[4]The employed notion of an increase in ambiguity is the expansion of the core of a convex capacity. But, this includes an increase in ambiguity aversion as well as ambiguity itself. These two cannot be separated in the framework of Choquet/Maxmin expected utility.

[5]Focusing on such a situation is meaningful because populism emerges only when controlling elites is hard.

On the other hand, this mechanism no longer works in the case of ambiguity.The voter is pessimistic so that s/he evaluates the utility using the prior that gives her/him the worst payoff. Thus, roughly speaking, the voter's expected payoff when electing a biased elite is evaluated using the prior that assigns the largest value to the probability that the degree of bias is quite high. After an increase in ambiguity, the set of candidates of the true distribution enlarges. Thus, the voter evaluates her/his payoff using the prior that assigns a larger value to the probability that the degree of bias is quite high after an increase in ambiguity. Therefore, the voter's expected payoff when electing a biased elite decreases with the degree of ambiguity. In contrast, the payoff when electing an unbiased non-elite remains the same because there is no ambiguity about an unbiased non-elite's type. As a result, the voter becomes reluctant to vote for a biased elite: an increase in ambiguity induces populism. Here, the probability that the degree of bias is low does not increase after an increase in ambiguity in contrast to that after an increase in risk. Thus, the mechanism that the voter prefers more uncertainty thanks to the possibility of replacement does not work at all. In summary, an increase in ambiguity rather than risk is a significant source of populism.

The implications of this result are the following. When the policy issue at the center of politics is traditional one, voters would know the distribution of elites' preferences over the issue because there have been a lot of opportunities for learning. In such a case, voters face risk. In contrast, when a new policy issue arises at the center of politics, voters would not know even the distribution of elites' preference because information is too imprecise to be described by a single prior. In such a case, voters face ambiguity. Therefore, the result suggests that the emergence of a new policy issue, which has never been prioritized, can be a source of populism.

In addition, a severe change in society implies that a new policy issue arises. Thus, the result also suggests that a significant change in society can be a sourse of populism because it makes the distribution of elites' policy preferences more ambiguous. This implication provides one explanation for the reality that populism is likely to arise after a large change in society. Funke et al. (2016) empirically show that populism is likely to emerge after economic crises.[6]

Lastly, I mention a difficulty in the analysis of a dynamic incomplete information game under ambiguity. In the case of ambiguity, dynamic consistency does not necessarily hold under simple updating rules corresponding to Bayes rule. This makes the analysis difficult. Nonetheless, the present study succeeds in resolving this difficulty in a tractable manner. It is known that when

[6] Another possible explanation is that a severe change in society suggests that elites did not work well, and so populism arises as punishment for their deviation. The explanation the present result provides is interesting compared to this alternative explanation in that a significant change in society itself (not as a signal that elites were biased in the previous period) can be a source of populism.

rectangularity is satisfied, dynamic consistency holds in the framework of Maxmin expected utility (Epstein and Schneider 2003a). In the analysis, I restrict my attention to stationary strategies. Thanks to this restriction, rectangularity holds for the voter given the others' stationary strategies. As a result, I do not have to care about dynamic inconsistency and updating rules.

The remainder of the paper proceeds as follows. Section 2 reviews the related literature. Section 3 describes the model. Section 4 derives the equilibrium. Section 5 defines an index to measure the likelihood of the emergence of populism. Section 6 analyzes the effect of an increase in uncertainty. Section 7 discusses an extension. Section 8 concludes.

2 Related Literature

■ **Formal model of populism:** The literature is divided mainly into two directions: signaling and pandering.[7] In the signaling literature, voters choose a politician who implements an extreme policy since it is a signal of the politician's good characteristics (e.g., Acemoglu, Egorov, and Sonin 2013).[8] In the pandering literature, voters have a belief about what is a good policy. In order to maintain reputation, politicians implement a policy consistent with the voters' belief even though they know that the policy is harmful (e.g., Frisell 2009; Jennings 2011; Binswanger and Prufer 2012).[9]

The aspect of populism captured in these models is different from that captured in my model. In these models, populism is a phenomenon that politicians choose policies hurting the interests of the majority of voters, but the majority of voters supports such politicians. It is not described as a phenomenon that voters vote for a non-elite, who has only limited knowledge and ability, instead of an elite because they distrust elites. As a result, the aspect of populism such that it is harmful to voters is highlighted.

■ **Dynamic elections with adverse selection:** In the model, the voter elects one candidate among the incumbent, a new unbiased non-elite, and new biased elites. Here, a new biased elite's degree of the bias is drawn from a distribution. This model has a common feature with dynamic elections models with adverse selection. Duggan (2000) develops an infinite horizon model in which voters choose the representative among citizen candidates. In his model, candidates' policy preferences are unobservable, and the voters decide whether to replace the incumbent by a new

[7] Beyond the two branches, there are other unmentioned papers which are possibly related: studies on populist economic policies (e.g., Mejía and Posada 2007; Leon 2014; Chesterley and Roberti 2016) and studies on policy divergence in electoral competition (e.g., Boleslavsky and Cotton 2015).

[8] This signaling literature begins with the study of Kartik and McAfee (2007), which does not mention populism.

[9] The studies above relate pandering to populism. Pioneering studies in the literature are those by Harrington (1993), Canes-Wrone, Herron, and Shotts (2001), and Maskin and Tirole (2004) though they do not refer to populism.

candidate, whose policy preference is drawn from a distribution.[10] Then, he characterizes a simple equilibrium, where politicians are divided into three types: (i) centrists whose policy preferences are close to a median voter's one and who implement their own ideal policy and can be reelected, (ii) moderates who adopt the most extreme policy that allows them to be reelected, and (iii) extremists whose policy preferences are far from a median voter's one and who implement their own ideal policy and cannot be reelected. There are many subsequent studies (e.g., Banks and Duggan 2008; Bernhard et al. 2009; Bernhard, Câmera and Squintani 2011; Câmera and Bernhard 2015), and I follow this literature.

However, there are several differences from the literature. First, no paper analyzes the effect of an increase in uncertainty.[11] Second, the focus differs. In the literature, all the candidates are citizen candidates (i.e., ordinary citizens). In contrast, I introduce a distinction between a non-elite candidate, who is an ordinary citizen, and an elite candidate, who is not an ordinary citizen. Then, I analyze the trade-off between them and succeed in analyzing populism in the framework of dynamic elections. To my knowledge, this is the first paper in which a distinction between elites and non-elites exist, and populism is analyzed.

■ **Political economics and ambiguity:** There are only limited studies that deal with ambiguity in political economics (Berliant and Konishi 2005; Ghirardato and Katz 2006; Bade 2011; 2016; Baumann and Svec 2016; Ellis 2016; Yang 2016; Nakada, Nitzan, and Ui 2017). There is no analysis on populism. Furthermore, they consider not dynamic but static situations.

■ **Optimal stopping problem under ambiguity:** In the model, the voter's optimization problem is finally reduced to be a variant of optimal stopping problems. In this sense, the present study is related to the studies on optimal stopping problems under ambiguity. Nishimura and Ozaki (2004) analyze a simple one-sided labor search model. They show that the effect of an increase in ambiguity can be different from that of an increase in risk in the model. After this work, several studies derive a similar result in a different or general setting (e.g., Nishimura and Ozaki 2007; Miao and Wang 2011).[12] I employ Choquet expected utility as Nishimura and Ozaki (2004) do, and follow several assumptions introduced by them.

There are several differences from this literature.[13] First, there is no analysis on political phe-

[10] Banks and Sundaram (1993) also analyze a similar situation, but their focus is effort-choice by a politician.

[11] Câmera and Bernhard (2015) analyze a change of the distribution about policy preferences. However, they consider the effect of not an increase in uncertainty but a decrease in polarization.

[12] Riedel (2009) analyzes the optimal stopping problem under ambiguity with discrete time generally although he does not focus on the effect of an increase in ambiguity.

[13] There is also a technical difference from a typical optimal stopping problem. In the present paper, the voter chooses an action among three alternatives in each period, and nothing is irreversible, while there are only two alternatives of

nomena. The present study is the first application of an optimal stopping problem under ambiguity to the analysis on political phenomena.

Second, even in a theoretical point of view, the present study has an important feature the existing literature does not have: I analyze a dynamic game with information asymmetry whereas the literature analyzes a one-person decision making problem. The largest difficulty in the analysis of a dynamic situation under ambiguity is a dynamic inconsistency problem. One easy approach to guarantee dynamic consistency in a one-person decision making problem is to assume an independent and indistinguishable distribution proposed by Epstein and Schneider (2003b).[14] Since information is completely not revealed under this assumption, rectangularity trivially holds, and so dynamic consistency is ensured (e.g., Nishimura and Ozaki 2004; 2007; Miao and Wang 2011). However, even under this assumption, rectangularity may fail in a dynamic incomplete information game. This is because belief updating depends on players' strategies, which are endogenous, as well as an exogenous stochastic process. Thus, this approach cannot be applied to the analysis on a dynamic incomplete information game straightforwardly. Therefore, to analyze a dynamic incomplete information game under ambiguity is quite difficult. Nonetheless, I succeed in solving the dynamic incomplete information game in a tractable manner by focusing on stationary equilibria. This is a novelty of the present study.[15]

3 The Model

3.1 Setting

Construct a dynamic elections model with information asymmetry. The model is infinite horizon: $t = 0, 1, \ldots$. The society consists of ordinary citizens and elites. The ordinary citizens are homogenous in terms of policy preferences, and they have power to choose who conducts policymaking. Due to the homogeneity, it suffices to focus on a representative voter (hereafter, the voter). Each player has a policy preference on a unidimensional policy space $(-\infty, \infty)$. In period t, the voter chooses who s/he will elect as the representative (i.e., policymaker). After that, the elected representative chooses policy x_t. This sequential game is infinitely repeated.

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the action, and an irreversible choice exists in a typical optimal stopping problem.

[14] Another approach is to employ a complicated updating rule under which dynamic consistency holds.

[15] There is a paper that analyzes a game theoretic situation in this literature: Boyarchenko and Levendorskiĭ (2012) analyze a preemption game under ambiguity. However, their game is without information asymmetry. Under such a setting, the belief is only based on the exogenous stochastic process. In this sense, their model is close to a one-person decision making problem in terms of belief formation.

3.1.1 Voter

Let the voter's ideal policy in period t be $\hat{x}_t$. The desirable policy varies depending on external circumstances. Thus, $\hat{x}_t$ is a stochastic variable, and different across time. It is independently drawn from a probability distribution F, whose density function is denoted by f. F is assumed to be symmetric: $f(a) = f(-a)$ for any $a \in [0, \infty)$. Since the voter is not familiar with policy issues, the value of $\hat{x}_t$ is unknown to the voter while the distribution F is known.

The voter's payoff in each period is represented by a linear loss function: $-|x_t - \hat{x}_t|$ i.e., the voter is risk-neutral. Note that a recent empirical study shows that a voter's loss function is almost linear (Berinsky and Lewis 2007), and so this assumption is consistent with the reality. The discussion on risk-aversion will be seen in Section 7. As I see later, the value of $-|x_t - \hat{x}_t|$ is unobservable to the voter until the game is ended so long as monitoring is not succeeded.

3.1.2 Politicians

There are two types of politicians in each period: a biased elite and an unbiased non-elite. A biased elite observes the value of $\hat{x}_t$ perfectly after s/he was elected as the representative. In this sense, a biased elite has ability to act for the voter. However, s/he has a policy preference different from the voter's, and so s/he may implement a policy different from $\hat{x}_t$.

Each biased elite's policy preference is biased compared to the voter's one:

1. Right biased elite: the ideal policy is $\hat{x}_t^r(\beta) = \hat{x}_t + \beta$, where $\beta \in [0, \bar{\beta}]$.

2. Left biased elite: the ideal policy is $\hat{x}_t^l(\beta) = \hat{x}_t - \beta$, where $\beta \in [0, \bar{\beta}]$.

Here, $\bar{\beta} \in (0, \infty)$. When a biased elite is elected as the representative in period t, s/he receives payoff from policymaking and office-seeking motivation. The payoff due to the policy mismatch in period t is $-|x_t - \hat{x}_t^i(\beta)|$, where $i = r, l$. In addition to this, during the period in the office, s/he receives $\rho \in (0, \infty)$ which represents the office-seeking motivation. In summary, during the period in the office, a biased elite receives $-|x_t - \hat{x}_t^i(\beta)| + \rho$. On the other hand, I assume that while a biased elite is not in the office, her/his payoff is zero since the mental cost due to policy mismatch would be quite small during the period not in office compared to that during the period in office.[16]

[16]Remember that a biased elite is different from an ordinary citizen. Therefore, her/his economic utility would not be affected by government policies compared to an ordinary citizen's utility. For example, regulation on minimum wage would not affect economic utility of a person who has high ability and obtains high income. In this sense, a biased elite's disutility from policy mismatch is due to the mental cost rather than the economic or physical cost. During the period in office, the mental cost would be large compared to during that not in office since the policy mismatch is implemented by her/himself during the period in office. The setting above captures this reality.

Next, consider the setting of an unbiased non-elite. This type of politician can observe the value of $\hat{x}_t$ after elected with only probability $\phi \in [0, 1)$. Note that this is assumed to be independent of the value of $\hat{x}_t$. Thus, an unbiased non-elite has only limited ability to act for the voter. The advantage of an unbiased non-elite is literally unbiasedness of her/his policy preference. The payoff due to the policy mismatch in period t is exactly the same as the voter's one: $-|x_t - \hat{x}_t|$. During the period in the office, an unbiased non-elite receives $-|x_t - \hat{x}_t| + \rho$ while her/his payoff is zero during the period not in the office.[17]

Lastly, a politician who loses an election will never never stand as a candidate.

3.1.3 Information Asymmetry and Voter's Decision

At the beginning of period t, there are four (three) politicians when $t \geq 1$ ($t = 0$): (i) the incumbent who was elected as the representative in period $t-1$, and (ii) alternative candidates consisting of a right biased elite, a left biased elite, and an unbiased non-elite (when $t = 0$, the incumbent does not exist). Note that the degree of biases of a right and left biased elite, who are alternative candidates in period t, are drawn from the same distribution, and this is common-knowledge. The voter elects one of them as the representative in each period.[18]

The voter can distinguish between a right biased elite, a left biased elite, and an unbiased non-elite. But, the voter is uncertain of a biased elite's degree of the bias. This is the first information asymmetry and induces adverse selection. In addition, the voter cannot observe the implemented policy x_t and the desirable policy $\hat{x}_t$. As a result, the voter cannot observe the implemented policy mismatch $|x_t - \hat{x}_t|$ in principle. This is the second information asymmetry and induces moral hazard.

However, these information asymmetries can be resolved directly or indirectly owing to a possibility of monitoring. I suppose that the voter observes the implemented policy mismatch $|x_t - \hat{x}_t|$ with probability $q \in (\underline{q}, \bar{q})$, where $0 < \underline{q} < \bar{q} \leq 1$. Note that this is independent across time. This can be regarded as monitoring by the mass media since media outlets gather news and report the incumbent's performance with some probability in reality. Thanks to this monitoring, both infor-

[17]Once the interpretation discussed in the former footnote is taken into account, an unbiased non-elite's payoff during the period not in office may be dependent of policy mismatch, since the unbiased non-elite is an ordinary citizen differently from a biased elite. Under such a setting, the same result holds since Lemma 1 does not change.

[18]An alternative setting is that there are both a large election and preliminary elections. Suppose that there are two parties: right-wing and left-wing. The right (left)-wing party endorses a right (left) biased elite. The party, the incumbent belongs to, endorses the incumbent automatically. The opposite party endorses a biased elite drawn from a distribution. Thus, there are the incumbent and a biased elite who are endorsed by each party in a large election. In addition, an unbiased non-elite exists as a candidate in a large election. In summary, there are three candidates in each large election. This setting is basically the same as that of Bernhard et al. (2009) except for the existence of an unbiased non-elite. Under this setting, the number of candidates is reduced to three without changing any result.

mation asymmetries can be partially resolved. Since $|x_t - \hat{x}_t|$ is observed after monitoring, the second information asymmetry can be resolved. In addition, after $|x_t - \hat{x}_t|$ is observed, the voter may be able to infer the degree of the incumbent biased elite's bias. Thus, the first information asymmetry can be also resolved. Note that whether the monitoring is succeeded is observable to politicians as well as the voter.

Due to these information asymmetries, the voter faces a trade-off between a biased elite and an unbiased non-elite. A biased elite has enough ability to act for the voter, but s/he may choose a policy far from $\hat{x}_t$. On the other hand, an unbiased non-elite has only limited ability, but there is no conflict of interests. I say "populism emerges" when the voter chooses an unbiased expert as the representative. To avoid unnecessary complication, I suppose that the voter votes for not an unbiased non-elite but a biased elite when both are indifferent for the voter.

3.1.4 Timing

In order to consider a situation where the implemented policy mismatch is unobservable with some probability, the voter's payoff due to the policy mismatch should not be realized in each period. To deal with this issue, suppose that the game ends at the end of each period independently with probability $1 - \delta$, where $\delta \in (0, 1)$. When the game ends, the voter's payoff is realized. Here, the innate discount rate is zero, so that the discount factor is δ.

The timing of each stage game is as follows:

1. Nature draws the values of β of biased elites who are alternative candidates from a distribution.

2. The voter decides the candidate s/he votes for.

3. The elected politician observes $\hat{x}_t$ with probability one if s/he is a biased elite, and with probability ϕ if s/he is an unbiased non-elite.

4. The elected politician chooses policy x_t.

5. The voter observes $|x_t - \hat{x}_t|$ with probability q.

3.2 Ambiguity on Politicians' Types

The voter does not know a biased elite's degree of the bias. I allow a situation where even the distribution of β, from which the degrees of biases of a new left biased elite and a new right biased elite is drawn, is unknown. In this subsection, I describe this ambiguity.

Let $(B, \mathcal{F}_B)$ be a measurable space, where $B = [0, \bar{\beta}]$, and $\mathcal{F}_B$ is the Borel $\sigma-$algebra on B. Each element $\beta \in B$ represents the degree of the bias of a biased elite. For any $t \geq 0$, I construct the t-dimensional product measurable space $(B^t, {\mathcal{F}_B}^t)$ (let ${\mathcal{F}_B}^0 \equiv \{\emptyset, B^\infty\}$) and embed it in the infinite-dimensional product measurable space $(B^\infty, \mathcal{F}_B^\infty)$.

3.2.1 Belief

I need to consider two types of the voter's belief: (i) about the degree of the bias of a biased elite who is an alternative candidate, and (ii) about the degree of the bias of the incumbent biased elite. Suppose that the voter elected a biased elite as the representative in the former period. Then, in the next election, the voter must decide whether to reelect this incumbent. Here, the voter should take into account the degree of the bias of (i) alternative candidates, and (ii) the incumbent. Thus, the both must be specified.

The belief formation is based on a history. At the end of period t, the voter observes $s_t \in S_t = D_t \times A_t$. First, $D_t = \{[0, \infty), \emptyset\}$ with its generic element d_t, and this represents information about the implemented policy mismatch in period t (i.e., $|x_t - \hat{x}_t|$). $d_t = d \in [0, \infty)$ means that the voter finds out that $|x_t - \hat{x}_t| = d$. Also, $d_t = \emptyset$ means that the voter does not find out the value of $|x_t - \hat{x}_t|$. Second, $A_t = \{0, r, l, u\}$ when $t \geq 1$, and $A_t = \{r, l, u\}$ when $t = 0$. This represents a voting action in period t. 0 represents reelecting the incumbent, r (l) represents electing a new right (left) biased elite, and u represents electing a new unbiased non-elite. The history, which has been observed by the voter until the beginning of period $t \geq 1$, is $s^{t-1} = (s_0, s_1, ..., s_{t-1}) \in S^{t-1} = \Pi_{\tau=0}^{\tau=t-1} S_\tau$. The null history S^{-1} is set to be $\{\emptyset\}$.

Consider (i). Let $\theta_t : S^{t-1} \times \mathcal{F}_B \to [0, 1]$, and call this a capacity kernel.[19] For any $A \in \mathcal{F}_B$, $\theta_{t, s^{t-1}}(A)$ represents a capacity such that the degree of a new biased elite, who is an alternative candidate at the beginning of period t, is in A, given the history s^{t-1}. This construction of the capacity kernel allows the voter to update θ_0 based on the past history.

I assume the following stochastic process. When θ is additive (i.e., in the case of risk), β follows an independent and identical distribution across time. When θ is non-additive (i.e., in the case of ambiguity), β follows an independent and indistinguishable distribution across time (Epstein and Schneider 2003b).[20] Then, $\theta_{t, s^{t-1}}$ is independent of t and s^{t-1}, and so θ_t is time-homogeneous. This

[19] Though the concept of a kernel is usually used to describe a Markov process, here I call the above a capacity kernel.

[20] Suppose that the data generating mechanism is independent and identical. When the voter knows the distribution, the voter does not update the belief since there is nothing left to learn. By contrast, when the voter does not know the distribution, the voter updates the belief. Thus, under ambiguity, IID assumption does not induce a time homogeneous capacity. However, there would be a situation in which the capacity is independent of t and s^{t-1}. This is independent

type of assumption has been used in many papers (e.g., Epstein and Wang 1994; 1995; Nishimura and Ozaki 2004; 2007; Miao and Wang 2011). Without notational abuse, $\theta_{t,s^{t-1}}$ is denoted by θ for all $t \geq 0$ and $s^{t-1} \in S^{t-1}$.

Next, consider (ii). Although the main focus is the incumbent biased elite, I consider the belief about the incumbent, which includes a case where the incumbent is an unbiased non-elite as well as a case where the incumbent is a biased elite, since it makes notation simple. Denote its capacity kernel by $\theta'_t : S^{t-1} \times \mathcal{F}_B \to [0, 1]$ for any $t \geq 1$. I will specify the details of this belief later because it depends on the strategy taken by the incumbent. But, now I specify the belief in the following two cases in which the belief does not depend on the strategy.

Let $\underline{\tau}(t)$ be the period when the incumbent at the beginning of period $t \geq 1$ was elected as the representative for the first time. Then, $d^t_{\underline{\tau}(t)} = D^{t-\underline{\tau}(t)+1}$ represents the history of the observed $|x_t - \hat{x}_t|$ which has been implemented by the incumbent until period t. When $d^t_{\underline{\tau}(t)} = \emptyset^{t-\underline{\tau}(t)+1}$ and $a_{\underline{\tau}(t)} \in \{r, l\}$ (i.e., when the voter has never observed the policy implemented by the incumbent, and the incumbent is a biased elite), the voter should not update the belief, and so in that case $\theta'_{t,s^{t-1}} = \theta$. Also, when $a_{\underline{\tau}(t)} = u$ (i.e., when the incumbent is an unbiased non-elite), the voter knows that the incumbent's degree of the bias is zero, and so $\theta'_{t,s^{t-1}}(\{0\}) = 1$.

Lastly, the capacity θ is assumed to be convex, continuous, and full-support on $[0, \bar{\beta}]$. Continuity guarantees the Fubini property (see Nishimura and Ozaki (2004)). Also, all the probability distribution functions in core(θ) are assumed to be continuous.[21] See Appendix A for the details about the assumptions.

3.2.2 Payoff and Equilibrium Concept

Define the voter's payoff by the iterated (i.e., recursive) Maxmin payoff whose set of priors in each period is equivalent to the core of the aforementioned capacity in each period. Thus, the voter's payoff is the iterated Choquet expected payoff based on the aforementioned capacity kernel. This equivalence comes from the following relationship: let u be bounded and measurable, and v be a convex capacity, then

$$\int u(\beta)dv = \min\left\{\int u(\beta)dG \middle| G \in \text{core}(v)\right\}.^{22} \tag{1}$$

and indistinguishable distribution. The voter thinks that the data generating process differs across time, but s/he does not understand how they differ. Thus, the voter learns nothing, but the belief is the same across time.

[21] Under this assumption, the existence of a solution to the Bellman equation is easily ensured because it is continuous (Lemma 8). Though it may be possible to guarantee the existence without continuity, I employ this assumption since the main purpose is to analyze the emergence of populism, and complicated technical issues are out of the scope.

[22] The minimum is attained since u is assumed to be bounded and measurable, and also core(θ) is weak* compact by the Alaoglu theorem. For the ease of notation, G represents not only a probability measure itself but also its probability

Note that the integral in the left-hand side is Choquet integral. Choquet expected utility with a convex capacity is equivalent to Maxmin expected utility whose set of priors is the core of the capacity. Here, a situation is reduced to be a decision making under risk when the capacity is additive (or equivalently when its core is a singleton).

For the equilibrium concept, I use the following one that is a natural analogue to Perfect Bayesian Equilibrium (PBE). I restrict my attention to pure strategies.

Definition 1 *The strategies of the voter, unbiased non-elites, biased elites, and the belief system* $(\theta, \{\theta'_t\}_{t=1}^{\infty})$ *constitute an equilibrium if*

1. *the strategies are sequentially rational for any* $t \geq 0$, *and*
2. *the belief system is consistent with the strategies in the sense that the belief is updated based on Naive Bayes rule*[23] *(i.e., full Bayesian updating rule in Maxmin expected utility) so long as it is possible.*

In the case of ambiguity, whether the definition above of the payoff and the equilibrium concept is appropriate is not clear.[24] This is due to a possibility of dynamic inconsistency. The recursive/iterated Maxmin payoff from period t is not necessarily equivalent to the Maxmin payoff that is not iterated payoff and is evaluated using only the core of the capacity in period t: the law of iterated expectation does not necessarily hold. To put it differently, when the latter payoff is employed, dynamic inconsistency can arise.

However, in the candidates of equilibrium I focus on (i.e., a kind of stationary equilibrium), dynamic consistency trivially holds since rectangularity, which guarantees dynamic consistency (Epstein and Schneider 2003a), is satisfied as I see later. Thus, the both payoffs are equivalent in the derivation of equilibrium. As a result, the distinction between them does not matter, and so I can employ the recursive payoff and use dynamic programming methods.[25]

To see this rigorously later, define the payoff that is calculated not recursively in a formal way. To this end, fix each player's strategy. Since politicians' strategies only depend on s^{t-1} and β (i.e., public history and type) in equilibria I focus on later, suppose that politicians' strategies only depend on them. Let p_t be a stochastic kernel for the belief about a new biased elite in period t, p'_t be a stochastic

distribution function in the following sections.

[23]Naive Bayes rule is $\theta(A|B) = \frac{\theta(A\cap B)}{\theta(B)}$. Other rules are possible so long as the belief in Section 4.2.3 is obtained.

[24]In general, the appropriate equilibrium concept is more complicated since the complicated updating rule that guarantees dynamic consistency should be used (see Hanany, Klibanoff, and Mukerji (2016)).

[25]The condition under which the payoff evaluated using the initial capacity is equivalent to the payoff calculated recursively is still unclear (see Yoo (1991) and Dominiak (2013)) though that in the framework of Maxmin expected utility is provided by Epstein and Schneider (2003a). The verification above is based on the framework of Maxmin expected utility theory. This is why Maximin expected utility is initially employed in the definition of the voter's payoff.

kernel for the belief about the incumbent biased elite in period t, and r_t be an objective stochastic kernel about the value of d_t i.e., $r_{t,\beta,s^{t-1}}$ is a probability measure about the value of d_t when the representative in period t has the degree of the bias β and the history is s^{t-1}.[26] Using p_τ ($\tau \geq t$), p'_t, and r_τ ($\tau \geq t$), one can construct a probability measure p^t on $(B^\infty, \mathcal{F}_B^\infty)$ about the sequence of the degree of the bias who is the representative in each period from period t, given the history s^{t-1}. Here, for each t, $p_{t,s^{\tau-1}} \in \text{core}(\theta)$, and $p'_{\tau,s^{\tau-1}} \in \text{core}(\theta'_{\tau,s^{\tau-1}})$ for any $s^{\tau-1} \in S^{\tau-1}$. Denote the set of p^t given s^{t-1} by $\mathcal{P}_{t,s^{t-1}}$. Then, the voter's non-iterated Maxmin payoff from period t conditional on s^{t-1} is

$$\inf_{p^t \in \mathcal{P}_{t,s^{t-1}}} EP_t(p^t, s^{t-1}),$$

where $EP_t(p^t, s^{t-1})$ is the expected payoff from period t conditional on s^{t-1} using p^t. Here, minimum takes only once, and the payoff is not calculated recursively. Later, I show that even if the payoff in Definition 2 is employed instead of the recursive payoff, the same result holds.[27]

4 Equilibrium

4.1 Benchmark: Payoff when an Unbiased Non-Elite is the Representative

To begin with, derive the voter's expected payoff when electing an unbiased non-elite as the representative in every period. In each period, an unbiased non-elite observes the value of $\hat{x}_t$ with probability ϕ. In this case, the unbiased non-elite implements policy $\hat{x}_t$. On the other hand, with probability $1 - \phi$, the unbiased non-elite cannot observe the value of $\hat{x}_t$. In this case, s/he chooses a policy that is the solution of the following problem so as to minimize the expected loss due to policy mismatch: $\int_{-\infty}^{\infty} |x_t - \hat{x}_t| dF$. It is well-known that the solution of this problem is the median of x_t. Thus, the unbiased non-elite chooses 0 as a policy since the median is zero from the symmetry of F. Therefore, when the value of $\hat{x}_t$ is unknown, the voter's expected payoff in period t is $\int_{-\infty}^{\infty} -|\hat{x}_t| dF = -2\int_0^\infty \hat{x}_t dF$.

In summary, I obtain the following lemma. All the proofs are contained in Appendix B.

Lemma 1 *The voter's expected payoff when electing an unbiased non-elite as the representative in every period is*

$$-\frac{2(1-\phi)}{1-\delta}\int_0^\infty \hat{x}_t dF. \tag{2}$$

[26] Given the strategy and the probability of the monitoring q, this probability can be identified straightforwardly.

[27] In this framework, the strategies and the belief system constitute an equilibrium if the strategies are sequentially rational, and $\text{core}(\theta'_t)$ is updated using full Bayesian updating rule so long as it is possible. Though I use full Bayesian updating rule, other updating rules are possible so long as the belief in Section 4.2.3 is obtained.

4.2 Payoff when the Voter Elects a Biased Elite as the Representative

In this subsection, I derive the voter's expected payoff when voting for a biased elite. Here, F is a symmetric distribution, and β of a new right biased elite and a new left biased elite are drawn from the same distribution. Thus, a right biased elite and a left biased elite are totally indifferent for the voter. Therefore, I do not distinguish whether a biased elite is left or right.[28]

4.2.1 Equilibrium Refinement

Due to the information asymmetry and repeated games structure, potentially there are a lot of equilibria, and it makes the analysis complicated. To avoid such complication, the literature has put restrictions on equilibrium strategies and beliefs, and focused on symmetric stationary equilibrium (Duggan 2000; Banks and Duggan 2008; Bernhard et al. 2009; Bernhard, Câmera and Squintani 2011; Câmera and Bernhard 2015).[29] I follow the same convention though the details are different.

To begin with, define $\tau^*(t)$ as follows: when $t \geq 1$

$$\tau^*(t) \equiv \begin{cases} \emptyset & ((\forall \tau \in \{\underline{\tau}(t), ..., t-1\}) \ \ d_\tau = \emptyset.) \\ \max\left\{\tau \in \{\underline{\tau}(t), ..., t-1\} \middle| d_\tau \neq \emptyset\right\} & (\text{otherwise}) \end{cases},$$

and when $t = 0$, $\tau^*(t) \equiv \emptyset$. Here, $\tau^*(t)$ is the latest period such that the policy mismatch, implemented by the incumbent in period t, was observed. $\tau^*(t) = \emptyset$ represents that the voter has never observed the degree of policy mismatch implemented by the incumbent. Using this expression and the definition of d_t, $d_{\tau^*(t)}$ is the policy mismatch observed in period $\tau^*(t)$ i.e., the latest observed policy mismatch implemented by the incumbent. Note that in the original definition of d_t, $d_{\tau^*(t)}$ is not well defined when $\tau^*(t) = \emptyset$. Thus, when $\tau^*(t) = \emptyset$, $d_{\tau^*(t)}$ is set to be $\emptyset$ without notational abuse. Using these notations, I introduce one class of voting strategy, satisfying stationarity, and assume that the voter's equilibrium strategy belongs to this class.

Assumption 1 *The voter's equilibrium strategy must satisfy the following. For any history, the voter decides whether to reelect the incumbent biased elite based on the same rule r, where the decision in period t only*

[28]One may think that whether to elect a left biased elite or a right biased elite would affect a biased elite's strategy since the incumbent right biased elite has less incentive to deviate if the voter elects a not right but left biased elite after her/his deviation. This is the essence of "party competition effect" Bernhardt et al. (2009) discuss. If this is the case, whether to elect a left biased elite or a right biased elite affects the voter's payoff. However, such a possibility does not exist in my model since the incumbent's payoff is zero after replaced by a new politician.

[29]Duggan (2014) who shows the Folk theorem is only one exception. However, even he points out "the application of dynamic electoral models will rely on equilibrium refinements (e.g., the common restriction to stationary equilibria) [.]" Moreover, the present model is more complicated than his model.

depends on the value of $d_{\tau^*(t)}$ *i.e.,* $r : [0, \infty) \cup \emptyset \to \{0, 1\}$*, where* 0 *represents reelection.*

This assumption has two requirements. First, the voter must decide whether to reelect the incumbent biased elite, only depending on the latest observed policy mismatch implemented by her/himself. Second, the voter must apply the same rule across time. Since voters have only bounded rationality in real world, to focus on this simple class of strategy is realistic.

Next, a biased elite's equilibrium strategy is assumed to satisfy both symmetry and stationarity.

Assumption 2 *A biased elite's equilibrium strategy must satisfy the following. Both a right biased elite and a left biased elite, whose degrees of bias* β *are the same, choose policies* x_t *whose* $|x_t - \hat{x}_t|$ *are the same. Also, a biased elite chooses* x_t *whose* $|x_t - \hat{x}_t|$ *is the same across time unless s/he has never observed the deviation from the equilibrium strategies since s/he became the representative.*

Since there is adverse selection, the voter must infer the incumbent's degree of the bias only through the observed $|x_t - \hat{x}_t|$. If the stationarity is not satisfied, inference becomes quite difficult. To avoid such difficulty, the stationarity is important. Assumptions 1 and 2 imply stationary equilibria. In this sense, these two restrictions can be regarded as equilibrium refinement.

Lastly, consider the voter's belief on off-equilibrium paths. In order to eliminate equilibria whose off-equilibrium belief is not plausible, I impose one restriction on the voter's belief.

Assumption 3 *The voter's belief that constitutes an equilibrium must satisfy the following. Suppose that the incumbent is a biased elite. When the voter observes* d*, which cannot be observed on the equilibrium path, for the first time, the voter believes that the incumbent elite's degree of the bias* β *is* $\min\{d, \bar{\beta}\}$*.*

This is about the belief about the incumbent when the the voter observes off-equilibrium policy mismatch. If the voter observes off-equilibrium policy mismatch d, which is smaller than or equal to $\bar{\beta}$, the voter should believe that the incumbent's degree of the bias is d. This is verified by assuming that with very small fraction, there is an extremely self-interested biased elite who always implements her/his own ideal policy. When the observed off-equilibrium policy mismatch d is larger than $\bar{\beta}$, there is no biased elite whose bias is d. In that case, this assumption requires that the voter believes that the degree of the bias is $\bar{\beta}$ which is closest to d.

4.2.2 Strategy

Given these restrictions, I derive the optimal strategies of the voter and politicians. First, I obtain the following lemma straightforwardly.

Lemma 2 *Suppose that there is an equilibrium which satisfies Assumptions 1-3. Denote the voter's payoff from period* 0*, when the voter elects a biased elite as the representative in period 0, and players follow the equilibrium strategies after the period 0 election, by* V*. Then,*

1. *the voter's payoff from period* $t \geq 1$ *when the voter votes for a new biased elite, who is an alternative candidate, in period* t*, and players follow the equilibrium strategies after the period t election, and*

2. *the voter's payoff from period* $t \geq 1$ *when the voter reelects the incumbent biased elite, whose implemented policy mismatch has never been observed in period* t*, and players follow the equilibrium strategies after the period t election*

are also V*.*

Therefore, if it is optimal for the voter to choose a biased elite as the representative in period 0, to choose a new biased elite or the incumbent biased elite as the representative is always better for the voter than to choose an unbiased non-elite. Thus, populism never emerges, in the sense that an unbiased non-elite is never elected to be the representative in every period, if and only if $V \geq$ (2) holds. For now, $V \geq$ (2) is assumed.

Here, I impose the following assumption.

Assumption 4 *The following inequality holds:*

$$\max\left\{\int_0^{\bar{\beta}} \beta dG \middle| G \in \text{core}(\theta)\right\} > 2(1-\phi)\int_0^{\infty} \hat{x}_t dF.$$

Suppose that every biased elite chooses her/his own ideal policy when s/he is elected as the representative, and the voter cannot replace the incumbent by another candidate. This is the worst scenario. Then, the expected payoff when the voter continues to elect a biased elite as the representative is

$$\frac{1}{1-\delta}\min\left\{-\int_0^{\bar{\beta}} \beta dG \middle| G \in \text{core}(\theta)\right\}. \tag{3}$$

If it is optimal for the voter to vote for a biased elite even under this worst scenario, the following analysis is meaningless since populism never arises. Thus, suppose that (3)< (2). This is Assumption 4. This holds when ϕ is large enough.

The first step to obtain V is to derive the equilibrium strategy of the voter. On this issue, I obtain the following lemma.

Lemma 3 *Every outcome sustained as equilibria satisfying Assumptions 1-4 (if exists) can be constructed by the voting strategy r having the following property: there is* $\beta^* \in [0, \bar{\beta})$ *such that the voter reelects the incumbent biased elite if* $d_{\tau^*(t)} \leq \beta^*$ *and does not reelect the incumbent biased elite if* $d_{\tau^*(t)} > \beta^*$.

Therefore, without loss of generality, I focus on the threshold voting strategy such that the voter reelects the incumbent biased elite if $d_{\tau^*(t)} \leq \beta^*$ and does not reelect her/him if $d_{\tau^*(t)} > \beta^*$.

The next step is to derive the strategy of a biased elite given the voter's strategy. Who has an incentive to implement a policy such that $|x_t - \hat{x}_t| \leq \beta^*$? Obviously, a biased elite, whose $\beta \leq \beta^*$, has an incentive to do so since s/he can be reelected after choosing his/her ideal policy $\hat{x}_t + (-)\beta$. In addition, even a biased elite, whose β is larger than β^*, may have an incentive to implement policy mismatch β^* for reelection since the benefit of reelection ρ exists. This incentive exists if and only if

$$\frac{\rho - (\beta - \beta^*)}{1-\delta} \geq \rho + \delta(1-q)\frac{\rho - (\beta - \beta^*)}{1-\delta} \Leftrightarrow \beta \leq \beta^{**} \equiv \beta^* + \frac{q\delta\rho}{1-(1-q)\delta}.$$

Since all the probability measures contained in core(θ) do not have atom at the point of β^{**} from the assumption on θ, whether a biased elite, whose degree of the bias is β^{**}, chooses the compromised policy mismatch β^* does not affect equilibrium outcome. Thus, I assume that such a biased elite chooses the policy mismatch β^*. Let β^{***} be $\min\left\{\beta^{**}, \bar{\beta}\right\}$. From the discussion above, a biased elite whose $\beta \in (\beta^*, \beta^{***}]$ will implement policy mismatch β^*. In summary, I obtain the following lemma.

Lemma 4 *A right (left) biased elite whose degree of the bias is* β *follows the strategy below:*

$$x_t = \begin{cases} \hat{x}_t + (-)\beta \ \ (\beta \in [0, \beta^*]) \\ \hat{x}_t + (-)\beta^* \ \ (\beta \in (\beta^*, \beta^{***}]) \\ \hat{x}_t + (-)\beta \ \ (\beta \in (\beta^{***}, \bar{\beta}]) \end{cases}.$$

Note that the discussion above does not depend on whether the voter reelects the incumbent biased elite when $d_{\tau^*(t)} = \emptyset$. To reelect the incumbent and to elect a new biased elite as the representative are indifferent for the voter since both payoffs are V from Lemma 2. Also, in either case, a biased elite's strategy is described by Lemma 4. Therefore, whether the voter reelects the incumbent biased elite when $d_{\tau^*(t)} = \emptyset$ does not affect equilibrium outcome. Due to this reason, I do not specify whether the voter reelects the incumbent biased elite when $d_{\tau^*(t)} = \emptyset$.

These derived strategies of the voter and a biased elite has a common feature with those derived in the previous literature of dynamic elections with information asymmetry though the details of the setting differ. Lastly, the following lemma is obtained.

Lemma 5 *If an equilibrium that satisfies Assumptions 1-4 exists, the following relationship holds in the equilibrium:* $-\frac{\beta^*}{1-\delta} = V$.

4.2.3 Belief

The next step is to derive the voter's belief. I specify the belief when the incumbent is a biased elite as follows:

1. When $d_{\tau^*(t)} = \emptyset$, $\theta'_{t,s^{t-1}} = \theta$.
2. When $d_{\tau^*(t)} = \beta \in [0, \bar{\beta}] \setminus \{\beta^*\}$, $\theta'_{t,s^{t-1}}(\{\beta\}) = 1$.
3. When $d_{\tau^*(t)} = \beta^*$, $\theta'_{t,s^{t-1}}([\beta^*, \beta^{***}]) = 1$.
4. When $d_{\tau^*(t)} = \beta \in (\bar{\beta}, \infty)$, $\theta'_{t,s^{t-1}}(\{\bar{\beta}\}) = 1$.

Consider the belief on the equilibrium path. 1 must hold since there is no information for updating. Also, when the voter has ever observed $\beta \in [0, \beta^*)$ or $\beta \in (\beta^{***}, \bar{\beta}]$ since the incumbent won the seat, the voter must believe that the incumbent's degree of the bias is β from the politician's strategy. 2 includes this. In addition, when the voter has ever observed β^*, the voter must believe that the incumbent's degree of the bias is in $[\beta^*, \beta^{***}]$ from the politician's strategy. 3 includes this. Note that I specified off equilibrium belief arbitrary. Although there are other off equilibrium beliefs, it does not matter in terms of equilibrium outcome so long as they are consistent with Assumptions 1-4.

There is one remark on the belief specified in 3. In 3, I specify only $\theta'_{t,s^{t-1}}([\beta^*, \beta^{***}])$ and do not specify $\theta'_{t,s^{t-1}}(A)$ for $A \subset [\beta^*, \beta^{***}]$. This is because which β among $[\beta^*, \beta^{***}]$ is the true degree of the bias of the incumbent is not payoff relevant for the voter. Since the voter receives the same payoff whatever value the true degree of the bias of the incumbent takes among $[\beta^*, \beta^{***}]$, the voter only uses $\theta'_{t,s^{t-1}}([\beta^*, \beta^{***}])$ when calculating her/his own payoff. Thus, this is enough.

Therefore, the payoff relevant information on the incumbent's degree of the bias is perfectly revealed or completely not revealed in candidates of equilibrium. It is well-known that rectangularity holds in such a case (Epstein and Schneider 2003a). Thus, given this belief and a biased elite's strategy, rectangularity holds. From this observation, the following lemma is obtained.

Lemma 6 *The set of equilibrium outcomes satisfying Assumptions 1-4 when the payoff is defined recursively is equivalent to that when the payoff is non-iterated one.*

In general, the iterated payoff is not necessarily equivalent to the non-iterated payoff. However, whichever payoff is employed, equilibrium outcome is the same because rectangularity holds when the properties of stationary equilibria are satisfied. This is the verification for using the recursive payoff. Thanks to this, dynamic programming methods can be used.

4.2.4 Payoff

From the discussion on strategies and beliefs, I obtain the following Bellman equation:

$$\begin{aligned} V = \min \Bigg\{ & \left[-\int_0^{\beta^*} \beta dG - \int_{\beta^*}^{\beta^{***}} \beta^* dG - \int_{\beta^{***}}^{\bar{\beta}} \beta dG \right] \\ & + \delta(1-q)V + \delta q \left[-\frac{1}{1-\delta} \int_0^{\beta^*} \beta dG - \frac{1}{1-\delta} \int_{\beta^*}^{\beta^{***}} \beta^* dG + \int_{\beta^{***}}^{\bar{\beta}} V dG \right] \Bigg| G \in \text{core}(\theta) \Bigg\}. \end{aligned} \tag{4}$$

Without loss of generality, focus on period 0. The expected payoff in period 0 by electing a biased elite as the representative is the first term. If the elected elite's degree of the bias is $\beta \notin [\beta^*, \beta^{***}]$, the elite just chooses her/his own ideal policy from Lemma 4. Thus, the loss for the voter is β. When the elected elite's degree of the bias is $\beta \in [\beta^*, \beta^{***}]$, the elite chooses policy β^* for reelection. Thus, the loss for the voter is β^*. The second and third terms represent the expected payoff from period 1. With probability $1-q$, the voter cannot observe the implemented policy mismatch. In this case, the voter reelects the incumbent biased elite or elects a new biased elite as the representative. Then, the expected payoff from period 1 is V from Lemma 2. This is the second term. On the other hand, with probability q, the voter observes the implemented policy mismatch. This is the third term. When the observed policy mismatch is smaller than or equal to β^*, the voter believes that the incumbent biased elite will continue to choose the same policy mismatch, and the voter reelects her/him from Lemmas 3 and 4. When the observed policy mismatch is larger than β^*, the voter replaces the incumbent by a new biased elite. In this case, the expected payoff is V from Lemma 2.

Note that in general, the equation $\int_a^c f(\beta) d\theta = \int_a^b f_1(\beta) d\theta + \int_b^c f_2(\beta) d\theta$ does NOT hold when $f(\beta) = f_1(\beta)$ if $\beta \in [a, b)$, and $f(\beta) = f_2(\beta)$ if $\beta \in (b, c]$ since the integral is Choquet integral. On the other hand, $\int_a^c f(\beta) d\theta = \min \left\{ \int_a^b f_1(\beta) dG + \int_b^c f_2(\beta) dG \middle| G \in \text{core}(\theta) \right\}$ holds from relationship (1). Therefore, the expression by Maximin expected utility is employed in the above.

4.3 Equilibrium

I derive an equilibrium. Since $V = -\frac{\beta^*}{1-\delta}$ holds from Lemma 5, equation (4) is equivalent to

$$-\frac{\beta^*}{1-\delta} = \min\left\{-\int_0^{\beta^*}\beta dG - \int_{\beta^*}^{\beta^{***}}\beta^* dG - \int_{\beta^{***}}^{\bar{\beta}}\beta dG + \delta q\left[-\frac{1}{1-\delta}\int_0^{\beta^*}\beta dG - \frac{1}{1-\delta}\int_{\beta^*}^{\bar{\beta}}\beta^* dG\right]\middle| G \in \mathrm{core}(\theta)\right\}$$
$$-\delta(1-q)\frac{\beta^*}{1-\delta}. \tag{5}$$

Thus, I finally obtain β^* and then V by solving equation (5).

The remained task is to show the existence of unique β^*. So far, I have assumed that there is an equilibrium satisfying Assumptions 1-4. However, such an equilibrium may not exist i.e., β^* satisfying equation (5) may not exist. Also, even if there is such β^*, there may be multiple equilibria since the solution to equation (5) may be multiple. Denote

$$h(\tilde{\beta}) = -(1-\delta(1-q))\frac{\tilde{\beta}}{1-\delta} - \min\left\{-\int_0^{\tilde{\beta}}\beta dG - \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta},\bar{\beta}\right\}}\tilde{\beta} dG - \int_{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta},\bar{\beta}\right\}}^{\bar{\beta}}\beta dG\right.$$
$$\left. + \delta q\left[-\frac{1}{1-\delta}\int_0^{\tilde{\beta}}\beta dG - \frac{1}{1-\delta}\int_{\tilde{\beta}}^{\bar{\beta}}\tilde{\beta} dG\right]\middle| G \in \mathrm{core}(\theta)\right\}. \tag{6}$$

This is the left-hand side minus the right-hand side of equation (5). When $h(\tilde{\beta}) = 0$, $\tilde{\beta}$ is equal to β^* that satisfies equation (5). I prove several lemmas about the properties of $h(\tilde{\beta})$.

Lemma 7 *$h(\tilde{\beta})$ is a decreasing function for $\tilde{\beta} \in \left[0, \bar{\beta} - \frac{\delta q\rho}{1-(1-q)\delta}\right]$, and $h(\tilde{\beta}) < 0$ holds for $\tilde{\beta} \in \left[\bar{\beta} - \frac{\delta q\rho}{1-(1-q)\delta}, \bar{\beta}\right]$.*

From the second part of this lemma, $\beta^* \in \left[0, \bar{\beta} - \frac{\delta q\rho}{1-(1-q)\delta}\right)$. Also, since $h(\tilde{\beta})$ is monotonically decreasing with $\tilde{\beta} \in \left[0, \bar{\beta} - \frac{\delta q\rho}{1-(1-q)\delta}\right]$, there is only a unique solution if β^* such that $h(\tilde{\beta}) = 0$ exists.

The next lemma is about the continuity of $h(\tilde{\beta})$.

Lemma 8 *$I(\tilde{\beta}) = \min\left\{J(\tilde{\beta}, G)\middle| G \in \mathrm{core}(\theta)\right\}$ is continuous with respect to $\tilde{\beta} \in (0, \bar{\beta})$ if J is continuous function.*

When core(θ) is a singleton, it is obvious that $h(\tilde{\beta})$ is continuous. However, its continuity is not necessarily obvious when core(θ) is not a singleton. Using this lemma, I have the continuity of $h(\tilde{\beta})$. The first term of h is obviously continuous. Thus, I can focus on the second term. For the second term of $h(\tilde{\beta})$,

$$J(\tilde{\beta}, G) = -\int_0^{\tilde{\beta}}\beta dG - \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta},\bar{\beta}\right\}}\tilde{\beta} dG - \int_{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta},\bar{\beta}\right\}}^{\bar{\beta}}\beta dG + \delta q\left[-\frac{1}{1-\delta}\int_0^{\tilde{\beta}}\beta dG - \frac{1}{1-\delta}\int_{\tilde{\beta}}^{\bar{\beta}}\tilde{\beta} dG\right].$$

Since $G \in \text{core}(\theta)$ is a continuous distribution function from the assumption, $J(\tilde{\beta}, G)$ is obviously continuous. Thus, the second term is also continuous from Lemma 8. In summary, $h(\tilde{\beta})$ is continuous. This property is used in order to show that a solution to $h(\tilde{\beta}) = 0$ exists.

Using Lemmas 7 and 8, I obtain the following result about the existence of unique β^*.

Lemma 9 *There always exists a unique* $\beta^* \in \left(0, \bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}\right)$ *that satisfies equation (5).*

Finally, I succeeded in deriving β^*, and as a result V. Then, I obtain the following theorem.

Theorem 1 ***(Equilibrium)***

Under Assumptions 1-4, (i) the voter elects not an unbiased non-elite but a biased elite in every period as the representative in any equilibria, and such an equilibrium exists if for β^* *that satisfies equation (5)*

$$\beta^* \leq \bar{\beta}^* \equiv 2(1-\phi) \int_0^\infty \hat{x}_t dF \tag{7}$$

holds, and (ii) otherwise, there exists no such equilibrium.

Under Assumptions 1-4, in any equilibria, where the voter elects a biased elite as the representative in every period, the voter's strategy must be described by Lemma 2, and a biased elite's strategy must be described by Lemma 3. And, β^* which characterizes this possible equilibrium is derived by solving $h(\tilde{\beta}) = 0$. This is what I have done so far. Therefore, the necessary and sufficient condition under which the voter votes for a biased elite is that V, which is equal to $-\frac{\beta^*}{1-\delta}$, is larger than or equal to the payoff when the voter elects an unbiased non-elite in every period as the representative (i.e., (2)). Condition (7) can be obtained by rewriting this condition.

Populism does not emerge if and only if in every period the voter does not vote for an unbiased non-elite. Thus, condition (7) is the condition that determines whether populism arises or not. When the value of β^* exceeds $\bar{\beta}^*$, populism arises. Otherwise, populism does not arise.

5 Monitoring Ability

In this section, I examine the effect of monitoring ability q on the emergence of populism.

To begin with, I obtain the following lemma.

Lemma 10 β^* *is decreasing with* q*.*

Therefore, as monitoring ability q increases, V becomes larger. This is because the two agency problems are mitigated by high monitoring ability. The first one is the moral hazard problem. The voter controls the incumbent biased elite by taking the strategy such that the voter does not reelect the incumbent if the observed policy mismatch is larger than β^*. Due to this, biased elites, whose bias is between β^* and β^{**}, choose policy mismatch β^*. The higher q is, the larger β^{**} is since the incumbent biased elite has less incentive for deviation. This is the first positive effect of an increase in q on the voter's payoff. The second one is the adverse selection problem. The voter may choose a high biased elite as the representative. When monitoring ability is high, the voter can detect the incumbent biased elite, whose β is high, with high probability. This mean that even if the voter elects a highly biased elite as the representative, s/he can replace this elite by a new biased elite with high probability. Thus, the concern about electing a highly biased elite becomes smaller as q increases. This is the second positive effect. Through these two paths, the value of voting for a biased elite increases with q.

Define q, which is nonnegative and where the solution to equation (5) is $\bar{\beta}^*$, by $\underline{q}^*$. Here, $\underline{q}^*$ is not necessarily in $(\underline{q}, \bar{q})$. Thus, let $\underline{q}^{**} \equiv \min\{\max\{\underline{q}, \underline{q}^*\}, \bar{q}\}$. From the definition of $\bar{\beta}^*$ and $\underline{q}^{**}$, and Lemma 10, I obtain the following proposition.

Proposition 1 ***(Effect of Monitoring Ability)***

There is unique $\underline{q}^{**}$.

(i) When $\underline{q}^{**} = \bar{q}$, condition (7) always does not hold.

(ii) When $\underline{q}^{**} \in (\underline{q}, \bar{q})$, condition (7) holds if and only if $q \in (\underline{q}^*, \bar{q})$.

(iii) When $\underline{q}^{**} = \underline{q}$, condition (7) always holds.

Thus, the monitoring ability must be high so that populism does not arise. Since q represents monitoring ability of the mass media, this proposition suggests that distrust of the mass media induces populism. This is consistent with the current situation where populism arises and trust for the mass media is undermined. Indeed, it has been pointed out that trust of the pubic for the mass media has been decreasing over time (Ladd 2011; Pew 2011).

Note that $\underline{q}^{**}$ represents the least requirement of the monitoring ability to prevent populism. Thus, a decrease in $\underline{q}^{**}$ means that populism is less likely to arise. In the next section, $\underline{q}^{**}$ is used as an index to measure the likelihood of the emergence of populism.

6 An Increase in Uncertainty

In this section, I examine how an increase in uncertainty on a biased elite's degree of the bias affects the emergence of populism by examining the effect on $\underline{q}^{**}$.

6.1 Effect of an Increase in Risk

I analyze the effect of an increase in uncertainty in the sense of *risk*. For this purpose, I employ a standard notion that measures the degree of risk: *mean-preserving spread*.

In the case of risk, the capacity θ is additive. Denote the additive capacity (i.e., probability measure) by G. I compare two probability distributions G_1 and G_2, and assume that both G_1 and G_2 are differentiable. The density function of each distribution is denoted by g_1 and g_2 respectively.

To begin with, I have a well-known property of a mean-preserving spread.

Lemma 11 *Suppose that probability distribution G_1 is a mean-preserving spread of probability distribution G_2. Then, for any $\tilde{\beta} \in [0, \bar{\beta}]$,*

$$\int_0^{\tilde{\beta}} G_1(\beta)d\beta \geq \int_0^{\tilde{\beta}} G_2(\beta)d\beta. \tag{8}$$

Since G_1 is a mean-preserving spread of G_2, G_2 second order stochastically dominates G_1. The property in Lemma 11 is the definition of the second order stochastically dominance.

The next lemma provides us with the alternative representation of $h(\tilde{\beta})$. Since the cumulative distribution function is assumed to be differentiable, this expression is possible.

Lemma 12 *$h(\tilde{\beta})$ can be rewritten as follows:*

$$h(\tilde{\beta}) = -\tilde{\beta} + \int_0^{\tilde{\beta}} \beta dG + \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta}, \bar{\beta}\right\}} (\beta - \tilde{\beta})dG - \frac{\delta q}{1-\delta}\int_0^{\tilde{\beta}} G(\beta)d\beta. \tag{9}$$

Using the preceding lemmas, I derive the proposition about the effect of an increase in risk.

Proposition 2 ***(Effect of an Increase in Risk)***

Suppose that probability distribution G_1 is a mean-preserving spread of probability distribution G_2.

(a) Suppose that inequality (8) holds with strong inequality when $\tilde{\beta} = \bar{\beta}^*$. Then, there is $\bar{\rho} > 0$ such that for $\rho \in (0, \bar{\rho})$, $\underline{q}^{**}(G_1) \leq \underline{q}^{**}(G_2)$.

(b) Suppose that both G_1 and G_2 are symmetric and unimodal distributions, and there is $\hat{\beta} \in (0, \bar{\beta})$ such that $g_1(\beta) > g_2(\beta)$ if $\beta \in (\hat{\beta}, \bar{\beta}/2)$, and $g_1(\beta) \leq g_2(\beta)$ if $\beta \in [0, \hat{\beta}]$.[30] *Then, $\underline{q}^{**}(G_1) \leq \underline{q}^{**}(G_2)$ holds if (i) $\hat{\beta} \leq \bar{\beta}^* \leq \bar{\beta}^* + \frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta} \leq \bar{\beta} - \hat{\beta}$, or (ii) $\hat{\beta} \geq \bar{\beta}^*$, and $\bar{\beta} - \hat{\beta} \leq \bar{\beta}^* + \frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta} \leq \bar{\beta} - \bar{\beta}^*$ is satisfied.*

It seems that the more uncertain an elite's bias is, the more reluctant the voter is to vote for the elite. However, Proposition 2 (a) argues that so long as uncertainty is risk, this is not the case

[30] This trivially holds when both are truncated normal distributions.

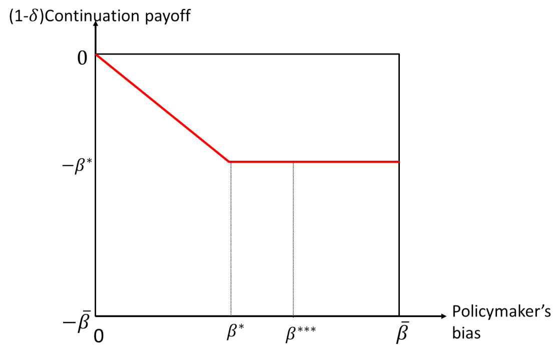

Figure 1: Continuation Payoff

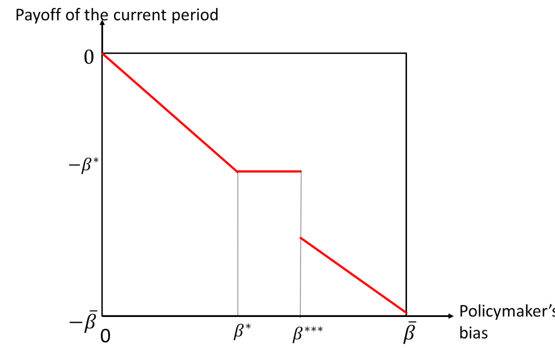

Figure 2: Payoff of the Current Period

when the reward and punishment mechanism to incentivize biased elites is limited (i.e., ρ is small). Rather, as the degree of risk increases, populism is less likely to arise. Since whether populism emerges matters only when it is difficult for the voter to control a biased elite (i.e., when the voter faces the severe agency problem), the result when ρ is small is meaningful.

The mechanism behind this result is as follows. Focus on the expected continuation payoff from period $t+1$ that is evaluated at period t given that the voter can observe the policy mismatch. That is

$$-\frac{1}{1-\delta}\int_0^{\beta^*}\beta dG-\frac{1}{1-\delta}\int_{\beta^*}^{\beta^{***}}\beta^* dG+\int_{\beta^{***}}^{\bar{\beta}}VdG=-\frac{1}{1-\delta}\int_0^{\beta^*}\beta dG-\frac{1}{1-\delta}\int_{\beta^*}^{\bar{\beta}}\beta^* dG$$

from equation (4) and Lemma 3. Thus, the payoff function of the continuation payoff when the policymaker in period t's degree of the bias is β is that in Figure 1. This function is convex. Thus, the voter can behave as if s/he was risk-lover i.e., the voter prefers more risky situation. As a result, an increase in risk can make the value of electing a biased elite larger. This convexity is created by the possibility of replacement. The voter can replace the incumbent by new one if the voter finds out that s/he is highly biased. Thus, even if the voter elects a highly biased elite whose degree of the bias is $\beta \in (\beta^{***}, \bar{\beta}]$, the voter can obtain V as the continuation payoff. This creates the convexity. In summary, the possibility of replacement thanks to the nature of dynamic elections makes populism less likely to arise after an increase in risk.

So far, I have focused on the effect on the continuation payoff. An increase in risk also affects the payoff of the current period. Figure 2 desribes the payoff function of the current period when the voter elects a elite whose degree of bias is β. Note that this is obtained from Lemma 4. In contrast to Figure 1, this payoff function is not convex. Thus, how a mean-preserving spread affects the expected payoff in the current period is not clear. There may be the negative effect. After a

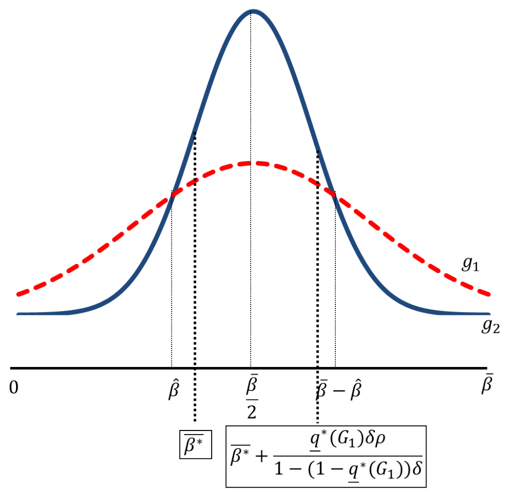

Figure 3: Condition (i) in Proposition 2 (b)

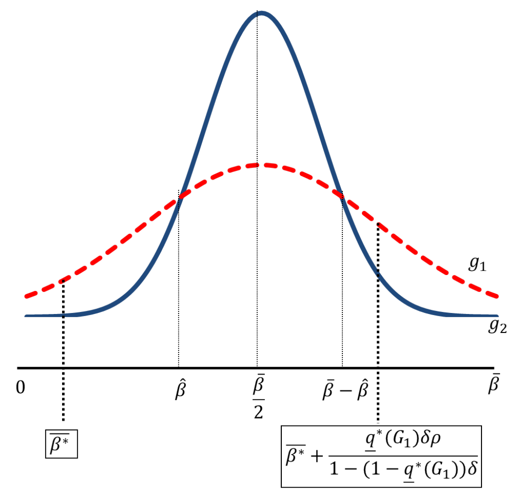

Figure 4: Condition (ii) in Proposition 2 (b)

mean-preserving spread, the probability that $\beta \in [\beta^*, \beta^{**}]$ may decrease while the probability that $\beta \in (\beta^{**}, \bar{\beta}]$ may increase. Thus, the expected payoff of the current period can be smaller after the mean-preserving spread. It depends on the distribution functions. However, when ρ is small, the probability that $\beta \in [\beta^*, \beta^{**}]$ is low since it becomes hard to control a policy implemented by a biased elite. As a result, the effect due to a decrease in the probability that $\beta \in [\beta^*, \beta^{**}]$ is negligible. Therefore, when ρ is small, the positive effect on the continuation payoff always dominates the possible negative effect on the payoff of the current period. This is the reason why sufficiently small ρ is necessary in Proposition 2 (a).

Lastly, it should be emphasized that sufficiently small ρ is a not necessary but sufficient condition. As pointed out, the effect of an increase in risk on the current period payoff is not clear. However, this does not mean that an increase in risk always decreases the current period payoff of the voter. When it increases the current payoff, an increase in risk always makes populism less likely to arise since an increase in risk makes both the current period and continuation payoffs when voting for a biased elite larger. When does such a situation arise? Proposition 2 (b) presents a simple case, where an increase in risk makes populism less likely to arise, even if ρ is not necessarily small. In the proposition, I consider symmetric distributions. Then, I show that when the values of $\bar{\beta}^*$ and $\bar{\beta}^* + \frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}$ satisfy a property, which can be seen in Figures 3 and 4, populism is less likely to arise as the degree of risk increases.

6.2 Effect of an Increase in Ambiguity

I showed that an increase in risk can make populism less likely to arise. Given this, it seems that that an increase in ambiguity also has a similar effect. Is this the case? The answer is NO. An increase in ambiguity makes populism more likely to arise.

To begin with, define an increase in ambiguity.

Definition 2 *θ_1 is more ambiguous than θ_2 if for any $A \in \mathcal{F}_B$, $\theta_1(A) \leq \theta_2(A)$ holds.*

This definition is also employed in the existing literature (Nishimura and Ozaki 2004; 2007; Miao and Wang 2011). Since both θ_1 and θ_2 are convex, this is equivalent to core(θ_1) $\supseteq$ core(θ_2). Remember relationship (1). The expansion of the core of a capacity means that the set of priors expands. Thus, this definition of an increase in ambiguity means that the set of candidates of the true distribution expands. Note that this definition includes an increase in uncertainty averse as well as that in ambiguity itself.[31] Though this should be noted as limitation, to disentangle an increase in ambiguity from that in uncertainty averse has never been succeeded in Choquet expected utility and Maxmin expected utility.[32]

Using this definition, I obtain the proposition on the effect of an increase in ambiguity.

Proposition 3 ***(Effect of an Increase in Ambiguity)***

Suppose that θ_1 is more ambiguous than θ_2. Then, $\underline{q}^{**}(\theta_1) \geq \underline{q}^{**}(\theta_2)$.

An increase in ambiguity raises the least requirement of monitoring ability $\underline{q}^{**}$ i.e., populism becomes more likely to arise. This is a contrast to the result obtained about the effect of an increase in risk. The effect of an increase in uncertainty about a biased elite's degree of the bias is totally different depending on which type of the uncertainty is involved.

Why does the result vary? The voter decides whether to reelect the incumbent e based on the evaluation about the value of electing a new biased elite. Here, remember that under ambiguity, a player evaluates the payoff using a probability measure that gives the lowest payoff among the core of a capacity. Thus, as ambiguity increases (i.e., the core of a capacity enlarges), the expected

[31]The behavioral foundation is provided by Ghirardato and Marinacci (2002). Let $\theta_1$and θ_2 be two (not necessarily convex) capacities, and let the preference relation be $\succ_i$ ($i = 1, 2$). Then, ($\forall A \in \mathcal{F}_B$) $\theta_2(A) \geq \theta_1(A)$ if and only if for any outcome x and act f, $x \succeq_2 f \Rightarrow x \succeq_1 f$ and $x \succ_2 f \Rightarrow x \succ_1 f$. They name this *more uncertainty averse*.

[32]Klibanoff, Marinacci, and Mukerji (2005: 1825) point out this problem: "such a separation is not evident in [...] the maximin expected utility [..]. and the Choquet expected utility model [.]" In smooth ambiguity model proposed by them, the separation is possible. However, in the model, people are assumed to have subjective probability over candidates of the true distribution, and in this sense, smooth ambiguity is different from a situation, where people do not have even subjective probability over candidates of the true distribution, which is my focus. Thus, I employ the framework of Choquet/Maxmin expected utility. Note that in the case of smooth ambiguity model, the same result would still hold. This can be seen from the observation such that $f(\bar{\beta}^*)$ is larger as the degree of ambiguity increases.

degree of the bias of a new biased elite becomes higher. Then, the voter is reluctant to replace the incumbent by a new biased elite even if the incumbent's degree of the bias β is high. Thus, threshold β^* weakly increases with the degree of ambiguity. Since $V = -\frac{\beta^*}{1-\delta}$ holds, V weakly decreases with the degree of ambiguity. Therefore, higher monitoring ability is necessary to prevent populism.

7 Risk-Averse Voter

So far, the voter has been assumed to be risk-neutral. In this section, I show that the same result still holds even under risk-aversion, so long as its degree is not high. Assume that the voter's payoff is $-|x_t - \hat{x}_t|^r$, where $r > 1$. Politicians' payoffs are defined similarly.

7.1 Equilibrium

To begin with, consider the voter's payoff when s/he elects an unbiased non-elite as the representative in every period. When the elected unbiased non-elite observes the value of $\hat{x}_t$, it chooses policy $\hat{x}_t$. When s/he does not observe the value of $\hat{x}_t$, it chooses policy x^* that minimizes $\int_{-\infty}^{\infty} |x_t - \hat{x}_t|^r dF$. Then, the voter's expected payoff when s/he elects an unbiased non-elite in every period is

$$-\frac{(1-\phi)}{1-\delta}\int_{-\infty}^{\infty} |x^* - \hat{x}_t|^r dF. \tag{10}$$

Assume the following corresponding to Assumption 4. Let it be Assumption 4′.

$$\max\left\{\int_0^{\bar{\beta}} \beta^r dG \middle| G \in \text{core}(\theta)\right\} > (1-\phi)\int_{-\infty}^{\infty} |x^* - \hat{x}_t|^r dF.$$

The voter's equilibrium strategy is the same as that in the basic model since it does not depend on $r = 1$. The only one change from the basic model is β^{**}. A biased elite, whose degree of the bias is β, has an incentive to choose policy mismatch β^{**} if and only if

$$\frac{\rho - (\beta - \beta^*)^r}{1-\delta} \geq \rho + \delta(1-q)\frac{\rho - (\beta - \beta^*)^r}{1-\delta} \Leftrightarrow \beta \leq \beta^{**} \equiv \beta^* + \left(\frac{q\delta\rho}{1-(1-q)\delta}\right)^{\frac{1}{r}}.$$

Given this, the correspondence to $h(\tilde{\beta})$ in the basic model is

$$h(\tilde{\beta}) = -(1-\delta(1-q))\frac{\tilde{\beta}^r}{1-\delta} - \min\left\{ -\int_0^{\tilde{\beta}} \beta^r dG - \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\left(\frac{q\delta\rho}{1-(1-q)\delta}\right)^{\frac{1}{r}}, \bar{\beta}\right\}} \tilde{\beta}^r dG \right.$$
$$\left. - \int_{\min\left\{\tilde{\beta}+\left(\frac{q\delta\rho}{1-(1-q)\delta}\right)^{\frac{1}{r}}, \bar{\beta}\right\}}^{\bar{\beta}} \beta^r dG + \delta q \left[-\frac{1}{1-\delta}\int_0^{\tilde{\beta}} \beta^r dG - \frac{1}{1-\delta}\int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta}^r dG \right] \middle| G \in \text{core}(\theta) \right\}.$$

Then, using the same procedure as in Lemma 7, the existence of unique $\beta^* \in \left(0, \bar{\beta} - \left(\frac{\delta q\rho}{1-(1-q)\delta}\right)^{\frac{1}{r}}\right)$, which is the solution to $h(\tilde{\beta}) = 0$, is shown. Finally, the correspondence to Theorem 1 is obtained.

Theorem 2 ***(Equilibrium under Risk-Aversion)***

Under Assumptions 1-4′, (i) the voter elects not an unbiased non-elite but a biased elite in every period as the representative in any equilibria, and such an equilibrium exists if for β^* that satisfies $h(\tilde{\beta}) = 0$,

$$\beta^* \leq \bar{\beta}^* \equiv (1-\phi)\int_{-\infty}^{\infty} |x^* - \hat{x}_t|^r dF \tag{11}$$

holds, and (ii) otherwise, there exists no such equilibrium.

7.2 Effect of an Increase in Risk

As in Section 5, define $\bar{\beta}^*$, $\underline{q}^*$, and $\underline{q}^{**}$. Then, the result corresponding to Proposition 1 is obtained. Given this, analyze the effect of an increase in risk.

Proposition 4 ***(Effect of an Increase in Risk under Risk-Aversion)***[33]

Suppose that probability distribution G_1 is a mean-preserving spread of probability distribution G_2, and inequality (8) holds with strong inequality when $\tilde{\beta} = \bar{\beta}^*$. Also, assume $\rho \in (0, \bar{\rho})$, where $\bar{\rho}$ is defined in Proposition 1. Then, there is $\bar{r} > 1$ such that for any $r \in (1, \bar{r})$, $\underline{q}^{**}(G_1) \leq \underline{q}^{**}(G_2)$ holds.

An increase in risk can encourage the voter to elect a biased elite as the representative even when the voter hates risk. When the degree of risk-aversion is not large, the positive effect of an increase in risk, which was obtained in Section 6.1, dominates the negative effect due to risk-aversion. As a result, an increase in risk makes populism less likely to arise.

[33]Though I showed only the correspondence to (a), the correspondence to (b) can be obtained in a similar way.

8 Concluding Remarks

In the present paper, populism is defined as a phenomenon such that voters vote for a politician, who does not have enough ability, but is not biased, instead of a biased elite. Given this concept of populism, I constructed an infinite horizon model, where the representative voter chooses a policymaker at the beginning of each period, and the elected politician implements a policy. Then, I analyzed how an increase in uncertainty on an elite's degree of the bias affects the emergence of populism. An increase in risk can make populism less likely to arise. In contrast, an increase in ambiguity makes populism more likely to arise. This result suggests that an increase in ambiguity rather than risk is a crucial source of populism.

Before closing this paper, let me see the remained challenges for the future researches. First, I focused on stationary equilibria. How the result changes, if non-stationary equilibria are taken into account, is an important question. Second, it may be promising to analyze learning process profoundly by assuming that the probability distribution is identical across time.

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Appendices

A Assumptions on Capacity

In Section 3.2.1, the several assumptions are imposed on the capacity θ. In this section, I explain the details about the assumptions.

The capacity θ is continuous if the following two conditions hold:

$$(\forall \langle A_i \rangle_i \subseteq \mathcal{F}_B) \;\; A_1 \subseteq A_2 \subseteq A_3 \subseteq \dots \Rightarrow \theta(\cup_i A_i) = \lim_{i \to \infty} \theta(A_i).$$

$$(\forall \langle A_i \rangle_i \subseteq \mathcal{F}_B) \;\; A_1 \supseteq A_2 \supseteq A_3 \supseteq \dots \Rightarrow \theta(\cap_i A_i) = \lim_{i \to \infty} \theta(A_i).$$

One example, where continuity does not hold, is $\varepsilon-$contamination, whose axiomatic foundation is given by Nishimura and Ozaki (2006) and Kopylov (2009): for any $A \in \mathcal{F}_B$,

$$\theta(A) = \begin{cases} (1-\varepsilon)P_0(A) \;\; (A \neq B) \\ 1 \;\; (A = B) \end{cases},$$

where $\varepsilon \in (0,1)$ and P_0 is a probability measure.

However, it should be emphasized that a non-continuous capacity can be approximated using a continuous capacity. To see this, consider the following approximation of $\varepsilon-$contamination, which is called $\delta-$approximation of $\varepsilon-$contamination and is provided by Nishimura and Ozaki (2004): for any $A \in \mathcal{F}_B$,

$$\theta_\delta(A) = \begin{cases} (1-\varepsilon)P_0(A) \;\; (P_0(A) \leq 1-\delta) \\ (1-\varepsilon)P_0(A) + \varepsilon[(P_0(A)-1)/\delta + 1] \;\; (P_0(A) > 1-\delta) \end{cases}.$$

(δ is different from the discount factor δ defined in Section 3.1.4.) When δ is sufficiently small, this capacity is an approximation of ε-contamination. And, this capacity satisfies continuity. In this sense, continuity is not that restrictive.

Lastly, I show one example which satisfies the assumption that all the probability distribution functions contained in the core of θ are continuous. The example is $\delta-$approximation of $\varepsilon-$contamination discussed above. In this case, the core of θ can be written as

$$\text{core}(\theta) = \left\{ (1-\varepsilon)P_0 + \varepsilon\mu \middle| \mu \in \mathcal{M}(P_0, \delta) \right\},$$

where

$$\mathcal{M}(P_0,\delta) = \left\{\mu \in \mathcal{M} \middle| (\forall A) \ \ \delta\mu(A) \le P_0(A)\right\}.$$

Note that $\mathcal{M}$ is the set of all probability measures. Therefore, all the probability measures contained in core(θ) assign the zero probability to any single point (i.e., continuous distribution function) so long as $\delta > 0$ and P_0 assigns the zero probability.

B Omitted Proofs

B.1 Proof of Lemma 1

From the argument, the expected payoff in a stage game when electing an unbiased non-elite as the representative in every period is $-2\int_0^\infty \hat{x}_t dF$. Thus, (2) obtains. ■

B.2 Proof of Lemma 2

(Strategy) A biased elite's strategy is the same across time from Assumption 2.

(Belief) From the capacity specified in Section 3.2.2, the beliefs about (i) a new biased elite, who is an alternative candidate, and (ii) the incumbent biased elite, whose implemented policy has never been observed are equal to θ. This is the same as the belief about a biased elite in period 0. In addition, how the belief about the biased elite is updated after period t is the same as that in period 0 since the initial capacity is the same, and a biased elite's strategy is also the same.

Therefore, the voter's payoff must be the same. ■

B.3 Proof of Lemma 3

Step. 1: The voter (does not) reelects the incumbent biased elite if $d_{\tau^*(t)} < \beta^*$ ($d_{\tau^*(t)} > \beta^*$)

Any strategy, such that there is no $d_{\tau^*(t)} > 0$ where the voter reelects the incumbent biased elite, trivially satisfies the property. Thus, I focus on a strategy such that there is $d_{\tau^*(t)} > 0$ where the voter reelects the incumbent biased elite. Denote such $d_{\tau^*(t)}$ by d^*.

(i) Case where $d^* \in (0, \bar{\beta}]$

I show that for any $d_{\tau^*(t)} \in [0, d^*]$, the voter reelects the incumbent biased elite on the equilibrium.

Since a biased elite can be reelected after implementing a policy such that $|x_t - \hat{x}_t| = d^*$, a biased elite whose bias β is d^* chooses a policy such that $|x_t - \hat{x}_t| = d^*$ on the equilibrium. Given this, from Assumption 2, d^* can be observed on the equilibrium path, and the voter expects that the incumbent biased elite will implement a policy such that $|x_t - \hat{x}_t| = d^*$ forever. Thus, the voter reelects her/him only if

$$-\frac{d^*}{1-\delta} \ge V. \tag{12}$$

Consider $d < d^*$. Suppose that there is $d < d^*$ such that the voter does not reelect the incumbent biased elite when $d_{\tau^*(t)} = d$. The voter expects that the incumbent biased elite's degree of the bias β is d when $d_{\tau^*(t)} = d$. Thus, when $d_{\tau^*(t)} = d$, the voter has no incentive to deviate from the equilibrium strategy and reelect the incumbent the biased elite only if $-d + \delta V \le V$. This can be rewritten as $-\frac{d}{1-\delta} \le V$. However, this contradicts inequality (12) since $d < d^*$. Thus, for any $d_{\tau^*(t)} \in [0, d^*]$, the voter reelects the incumbent biased elite on the equilibrium.

(ii) Case where $d^* \in (\bar{\beta}, \infty)$ and d^* can be chosen on the equilibrium path

From Assumption 2, the voter expects that the incumbent biased elite will implement a policy such that $|x_t - \hat{x}_t| = d^*$ forever so long as the incumbent has never observed the deviation since s/he became the representative. Thus, the voter reelects her/him only if inequality (12) holds.

Consider $d < d^*$. Suppose that there is $d < d^*$ such that the voter does not reelect the incumbent biased elite when $d_{\tau^*(t)} = d$. The voter expects that the incumbent biased elite's degree of the bias β is $\min\{d, \bar{\beta}\}$ when $d_{\tau^*(t)} = d$. Then, using the same procedure as in (i), I can show that for any $d_{\tau^*(t)} \in [0, d^*]$, the voter reelects the incumbent biased elite on the equilibrium.

(iii) Case where $d^* \in (\bar{\beta}, \infty)$ and d^* cannot be chosen on the equilibrium path

This implies that any biased elite has no incentive to choose a policy such that $|x_t - \hat{x}_t| = d^*$. Thus, the same outcome can be sustained by the voting strategy such that the voter does not reelect the incumbent biased elite when $d_{\tau^*(t)} = d^*$. Thus, it is unnecessary to take into account the case (iii).

From (i) to (iii), every outcome sustained by equilibria satisfying Assumptions 1-4 (if exists) can be constructed by the voting strategy such that he voter (does not) reelects the incumbent biased elite if $d_{\tau^*(t)} < \beta^* (d_{\tau^*(t)} > \beta^*)$.

Step. 2: $\beta^* < \bar{\beta}$ holds

I show by contradiction. When $\beta^* \geq \bar{\beta}$, all biased elites whose $\beta \in [0, \bar{\beta})$ choose their own ideal policies if elected. Thus, the voter's payoff when s/he follows this voting strategy is (3). From Assumption 4, (3) < (2), and so in this case, such voting strategy does not constitute an equilibrium. Therefore, $\beta^* < \bar{\beta}$ holds.

Step. 3: The voter reelects the incumbent biased elite if $d_{\tau^*(t)} = \beta^*$

I show by contradiction. Suppose that the voter does not reelect the incumbent biased elite if $d_{\tau^*(t)} = \beta^*$. Since $\rho, \delta, q > 0$ hold, there are biased elites whose $\beta > \beta^*$ and who choose a policy which is smaller than β^*. However, there is no optimal policy these biased elites should choose because for any policy which is smaller than β^*, there is a policy which is closer to β^* and is better for them. Thus, there is no such equilibrium. Therefore, in the equilibrium, the voter reelects the incumbent biased elite if $d_{\tau^*(t)} = \beta^*$.

Therefore, only the strategy specified in the lemma can constitute an equilibrium. ■

B.4 Proof of Lemma 5

(i) $-\frac{\beta^*}{1-\delta} < V$ does not hold.

The voter has no incentive to deviate from the strategy on the equilibrium path when $d_{\tau^*(t)} = \beta^*$ only if $-\frac{\beta^*}{1-\delta} \geq V$. Thus, $-\frac{\beta^*}{1-\delta} < V$ does not hold.

(ii) $-\frac{\beta^*}{1-\delta} > V$ does not hold.

When $-\frac{\beta^*}{1-\delta} > V$ holds, there is $\beta' \in (\beta^*, \beta^{**})$ such that $-\frac{\beta^*}{1-\delta} > V$ does not hold. Then, from Assumptions 3, the voter expects that the incumbent biased elite's bias is β' when $d_{\tau^*(t)} = \beta'$

and s/he has never observed deviation from equilibrium. Thus, by one-shot deviation, the voter obtains the utility: $-\beta' + \delta V$. This must be smaller than or equal to V i.e., $-\frac{\beta^*}{1-\delta} \leq V$ must hold. This is contradiction.

From (i) and (ii), $-\frac{\beta^*}{1-\delta} = V$. ■

B.5 Proof of Lemma 6

To begin with, show the following argument (i): if an outcome is sustained by an equilibrium when the recursive payoff is employed, it is also sustained by an equilibrium when the non-iterated payoff is employed. Every equilibrium outcome when the recursive payoff is employed can be created using the belief system specified in Section 4.2.3, which satisfies rectangularity. And, the recursive payoff is equivalent to the non-iterated payoff given the belief system and the politicians' strategies. Therefore, (i) is shown.

Next, show the following argument: (ii) if an outcome is sustained by an equilibrium when the non-iterated payoff is employed, it is also sustained by an equilibrium when the recursive payoff is employed. Observe that the proofs of Lemmas 2, 3, and 5 do not depend on the fact that the payoff is recursive. Therefore, the characterization of equilibrium discussed in Section 4.2.2 holds even when the non-iterated payoff is employed. As a result, every equilibrium outcome when the non-iterated payoff is employed can be created using the belief system specified in Section 4.2.3, which satisfies rectangularity. And, the non-iterated payoff is equivalent to the recursive payoff given the belief system and the politicians' strategies. Therefore, (ii) is shown.

In summary, Lemma 6 is proven. ■

B.6 Proof of Lemma 7

(i) $h(\tilde{\beta})$ is a decreasing function for $\tilde{\beta} \in \left[0, \bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}\right]$

Denote

$$J(\tilde{\beta}, G|q) \equiv \int_0^{\tilde{\beta}} \beta dG + \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta}, \bar{\beta}\right\}} \tilde{\beta} dG + \int_{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta}, \bar{\beta}\right\}}^{\bar{\beta}} \beta dG + \delta q \left[\frac{1}{1-\delta}\int_0^{\tilde{\beta}} \beta dG + \frac{1}{1-\delta}\int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG\right]. \tag{13}$$

Using this, let

$$G_{\tilde{\beta}+\varepsilon} \in \arg\min \left\{-J(\tilde{\beta}+\varepsilon|q) \middle| G \in \mathrm{core}(\theta)\right\}.$$

Then, for any $\varepsilon \in \left(0, \bar{\beta} - \frac{\delta q\rho}{1-(1-q)\delta}\right)$,

$$
\begin{aligned}
&h(\tilde{\beta}+\varepsilon) - h(\tilde{\beta}) \\
&< -(1-\delta(1-q))\frac{\varepsilon}{1-\delta} + \int_0^{\tilde{\beta}+\varepsilon} \beta dG_{\tilde{\beta}+\varepsilon} + \int_{\tilde{\beta}+\varepsilon}^{\tilde{\beta}+\frac{\delta q\rho}{1-(1-q)\delta}} (\tilde{\beta}+\varepsilon) dG_{\tilde{\beta}+\varepsilon} + \int_{\tilde{\beta}+\varepsilon+\frac{\delta q\rho}{1-(1-q)\delta}}^{\bar{\beta}} \beta dG_{\tilde{\beta}+\varepsilon} \\
&\quad + \frac{\delta q}{1-\delta}\int_0^{\tilde{\beta}+\varepsilon} \beta dG_{\tilde{\beta}+\varepsilon} + \frac{\delta q}{1-\delta}\int_{\tilde{\beta}+\varepsilon}^{\bar{\beta}} (\tilde{\beta}+\varepsilon) dG_{\tilde{\beta}+\varepsilon} - \int_0^{\tilde{\beta}} \beta dG_{\tilde{\beta}+\varepsilon} - \int_{\tilde{\beta}}^{\tilde{\beta}+\frac{\delta q\rho}{1-(1-q)\delta}} \tilde{\beta} dG_{\tilde{\beta}+\varepsilon} - \int_{\tilde{\beta}+\frac{\delta q\rho}{1-(1-q)\delta}}^{\bar{\beta}} \beta dG_{\tilde{\beta}+\varepsilon} \\
&\quad - \frac{\delta q}{1-\delta}\int_0^{\tilde{\beta}} \beta dG_{\tilde{\beta}+\varepsilon} - \frac{\delta q}{1-\delta}\int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG_{\tilde{\beta}+\varepsilon} \\
&< -(1-\delta(1-q))\frac{\varepsilon}{1-\delta} + (\tilde{\beta}+\varepsilon)\left[G_{\tilde{\beta}+\varepsilon}(\tilde{\beta}+\varepsilon) - G_{\tilde{\beta}+\varepsilon}(\tilde{\beta})\right] \\
&\quad + \tilde{\beta}\left[G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\varepsilon+\frac{\delta q\rho}{1-(1-q)\delta}\right) - G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\frac{\delta q\rho}{1-(1-q)\delta}\right)\right] - \tilde{\beta}\left[G_{\tilde{\beta}+\varepsilon}(\tilde{\beta}+\varepsilon) - G_{\tilde{\beta}+\varepsilon}(\tilde{\beta})\right] \\
&\quad + \varepsilon\left[G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\varepsilon+\frac{\delta q\rho}{1-(1-q)\delta}\right) - G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\varepsilon\right)\right] \\
&\quad - \left(\tilde{\beta}+\frac{\delta q\rho}{1-(1-q)\delta}\right)\left[G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\varepsilon+\frac{\delta q\rho}{1-(1-q)\delta}\right) - G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\frac{\delta q\rho}{1-(1-q)\delta}\right)\right] \\
&\quad + \frac{\delta q}{1-\delta}(\tilde{\beta}+\varepsilon)\left[G_{\tilde{\beta}+\varepsilon}(\tilde{\beta}+\varepsilon) - G_{\tilde{\beta}+\varepsilon}(\tilde{\beta})\right] - \frac{\delta q}{1-\delta}\tilde{\beta}\left[G_{\tilde{\beta}+\varepsilon}(\tilde{\beta}+\varepsilon) - G_{\tilde{\beta}+\varepsilon}(\tilde{\beta})\right] + \frac{\delta q}{1-\delta}\varepsilon\left[1 - G_{\tilde{\beta}+\varepsilon}(\tilde{\beta}+\varepsilon)\right] \\
&= \varepsilon\left\{\left[G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\varepsilon+\frac{\delta q\rho}{1-(1-q)\delta}\right) - G_{\tilde{\beta}+\varepsilon}(\tilde{\beta})\right] + \frac{\delta q}{1-\delta}\left[1 - G_{\tilde{\beta}+\varepsilon}(\tilde{\beta})\right] - \frac{(1-\delta(1-q))}{1-\delta}\right\} \\
&\quad - \frac{\delta q\rho}{1-(1-q)\delta}\left[G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\varepsilon+\frac{\delta q\rho}{1-(1-q)\delta}\right) - G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\frac{\delta q\rho}{1-(1-q)\delta}\right)\right]. \qquad (14)
\end{aligned}
$$

The first inequality comes from the nature of min{·}. Here, the first term of (14) is negative since

$$G_{\tilde{\beta}+\varepsilon}\left(\tilde{\beta}+\varepsilon+\frac{\delta q\rho}{1-(1-q)\delta}\right) - G_{\tilde{\beta}+\varepsilon}(\tilde{\beta}) + \frac{\delta q}{1-\delta}\left[1 - G_{\tilde{\beta}+\varepsilon}(\tilde{\beta})\right] - \frac{(1-\delta(1-q))}{1-\delta} < 1 + \frac{\delta q}{1-\delta} - \frac{(1-\delta(1-q))}{1-\delta} = 0.$$

Also, the second term is obviously negative. Thus, (14)<0, and so $h(\tilde{\beta}+\varepsilon) - h(\tilde{\beta}) < 0$. ■

(ii) $h(\tilde{\beta}) < 0$ for any $\tilde{\beta} \in \left[\bar{\beta} - \frac{\delta q\rho}{1-(1-q)\delta}, \bar{\beta}\right]$

For $\tilde{\beta} \in \left[\bar{\beta} - \frac{\delta q\rho}{1-(1-q)\delta}, \bar{\beta}\right]$,

$$
\begin{aligned}
h(\tilde{\beta}) &= -(1-\delta(1-q))\frac{\tilde{\beta}}{1-\delta} - \min\left\{ -\frac{1-\delta(1-q)}{1-\delta}\left[\int_0^{\tilde{\beta}} \beta dG + \int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG\right] \middle| G \in \text{core}(\theta) \right\} \\
&= -\frac{(1-\delta(1-q))}{1-\delta}\left\{ -\tilde{\beta} + \max\left\{ \int_0^{\tilde{\beta}} \beta dG + \int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG \middle| G \in \text{core}(\theta) \right\} \right\} < 0.
\end{aligned}
$$

The last inequality holds since

$$\tilde{\beta} < \max\left\{ \int_0^{\tilde{\beta}} \beta dG + \int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG \middle| G \in \text{core}(\theta) \right\}$$

holds because of the fact that G is full support. ■

B.7 Proof of Lemma 8

Pick up a $\tilde{\beta}$ and denote it by b. Then, what I should show is that the following holds:

$$(\forall \varepsilon > 0)\ \ (\exists \gamma > 0)\ \ (\forall \tilde{\beta} \in (0, \bar{\beta}))\ \ [|\tilde{\beta} - b| < \gamma \Rightarrow |I(\tilde{\beta}) - I(b)| < \varepsilon]. \tag{15}$$

Fix $\varepsilon > 0$.

(i) When $\tilde{\beta} > b$:

First, consider this case. Define $I'_a(\tilde{\beta})$ as $J(\tilde{\beta}, G_a)$ where $G_a \in \arg\min_{G \in \text{core}(\theta)} J(a, G)$. Then, since $I'_a(\tilde{\beta})$ is obviously a continuous function,

$$(\exists \gamma > 0)\ \ (\forall \tilde{\beta} \in (0, \bar{\beta}))\ \ [|\tilde{\beta} - b| < \gamma \Rightarrow |I'(\tilde{\beta}) - I'(b)| < \varepsilon]$$

holds. Denote this γ by $\bar{\gamma}$. Here,

$$|I'(\tilde{\beta}) - I'(b)| = |I'(\tilde{\beta}) - I(b)| \geq |I(\tilde{\beta}) - I(b)|.$$

Thus,

$$(\exists \bar{\gamma} > 0)\ \ (\forall \tilde{\beta} \in (b, \bar{\beta}))\ \ [|\tilde{\beta} - b| < \gamma \Rightarrow |I(\tilde{\beta}) - I(b)| < \varepsilon].$$

(ii) When $\tilde{\beta} < b$:

Next, consider the case where $\tilde{\beta}$ is smaller than b. Since I'_a is a continuous function, there is $a < b$ such that $I'_a(a) - I'_a(b) < \varepsilon$. Here, for any $\tilde{\beta} \in (b - \underline{\gamma}, b)$ where $\underline{\gamma} = b - a$,

$$|I'_a(a) - I'_a(b)| = |I(a) - I'_a(b)| \geq |I(a) - I(b)| \geq |I(\tilde{\beta}) - I(b)|$$

holds. Thus,

$$(\exists \underline{\gamma} > 0)\ \ (\forall \tilde{\beta} \in (0, b))\ \ [|\tilde{\beta} - b| < \gamma \Rightarrow |I(\tilde{\beta}) - I(b)| < \varepsilon].$$

Combining (i) and (ii),

$$(\exists \min\{\underline{\gamma}, \bar{\gamma}\} > 0)\ \ (\forall \tilde{\beta} \in (b, \bar{\beta}))\ \ [|\tilde{\beta} - b| < \gamma \Rightarrow |I(\tilde{\beta}) - I(b)| < \varepsilon].$$

Thus, (15) holds. ■

B.8 Proof of Lemma 9

To begin with, from Lemma 7, $h(\tilde{\beta}) < 0$ holds for any $\tilde{\beta} \in \left[\bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}, \bar{\beta}\right]$. Thus, $\beta^* < \bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}$. Therefore, I focus on $\tilde{\beta} \in \left[0, \bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}\right)$.

Here, $h(\tilde{\beta})$ is decreasing with $\tilde{\beta}$ for any $\tilde{\beta} \in \left[0, \bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}\right)$ from Lemma 7. And,

$$h(0) = \max \left\{ \int_{\frac{\delta q \rho}{1-(1-q)\delta}}^{\bar{\beta}} \beta dG \middle| G \in \text{core}(\theta) \right\} > 0.$$

Thus, from the continuity of h (Lemma 8), there is a unique $\beta^* \in \left(0, \bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}\right)$ which satisfies equation (5) if and only if $h\left(\bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}\right) < 0$. Actually this holds from Lemma 7. Therefore, there is a unique $\beta^* \in \left(0, \bar{\beta} - \frac{\delta q \rho}{1-(1-q)\delta}\right)$ which satisfies equation (5). ■

B.9 Proof of Theorem 1

Since $V = -\frac{\beta^*}{1-\delta}$ holds, $V \geq (2)$ can be rewritten as

$$-\frac{\beta^*}{1-\delta} \geq -\frac{2(1-\phi)}{1-\delta} \int_0^\infty \hat{x}_t dF.$$

This is equivalent to

$$\beta^* \leq 2(1-\phi) \int_0^\infty \hat{x}_t dF.$$

Given this, Theorem 1 holds from the preceding lemmas. ■

B.10 Proof of Lemma 10

Suppose that $q_1 > q_2$. The objective is to show that $\beta^*(q_1) < \beta^*(q_2)$ holds.

From Lemma 7, $h(\tilde{\beta}) < 0$ holds for any $\tilde{\beta} > \beta^*$. This implies that when $h(\tilde{\beta}|q^1) < 0$ is satisfied for any $\tilde{\beta} \geq \beta^*(q^2)$, $\beta^*(q^1) < \beta^*(q^2)$ holds. Therefore, my task is to show that $h(\tilde{\beta}|q^1) < 0$ is satisfied for any $\tilde{\beta} \geq \beta^*(q^2)$.

When $h(\tilde{\beta}|q_1) < h(\tilde{\beta}|q_2)$, this trivially holds since $h(\tilde{\beta}|q_2) \leq 0$. Let $G_{q_1} \in \arg\min \left\{ -J(\tilde{\beta}|q_1) \middle| G \in \text{core}(\theta) \right\}$, where $J(\tilde{\beta}|q)$ is defined by (13). Indeed, $h(\tilde{\beta}|q_1) < h(\tilde{\beta}|q_2)$ holds since

$$\begin{aligned}
h(\tilde{\beta}|q_1) - h(\tilde{\beta}|q_2) \leq & -\delta(q_1 - q_2)\frac{\tilde{\beta}}{1-\delta} + \int_0^{\tilde{\beta}} \beta dG_{q_1} + \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q_1)\delta},\bar{\beta}\right\}} \tilde{\beta} dG_{q_1} + \int_{\min\left\{\tilde{\beta}+\frac{q_1\delta\rho}{1-(1-q_1)\delta},\bar{\beta}\right\}}^{\bar{\beta}} \beta dG_{q_1} \\
& + \delta q_1 \left[\frac{1}{1-\delta}\int_0^{\tilde{\beta}} \beta dG_{q_1} + \frac{1}{1-\delta}\int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG_{q_1}\right] \\
& - \int_0^{\tilde{\beta}} \beta dG_{q_1} - \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\frac{q_2\delta\rho}{1-(1-q_2)\delta},\bar{\beta}\right\}} \tilde{\beta} dG_{q_1} - \int_{\min\left\{\tilde{\beta}+\frac{q_2\delta\rho}{1-(1-q_2)\delta},\bar{\beta}\right\}}^{\bar{\beta}} \beta dG_{q_1} \\
& - \delta q_2 \left[\frac{1}{1-\delta}\int_0^{\tilde{\beta}} \beta dG_{q_1} + \frac{1}{1-\delta}\int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG_{q_1}\right] \\
= & -\delta(q_1 - q_2)\frac{\tilde{\beta}}{1-\delta} + \int_{\min\left\{\tilde{\beta}+\frac{q_2\delta\rho}{1-(1-q_2)\delta},\bar{\beta}\right\}}^{\min\left\{\tilde{\beta}+\frac{q_1\delta\rho}{1-(1-q_1)\delta},\bar{\beta}\right\}} (\tilde{\beta} - \beta) dG_{q_1} + \frac{\delta(q_1 - q_2)}{1-\delta}\left[\int_0^{\tilde{\beta}} \beta dG_{q_1} + \int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG_{q_1}\right] \\
= & -\frac{\delta(q_1 - q_2)}{1-\delta}\left\{\tilde{\beta} - \left[\int_0^{\tilde{\beta}} \beta dG_{q_1} + \int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG_{q_1}\right]\right\} + \int_{\min\left\{\tilde{\beta}+\frac{q_2\delta\rho}{1-(1-q_2)\delta},\bar{\beta}\right\}}^{\min\left\{\tilde{\beta}+\frac{q_1\delta\rho}{1-(1-q_1)\delta},\bar{\beta}\right\}} (\tilde{\beta} - \beta) dG_{q_1} \qquad (16)
\end{aligned}$$

Here, in the second equality, I use the fact that

$$\tilde{\beta} + \frac{q_2\delta\rho}{1-(1-q_2)\delta} < \tilde{\beta} + \frac{q_1\delta\rho}{1-(1-q_1)\delta}.$$

Then, the first term of (16) is negative since G_{q_1} has full-support and the second term of (16) is obviously non-positive. In summary, $h(\tilde{\beta}|q_1) - h(\tilde{\beta}|q_2) < 0$.

Therefore, $\beta^*(q_1) < \beta^*(q_2)$ holds. ■

B.11 Proof of Proposition 1

I show only that there is unique $\underline{q}^{**}$ because the other part trivially holds given Lemma 11. To prove this, it suffices to show the existence of unique $\underline{q}^*$.

$h(\bar{\beta}^*) > 0$ when $q = 0$ since

$$h(\bar{\beta}^*|q=0) = -(1-\delta)\frac{\bar{\beta}^*}{1-\delta} - \min\left\{-\int_0^{\bar{\beta}} \beta dG \middle| G \in \mathrm{core}(\theta)\right\} = -\bar{\beta}^* + \max\left\{\int_0^{\bar{\beta}} \beta dG \middle| G \in \mathrm{core}(\theta)\right\} > 0.$$

The last inequality comes from Assumption 4. And, $h(\tilde{\beta})$ is decreasing with $q \geq 0$ as in the proof of Lemma 10, and obviously $h(\tilde{\beta})$ is continuous with respect to q. Thus, there is unique $\underline{q}^* \geq 0$. ■

B.12 Proof of Lemma 12

$$h(\tilde{\beta}) = -\frac{1-\delta(1-q)}{1-\delta}\tilde{\beta} + \int_0^{\bar{\beta}} \beta dG - \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}-\frac{q\delta\rho}{1-(1-q)\delta},\bar{\beta}\right\}} (\beta - \tilde{\beta}) dG + \frac{\delta q}{1-\delta}\left[\int_0^{\tilde{\beta}} \beta dG + \int_{\tilde{\beta}}^{\bar{\beta}} \tilde{\beta} dG\right]. \qquad (17)$$

Here,

$$\int_0^{\tilde{\beta}} \beta dG = \beta G(\beta)|_0^{\tilde{\beta}} - \int_0^{\tilde{\beta}} G(\beta)d\beta = \tilde{\beta}G(\tilde{\beta}) - \int_0^{\tilde{\beta}} G(\beta)d\beta. \tag{18}$$

since G is differentiable. Thus, by substituting (18) into (17),

$$(17) = -\tilde{\beta} + \int_0^{\tilde{\beta}} \beta dG + \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}-\frac{q\delta\rho}{1-(1-q)\delta},\bar{\beta}\right\}} (\beta - \tilde{\beta})dG - \frac{\delta q}{1-\delta}\int_0^{\tilde{\beta}} G(\beta)d\beta. \blacksquare$$

B.13 Proof of Proposition 2

B.13.1 Proof of Proposition 2 (a)

To begin with, I show that if $f(\bar{\beta}^*|q, G_1) < f(\bar{\beta}^*|qG_2)$ holds for any $q \in [\underline{q}, 1]$, $\underline{q}^{**}(G_1) \leq \underline{q}^{**}(G_2)$.

(i) Case where $\underline{q}^{**}(G_2) = \bar{q}$:

In this case, $\underline{q}^{**}(G_1) \leq \underline{q}^{**}(G_2)$ always holds by definition.

(ii) Case where $\underline{q}^{**}(G_2) \in (\underline{q}, \bar{q})$:

For any $q \in [0, \underline{q}^*(G_1)]$, $h(\bar{\beta}^*|q, G_1) \geq 0$ since $h(\tilde{\beta})$ is decreasing with q. Thus, when $h(\bar{\beta}^*|\underline{q}^*(G_2), G_1) < 0$, $\underline{q}^{**}(G_1) < \underline{q}^{**}(G_2)$ holds. Here, $h(\bar{\beta}^*|\underline{q}^*(G_2), G_1) < 0$ is equivalent to $h(\bar{\beta}^*|\underline{q}^*(G_2), G_1) < h(\bar{\beta}^*|\underline{q}^*(G_2), G_2)$ since $h(\bar{\beta}^*|\underline{q}^*(G_2), G_2) = 0$. Therefore, when $h(\bar{\beta}^*|q, G_1) < h(\bar{\beta}^*|qG_2)$ holds for any $q \in [\underline{q}, 1]$, $h(\bar{\beta}^*|\underline{q}^*(G_2), G_1) < 0$ is satisfied.

(iii) Case where $\underline{q}^{**}(G_2) = \underline{q}$:

$\underline{q}^{**} = \underline{q}$ holds if and only if $h(\tilde{\beta}|q, G) < 0$ holds for any $q \in (\underline{q}, 1]$. Thus, $h(\tilde{\beta}|q, G_2) < 0$ holds for any $q \in (\underline{q}, 1]$. Therefore, when $h(\tilde{\beta}|q, G_1) < h(\bar{\beta}^*|q, G_2)(< 0)$ is satisfied for any $q \in (\underline{q}, 1]$, $\underline{q}^{**}(G_1) = \underline{q} = \underline{q}^{**}(G_2)$ is obtained.

From (i) to (iii), if $h(\bar{\beta}^*|q, G_1) < h(\bar{\beta}^*|q, G_2)$ holds for any $q \in [\underline{q}, 1]$, $\underline{q}^{**}(G_1) \leq \underline{q}^{**}(G_2)$. Threfore, it suffices to show that $h(\bar{\beta}^*|q, G_1) < h(\bar{\beta}^*|q, G_2)$ holds for any $q \in [\underline{q}, 1]$.

Using the expression of $h(\tilde{\beta})$ derived in Lemma 12,

$$\begin{aligned} h(\bar{\beta}^*|q, G_1) - h(\bar{\beta}^*|q, G_2) = &-\frac{\delta q}{1-\delta}\left[\int_0^{\tilde{\beta}} G_1(\beta)d\beta - \int_0^{\tilde{\beta}} G_2(\beta)d\beta\right] \\ &+ \left[-\int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta},\bar{\beta}\right\}} (\beta - \bar{\beta})dG_1 + \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta},\bar{\beta}\right\}} (\beta - \bar{\beta})dG_2\right]. \end{aligned} \tag{19}$$

Note that $\int_0^{\bar{\beta}} \beta dG_1 = \int_0^{\bar{\beta}} \beta dG_2$ from the definition of mean-preserving spread. I use this fact in the above.

Here,

$$(19) \leq -\frac{\delta q}{1-\delta}\left[\int_0^{\tilde{\beta}} G_1(\beta)d\beta - \int_0^{\tilde{\beta}} G_2(\beta)d\beta\right] + \int_{\tilde{\beta}}^{\min\left\{\tilde{\beta}+\frac{q\delta\rho}{1-(1-q)\delta},\bar{\beta}\right\}} (\beta - \bar{\beta})|g_1(\beta) - g_2(\beta)|d\beta. \tag{20}$$

The first term of (20) is independent of ρ and negative. On the other hand, the second term of (20) is weakly decreasing with ρ, and it goes to zero as ρ goes to zero since

$$\tilde{\beta} + \frac{q\delta\rho}{1-(1-q)\delta} \to \tilde{\beta}.$$

Thus, for each $q \in [\underline{q}, 1]$, there is $\bar{\rho}(q) > 0$ such that for $\rho \in (0, \bar{\rho}(q)]$, $h(\bar{\beta}^*|q, G_1) < h(\bar{\beta}^*|qG_2)$ holds. Take the minimum of $\bar{\rho}(q)$, and denote it by $\bar{\rho}$. Then, for $\rho \in (0, \bar{\rho}]$, $h(\bar{\beta}^*|q, G_1) < h(\bar{\beta}^*|qG_2)$ holds for any $q \in [\underline{q}, 1]$. Therefore, for $\rho \in (0, \bar{\rho}]$, $\underline{q}^{**}(G_1) \leq \underline{q}^{**}(G_2)$. ■

B.13.2 Proof of Proposition 2 (b)

As in the proof of Proposition 2 (a), if $h(\bar{\beta}^*|\underline{q}^*(G_1), G_1) \geq h(\bar{\beta}^*|\underline{q}^*(G_1), G_2)$ holds, $\underline{q}^*(G_1) \geq \underline{q}^*(G_2)$. Thus, it suffices to prove that $h(\bar{\beta}^*|\underline{q}^*(G_1), G_1) \geq h(\bar{\beta}^*|\underline{q}^*(G_1), G_2)$ holds. Here, from equation (19), if

$$\int_{\bar{\beta}^*}^{\min\left\{\bar{\beta}^*+\frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}, \bar{\beta}\right\}} (\beta - \bar{\beta}^*)dG_2 \geq \int_{\bar{\beta}^*}^{\min\left\{\bar{\beta}^*+\frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}, \bar{\beta}\right\}} (\beta - \bar{\beta}^*)dG_1 \tag{21}$$

holds, $h(\bar{\beta}^*|\underline{q}^*(G_1), G_1) \geq h(\bar{\beta}^*|\underline{q}^*(G_1), G_2)$ automatically holds.

(i) Case where condition (i) is satisfied:

In this case, inequality (21) is equivalent to

$$\int_{\bar{\beta}^*}^{\bar{\beta}^*+\frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}} (\beta - \bar{\beta}^*)dG_2 \geq \int_{\bar{\beta}^*}^{\bar{\beta}^*+\frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}} (\beta - \bar{\beta}^*)dG_1. \tag{22}$$

Here, from condition (i), $g_2(\beta) \geq g_1(\beta)$ holds for any $\beta \in \left[\bar{\beta}^*, \bar{\beta}^* + \frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}\right]$. Therefore, inequality (22) holds since $\beta - \beta^* \geq 0$.

(ii) Case where condition (ii) is satisfied

In this case, inequality (21) is also equivalent to inequality (22). Here,

$$\int_{\bar{\beta}^*}^{\bar{\beta}^*+\frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}} (\beta - \bar{\beta}^*)dG = \int_0^{\bar{\beta}} (\beta - \bar{\beta}^*)dG - \int_0^{\bar{\beta}^*} (\beta - \bar{\beta}^*)dG - \int_{\bar{\beta}^*+\frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}}^{\bar{\beta}} (\beta - \bar{\beta}^*)dG.$$

Given the fact that $\int_0^{\bar{\beta}} (\beta - \bar{\beta}^*)dG_1 = \int_0^{\bar{\beta}} (\beta - \bar{\beta}^*)dG_2$ from the definition of the mean-preserving spread, inequality (22) holds if and only if

$$\int_0^{\bar{\beta}^*} (\beta - \bar{\beta}^*)dG_1 + \int_{\bar{\beta}^*+\frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}}^{\bar{\beta}} (\beta - \bar{\beta}^*)dG_1 \geq \int_0^{\bar{\beta}^*} (\beta - \bar{\beta}^*)dG_2 + \int_{\bar{\beta}^*+\frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}}^{\bar{\beta}} (\beta - \bar{\beta}^*)dG_2. \tag{23}$$

Here, from condition (ii), $g_2(\beta) \leq g_1(\beta)$ and $\beta - \bar{\beta}^* > 0$ hold for any $\beta \in \left[\bar{\beta}^* + \frac{\underline{q}^*(G_1)\delta\rho}{1-(1-\underline{q}^*(G_1))\delta}, \bar{\beta}\right]$.

Thus, inequality (23) holds if

$$\int_0^{\bar{\beta}^*}(\beta-\bar{\beta}^*)dG_1+\int_{\bar{\beta}-\bar{\beta}^*}^{\bar{\beta}}(\beta-\bar{\beta}^*)dG_1 \geq \int_0^{\bar{\beta}^*}(\beta-\bar{\beta}^*)dG_2+\int_{\bar{\beta}-\bar{\beta}^*}^{\bar{\beta}}(\beta-\bar{\beta}^*)dG_2. \tag{24}$$

Here, due to the symmetry of G, $g(\beta) = g(\bar{\beta}-\beta)$ holds for any $\beta \in [0, \bar{\beta}^*]$. Using this, the left-hand side minus the right-hand side of inequality (24) is equivalent to

$$\int_0^{\bar{\beta}^*}(\beta-\bar{\beta}^*)(g_1(\beta)-g_2(\beta))d\beta+\int_0^{\bar{\beta}^*}(\bar{\beta}-\beta-\bar{\beta}^*)(g_1(\beta)-g_2(\beta))d\beta.$$

Now, this is non-negative since $|\beta-\bar{\beta}^*| < |\bar{\beta}-\beta-\bar{\beta}^*|$ for any $\beta \in [0, \bar{\beta}^*]$ from condition (ii). Therefore, inequality (24) holds. As a result, I have inequality (23). ■

B.14 Proof of Proposition 3

As in the proof of Proposition 2, if $h(\bar{\beta}^*|\underline{q}^*(\theta_2), \theta_1) \geq h(\bar{\beta}^*|\underline{q}^*(\theta_2), \theta_2)$ holds, $\underline{q}^*(\theta_1) \geq \underline{q}^*(\theta_2)$. Thus, it suffices to prove that $h(\bar{\beta}^*|\underline{q}^*(\theta_2), \theta_1) \geq h(\bar{\beta}^*|\underline{q}^*(\theta_2), \theta_2)$ holds.

Then, using $J(\tilde{\beta}|q)$ defined by (13),

$$\begin{aligned} h(\bar{\beta}^*|\underline{q}^*(\theta_2),\theta_1)-h(\bar{\beta}^*|\underline{q}^*(\theta_2),\theta_2) &= -\min\left\{-J(\bar{\beta}^*|\underline{q}^*(\theta_2))\middle|G\in\text{core}(\theta_1)\right\}+\min\left\{-J(\bar{\beta}^*|\underline{q}^*(\theta_2))\middle|G\in\text{core}(\theta_2)\right\} \\ &= \max\left\{J(\bar{\beta}^*|\underline{q}^*(\theta_2))\middle|G\in\text{core}(\theta_1)\right\}-\max\left\{J(\bar{\beta}^*|\underline{q}^*(\theta_2))\middle|G\in\text{core}(\theta_2)\right\} \geq 0 \end{aligned}$$

The last inequality comes from the fact that $\text{core}(\theta_1) \supseteq \text{core}(\theta_2)$ and $J(\bar{\beta}^*|\underline{q}^*(\theta_2)) > 0$.

Therefore, $\underline{q}^*(\theta_1) \geq \underline{q}^*(\theta_2)$. ■

B.15 Proof of Proposition 4

As in the proof of Proposition 2, it suffices to prove that $h(\bar{\beta}^*|q, G_1) < h(\bar{\beta}^*|q, G_2)$ holds for any $q \in [\underline{q}, 1]$.

Denote

$$\begin{aligned} H(G,r) = &\int_0^{\bar{\beta}^*}(\beta^r-\beta)dG+\int_{\bar{\beta}^*}^{\min\left\{\bar{\beta}^*+\frac{q\delta\rho}{1-(1-q)\delta},\bar{\beta}\right\}}((\bar{\beta}^*)^r-\bar{\beta}^*)dG+\int_{\min\left\{\bar{\beta}^*+\frac{q\delta\rho}{1-(1-q)\delta},\bar{\beta}\right\}}^{\min\left\{\bar{\beta}^*+\left(\frac{q\delta\rho}{1-(1-q)\delta}\right)^{\frac{1}{r}},\bar{\beta}\right\}}((\bar{\beta}^*)^r-\beta)dG \\ &+\int_{\min\left\{\bar{\beta}^*+\left(\frac{q\delta\rho}{1-(1-q)\delta}\right)^{\frac{1}{r}},\bar{\beta}\right\}}^{\bar{\beta}}(\beta^r-\beta)dG+\frac{\delta q}{1-\delta}\int_0^{\bar{\beta}^*}(\beta^r-\beta)dG+\frac{\delta q}{1-\delta}\int_{\bar{\beta}^*}^{\bar{\beta}}((\bar{\beta}^*)^r-\bar{\beta}^*)dG. \end{aligned}$$

Then,

$$h(\bar{\beta}^*|q,G_1)-h(\bar{\beta}^*|q,G_2) = (19)+H(G_1,r)-H(G_2,r). \tag{25}$$

In the above, $h(\bar{\beta}^*|q, G_1) - h(\bar{\beta}^*|q, G_2)$ is approximated by (19). As a matter of fact, $h(\bar{\beta}^*|q, G_1) - h(\bar{\beta}^*|q, G_2) = (19)$ when $r = 1$. Here, $H(G_1, r) - H(G_2, r)$ represents the approximation error.

From the assumption, (19)<0 is negative, and (19) is independent of r. On the other hand, $H(G_1, r) - H(G_2, r)$ is continuous with respect to r, and zero when $r = 1$. Therefore, there is $\bar{r} > 1$ such that for any $r \in (1, \bar{r})$, the right-hand side of equation (25) is negative. ■