Optimal Objective Function

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PRELIMINARY. ANY COMMENTS APPRECIATED.

1 Introduction

In a framework of industrial organization we basically assume firms' objective is maximizing its profit. It leads to analyses of their optimizing behavior for a profit function, seemingly without loss of generality. Since Fershtmand and Judd (1987), however, it became well-known that optimizing for a profit function does not necessarily maximize the profit. On the flip side of the same coin, there are situations where if firms are profit maximizers, they behave in non-profit maximizing way. The canonical example is called delegation. It is the situation where an owner of a firm hires outside CEO with a contract specifying that the reward depends not only on the profit but, say, sales as well. With the contract, the CEO can credibly commit himself to produce more than Cournot optimum. The commitment intimidates its competitors into less production and ends up bringing higher profit for the CEO and owner. While this is an underlying incentive for setting non-profit function as an objective to maximize profit, the game structure becomes prisoners' dilemma once the competitors can also make a contract with a mixture of profit and sales.

As researchers pile up analyses in the literature of delegation, combinations of profit and social welfare, consumer surplus ,or competitors' profits, instead of sales, have been investigated. Though the elements are different, the basic effect of the contracts in common is a commitment to increasing production in Cournot setting, so what kind of function for the researchers to use is on its situations. For models of partial privatization, social welfare or consumer surplus is called, for instance. Other examples include, but are not limited to, those of corporate social responsibility acts, relative profits, or self-managed firms.

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Despite the variety of papers, how the adopted element increases production is hardly inspected (Here we mean how in mathematical way. The meaning get clear in the following sections). One reason of the absence is symmetry of firms in the models.

2 Setting

The game is a two-stage, pure private duopolistic delegation then Cournot game. Fix firm 1's objective function in the second stage as

$$O_i = \alpha_i * \pi_i + (1 - \alpha_1) * S_i$$

$$where\ \pi_i = S_i - c * q_i,\ S_i = (a - q_1 - q_2)q_i \quad (i = 1). \tag{1}$$

In the first round, firm 1 chooses a value of α_1 to maximize its profit, simultaneously firm 2 choosing its counterpart. In the following we compare candidates of firm 2's second stage objective function. We for now do not take care with a domain of α_is to shed light just on roles of different functions to profits. It, for instance, turns out that mixing consumer surplus with original profit function defeats the above firm 1's functional form, at the same time the opposite holds when the firm 2 mixing social surplus.

3 Benchmark Case

Both firms maximize a linear combination of profit and sales function in the second stage, and choose the degree of mixing in the first.

$$\max_{q_i}\ \alpha_i\pi_i + (1 - \alpha_i)S_i \qquad (i = 1, 2). \tag{2}$$

Reaction functions and equilibrium quantities are

$$q_i(q_j) = \frac{a - \alpha_i c - q_j}{2} \qquad (i = 1, 2,\ i \neq j). \tag{3}$$

$$q_i^*(\alpha_i, \alpha_j) = \frac{a + c(\alpha_j - 2\alpha_i)}{3}. \tag{4}$$

Anticipating this equilibria in the second stage, both choose the degrees of its linear combination, α_i. The problem and result are,

$$\max_{\alpha_i}\ \pi_i(\alpha_i, \alpha_j) \tag{5}$$

$$where \quad \pi_i(\alpha_i, \alpha_j) = \pi_i(q_i^*(\alpha_1, \alpha_2), q_j^*(\alpha_1, \alpha_2)).$$

$$\alpha_i(\alpha_j) = -\frac{a + (\alpha_j - 6)c}{4c} \tag{6}$$

$$\alpha_i^* = \frac{6}{5} - \frac{a}{5c} \tag{7}$$

$$\pi_i^* = \frac{2(a-c)^2}{25}, \quad q_i^* = \frac{2(a-c)}{5}. \tag{8}$$

For the exposition afterward there are several things worth noting. First, this game indeed has a prisoners' dilemma structure, i.e.

$$\pi_i(\alpha_i(1), 1) > \pi_i(1, 1) > \pi_i(\alpha_i^*, \alpha_j^*). \tag{9}$$

Second, it is apparent from the equation (4), tuning up α_i causes shift move of $q_i(q_j)$. Third, reaction functions of both α_is and q_is show strategic substitution relationship. The intuition behind α_is strategic substitution is the same as q_is. Fourth, since $q_i^*(\alpha_1, \alpha_2)$ is a function of α_is, reaction curves of α_is (7) can be mapped onto $q_1 - q_2$ coordinate plane.

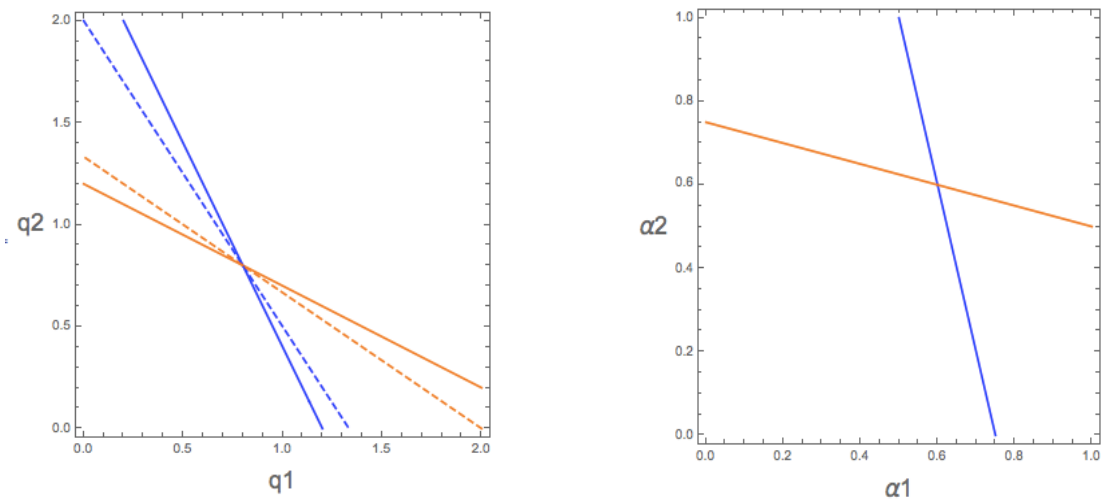

Figure 1: Left: Reaction curves of q_i ($\alpha_i = \alpha_i^*$) and loci of the intersection along with reaction curves of α_i (dashed), Right: Reaction curves of α_i (Example values: $a = 3$, $c = 1$).

4 Welfare Function

As one kind of comparative statics, we introduce new element of objective function instead of a sales function only to the firm 2. The first example is a social welfare function. Thus the objective function in the second stage becomes to

$$O_2 = \alpha_2 * \pi_2 + (1 - \alpha_2) * W$$

$$where\ W = CS + \pi_1 + \pi_2,\ CS = \frac{(q_1 + q_2)^2}{2}. \tag{10}$$

Under this setting, the firm 2's quantitative reaction functions and the equilibria are,

$$q_2(q_1) = \frac{a - c - q_1}{1 + \alpha_2}, \tag{11}$$

$$q_1^* = \frac{a\alpha_1 + (1 - \alpha_1 - \alpha_1\alpha_2)c}{1 + 2\alpha_2}, \quad q_2^* = \frac{a + (\alpha_1 - 2)c}{1 + 2\alpha_2}. \tag{12}$$

Then, the first stage generates,

$$\alpha_1(\alpha_2) = \frac{(2\alpha_2 + 3)c - a}{(2\alpha_2 + 2)c}\ , \quad \alpha_2(\alpha_1) = \frac{1}{2} \tag{13}$$

$$(\alpha_1^*,\ \alpha_2^*) = \left(\frac{4c - a}{3c}\ ,\ \ \frac{1}{2}\right) \tag{14}$$

$$(\pi_1^*,\ \pi_2^*) = \left(\frac{(a - c)^2}{12}\ ,\ \ \frac{(a - c)^2}{18}\right) \tag{15}$$

$$(q_1^*,\ q_2^*) = \left(\frac{(a - c)}{2}\ ,\ \ \frac{(a - c)}{3}\right) \tag{16}$$

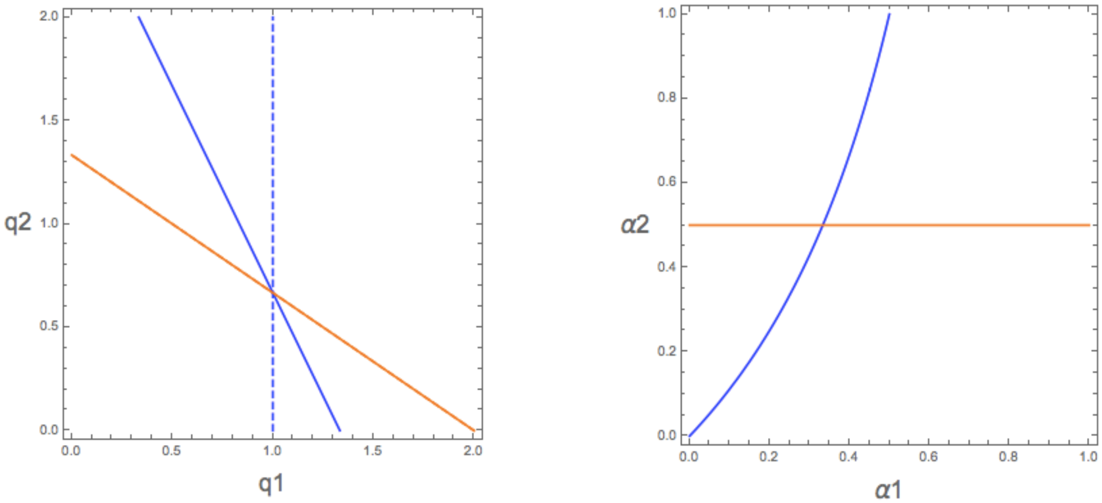

Figure 2: Left: Reaction curves of q_i ($\alpha_i = \alpha_i^*$) and loci of the intersection along with reaction curves of α_i (dashed), Right: Reaction curves of α_i (Example values: $a = 3$, $c = 1$).

Prisoners' dilemma structure still stands, but other profit comparison is not symmetric, especially,

$$\pi_1(\alpha_1^*, \alpha_2^*) > \pi_1(1, \alpha_2(1)) = \pi_2(\alpha_1(1), 1) > \pi_2(\alpha_1^*, \alpha_2^*) \tag{17}$$

Second, this time tuning up α_2 causes slope change to $\alpha_2(\alpha_1)$. Third, strategic substitute relationship of α_is turns to strategic complement. Fourth, the one locus of intersections of q_is perfectly coincides with the equilibrium reaction curve of $q_2(q_1)$.

5 Generalizing Linear Reaction Function

As shown above, when the firm 1, adopting a linear combination of its profit and sales, competes against the firm 2, combination of profit and welfare, the firm 1 dominates. In this section we try to shed light on exactly which characteristics of welfare or sales function play a crucial role in the difference. To do that, we generalize a reaction function of firm 2's quantity, $q_2(q_1)$, in two ways: different slopes and adjusting an intercept, adjusting a slope and different intercepts.

5.1 Different Slopes and Adjusting an Intercept

This way of generalizing corresponds to the benchmark case, because both firms can shift its own quantity reaction function by adjusting intercepts and holding slopes as given. The firm 1's maximizing problem is the same as before specified in the equation (2). Thus the firm 1's reaction function $q_1(q_2)$ is as (3). The firm 2's problem is implicitly defined by its reaction function,

$$q_2(q_1) = -d * q_1 + f(\alpha_2) \tag{18}$$

where f is monotonic and differentiable.

In the benchmark case,

$$d = \frac{1}{2}, \quad f = \frac{a - \alpha_2 * c}{2}.$$

Under this setting, the equilibrium is,

$$\alpha_1^* = \frac{ad + 2c(d-2)}{c(3d-4)}, \quad f(\alpha_2^*) = \frac{(d-2)(a-c)}{3d-4}, \tag{19}$$

$$\pi_1^* = -\frac{(d-1)(a-c)^2}{(4-3d)^2}, \quad \pi_2^* = \frac{2(d-1)^2(a-c)^2}{(4-3d)^2} \tag{20}$$

Proposition 1 *Under this setting,*

$$\pi_1^* \gtrless \pi_2^* \Leftrightarrow d \gtrless \frac{1}{2} \tag{21}$$

5.2 Adjusting a Slope, Different intercepts

This way of generalizing corresponds to the welfare function case, because the firm 2 can change its slope of quantity reaction curve and holding its intercept as given. The firm 2's problem is implicitly defined by its reaction function,

$$q_2(q_1) = -g(\alpha_2) * q_1 + h \tag{22}$$

where g is monotonic and differentiable.

Under this setting, the equilibrium is,

$$\alpha_1^* = \frac{a - 2h}{c}, \quad g(\alpha_2^*) = \frac{2(a-c-2h)}{a-c-3h}, \tag{23}$$

$$\pi_1^* = -\frac{(a-c-3h)(a-c-h)}{4}, \quad \pi_2^* = \frac{(a-c-h)^2}{2} \tag{24}$$

Proposition 2 *Under this setting,*

$$\pi_1^* > \pi_2^* \Leftrightarrow \frac{3(a-c)}{5} < h < a - c \tag{25}$$

6 Consumer Surplus Function

As one kind of comparative statics, we introduce new element of objective function instead of a sales function only to the firm 2. The second example is a consumer surplus (CS henceforth) function, minding the firm engaged to CSR activity. Thus the objective function in the second stage becomes to

$$O_2 = \alpha_2 * \pi_2 + (1 - \alpha_2) * CS. \tag{26}$$

Under this setting, the firm 2's quantitative reaction functions and the equilibria are,

$$q_2(q_1) = \frac{\alpha_2(a-c) - 2\alpha_2 q_1 + q_1}{3\alpha_2 - 1} \tag{27}$$

$$q_1^* = \frac{a(2\alpha_2 - 1) + c(-3\alpha_1\alpha_2 + \alpha_1 + \alpha_2)}{4\alpha_2 - 1}, \quad q_2^* = \frac{a + c(\alpha_1(2\alpha_2 - 1) - 2\alpha_2)}{4\alpha_2 - 1}. \tag{28}$$

Then, the first stage generates,

$$\alpha_1(\alpha_2) = \frac{4a{\alpha_2}^2 - 4a\alpha_2 + a - 10{\alpha_2}^2 c + 6\alpha_2 c - c}{2\alpha_2 c - 6{\alpha_2}^2 c}\ , \quad \alpha_2(\alpha_1) = \frac{3a - (\alpha_1 + 2)c}{4(a-c)} \tag{29}$$

$$(\alpha_1^*,\ \alpha_2^*) = \left(\frac{-2\sqrt{3}a + 3a + 2\sqrt{3}c}{3c}\ ,\quad \frac{3+\sqrt{3}}{6} \right) \tag{30}$$

$$(\pi_1^*,\ \pi_2^*) = \frac{\left(2\sqrt{3}-3\right)(a-c)^2}{6}\ ,\quad \frac{\left(-\sqrt{3}+2\right)(a-c)^2}{3} \tag{31}$$

$$(q_1^*,\ q_2^*) = \frac{\left(\sqrt{3}-1\right)(a-c)}{2}\ ,\quad \frac{\left(-\sqrt{3}+3\right)(a-c)}{3} \tag{32}$$

Prisoners's dilemma again holds, and the other profit order is,

$$\pi_2(\alpha_1^*, \alpha_2^*) > \pi_1(\alpha_1^*, \alpha_2^*) > \pi_1(1, \alpha_2(1)) = \pi_2(\alpha_1(1), 1) \tag{33}$$

Appendices

Appendix A. We formulate variant elasticity model. The demand system is given by $p = a - q_1 - \delta q_2 \ \ (\delta > 0)$. Both firms maximize a linear combination of profit and sales function in the second stage, and choose the degree of mixing in the first.

$$\max_{q_i}\ \alpha_i \pi_i + (1-\alpha_i) S_i \qquad (i = 1, 2). \tag{34}$$

Reaction functions and equilibrium quantities are

$$q_1(q_2) = -\frac{\delta q_2 + \alpha_1 c - a}{2}, \tag{35}$$

$$q_2(q_1) = -\frac{q_1 + \alpha_2 c - a}{2\delta} \tag{36}$$

$$q_1^*(\alpha_1, \alpha_1) = \frac{(\alpha_2 - 2\alpha_1)\, c + a}{3}, \tag{37}$$

$$q_2^*(\alpha_1, \alpha_2) = -\frac{(2\alpha_2 - \alpha_1)\, c - a}{3\delta}. \tag{38}$$

Anticipating this equilibria in the second stage, both choose the degrees of its linear combination, α_i. The problem and result are,

$$\max_{\alpha_i} \ \pi_i(\alpha_i, \alpha_j) \tag{39}$$

$$where \ \ \pi_i(\alpha_i, \alpha_j) = \pi_i(q_i^*(\alpha_1, \alpha_2), q_j^*(\alpha_1, \alpha_2)).$$

$$\alpha_i(\alpha_j) = -\frac{(\alpha_j - 6)\, c + a}{4c}, \tag{40}$$

$$\alpha_i^* = \frac{6c - a}{5c} \tag{41}$$

$$(\pi_1^*, \ \pi_2^*) = \left(\frac{2(a-c)^2}{25} \ , \quad \frac{2(a-c)^2}{25\delta} \right) \tag{42}$$

$$(q_1^*, \ q_2^*) = \left(\frac{(a-c)}{5} \ , \quad \frac{(a-c)}{5\delta} \right) \tag{43}$$