Informative Campaign under Multidimensional Politics: A Role of Naive Voters [*]

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Abstract

We construct a model of election in which an incumbent and a challenger decide how to allocate campaign resources to two types of campaign (policy and ability), and after that, a media outlet decides whether to gather news. We show that the allocation of campaign resources (i.e., which issue a candidate emphasizes) conveys truthful information only when sophisticated voters and naive voters coexist. In addition, we show that in any separating equilibria, negative campaign against the incumbent's ability occurs as a signal of the incumbent's low ability. Overall, these results imply that (i) a candidate's campaign allocation over issues (i.e., campaign message) can be informative, and (ii) the relationship between the existence of a separating equilibrium and the number of naive voters is non-monotonic.

Keywords: Political campaign; negative campaign; issue selection; Bayesian irrationality; mass media

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1. Introduction

Voters are not necessarily familiar with policy issues and candidates' characteristics. Thus, so as to maintain the responsiveness of the representative democracy, information transmission from agents, such as candidates, interest groups, and mass media, to voters is quite important. One of significant paths of such information transmission is political campaign, including advertising, speeches, and debates in elections. However, such campaign does not necessarily convey the truth because information is often soft, and there are conflicts of interests between senders of information and voters. When does campaign convey the truth? The present paper contributes to the research about campaign under soft information.

In the 1996 U.S. presidential election, William Jefferson Clinton (the incumbent) and Robert Joseph Dole (a challenger) competed with each other. Their strategies in the first presidential debate on October 6, 1996 were quite different. Dole attacked Clinton's character. On the contrary, Clinton focused on policy issues, and shifted the discussion away from the question about his character (Benoit 2007: 45).

As seen in this example, candidates' campaign strategies are different. It suggests that a candidate strategically chooses the allocation of campaign resources (e.g., the allocation of campaign spending in advertising, the allocation of time in speeches and debates) between campaign on policy and that on characters such as ability. This strategic choice would depend on the situation the candidate faces, especially private information the candidate has. If this is the case, the allocation of campaign resources (i.e., which of policy and character the candidate emphasizes) sends the information the candidate has, to voters. In other words, even if messages are soft information, a candidate can send truthful information. The purpose of the present paper is to investigate this signaling role.

To this end, we construct a model in which voters, an incumbent, a challenger, and a media outlet exist. Each candidate competes with each other in two dimensions: policy and ability. The incumbent's ability is known to the both candidates while the voters do not know it. The challenger's ability is unobservable to all the players since whether s/he has sufficient ability is realized only after s/he has a seat. Due to this information structure each candidate allocates one unit of resources to campaign on policy and campaign on the incumbent's ability. After that, the mass media decides whether to gather news about the incumbent's ability to maximize its profit. Then, the voters vote for one of the two candidates. Here, the voters are divided into two types: sophisticated (and informed) voters and naive (and uninformed) voters. A sophisticated voter knows which one's policy is good for her/himself, and s/he is sophisticated in that her/his belief is updated following the Bayes rule. In contrast, a naive voter does not know whose policy is good, and s/he is just persuaded by campaign.

We show that the allocation of campaign resources can send truthful information to the sophisticated voters. In other words, separating equilibria, where campaign is informative, exist under several conditions. In addition, in every such equilibrium, the challenger increases the fraction of campaign on the incumbent's ability if and only if the incumbent's ability is low. Since campaign on the incumbent's ability can be regarded as negative campaign, this result argues that negative campaign provided by the challenger arises as a signal of the incumbent's low ability.

Why can the challenger's allocation of campaign resources be informative? Messages itself are costless, and so the game is basically a cheap-talk game. However, messages change the naive voters' voting behaviors so that the game is reduced to a costly signaling game. As a result, informative equilibria are sustained.

To see this, consider how campaign on policy and that on ability persuade the naive voters. When the challenger succeeds in persuading a naive voter that the challenger's policy is good, the voter is persuaded that the incumbent's policy would be bad at the same time. This is because the incumbent's policy is logically not good when the challenger's policy is desirable, so long as the two policies are about the same issue and they are different. In contrast, when the challenger succeeds in persuading a naive voter that the incumbent's ability is low, the voter is not persuaded that the challenger's ability is high. This is because there is a case where both candidates have only low ability. In other words, there is a logical connection between the evaluations of the candidates' policies, whereas one's ability is logically independent of the other's ability. As a result, the productivity of campaign differs. Campaign on policy can mobilize more naive voters than campaign on ability can. This implies that allocating a large amount of resources to campaign on ability is costly for the challenger.[1] As a result, the cheap-talk game becomes a costly signaling game given the naive voters' behaviors. Therefore, by increasing the fraction of campaign on ability, the challenger can send a credible signal.

Since this is the key which crates separating equilibria, the existence of the naive voters is crucial. We obtain the non-monotonic relationship between the existence of informative campaign and the number of the naive voters.

While the candidates cannot persuade the sophisticated voters without credibility of campaign, they can persuade the naive voters even if the truth is totally different from what campaign argues. Thus, it seems that the existence of the naive voters makes candidates conduct campaign which is not based on the truth. Nonetheless, an increase in the number of the naive voters does not necessarily undermine the credibility of a message campaign conveys. Rather, without naive voters, campaign cannot be informative. This result is obtained because the cost of campaign allocation is created by the existence of the naive voters. In the mechanism above, increasing the fraction of campaign on ability is costly for the challenger since the effect of campaign about ability on the naive voters is smaller than that of campaign on policy. Therefore, the existence of

[1] This is consistent with the empirical result that policy is a more frequent topic of campaign messages than character, and winners are more likely to emphasize policy than character (Benoit 2007).

the naive voters is crucial.

Notice that this mechanism is still not enough to create separating equilibria. The mass media is also essential. The net benefit of allocating a large amount of resources to campaign on the incumbent's ability must depend on whether the incumbent's ability is low. However, the cost of campaign due to the above mechanism is independent of the incumbent's ability. Thus, so that an increase in campaign on ability is a credible signal, its benefit must depend on the incumbent's ability. To this end, the mass media plays a role. Suppose that the challenger allocates a large amount of resources to campaign on the incumbent's ability. Then, the mass media suspects that the incumbent's ability would be low. Thus, it gathers news to report this scandal. Here, suppose that the challenger increases negative campaign in spite of the incumbent's high ability. Then, the mass media would find out the truth with some probability and report it. After the news is reported, the challenger cannot win an election since the sophisticated voters know that the incumbent's ability is high. Thus, so long as the incumbent's ability is high, the challenger may not win an election even if s/he allocates a large amount of resources to the negative campaign. As a result, the net benefit of the negative campaign is smaller when the incumbent's ability is high than that when the incumbent's ability is low. This makes the negative campaign a credible signal.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 describes the model. Section 4 derives equilibria. Section 5 concludes.

2. Related Literature

There are mainly three modeling approaches on how to describe information transmission through political campaign, depending on whether voters are sophisticated and whether information is soft. The first approach considers a situation in which there are some fractions of naive voters who are just persuaded by political campaign (e.g., Baron 1994; Mueller and Stramann 1994; Grossman and Helpman 1996). In contrast, the second approach considers a situation in which voters are sophisticated and information is soft (e.g., Potters, Sloof, and Van Winde 1997; Prat 2002a, b; Panova 2011). In the last approach, information is assumed to be hard (e.g., Coate 2004a, b; Ashworth 2006).

The closest one is the second approach, mainly beginning with Prat (2002a, b). In his model, the following three are the key: (i) not candidates but interest groups send a signal, (ii) not a message campaign conveys but the amount of campaign spending matters, and (ii) voters are sophisticated. It is meaningless for an interest group to provide campaign spending to a candidate who is less likely to win an election. Thus, the interest group would provide campaign spending only with a candidate whose characteristics seem to be good. Therefore, the amount of campaign spending is informative. However, this mechanism does not work when candidates send messages.[2] This is (i). Here, the amount of campaign spending is a credible signal. This is (ii). Lastly, in this mechanism, the rationality of voters is important. If voters are naive, even a candidate whose characteristics are bad can win an election after large amount of campaign. This is (iii).

Our approach combines the second approach with the first approach in that campaign can send a signal to sophisticated voters while it persuades naive voters. The present study has three contributions to the literature, each of which corresponds to (i)-(iii) in the above.

The first contribution is to show that even a candidate can send a credible message through campaign.[3] The related work is that of Daley and Snowberg (2011) that show the role of a candidate's fund-raising as a credible signal. In their model, a candidate's ability is negatively correlated with the cost of fund-raising. Thus, raising a lot of campaign contribution is a signal of high ability. Therefore, the mechanism creating the credible message is different, first. Furthermore, a message a candidate sends is also different. Whereas a candidate sends a message about her/his own type in their model, a candidate sends a message about the opponent's type (i.e., negative campaign can be a credible message) in our model.

The second contribution is to show that not the amount of campaign spending but messages campaign conveys (i.e., which of policy and ability a candidate emphasizes) can be informative.

The third contribution is to show that an increase in the number of naive voters does not necessarily undermine the credibility of a message campaign conveys. The last two contributions are discussed for the details in the below

■ **Strategic choice of campaign issues and information transmission:** In the traditional literature of political campaign, what issue campaign emphasizes (i.e., the contents of messages) has not been examined enough. In contrast, the present study examines a strategic choice about the resource allocation between campaign on policy and campaign on ability, and shows that such a choice can be informative for voters.

Recently, there is a burgeoning literature that examines a strategic choice of campaign issues and information transmission through it (e.g., Polborn and David 2006; Hao and Li 2013; Egorov 2015; Zhang 2015; Bhattacharya 2016; Dragu and Fan 2016; Shipper and Woo 2016). Especially, Polborn and David (2006), Hao and Li (2013), Bhattacharya (2016), and Shipper and Woo (2016) analyze negative campaign like us.[4]

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[2] A candidate's objective is not to provide campaign with a candidate who is likely to win an election, but to win an election. In addition, campaign is costless for candidates because the sources are provided by interest groups. Thus, the mechanism above does not work.

[3] Potters, Sloof, and Van Winde (1997) show that a candidate's message can be truth-telling, but in their model, the cost of sending a signal is exogenously given i.e., the cost is an adhoc assumption.

[4] The studies of the strategic choice between positive campaign and negative campaign exist (e.g., Skaperdas and Grofman 1995; Harrington and Hess 1996; Polborn and David 2006; Lovett and Shachar 2011; Hao and Li 2013; Kasamatsu 2017).

The present study has a large difference from this literature in terms of information structure. In most of the existing literature, either (i) voters are non-Bayesian updaters, or (ii) the condition, under which information campaign argues is transmitted, is exogenously given. For example, Dragu and Fan (2016) assume that voters are naively persuaded by campaign on each issue's salience. In other studies (e.g., Polborn and David 2006; Hao and Li 2013; Egorov 2015; Bhattacharya 2016; Shipper and Woo 2016), when the information campaign argues is transmitted is exogenously given, and it is assumed to be hard.[5] In contrast, in our model, (a) there are Bayesian updaters (sophisticated voters), and (b) when information is transmitted depends on endogenous players' strategies, and information is soft.

To our knowledge, the one exception satisfying (a) and (b) is the study by Zhang (2016). Her focus is different from ours. She analyzes a situation in which there are multiple issues and a candidate decides whether to reveal information about each issue. Thus, there is no trade-off between campaigns on issues: to reveal information about one issue does not mean not to reveal information about another issue. In reality, campaign resource is limited, and thus allocation matters. We analyze the allocation of campaign resources given fixed budget.

■ **Naive voters and information transmission:** As pointed out, there is a non-monotonic relationship between the number of naive voters and the existence of separating equilibria. This result suggests that the existence of naive voters can play a positive role in enhancing voters' welfares. To our knowledge, there is no such discussion in the literature of political campaign.[6]

In the literature of cheap talk games, Ottaviani and Squintani (2006) and Kartik, Ottaviani and Squintani (2007) examine the possibility of a naive receiver. They show that the larger the probability that a receiver is naïve is, the more information is transmitted. In their model, the existence of a naive receiver creates lying cost similarly with our model. The difference is in the conflicts of interests. In their model, there is a correlation between the sender's ideal point and the receiver's one similarly à la Crawford and Sobel's (1982) model, and this plays a crucial role. In contrast, there is no such correlation in our model i.e., no common interests.

The related study in the literature of political campaign is that of Grillo (2016). He analyzes electoral campaign with two candidates when voters have reference dependence utilities with loss aversion. Then, he shows that such a behavioral anomaly can push the candidates to be truth-full. In particular, the anomaly creates lying cost under a possibility of detection. As a result, each candidate sends a truth-telling message about her/his own ability. Our study is different in the

[5] For example, Egorov (2015) assumes that the truth is revealed only when both candidates campaign the same issue. It implies that information is hard when it is transmitted. Note that this does not mean that updating does not matter. Each candidate can hide information, and so voters update the belief based on whether information is revealed.

[6] Even if we look at studies beyond the literature of campaign, studies showing a positive role of voters' irrationality are limited. Several studies (Ashworth and De Mesquita 2014; Levy and Razin 2015; Lockwood 2017) show such possibilities, but the irrationalities they focus on are different from ours.

following aspects. First, irrationality we focus on is about not attitudes toward uncertainty but strategic reasoning. Second, we consider campaign under multidimensional politics (i.e., issue-selection) whereas he considers campaign under one dimensional politics. As a result of these differences, we find out a new mechanism in which campaign becomes informative owing to the existence of anomalies/ irrationalities of voters.[7]

■ **Mass media and challenger's message:** One key ingredient to create a credible signal is the mass media in our model. Though political campaign and the mass media have been considered to be related to each other, this is the first paper that explicitly shows the role of the mass media to make campaign informative, so far as we know.[8] Beyond the literature of political campaign, there are two papers which show that the opposition party's message about the majority party through a filibuster can be informative thanks to the existence of the mass media (Stone 2013; Kishishita 2017). We introduce a setting similar with that of Kishishita (2017), and create a role of the mass media as watchdog though the context is different.

3. The Model

The model is a four-stage game. There exist two candidates (incumbent A and challenger B), sophisticated voters, and naive voters. Voters are continuum of measure one. The *sophisticated voters* are those who know whose policy is best, and update their beliefs about the candidates' characteristics using the Bayes rule. To put it differently, they are informed and Bayesian sophisticated. Let the fraction of the sophisticated voters be $\gamma \in (0, 1)$. The *naive voters* are those who do not know a good policy for themselves, and are persuaded by campaign naively. They are uninformed and non-Bayesian updaters. The fraction of the naive voters is $1 - \gamma$.

3 .1 Candidates' Characteristics

Candidate $k \in \{A, B\}$ is characterized along two dimensions: her/his ideal policy x_k, and her/his ability θ_k.

There is a policy issue which is central in the election. About this issue, each candidate cannot commit her/his own policy such as in the citizen candidate model (Osborne and Slivinski 1996). Therefore, the policy that will be implemented if a candidate wins the election is her/his ideal policy. The value of x_k is assumed to be common-knowledge, and $x_A \neq x_B$.

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[7] Another difference is about a game structure. In our model, both candidates know the incumbent's ability so that the multiple senders send a message about the same state. On the other hand, in their model, each candidate knows only her/his own ability so that the game is that with one sender.

[8] Polborn and David (2006) and Bhattacharya (2016) implicitly assume the fact-checking by the mass media to verify their information revelation protocol. However, in their models, the mass media does not exist as a self-interested player, and so they do not incorporate its endogenous decision.

θ_k captures candidate k's ability. Since how well a candidate can conduct the job as the policymaker would be unclear before s/he obtains a seat, we assume that a candidate's ability is revealed only after s/he becomes the policymaker. More specifically, the challenger's ability is unobservable to anyone including the challenger. In contrast, the incumbent's ability has already been revealed, and the challenger as well as the incumbent knows the ability. θ_k is $g(> 0)$ (i.e., high) with probability 0.5 and 0 (i.e., low) with probability 0.5. Note that the value of θ_A and θ_B are independently determined. This prior is common knowledge.

Both candidates know the incumbent's ability, but the voters do not know it. On the contrary, all the players do not know the challenger's ability as discussed in the above.

3. 2 Voters' Utility

Denote the set of all voters by $I \subseteq \mathbb{R}$. Each voter votes for one of the two candidates in an election i.e., there is no abstention.[9] The voters take into account each candidate k's policy x_k and ability θ_k, when they vote in the election. Each voter $i \in I$ has an ideal policy $\hat{x}_i \in \{x_A, x_B\}$, and the degree of the feeling toward incumbent A denoted by ε_i . Here, $\hat{x}_i = x_A$ with probability ρ and $\hat{x}_i = x_B$ with probability $1 - \rho$. In addition, ε_i follows an independent and identical distribution whose cumulative distribution function is Φ, and density function is φ. This is symmetric in that for any $\varepsilon \in [0, \infty), \phi\,(\varepsilon) = \phi(-\varepsilon)$. Note that the value of ε_i is determined independently of the value of $\hat{x}_i$, and the voter knows her/his value of ε_i.

Since each voter has no strategic power, we consider sincere voting. Voter i's payoff when candidate k wins conditional on x_k and θ_k is

$$u_i(k) = -\alpha v(|x_k - \hat{x}_i|) + \beta\theta_k + \mathbf{1}\{k = A\}\varepsilon_i.$$

Here, $\alpha, \beta > 0$,

$$v(|x_k - \hat{x}_i|) = \begin{cases} 0 \text{ if } \hat{x}_i = x_k \\ -d \text{ otherwise} \end{cases},$$

where $d > 0$, and $\mathbf{1}\{k = A\}$ is the indicator function which takes one when $k = A$, and zero otherwise.

If the expected utility when candidate k wins the election is higher than that when the other candidate wins the election, voter i votes for candidate k. When the both candidates are indifferent, s/he votes for incumbent A with probability a half. We assume that $\alpha d > g\beta/2$. Since αd is the loss when the bad policy is implemented, and $g\beta$ is the benefit when a candidate whose ability is high is elected, this assumption means that the importance of ability for the voters is smaller than that twice as large as the importance of policy for them. Thus, this assumption holds so long as the importance of ability is not too large compared to the importance of policy.

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[9] Thus, campaign affects not the decision to vote but the decision about whom to vote for. This is a usual setting in the literature.

3.3 Sophisticated Voters

Each sophisticated voter i knows the value of $\hat{x}_i$ because s/he has sufficient knowledge. On the other hand, even sophisticated voters do not know the incumbent's ability θ_A since it is private information. Thus, they infer it using each candidate's campaign allocation (C^A, C^B) and the prior distribution of the ability. Denote the belief about θ_A given (C^A, C^B) by $\pi(\theta_A = g \mid (C^A, C^B))$. Note that we assume that the sophisticated voters and the mass media have the same belief about θ_A.

Therefore, sophisticated voter i decides whom to vote for based on her/his ideal policy $\hat{x}_i$, the candidates' policies x_A and x_B, and their belief about the incumbent's ability π. Then, the number of sophisticated voters whose ideal policy is x_A and who vote for incumbent A is given by

$$\gamma \times \rho \times \Phi\big(\alpha d + \beta\big(\pi\big(\theta_A = g \mid (C^A, C^B)\big) - 0.5\big)g\big).$$

Similarly, the number of sophisticated voters whose ideal policy is x_B and who votes for incumbent A is given by

$$\gamma \times (1-\rho) \times \Phi\big(-\alpha d + \beta\big(\pi\big(\theta_A = g \mid (C^A, C^B)\big) - 0.5\big)g\big).$$

3.4 Naive Voters

Since the naive voters have only limited knowledge, they do not know what is a good policy for themselves. Each naive voter i does not know ideal policy $\hat{x}_i$. For example, suppose that trade liberalization is the central issue. This represents that a naive voter does not know the effect of trade liberalization on her/his economic situation, and so s/he does not know whether trade should be liberalized. Also, s/he does not know the incumbent's ability similarly with that of sophisticated voters. In addition, they are persuaded perfectly by candidates' campaign about policy and ability. The effect of campaign on the naive voters will be discussed in the next subsection.

3.5 Campaign

Each candidate has one unit of campaign resources. A candidate's objective is to maximize her/his expected number of obtained votes.[10] Thus, each candidate allocates the campaign resource to campaign on policy and that on ability (e.g., how allocates time of speeches to policy issues and ability issues) so as to maximize the number of obtained votes after they observe the incumbent's ability. In other words, each candidate determines the fraction of campaign on policy C^k and the fraction of campaign on ability $1 - C^k$.

[10] Since there is no abstention, maximizing the number of votes is the same as maximizing the vote share. Note that this objective function is realistic because the vote share affects post-election policy making though it is enough to obtain a half of votes to win the election.

For simplicity, assume that each candidate chooses the fraction of campaign on policy from $\{C_H, C_L\}$, where $0 < C_L < C_H < 1$.[11] Campaign on policy persuades a naive voter that the voter's ideal policy is the candidate's policy. The challenger's (incumbent's) campaign on ability persuades a naive voter that the incumbent's ability is low (high).

The amount of voters a candidate can persuade is represented by $p_k(C^k, C^{-k})$ and $n_k(1 - C^k, 1 - C^k)$ given each candidate's campaign allocation. Candidate k persuades $p_k(C^k, C^{-k}) \times 100$ percent of the naive voters that candidate k 's policy is good. And, candidate A (B) persuades $n_A(1 - C^A, 1 - C^B) \times 100$ $(n_B(1 - C^B, 1 - C^A) \times 100)$ percent of the naive voters that the incumbent's ability is high (low). Therefore, the challenger's campaign on ability represents negative campaign on the incumbent's ability. Notice that the challenger cannot persuade voters that her/his ability is high because the ability has not been realized.

We assume that p_k and n_k satisfy the following conditions.

Assumption 1

i. *(Symmetry)* $p_k(x, y) = p_{-k}(x, y) = p(x, y)$ and $n_k(1 - x, 1 - y) = n_{-k}(1 - x, 1 - y) = n(1 - x, 1 - y)$ for all $k \in \{A, B\}$, where x (y) represents a candidate's own (the opponent's) amount of campaign on policy.

ii. *(Full persuasion)* For each $k \in \{A, B\}$, $p_k(C^k, C^{-k}) + p_{-k}(C^{-k}, C^k) = 1$ and $n_k(1 - C^k, 1 - C^{-k}) + n_{-k}(1 - C^{-k}, 1 - C^k) = 1$ for any $C^k, C^{-k} \in \{C_H, C_L\}$.

iii. *(Monotonicity)* For each $k \in \{A, B\}$, $p_k(C_H, C^{-k}) > p_k(C_L, C^{-k})$ and $n_k(1 - C_H, 1 - C^{-k}) < n_k(1 - C_L, 1 - C^{-k})$ for any $C^{-k} \in \{C_H, C_L\}$, and $p_k(C^k, C_H) < p_k(C^k, C_L)$ and $n_k(1 - C^k, 1 - C_H) > p_k(1 - C^k, 1 - C_L)$ for any $C^k \in \{C_H, C_L\}$.

To put it differently, we assume that (i) the effect of campaign on the naive voters is the same across candidates, (ii) each naive voter is persuaded by campaign about policy (ability) provided by either the incumbent or the challenger, and (iii) a candidate succeeds in persuading a larger amount of the naive voters on a dimension as the amount of campaign about the dimension s/he (the opponent) provides increases (decreases).

3.6 Mass Media

Consider the setting of the mass media similar to that of Kishishita (2017), originally based on the setting of Besley and Prat (2006). There is one media outlet. The media outlet can observe the value of θ_A with probability $\delta \in (0, 1)$ by spending cost m. The media outlet reports the news if and only if it observes the truth.[12] It cannot report news that is not true. Also, we assume that

[11] In reality, voters would not be able to distinguish a small difference in allocations i.e., even sophisticated voters would observe only which of policy and ability a candidate emphasizes. Thus, considering a binary choice is meaningful to investigate the signaling role of campaign allocation.

[12] This setting implies that (i) news is hard information (the media outlet cannot tell a lie) and (ii) the media

only the sophisticated voters receive information through news.[13]

Next, define the profit of the outlet. We suppose that only news that incumbent's ability is low is profitable.[14] If the media outlet reports such news, it obtains the revenue $a(>0)$, which represents the sum of audience-related benefits including sales. If not, it obtains zero revenue. Thus, when the media outlet reports news that the incumbent's ability is low, its profit is $a-m$. The media outlet gathers news by spending cost m if and only if the expected profit is non-negative. We assume that $\delta a > m$ i.e., the news that the incumbent ability is low is sufficiently profitable so that the media outlet has an incentive to gather news when it believes that the incumbent's ability is low with probability one.

This setting implies that negative news is still profitable even after people have already known that the incumbent's ability is low through campaign. Here, we implicitly assume that news conveys not only whether the incumbent's ability is high, but also details information about how and why the incumbent ability is low, which cannot be obtained from campaign. Thus, news is still valuable for the voters since it conveys additional information.

3.7 Timing of the Game

The timing of the game is as follows:

1. Nature chooses $\{\theta_k\}_{k\in\{A,B\}}, \{\hat{x}_i\}_{i\in I}$ and $\{\varepsilon_i\}_{i\in I}$ independently.
2. Each candidate observes the incumbent's ability θ_A and chooses the fraction of campaign on policy $C^k \in \{C_H, C_L\}$ simultaneously. Then, the media outlet and the sophisticated voters observe $\{C^k\}_{k\in\{A,B\}}$.
3. The media outlet decides whether to spend costs m and gather news to observe the incumbent's ability θ_A. If it spends costs, it can observe θ_A with probability δ and then reports the observed value of θ_A to the sophisticated voters.
4. Each voter votes for either the incumbent or the challenger in the election.

The solution concept is a sequential equilibrium.[15]

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outlet does not selectively withhold the news. The first one has been widely used in the previous literature (e.g., Besley and Prat 2006; Bernhardt, Krasa, and Polborn 2008; Warren 2012). The second one means no possibility of media capture, and so the media outlet always reports news so long as it obtains the news. However, as Besley and Prat (2006) show, politicians could capture the mass media and make it hide the news. Even if we introduce such a possibility, the result obtained in the present paper would hold under several conditions. This is because the logic behind the result of Kishishita (2017) that the opposition party's whistleblowing through a filibuster is still robust under a possibility of media capture can be applied to our model.

[13] Even if we assume that the naive voters also receive news, a similar result still holds.

[14] Many empirical results show that negative news tends to be reported more than positive news (e.g., Harrington 1989; Patterson 1997; Soroka 2006; Ju 2008). Note that even if positive news (i.e., news that the incumbent's ability is low) is profitable, the result holds as long as the profit of reporting positive news is so small that the media outlet has no incentive to spend cost m if it expects that the probability that the incumbent ability is low is zero. Technically, this condition can be written as $\delta a' < m$, where a' is the sum of audience related benefit by reporting positive news.

[15] Strategies and belief system constitute a sequential equilibrium if and only if (i) each player's strategy

4. Equilibrium

In this section, we derive equilibria. In Section 4.1, we derive the number of the naive voters who vote for the incumbent given (C^A, C^B), for the preparation. In Sections 4.2 and 4.3, we derive the necessary and sufficient conditions for the existence of two classes of separating equilibria. Then, we prove that the other separating equilibria do not exist, and characterize the necessary and sufficient condition for the existence of separating equilibria in Section 4.4. For the last step, we eliminate pooling equilibria using the intuitive criterion in Section 4.5.

4.1 Voting Behavior of an Naive Voter

To begin with, we derive the number of the naive voters who vote for the incumbent given (C^A, C^B). From Assumption 1, the naive voters are divided into the following four types: those who believe that (1) the incumbent's policy and ability are good, (2) the challenger's policy and the incumbent's ability are good, (3) the incumbent's policy is good and her/his ability is low, (4) and the challenger's policy is good and the incumbent's ability is low.

From the distribution of ε_i, the fraction of those who vote for the incumbent among (1) is $\Phi(\alpha d + 0.5\beta g)$, that among (2) is $\Phi(-\alpha d + 0.5\beta g)$, that among (3) is $\Phi(\alpha d - 0.5\beta g)$, and that among (4) is $\Phi(-\alpha d - 0.5\beta g)$. Denote each of them by Φ_{HH}, Φ_{LH}, Φ_{HL}, and Φ_{LL} respectively for the ease of notation. We obtain the following lemma.

Lemma 1 $\Phi_{HH} > \Phi_{HL} > \Phi_{LH} > \Phi_{LL}$ holds.

Proof Since Φ is a strictly increasing function by definition, $\Phi_{HH} > \Phi_{HL}$ and $\Phi_{LH} > \Phi_{LL}$ holds. In addition, we have $\alpha d - 0.5\beta g > \alpha d + 0.5\beta g$ from the assumption that $\alpha d > 0.5\beta g$. Thus, $\Phi_{HL} > \Phi_{LH}$ holds. In summary, we obtain Lemma 1. ■

Then, the number of (1) who vote for the incumbent is $p(C^A, C^B)n(C^A, C^B)\Phi_{HH}$, that of (2) who vote for the incumbent is $\left(1 - p(C^A, C^B)\right)n(C^A, C^B)\Phi_{LH}$, that of (3) who vote for the incumbent is $p(C^A, C^B)\left(1 - n(C^A, C^B)\right)\Phi_{HL}$, and that of (4) who vote for the incumbent is $\left(1 - p(C^A, C^B)\right)\left(1 - n(C^A, C^B)\right)\Phi_{LL}$.

We finally obtain the number of the naive voters who vote for the incumbent. That is

$$(1-\gamma) \times \underbrace{\begin{bmatrix} p(C^A, C^B)n(C^A, C^B)(\Phi_{HH} - \Phi_{LH} - \Phi_{HL} + \Phi_{LL}) + p(C^A, C^B)(\Phi_{HL} - \Phi_{LL}) \\ +n(C^A, C^B)(\Phi_{LH} - \Phi_{LL}) + \Phi_{LL} \end{bmatrix}}_{\equiv F(C^A, C^B)}.$$

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is sequentially sophisticated given each belief, and (ii) beliefs of the sophisticated voters and the media outlet are consistent with the strategies. Since the naive voters are non-Bayesian updaters, their belief can be inconsistent with the strategies.

Throughout the paper, we impose the following assumption on the efficiency of campaign.

Assumption 2 The following inequality (*) holds:

$$\left(\frac{\Phi_{HL} - \Phi_{LH}}{\Phi_{LH} - \Phi_{LL}} + 1\right)\left(p(C_H, C_H) - p(C_L, C_H)\right) > n(1 - C_L, 1 - C_H) - n(1 - C_H, 1 - C_H).$$

$p(C_H, C_H) - p(C_L, C_H)$, the left-hand side of inequality (*), represents the marginal effect of an increase in campaign about policy on the naive voters. Also, $n(1 - C_L, 1 - C_H) - n(1 - C_H, 1 - C_L)$, the right-hand side, represents the marginal effect of an increase in campaign about ability on the naive voters. Thus, inequality (*) holds when the marginal effect of an increase in campaign about ability is not so large compared to an increase in campaign on policy. Since it is unnatural that both effects are totally different, Assumption 2 is not that restrictive.

A straightforward example satisfying this condition is that $p(C_H, C_L) - p(C_H, C_H) = n(1 - C_H, 1 - C_H) - n(1 - C_H, 1 - C_L)$.[16] More specifically, the following is an example satisfying inequality (*).

Example When $p(x, y) = n(x, y)$, and $C_H + C_L = 1$, inequality (*) holds.

Assumption 2 is powerful because it guarantees that campaign on ability is less efficient compared to campaign on policy. To see this, we obtain the next lemma. The omitted proofs are contained in Appendix.

Lemma 2 Under Assumptions 1 and 2,

$$\begin{aligned} F(C_H, C_H) - F(C_L, C_H) &= F(C_H, C_L) - F(C_L, C_L) = F(C_H, C_L) - F(C_H, C_H) \\ &= F(C_L, C_L) - F(C_L, C_H) > 0. \end{aligned}$$

Because of Assumption 1, the marginal effect of an increase (decrease) in campaign on policy by a candidate (the opponent) is the same independently of the opponent (the candidate)'s allocation of campaign resources, and the marginal effect of an increase in campaign on policy by a candidate is the same as the marginal effect of a decrease in campaign on policy by the opponent. Thus, all the four values above are the same. This is the first part of Lemma 2.

Moreover, because of Assumption 2, these effects are always positive. This is the second part of Lemma 2. This second part is the reason why Assumption 2 is useful. The second part means that the marginal effect of an increase in campaign on policy is strictly positive. In other words, Assumption 2 guarantees that campaign on policy is more efficient than campaign on ability in

[16] From Lemma 1, $(\Phi_{HL} - \Phi_{LH})/(\Phi_{LH} - \Phi_{LL}) + 1 > 1$. Thus, $p(C_H, C_L) - p(C_H, C_H) = n(1 - C_H, 1 - C_H) - n(1 - C_H, 1 - C_L)$ implies that inequality (*) holds.

terms of mobilizing the naive voters.

The reason why Assumption 2 guarantees this property is as follows. Policy and ability have different features. When the two candidates propose different policies on the same issue, the fact that one candidate's policy is good for voters implies that the other candidate's policy is bad. For example, suppose that trade reform is a central issue in an election, and one candidate promises trade protection while the other promises trade liberalization. In such a case, if a voter believes that trade protection is good, s/he also believes that trade liberalization is bad. Thus, when a candidate succeeds in persuading a voter that the policy the candidate promises is good for the voter, it also implies that the voter is persuaded that the policy the other candidate promises is bad for the voter. In contrast, such an effect does not exist in campaign about ability. Even if a candidate has a high ability, it does not mean that the other candidate has a low ability because the both can have high abilities. Thus, campaign on ability is less efficient than campaign on policy even under the weak condition (Assumption 2). This is consistent with the empirical result that policy is a more frequent topic of campaign messages than character, and winners are more likely to emphasize policy than character (Benoit 2007).

4.2 Negative Campaign Equilibrium [I]

In this subsection, we derive the necessary and sufficient condition under which there is at least one equilibrium in which (i) the challenger increases negative campaign on ability if and only if the incumbent's ability is low, and (ii) the incumbent emphasizes campaign on policy independently of the incumbent's ability. From now on, we call this class of equilibria *negative campaign equilibrium [I]*.

Suppose that there is negative campaign equilibrium [I]. Then, candidates' strategies, the sophisticated voters and the media outlet's belief and strategy should satisfy the following.

(1) *The incumbent always chooses* $C^A = C_H$.

(2) *If the incumbent's ability is high, the challenger chooses* $C^B = C_H$*. And if the incumbent's ability is low, the challenger chooses* $C^B = C_L$.

(3) *The media outlet and the sophisticated voters' belief after observing the amount of campaign:*

$$\pi\big(\theta_A = g \mid (C_H, C^B)\big) = \begin{cases} 1 \text{ if } C^B = C_H \\ 0 \text{ if } C^B = C_L \end{cases}.$$

(4) *When* $C^A = C_H, C^B = C_L$*, the media outlet spends costs, and observes and reports the true type of the incumbent's ability with probability* δ*. When* $C^A = C_H, C^B = C_H$*, the media outlet does not gather news.*[17]

[17] When $\theta_A = g$, the media outlet cannot obtain news such that the incumbent's ability is bad. Therefore, the media outlet does not gather news when it believes that $\theta_A = g$. When the media outlet believes that $\theta_A = 0$, the media outlet gathers news because $\delta a > m$.

(5) *Sophisticated voter* i *votes for the incumbent (the challenger) if*

$$\alpha[v(|x_A - \hat{x}_i|) - v(|x_B - \hat{x}_i|)] + \beta\big[\pi\big(\theta_A = g|\ (C^A, C^B)\big) - 0.5\big]g$$

is positive (negative). And s/he votes for the incumbent with probability 0.5 *if her/his expected relative utility is equal to be zero.*

Given this, we obtain the following proposition about the necessary and sufficient condition for the existence of negative campaign equilibrium [I].

Proposition 1 There exists a separating equilibrium in which (i) $C^B = C_L$ if and only if $\theta_A = 0$, and (ii) C^A is independent of the value of θ_A, if and only if $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$, where

$$\underline{\gamma} \equiv \frac{F(C_H, C_L) - F(C_H, C_H)}{F(C_H, C_L) - F(C_H, C_H) + [\rho(\Phi_{HH} - \Phi_{HL}) + (1-\rho)(\Phi_{LH} - \Phi_{LL})]};$$

$$\overline{\gamma} \equiv \frac{F(C_H, C_L) - F(C_H, C_H)}{F(C_H, C_L) - F(C_H, C_H) + (1-\delta)[\rho(\Phi_{HH} - \Phi_{HL}) + (1-\rho)(\Phi_{LH} - \Phi_{LL})]}.$$

In addition, $\underline{\gamma}, \overline{\gamma} \in (0, 1)$.

From this proposition, we can conclude that negative campaign about the incumbent's ability can be a credible signal. It is well known that so as to make a message a credible signal, sending the message must be costly. Here, campaign itself is assumed to be totally costless. However, the allocation of campaign resources change the naive voters' voting behaviors. In this aspect, negative campaign about the incumbent's ability is costly for the challenger. This is because of Lemma 2. Campaign on ability is less efficient than campaign on policy to mobilize the naive voters. In this sense, campaign on ability is costly. Therefore, the cheap talk game is reduced to a costly signaling game, and as a result, negative campaign about the incumbent's ability can be a credible signal.

The proposition above also characterizes the number of the sophisticated voters under which negative campaign equilibrium [I] exists. When the number of the naive voters who can be persuaded is large, a candidate can win the election by providing a large amount of campaign independently of whether campaign contains truth-telling information. Thus, it seems that the existence of the naive voters makes campaign less credible (i.e., less informative). In the result above, there is an upper bound of the number of the naive voters so that campaign becomes informative. Thus, our model derives this negative role of the naive voters. Surprisingly, in our model, there is also a lower bound of the number of the naive voters so that campaign becomes informative. In other words, an increase in the number of the naive voters can enhance the credibility of a message campaign conveys. There is a positive role of the naive voters. In summary, there is a non-monotonic relationship between the number of the naive voters and the credibility of a message campaign conveys.

Why do we obtain this surprising result? Remember the mechanism that creates informative campaign. An increase in the amount of negative campaign on the incumbent's ability becomes a credible signal because campaign on ability is costly in the sense that it can mobilize more naive voters than campaign on ability can. Thus, the effect of campaign on the naive voters creates the cost of negative campaign on the incumbent's ability. Therefore, some fraction of the naive voters is essential to making campaign informative.

Lastly, we mention the role of the mass media in creating a credible signal. As explained, campaign on ability is costly for the challenger. Unfortunately, this is not enough to create a credible signal. The cost of an increase in the fraction of campaign on ability is independent of the incumbent's ability. However, its net benefit must depend on the incumbent's ability. To do so, the mass media is necessary. Suppose that the challenger increases campaign on the incumbent's ability though the ability is high. Then, the mass media tries to gather news, and finds out that the ability is high, with some probability. As a result, the news that the ability is high is reported, and the sophisticated voters find out that the message the challenger sends is wrong. In other words, the challenger's lie is detected and the number of her/his obtained votes becomes quite small with some probability even if s/he increases the fraction of campaign on the incumbent's ability. Therefore, the benefit of campaign on ability is smaller for the challenger when the ability is high than when the ability is low. Thanks to this nature, separating equilibria can be constructed. The role of the mass media can be seen in the value of $\bar{\gamma}$. When $\delta = 0$ (i.e., the media plays no role), $\bar{\gamma} = \underline{\gamma}$, and so negative campaign equilibrium [I] is almost impossible to be constructed.

4.3 Negative Campaign Equilibrium [II]

Next, we examine another class of separating equilibria where negative campaign on the incumbent's ability provides the truthful information about the incumbent's ability. That is the equilibrium in which (i) the challenger increases negative campaign on ability if and only if the incumbent's ability is low, and (ii) the incumbent increases campaign on ability if and only if the incumbent's ability is high. We call this class of equilibria *negative campaign equilibrium [II]*. The difference from the class of equilibria discussed in the former subsection is that this is a separating equilibrium with multiple senders. In the equilibrium we focus on in this subsection, the incumbent as well as the challenger send signals to the sophisticated voters.

Define $p^* \equiv 1 - m/(\delta a)$. Note that $p^* \in (0, 1)$. This is the threshold value of p where the media outlet spends cost m if and only if $\pi \leq p^*$. In this equilibrium, the strategies and beliefs are the following:

(1) *If the incumbent's ability is high, the incumbent chooses $C^A = C_L$. If the incumbent's ability is low, the incumbent chooses $C^A = C_H$.*

(2) *If the incumbent's ability is high, the challenger chooses $C^B = C_H$. I the incumbent's ability*

is low, the challenger chooses $C^B = C_L$.

(3) The media outlet and the sophisticated voters' belief after observing the amount of campaign:

$$\pi(\theta_A = g|(C^A, C^B)) = \begin{cases} 1 \text{ if } (C^A, C^B) = (C_L, C_H) \\ 0 \text{ if } (C^A, C^B) = (C_H, C_L) \\ p_{HH} \text{ if } (C^A, C^B) = (C_H, C_H) \\ p_{LL} \text{ if } (C^A, C^B) = (C_L, C_L) \end{cases}.$$

(4) When $(C^A, C^B) = (C_L, C_H)$*, the media outlet spends costs, and observes and reports the true type of the incumbent's ability with probability* δ*. When* $(C^A, C^B) = (C_H, C_L)$*, the media outlet does not gather news. When* $(C^A, C^B) = (C_H, C_H)$ *(*$(C^A, C^B) = (C_L, C_L)$*), the media outlet spends costs, and observes and reports the true type of the incumbent's ability with probability* δ *if and only if* $p_{HH} \le p^*$ *(*$p_{LL} \le p^*$*).*

(5) Sophisticated voter i *votes for the incumbent (the challenger) if*

$$\alpha[v(|x_A - \hat{x}_i|) - v(|x_B - \hat{x}_i|)] + \beta[\pi(\theta_A = g|\,(C^A, C^B)) - 0.5]g$$

is positive (negative). And s/he votes for the incumbent with probability 0.5 *if her/his expected relative utility is equal to be zero.*

Given this, we obtain the following lemma about the necessary and sufficient condition under which the strategies and beliefs above constitute a sequential equilibrium.

Lemma 3 The strategies and beliefs above constitute a sequential equilibrium if and only if the following condition holds:

(1) p_{HH} satisfies either (1-1) or (1-2):

(1-1) $p_{HH} \le p^*$, and $\gamma \ge \gamma_M^-(p_{HH}) \equiv \max\{\gamma_1^-(p_{HH}), \gamma_2^-(p_{HH})\}$

(1-2) $p_{HH} > p^*$, and $\gamma \ge \gamma_M^+(p_{HH}) \equiv \max\{\gamma_1^+(p_{HH}), \gamma_2^+(p_{HH})\}$.

(2) p_{LL} satisfies either (2-1) or (2-2):

(2-1) $p_{LL} \le p^*$, and $\gamma \le \gamma_m^-(p_{LL}) \equiv \min\{\gamma_1^-(p_{LL}), \gamma_2^-(p_{LL})\}$

(2-2) $p_{LL} > p^*$, and $\gamma \le \gamma_m^+(p_{LL}) \equiv \min\{\gamma_1^+(p_{LL}), \gamma_2^+(p_{LL})\}$.

Here,

$$\gamma_1^-(p) \equiv \frac{F(C_H, C_H) - F(C_L, C_H)}{F(C_H, C_H) - F(C_L, C_H) + (1-\delta)[\rho(\Phi_{HH} - \Phi(\alpha d + (p-0.5)\beta g)) + (1-\rho)(\Phi_{LH} - \Phi(-\alpha d + (p-0.5)\beta g))]};$$

$$\gamma_1^+(p) \equiv \frac{F(C_H, C_H) - F(C_L, C_H)}{F(C_H, C_H) - F(C_L, C_H) + \rho(\Phi_{HH} - \Phi(\alpha d + (p-0.5)\beta g)) + (1-\rho)(\Phi_{LH} - \Phi(-\alpha d + (p-0.5)\beta g))};$$

$$\gamma_2^-(p) \equiv \frac{F(C_H, C_H) - F(C_L, C_H)}{F(C_H, C_H) - F(C_L, C_H) + (1-\delta)[\rho(\Phi(\alpha d + (p-0.5)\beta g) - \Phi_{HL}) + (1-\rho)(\Phi(-\alpha d + (p-0.5)\beta g) - \Phi_{LL})]};$$

$\gamma_2^+(p)$

$$\equiv \frac{F(C_H, C_H) - F(C_L, C_H)}{F(C_H, C_H) - F(C_L, C_H) + \rho(\Phi(\alpha d + (p - 0.5)\beta g) - \Phi_{HL}) + (1 - \rho)(\Phi(-\alpha d + (p - 0.5)\beta g) - \Phi_{LL})}.$$

In addition, $\gamma_1^-(p), \gamma_1^+(p), \gamma_2^-(p), \gamma_2^+(p) \in (0, 1)$ holds.[18]

If and only if there exist p_{HH} and p_{LL} satisfying the conditions in Lemma 3, negative campaign equilibrium [II] exists. Thus, in order to derive the necessary and sufficient condition for the existence of the equilibrium, it suffices to derive the necessary and sufficient condition under which there exist p_{HH} and p_{LL} satisfying the conditions in Lemma 3. For this purpose, we obtain several lemmas. Define

$$\gamma_M(p)\left(\gamma_m(p)\right) \equiv \begin{cases} \gamma_M^-(p)\ \left(\gamma_m^-(p)\right) \text{ if } p \leq p^* \\ \gamma_M^+(p)\ \left(\gamma_m^+(p)\right) \text{ if } p > p^* \end{cases}.$$

Lemma 4 $\gamma_1^+(p)$ and $\gamma_1^-(p)$ ($\gamma_2^+(p)$ and $\gamma_2^-(p)$) are increasing (decreasing) in p. In addition, there exists a unique solution $\hat{p}$ that satisfies $\gamma_M(\hat{p}) = \gamma_m(\hat{p})$.

Lemma 5 The following equation holds:

$$0.5[\rho(\Phi_{HH} - \Phi_{HL}) + (1 - \rho)(\Phi_{LH} - \Phi_{LL})]$$
$$= \rho[\Phi_{HH} - \Phi(\alpha d + (\hat{p} - 0.5)\beta g)] + (1 - \rho)[\Phi_{LH} - \Phi(-\alpha d + (\hat{p} - 0.5)\beta g)].$$

Proof From the definition of γ_1 and γ_2, this is straightforwardly obtained. ■

Given the lemmas above, we finally obtain the necessary and sufficient condition under which there exist p_{HH} and p_{LL} satisfying the conditions in Lemma 3. To put it differently, we obtain the necessary and sufficient condition for the existence of negative campaign equilibrium [II].

Proposition 2

(a) Suppose that $\hat{p} > p^*$. There exists a separating equilibrium in which (i) when $\theta_A = 0$, $(C^A, C^B) = (C_H, C_L)$, and (ii) when $\theta_A = g$, $(C^A, C^B) = (C_L, C_H)$,

1. if and only if $\gamma_1^+(\hat{p}) \leq \gamma \leq \gamma_1^-(p^*)$, when $\gamma_1^+(\hat{p}) < \gamma_1^-(p^*)$, and
2. if and only if $\gamma = \gamma_1^+(\hat{p})$, when $\gamma_1^+(\hat{p}) \geq \gamma_1^-(p^*)$.

(b) Suppose $\hat{p} \leq p^*$. There exists a separating equilibrium, in which (i) when $\theta_A = 0$, $(C^A, C^B) = (C_H, C_L)$, and (ii) when $\theta_A = g$, $(C^A, C^B) = (C_L, C_H)$,

1. if and only if $\gamma_1^+(p^*) < \gamma \leq \gamma_1^-(\hat{p})$, when $\gamma_1^-(\hat{p}) > \gamma_1^+(p^*)$, and

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[18] The role of the mass media examined in negative campaign equilibrium [I] can be seen also in negative campaign equilibrium [II]. Suppose that $\delta = 0$. Since the media outlet always does not gather news, $\gamma_M^+(p_{HH}) \leq \gamma \leq \gamma_m^+(p_{LL})$ must hold to sustain an equilibrium from Lemma 3. Such γ exists only when $p_{HH} = p_{LL} = 1$. Moreover, in this case, $\gamma_M^+(p_{HH}) = \gamma_m^+(p_{LL}) = \underline{\gamma}$. In summary, when $\delta = 0$, negative campaign equilibrium [II] does not exist so long as $\gamma \neq \underline{\gamma}$.

2. if and only if $\gamma = \gamma_1^-(\hat{p})$, when $\gamma_1^+(p^*) \leq \gamma_1^-(\hat{p})$.

When $\hat{p} > p^*$ and $\gamma_1^+(\hat{p}) \geq \gamma_1^-(p^*)$, or $\hat{p} \leq p^*$ and $\gamma_1^+(p^*) \leq \gamma_1^-(\hat{p})$, the equilibrium exists only when $\gamma = \gamma_1^+(\hat{p})$. In such cases, the equilibrium almost always does not exist. The meaningful cases are those under which $\hat{p} > p^*$ and $\gamma_1^+(\hat{p}) < \gamma_1^-(p^*)$, or $\hat{p} \leq p^*$ and $\gamma_1^+(\hat{p}) > \gamma_1^-(p^*)$. In such cases, the necessary and sufficient condition for the existence of the equilibrium is given by the interval of the value of γ. It means that there is an upper bound and lower bound of the number of sophisticated voters. This is the same as the condition for the existence of negative campaign equilibrium [I]. Again, the sufficiently large number of the naive voters is necessary (i.e., γ must be lower than the upper bound) because the existence of the naive voters creates the cost of campaign on ability. In addition, the sufficiently large number of the sophisticated voters is necessary (i.e., γ must be higher than the lower bound) because the cost of campaign on ability would be too high to send a signal when there is an extremely large number of the naive voters.

4.4 Characterization and Existence of Separating Equilibria

So far, we have focused on two classes of separating equilibria in which negative campaign on the incumbent's ability by the challenger is a signal of the incumbent's low ability. However, potentially, there may exist other separating equilibria. In Table 1, we can see twelve candidates of separating equilibria. In this subsection, we show the non-existence of the other separating equilibria, and characterize the necessary and sufficient condition about the fraction of the sophisticated voters under which a separating equilibrium exists.

To begin with, we obtain the following lemma that shows that there is no equilibrium similar with negative campaign equilibrium [I] except that the incumbent chooses C_L independently of her/his ability.

Lemma 6 There is no sequential equilibrium where $C^A = C_L$ independently of θ_A, and $C^B = C_H$ if and only if $\theta_A = 0$.

To increase the amount of campaign on ability is costly for the incumbent. Suppose that the incumbent's ability is low. In such a case, the incumbent's low ability is uncovered by the challenger's negative campaign. Given this, for the incumbent, there is no incentive to choose costly campaign allocation.

	$\boldsymbol{\theta_A = g}$	$\boldsymbol{\theta_A = 0}$		$\boldsymbol{\theta_A = g}$	$\boldsymbol{\theta_A = 0}$		$\boldsymbol{\theta_A = g}$	$\boldsymbol{\theta_A = 0}$
$\boldsymbol{C^B}$	C_H	C_L	$\boldsymbol{C^B}$	C_H	C_L	$\boldsymbol{C^B}$	C_H	C_L
$\boldsymbol{C^A}$	C_H	C_H	$\boldsymbol{C^A}$	C_L	C_L	$\boldsymbol{C^A}$	C_H	C_L

Case 1			**Case 2**			**Case 3**		
	$\boldsymbol{\theta_A = g}$	$\boldsymbol{\theta_A = 0}$		$\boldsymbol{\theta_A = g}$	$\boldsymbol{\theta_A = 0}$		$\boldsymbol{\theta_A = g}$	$\boldsymbol{\theta_A = 0}$
$\boldsymbol{C^B}$	C_H	C_L	$\boldsymbol{C^B}$	C_L	C_H	$\boldsymbol{C^B}$	C_L	C_H
$\boldsymbol{C^A}$	C_L	C_H	$\boldsymbol{C^A}$	C_H	C_H	$\boldsymbol{C^A}$	C_L	C_L
Case 4			**Case 5**			**Case 6**		
	$\boldsymbol{\theta_A = g}$	$\boldsymbol{\theta_A = 0}$		$\boldsymbol{\theta_A = g}$	$\boldsymbol{\theta_A = 0}$		$\boldsymbol{\theta_A = g}$	$\boldsymbol{\theta_A = 0}$
$\boldsymbol{C^B}$	C_L	C_H	$\boldsymbol{C^B}$	C_L	C_H	$\boldsymbol{C^B}$	C_H	C_H
$\boldsymbol{C^A}$	C_H	C_L	$\boldsymbol{C^A}$	C_L	C_H	$\boldsymbol{C^A}$	C_H	C_L
Case 7			**Case 8**			**Case 9**		
	$\boldsymbol{\theta_A = g}$	$\boldsymbol{\theta_A = 0}$		$\boldsymbol{\theta_A = g}$	$\boldsymbol{\theta_A = 0}$		$\boldsymbol{\theta_A = g}$	$\boldsymbol{\theta_A = 0}$
$\boldsymbol{C^B}$	C_H	C_H	$\boldsymbol{C^B}$	C_L	C_L	$\boldsymbol{C^B}$	C_L	C_L
$\boldsymbol{C^A}$	C_L	C_H	$\boldsymbol{C^A}$	C_H	C_L	$\boldsymbol{C^A}$	C_L	C_H
Case 10			**Case 11**			**Case 12**		

Table 1: Candidates of Separating Equilibria

Next, we obtain the lemma that shows that there is no equilibrium in which the challenger increases the amount of negative campaign if and only if the incumbent's ability is high.

Lemma 7 There is no sequential equilibrium where $C^B = C_L$ if and only if $\theta_A = g$.

Since campaign on ability is costly, the challenger has no incentive to do so when the information it conveys is harmful to her/himself (i.e., it makes the sophisticated voters think that the incumbent's ability is high).

Next, we obtain the following two lemmas that show that there is no equilibrium where only the incumbent's allocation of campaign resources is informative.

Lemma 8 There is no sequential equilibrium where $C^A = C_H$ if and only if $\theta_A = g$ while the challenger's campaign is independent of θ_A.

Lemma 9 There is no sequential equilibrium where $C^A = C_L$ if and only if $\theta_A = g$ while the challenger's campaign is independent of θ_A.

This is because the mass media does not work as the watchdog. Suppose that the incumbent sends a message that her/his ability is high despite of the low ability. Since the challenger does not send any information, the mass media believes that the incumbent's ability is high. Then, the media does not gather news because there is no possibility that it finds out the low ability (i.e., profitable news). Therefore, the incumbent can deceive sophisticated voters perfectly since the mass media no longer works as the fact-checker. As a result, the incumbent's campaign is nothing informative.

Lastly, we obtain the lemma that shows that there is no equilibrium where the incumbent increases the amount of campaign on ability only when her/his ability is high even if the challenger's campaign is informative. Since campaign on ability is costly, the incumbent has no incentive to increase its amount when it conveys negative information about the incumbent.

Lemma 10 There is no sequential equilibrium where $C^A = C_L$ if and only if $\theta_A = 0$, and $C^B = C_L$ if and only if $\theta_A = 0$.

Given these lemmas, only equilibria examined in Sections 4.2 and 4.3 can constitute a separating equilibrium.

Proposition 3 If a separating equilibrium exists, that is either negative campaign equilibrium [I] or [II].

Proof From Lemma 6, Case 2 in Table 1 cannot constitute an equilibrium. From Lemma 7, Cases 5-8 cannot constitute an equilibrium. From Lemma 8, Cases 9 and 11 cannot constitute any equilibrium. From Lemma 9, Cases 10 and 12 cannot constitute any equilibrium. From Lemma 10, Case 3 cannot constitute any equilibrium. In summary, only Cases 1 and 4 can constitute a separating equilibrium. Therefore, we obtain the result. ■

Therefore, in any separating equilibrium, the challenger increases the amount of campaign on the incumbent's ability if and only if the incumbent's ability is low. In other words, in any separating equilibrium, negative campaign on the incumbent's ability arises as a signal of the incumbent's low ability.

From Propositions 1-3, we finally obtained the full characterization of separating equilibria. The remaining task is to derive the necessary and sufficient condition under which there is at least one separating equilibrium, using the conditions in Propositions 1 and 2. To this end, we need to examine the magnitude relation between the upper and lower bounds of γ derived in Proposition 1 and those derived in Proposition 2. We obtain the lemma about this relation.

Lemma 11 For any $p \in (0,1)$, $\gamma_1^+(p), \gamma_2^+(p) > \underline{\gamma}$ and $\gamma_1^-(p), \gamma_2^-(p) > \overline{\gamma}$.

Therefore, both the upper and lower bounds of γ for the existence of negative campaign equilibrium [II] are higher than those for the existence of negative campaign equilibrium [I]. In negative campaign equilibrium [I], the incumbent as well as the challenger send a signal through the allocation of campaign resources. This creates the property above.

To see this, first, consider the challenger's deviation incentive when the incumbent's ability is high. In such a case, the challenger has an incentive to increase the amount of campaign on ability and sends a signal that the incumbent's ability is low to the sophisticated voters. When only the challenger sends a signal (i.e., in negative campaign equilibrium [I]), this deviation is succeeded, and the sophisticated voters believe the challenger's lie so long as monitoring by the mass media is not succeeded. On the contrary, when the incumbent as well as the challenger send signals (i.e., in negative campaign equilibrium [II]), the sophisticated voters find out that either the incumbent or the challenger tells a lie after the challenger's unilateral deviation. Thus, the sophisticated voters do not fully believe the challenger's message. Therefore, the deviation incentive of the challenger is smaller when the incumbent as well as the challenger send signals than when only the challenger sends a signal. Since the existence of the naive voters creates the cost of campaign on ability (i.e., sending a signal), the necessary number of the naive voters is lower (the upper bound of γ is higher) as the deviation incentive above is smaller. Therefore, the upper bound of γ is higher in negative campaign equilibrium [II] than negative campaign equilibrium [I].

Second, examine the lower bound of γ. Consider the challenger's deviation incentive when the incumbent's ability is low. Since campaign on ability is costly, the challenger has an incentive to allocate only the small amount of campaign resources to campaign on ability even if the incumbent's ability is low. When only the challenger sends a signal, the sophisticated voters fully believe that the incumbent's ability is low after this deviation. Thus, the loss due to decreasing the amount of campaign on ability is large. On the other hand, when not only the challenger but also the incumbent send signals, after the deviation, the sophisticated voters think that either one deviates so that they do not fully believe that the incumbent's ability is low. Thus, the loss due to decreasing the amount of campaign on ability is smaller in this case than when only the challenger sends a signal. Therefore, in order to prevent this type of deviation, the cost of campaign on ability must be small when both candidates send signals. As a result, the necessary number of the sophisticated voters is higher (the lower bound of γ is higher) in negative campaign equilibrium [II] than negative campaign equilibrium [I].

Though the results in Lemma 11 are independent of the values of parameters, we obtain an additional result depending on the value of δ.

Lemma 12 Fix p^* and $\hat{p}$. If and only if $\delta \geq 0.5$, $\gamma_1^+(\hat{p}) \leq \overline{\gamma}$. In addition, there is $\bar{\delta} \in [0,1)$ such that if and only if $\delta \geq \bar{\delta}$, $\gamma_1^+(p^*) \leq \overline{\gamma}$.[19]

[19] Notice that $\bar{\delta}$ depends on p^* and as a result δ. The second part of Lemma 12 does not necessarily imply that $\gamma_1^+(p^*) \leq \overline{\gamma}$ is more likely to hold as δ increases. Fix the value of p^*, and consider the set $K(p^*) \equiv \{(\delta, a/m) | \, 1 - m/(\delta a) = p^*\}$. The precise implication of the second half is that for $(\delta, a/m)$ and $(\delta', a'/m') \in K(p^*)$ such that $\delta > \delta'$, $\gamma_1^+(p^*) \leq \overline{\gamma}$ is more likely to hold under $(\delta, a/m)$ than under $(\delta', a'/m')$.

Finally, we obtain the necessary and sufficient condition under which at least one separating equilibrium exists.

Theorem 1 Suppose that either (i) $\hat{p} > p^*$ and $\gamma_1^+(\hat{p}) < \gamma_1^-(p^*)$, or (ii) $\hat{p} \le p^*$ and $\gamma_1^+(\hat{p}) > \gamma_1^-(p^*)$ holds. Then, at least one separating equilibria exists if and only if the following condition is satisfied:

1. When $p^* < \hat{p}$ and $\delta \ge 0.5$, $\underline{\gamma} \le \gamma \le \gamma_1^-(p^*)$ is satisfied.
2. When $p^* < \hat{p}$ and $\delta < 0.5$, either $\gamma_1^+(\hat{p}) < \gamma \le \gamma_1^-(p^*)$ or $\underline{\gamma} \le \gamma \le \overline{\gamma}$ is satisfied
3. When $p^* \ge \hat{p}$ and $\delta \ge \bar{\delta}$, $\underline{\gamma} \le \gamma \le \gamma_1^-(\hat{p})$ is satisfied.
4. When $p^* \ge \hat{p}$ and $\delta < \bar{\delta}$, either $\gamma_1(p^*) \le \gamma \le \gamma_1^-(\hat{p})$ or $\underline{\gamma} \le \gamma \le \overline{\gamma}$ is satisfied.

Proof Combining Propositions 1-3, and Lemmas 11 and 12, we have this argument. ■

As discussed in Proposition 2, if neither (i) nor (ii) holds, negative campaign equilibrium [II] almost always does not exist (i.e., γ satisfying the conditions is just one point). In such cases, the necessary and sufficient condition for the existence of a separating equilibrium is almost the same as that for the existence of negative campaign equilibrium [I]. Therefore, in the theorem above, we focus on the case where either (i) or (ii) holds. Then, as in Propositions 1 and 2, each condition for the existence of negative campaign equilibrium [I] and [II] respectively is represented by the interval of γ. One question is whether each interval is overlapped (i.e., whether the condition for the existence of a separating equilibrium is also represented by a single interval). There are some cases where the two intervals are not overlapped. That is 2 and 4 in Theorem 1. However, when the probability that the media outlet succeeds in finding out the truth δ is high, the two intervals are overlapped, and as a result, the condition for the existence of a separating equilibrium is characterized by a single interval. That is 1 and 3 in Theorem 1.

4.5 Elimination of Pooling Equilibria

While we have examined separating equilibria so far, pooling equilibria in which campaign does not convey any information could exist. Indeed, the following lemma shows the existence of a pooling equilibrium. Throughout this section, we assume that $p^* < 0.5$ holds i.e., the mass media does not gather news in pooling equilibria.

Lemma 13 There is a sequential equilibrium where $(C^A, C^B) = (C_H, C_H)$ independently of θ_A.

Due to this nature, we need to examine the condition under which negative campaign equilibria [I] and [II] are more plausible than pooling equilibria. To this end, we extend the intuitive criterion proposed by Cho and Kreps (1987) to the case where there are two senders.[20]

Consider the following general model.[21] There exist two senders $s = 1, 2$ and one receiver r. Each player takes action $a_i \in A_i$ $(i = 1, 2, r)$. Define $A \equiv \times_i A_i$. In addition, there is a state space $\Theta = \{\theta_1, \theta_2\}$ where its generic element is denoted by θ. Player i's payoff is $u_i: \Theta \times A \to \mathbb{R}$. The timing of the game is as follows. First, only players 1 and 2 observe θ.Then, players 1 and 2 choose their actions simultaneously. After observing their actions, player r chooses her/his action. Note that we focus on pure strategies.

Introduce some notations.[22] Denote the expected equilibrium payoff of player s given θ by $u_s^*(\theta)$. Let player s's pure strategy given θ by $m_s^*(\theta)$. Let the belief of player r on the state given a_1, a_2 by π. Using this, define the set of best response actions of player r given π and a_1, a_2, by $BR_r(\pi, a_{-r})$. Then, for any set T of states, define

$$BR_r(T, a_{-r}) \equiv \bigcup_{\{\pi:\pi(T)=1\}} BR_r(\pi, a_{-r}).$$

For $s = 1, 2$, let

$$\Theta^s(a_1, a_2) \equiv \left\{\theta \in \Theta \middle| m_{-s}^*(\theta) = a_{-s}, u_s^*(\theta) \leq \max_{a_r \in BR_r(\pi, a_{-r})} u_1(a_1, a_2, a_r, \theta)\right\}$$

if $\max_{a_r \in BR_r(\pi, a_{-r})} u_1(a_1, a_2, a_r, \theta)$ exists, and

$$\Theta^s(a_1, a_2) \equiv \left\{\theta \in \Theta \middle| m_{-s}^*(\theta) = a_{-s}, u_s^*(\theta) < \sup_{a_r \in BR_r(\pi, a_{-r})} u_1(a_1, a_2, a_r, \theta)\right\}$$

otherwise.

Lastly, we define the off-path of the pair of actions taken by the senders and the off-path of the action taken by a sender, respectively. We say "(a_1, a_2) is off-path" if there is no $\theta \in \Theta$ such that $(a_1, a_2) = \big(m_1^*(\theta), m_2^*(\theta)\big)$, and say "$a_s$ is off-path" if there is no $\theta \in \Theta$ such that $a_s = m_s^*(\theta)$.

Given these notations, we introduce the concept of the intuitive criterion in our frame work.

Definition *A sequential equilibrium with the belief system π^* satisfies the intuitive criterion if the following conditions are satisfied for each off-equilibrium path (a_1, a_2):*

1. *If a_s is off-path, but a_{-s} is on-path, $\pi^*(a_1, a_2) \in \Delta(\Theta^s(a_1, a_2))$ so long as $\Theta^s(a_1, a_2)$ is non-empty.*
2. *If a_1 and a_2 are on-path, $\pi^*(a_1, a_2) \in \Delta(\Theta^1(a_1, a_2) \cup \Theta^2(a_1, a_2))$ so long as*

[20] Our refinement is similar with those employed by Bagwell and Ramey (1991), Schultz (1996), Zhang (2016), and so on. For the verification of this refinement, see those papers.

[21] To be precise, the following model does not include the present model since the mass media and the sophisticated voters exist as player r in our model. However, the criterion can be straightforwardly extended to our model. Thus, we employ the following simple model for the ease of expositions.

[22] $-s$ represents a sender who is not s i.e., $-s = 2$ if $s = 1$, and $-r$ represents the two senders.

$\Theta^1(a_1, a_2) \cup \Theta^2(a_1, a_2)$ *is non-empty.*

Using this criterion, we derive the necessary and sufficient condition under which only negative campaign equilibria are sequential equilibria satisfying the intuitive criterion.

As in Lemma 13, there is always a pooling equilibrium in which both candidates choose C_H independently of the incumbent's ability. The first task is to derive the necessary and sufficient condition under which this equilibrium does not satisfy the intuitive criterion. Let

$$I(\delta) \equiv F(C_H, C_L) - F(C_H, C_H) + \rho\Phi(\alpha d) + (1-\rho)\Phi(-\alpha d) - (1-\delta)[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}] - \delta[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}].$$

Lemma 14 Sequential equilibria in which $(C^A, C^B) = (C_H, C_H)$ independently of θ_A do not satisfy the intuitive criterion if and only if the following condition is satisfied.

1. When $I(\delta) > 0$, $\gamma_L < \gamma < \gamma_H$ and $\gamma \leq \gamma_H{}'$ hold.
2. When $I(\delta) \leq 0$, $\gamma_L < \gamma \leq \gamma_H{}'$ holds.

Here,

$$\gamma_L \equiv \frac{F(C_H, C_L) - F(C_H, C_H)}{F(C_H, C_L) - F(C_H, C_H) + \rho\big(\Phi(\alpha d) - \Phi_{HL}\big) + (1-\rho)(\Phi(-\alpha d) - \Phi_{LL})};$$

$$\gamma_H \equiv \frac{F(C_H, C_L) - F(C_H, C_H)}{I(\delta)} \quad (\text{if } I(\delta) > 0);$$

$$\gamma_H{}' \equiv \frac{F(C_H, C_L) - F(C_H, C_H)}{F(C_H, C_L) - F(C_H, C_H) + \rho\big(\Phi(\alpha d) - \Phi(\alpha d + (p^* - 0.5)\beta g)\big) + (1-\rho)\big(\Phi(-\alpha d) - \Phi(-\alpha d + (p^* - 0.5)\beta g)\big)}.$$

In addition, γ_L, $\gamma_H{}' \in (0, 1)$, and $\gamma_H > 0$.

In this lemma, we derived the necessary condition under which the pooling equilibria considered in Lemma 13 do not satisfy the intuitive criterion. However, this is not enough because there may exist another pooling equilibrium: an equilibrium in which $(C^A, C^B) = (C_L, C_L)$ independently of θ_A. The following lemma argues that such an equilibrium does not exist under the condition in Lemma 14.

Lemma 15 There is no sequential equilibrium in which $(C^A, C^B) = (C_L, C_L)$ independently of θ_A if the condition in Lemma 14 is satisfied.

Therefore, the condition in Lemma 14 is a necessary and sufficient condition for eliminating pooling equilibria. However, if the condition is not satisfied for any γ under which a negative campaign equilibrium exists, there is no γ for which only negative campaign equilibria satisfy the intuitive criterion. To avoid such cases, we need a condition about the value of δ, given by the following lemma.

Lemma 16 There is $\bar{\bar{\bar{\delta}}} \in [0,1)$ such that for any $\delta \in (\bar{\bar{\bar{\delta}}}, 1)$, $\min\{\gamma_1^-(p^*),\ \gamma_1^-(\hat{p})\} > \gamma_L$.
Proof γ_L is independent of δ whereas $\min\{\gamma_1^-(p^*),\ \gamma_1^-(\hat{p})\}$ is increasing with δ. In addition, as $\delta \to 1$, $\min\{\gamma_1^-(p^*),\ \gamma_1^-(\hat{p})\} \to 1$. Thus, as $\delta \to 1$, $\min\{\gamma_1^-(p^*),\ \gamma_1^-(\hat{p})\} > \gamma_L$. Combining these facts, we complete the proof. ■

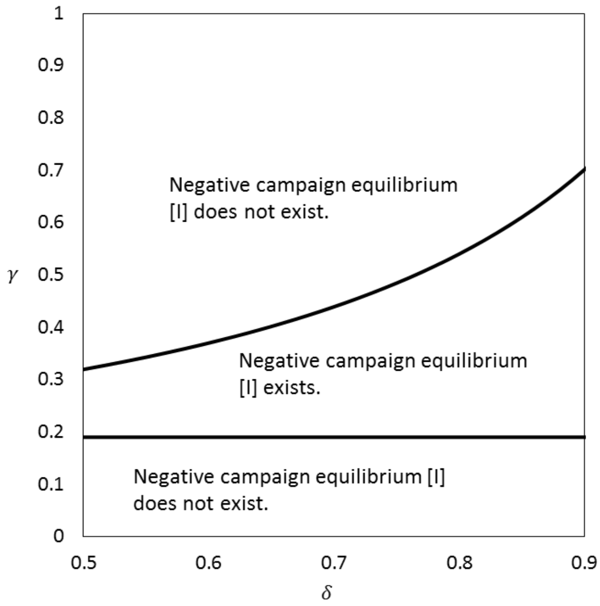
As seen in the following theorem, if the condition in Lemma 16 holds, there exists γ for which the necessary and sufficient condition, under which only negative campaign equilibria are sequential equilibria satisfying the intuitive criterion, is satisfied.

Theorem 2 Assume that $\delta > \bar{\bar{\bar{\delta}}}$ and $I(\delta) > 0$. In addition, suppose that either (i) $\hat{p} > p^*$, $\delta \geq 0.5$, and $\gamma_1^+(\hat{p}) < \gamma_1^-(p^*)$, or (ii) $\hat{p} \leq p^*$, $\delta \geq \bar{\delta}$, and $\gamma_1^+(\hat{p}) > \gamma_1^-(p^*)$ holds. Then, at least one separating equilibrium in which the challenger chooses C_L if and only if $\theta_A = 0$, satisfies the intuitive criterion, and all the other equilibria (pooling equilibria) do not satisfy the intuitive criterion, if and only if the following conditions hold.

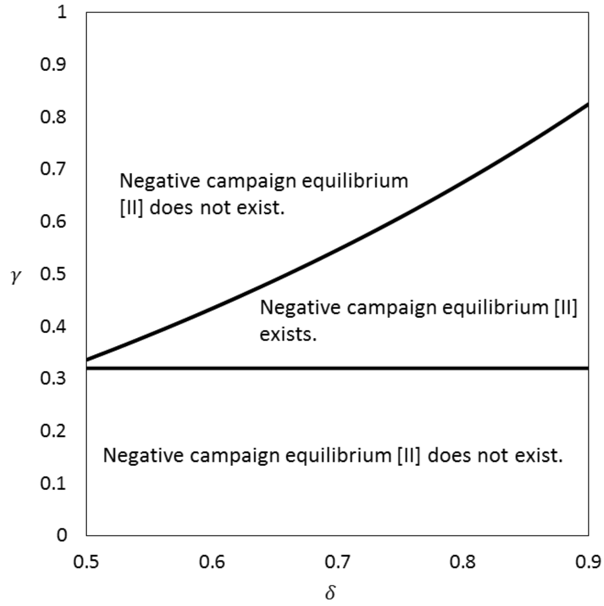
1. When $p^* < \hat{p}$ and $\min\{\gamma_1^-(p^*), \gamma_H, \gamma_H'\} = \gamma_H$, $\gamma_L < \gamma < \gamma_H$ is satisfied.
2. When $p^* < \hat{p}$ and $\min\{\gamma_1^-(p^*), \gamma_H, \gamma_H'\} \neq \gamma_H$, $\gamma_L < \gamma \leq \min\{\gamma_1^-(p^*),\ \gamma_H'\}$ is satisfied.
3. When $p^* \geq \hat{p}$ and $\min\{\gamma_1^-(\hat{p}), \gamma_H, \gamma_H'\} = \gamma_H$, $\gamma_L < \gamma < \gamma_H$ is satisfied.
4. When $p^* \geq \hat{p}$ and $\min\{\gamma_1^-(\hat{p}), \gamma_H, \gamma_H'\} \neq \gamma_H$, $\gamma_L < \gamma \leq \min\{\gamma_1^-(\hat{p}),\ \gamma_{\mathrm{H}}'\}$ is satisfied.

Here, for the ease of expositions, we focus on the cases where the condition for the existence of separating equilibria is characterized by a single interval (i.e., 1 and 3 in Theorem 1), and $I(\delta) > 0$. Though the condition for the elimination of pooling equilibria is stricter than that for the existence of separating equilibria, we, again, obtain a single interval of γ.

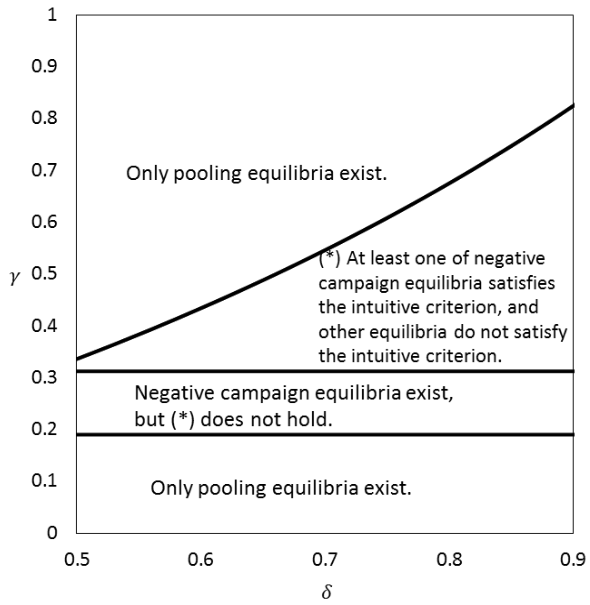
Lastly, to summarize our results, see a numerical example in Figure 1. In (a) ((b)), we can see that there is both an upper bound and a lower bound for the existence of negative campaign equilibrium [I] ([II]). The upper and lower bounds for the existence of negative campaign equilibrium [II] is higher than those for the existence of negative campaign equilibrium [I]. In (c), we can see that the condition for the elimination of pooling equilibria is characterized by a single interval of γ. As discussed, this interval is not necessarily equivalent to the interval for the existence of negative campaign equilibria. There is a region in which negative campaign equilibria exist, but the condition in Theorem 2 is not satisfied.



(a) Negative Campaign Equilibrium [I]



(b) Negative Campaign Equilibrium [II]



(c) Equilibrium Refinement

Figure 1: Numerical Example

$\alpha d = 0.8, \beta g = 0.9, \Phi = N(0,1), \rho = 0.7, \frac{a}{m} = 0.4662,\ p(x,y) = n(x,y) = \frac{x}{x+y}, C_H = 0.8, C_L = 0.2.$

5. Concluding Remarks

We constructed a model where an incumbent and a challenger compete with each other in an election, and each candidate decides how to allocate her/his own resources to two types of campaign: policy and ability. In the model, sophisticated voters, naive voters, and a media outlet exist. Then, we showed that there is a separating equilibrium such that campaign allocation is informative so long as sophisticated voters and naive voters coexist. In addition, we showed that in any separating equilibria, the challenger increases the fraction of campaign on the incumbent's ability if and only if the incumbent's ability is low (i.e., negative campaign against the incumbent arises as a signal). Overall, this study demonstrated that (i) using campaign allocation over multidimensional subjects, a candidate sends a credible message to sophisticated voters, and (ii) there is a non-monotonic relationship between the credibility of a message campaign conveys and the number of naive voters.

Before closing this paper, let us see the remaining challenges for the future researches. First, in our model, policies and abilities are binary. To examine the case with continuous variables may be promising. Second, candidates do not choose their policies in our model. However, in reality, they can commit policies to some extent, and so policies can be strategic choice variables. Such cases should be examined in the future.

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Appendix: Omitted Proofs

A.1 Proof of Lemma 2

$$
\begin{aligned}
&F(C_H, C_L) - F(C_H, C_H) \\
&\quad = \big(p(C_H, C_L)n(1 - C_H, 1 - C_L) - p(C_H, C_H)n(1 - C_H, 1 - C_H)\big)(\Phi_{HH} - \Phi_{LH} \\
&\quad\quad - \Phi_{HL} + \Phi_{LL}) + \big(p(C_H, C_L) - p(C_H, C_H)\big)(\Phi_{HL} - \Phi_{LL}) \\
&\quad\quad + \big(n(1 - C_H, 1 - C_L) - n(1 - C_H, 1 - C_H)\big)(\Phi_{LH} - \Phi_{LL}),
\end{aligned}
$$

$$
\begin{aligned}
&F(C_L, C_L) - F(C_L, C_H) \\
&\quad = \big(p(C_L, C_L)n(1 - C_L, 1 - C_L) - p(C_L, C_H)n(1 - C_L, 1 - C_H)\big)(\Phi_{HH} - \Phi_{LH} \\
&\quad\quad - \Phi_{HL} + \Phi_{LL}) + \big(p(C_L, C_L) - p(C_L, C_H)\big)(\Phi_{HL} - \Phi_{LL}) \\
&\quad\quad + \big(n(1 - C_L, 1 - C_L) - n(1 - C_L, 1 - C_H)\big)(\Phi_{LH} - \Phi_{LL}).
\end{aligned}
$$

$$
\begin{aligned}
&F(C_H, C_H) - F(C_L, C_H) \\
&\qquad = \big(p(C_H, C_H)n(1 - C_H, 1 - C_H) - p(C_L, C_H)n(1 - C_L, 1 - C_H)\big)(\Phi_{HH} - \Phi_{LH} \\
&\qquad - \Phi_{HL} + \Phi_{LL}) + \big(p(C_H, C_H) - p(C_L, C_H)\big)(\Phi_{HL} - \Phi_{LL}) \\
&\qquad + \big(n(1 - C_H, 1 - C_H) - n(1 - C_L, 1 - C_H)\big)(\Phi_{LH} - \Phi_{LL}), \\
&F(C_H, C_L) - F(C_L, C_L) \\
&\qquad = \big(p(C_H, C_L)n(1 - C_H, 1 - C_L) - p(C_L, C_L)n(1 - C_L, 1 - C_H)\big)(\Phi_{HH} - \Phi_{LH} \\
&\qquad - \Phi_{HL} + \Phi_{LL}) + \big(p(C_H, C_L) - p(C_L, C_L)\big)(\Phi_{HL} - \Phi_{LL}) \\
&\qquad + \big(n(1 - C_H, 1 - C_L) - n(1 - C_L, 1 - C_L)\big)(\Phi_{LH} - \Phi_{LL}).
\end{aligned}
$$

Note that by the symmetry of the density function ϕ, the following equality holds:

$$
\Phi_{HH} - \Phi_{LH} - \Phi_{HL} + \Phi_{LL} = \int_{\alpha d - 0.5\beta g}^{\alpha d + 0.5\beta g} \phi(x)dx - \int_{-(\alpha d + 0.5\beta g)}^{-(\alpha d - 0.5\beta g)} \phi(x)dx = 0.
$$

Therefore,

$$
\begin{aligned}
&F(C_H, C_L) - F(C_H, C_H) \\
&\qquad = \big(p(C_H, C_L) - p(C_H, C_H)\big)(\Phi_{HL} - \Phi_{LL}) \\
&\qquad + \big(n(1 - C_H, 1 - C_L) - n(1 - C_H, 1 - C_H)\big)(\Phi_{LH} - \Phi_{LL}), \\
&F(C_L, C_L) - F(C_L, C_H) \\
&\qquad = \big(p(C_L, C_L) - p(C_L, C_H)\big)(\Phi_{HL} - \Phi_{LL}) \\
&\qquad + \big(n(1 - C_L, 1 - C_L) - n(1 - C_L, 1 - C_H)\big)(\Phi_{LH} - \Phi_{LL}). \\
&F(C_H, C_H) - F(C_L, C_H) \\
&\qquad = \big(p(C_H, C_H) - p(C_L, C_H)\big)(\Phi_{HL} - \Phi_{LL}) \\
&\qquad + \big(n(1 - C_H, 1 - C_H) - n(1 - C_L, 1 - C_H)\big)(\Phi_{LH} - \Phi_{LL}), \\
&F(C_H, C_L) - F(C_L, C_L) \\
&\qquad = \big(p(C_H, C_L) - p(C_L, C_L)\big)(\Phi_{HL} - \Phi_{LL}) \\
&\qquad + \big(n(1 - C_H, 1 - C_L) - n(1 - C_L, 1 - C_L)\big)(\Phi_{LH} - \Phi_{LL}).
\end{aligned}
$$

Here, for any $x \in \{C_H, C_L\}$, $p(x, x) = n(x, x) = 0.5$ and for any $x, y \in \{C_H, C_L\}$, $p(x, y) = 1 - p(y, x)$ and $n(x, y) = 1 - n(y, x)$ because of Assumption 1. Thus, $F(C_H, C_L) - F(C_H, C_H) = F(C_L, C_L) - F(C_L, C_H) = F(C_H, C_H) - F(C_L, C_H) = F(C_H, C_L) - F(C_L, C_L)$.

Finally, $F(C_H, C_L) - F(C_H, C_H) > 0$ can be rewritten as:

$$
\left(\frac{\Phi_{HL} - \Phi_{LH}}{\Phi_{LH} - \Phi_{LL}} + 1\right)\big(p(C_H, C_L) - p(C_H, C_H)\big) > n(1 - C_H, 1 - C_H) - n(1 - C_H, 1 - C_L).
$$

Therefore, if and only if

$$
\left(\frac{\Phi_{HL} - \Phi_{LH}}{\Phi_{LH} - \Phi_{LL}} + 1\right)\big(p(C_H, C_L) - p(C_H, C_H)\big) > n(1 - C_H, 1 - C_H) - n(1 - C_H, 1 - C_L)
$$

holds, $F(C_H, C_L) - F(C_H, C_H) > 0$ holds. ■

A.2 Proof of Proposition 1

(1) "Only if" part

Consider the incentive compatibility condition of challenger B. Note that we focus on the equilibrium where $C^A = C_H$ because the condition is the same independently of C^A from the property in Lemma 2.

(i) When $\theta_A = g$:

If and only if the number of voters who votes for the incumbent when $(C^A, C^B) = (C_H, C_H)$ is less than or equal to that when $(C^A, C^B) = (C_H, C_L)$, the incumbent chooses C_H when $\theta_A = g$. Derive this condition.

The sophisticated voters and the media outlet believe that $\theta_A = g$ when $(C^A, C^B) = (C_H, C_H)$. As a result, in the election, the sophisticated voters believe that $\theta_A = g$ when $(C^A, C^B) = (C_H, C_H)$.

On the other hand, the sophisticated voters and the media outlet believe that $\theta_A = 0$ when $(C^A, C^B) = (C_H, C_L)$. Thus, when $(C^A, C^B) = (C_H, C_L)$, the media outlet gathers news and reports the news such that the incumbent's ability is high with probability δ. In summary, in the election, with probability δ, the sophisticated voters believe that $\theta_A = g$, while with probability $1-\delta$, they believe that $\theta_A = 0$ when $(C^A, C^B) = (C_H, C_L)$.

From this discussion, the condition is given by

$$\begin{aligned}[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}] + (1-\gamma)F(C_H, C_H) \\ \le \gamma\{(1-\delta)[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}] + \delta[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}]\} \\ + (1-\gamma)F(C_H, C_L).\end{aligned}$$

By rewriting this condition, we have $\gamma \le \bar{\gamma}$.

(ii) When $\theta_A = 0$:

If and only if the number of voters who vote for the incumbent when $(C^A, C^B) = (C_H, C_L)$ is less than or equal to that when $(C^A, C^B) = (C_H, C_H)$, the incumbent chooses C_H when $\theta_A = 0$. Derive this condition.

The sophisticated voters and the media outlet believe that $\theta_A = g$ when $(C^A, C^B) = (C_H, C_H)$. Thus, the media outlet does not gather news. As a result, in the election, the sophisticated voters believe that $\theta_A = g$ when $(C^A, C^B) = (C_H, C_H)$.

On the other hand, the sophisticated voters and the media outlet believe that $\theta_A = 0$ when $(C^A, C^B) = (C_H, C_L)$. As a result, in the election, the sophisticated voters believe that $\theta_A = 0$ when $(C^A, C^B) = (C_H, C_L)$.

From this discussion, the condition is given by

$$\gamma[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}] + (1-\gamma)F(C_H, C_L) \le \gamma[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}] + (1-\gamma)F(C_H, C_H).$$

By rewriting this condition, we have $\gamma \ge \underline{\gamma}$.

From (i) and (ii), $\underline{\gamma} \le \gamma \le \bar{\gamma}$ is the necessary condition for the existence of the equilibrium.

(2) "If" part

We show that if $\underline{\gamma} \leq \gamma \leq \bar{\gamma}$ holds, at least one such equilibrium exists. For this purpose, we show that the following specific equilibrium always exists so long as $\underline{\gamma} \leq \gamma \leq \bar{\gamma}$ holds.

(1) *The media outlet and the sophisticated voters' belief after observing the amount of campaign:*

$$\pi\big(\theta_A = g \mid (C^A, C^B)\big) = \begin{cases} 1 \text{ if } C^B = C_H \\ 0 \text{ if } C^B = C_L \end{cases}$$

(2) *Sophisticated voter* i *votes for the incumbent (the challenger) if*

$$\alpha[v(|x_A - \hat{x}_i|) - v(|x_B - \hat{x}_i|)] + \beta\big[\pi\big(\theta_A = g \mid (C^A, C^B)\big) - 0.5\big]g$$

is positive (negative). And s/he votes for the incumbent with probability 0.5 *if her/his expected relative utility is equal to be zero.*

(3) *The incumbent always chooses* $C^A = C_H$.

(4) *If the incumbent's ability high, the challenger chooses* $C^B = C_H$. *And if the incumbent's ability is bad, the challenger chooses* $C^B = C_L$.

(5) *When the challenger chooses* $C^B = C_L$ *the media outlet spends costs, and observes and reports the true type of the incumbent's ability with probability* δ. *Otherwise, the media outlet does not spend costs.*

Belief

Since the naive voters are not Bayesian updaters, only the belief of the sophisticated voters and the media outlet must be consistent with the strategies.

Their belief just after observing (C_A, C_B) is obvious consistent with the strategies. After that, the media outlet's (the sophisticated voters') belief is updated based on the outcome of gathering news.

Strategy

The specified strategies of both the sophisticated and the naive voters are optimal for themselves given their beliefs. From now on, we examine the incentive compatibility conditions of each candidate and the media outlet.

(i) The incentive compatibility of incumbent A:

(i-1) When $\theta_A = g$:

In this case, the challenger chooses C_H.

If and only if the expected number of the voters who vote for the incumbent when $(C^A, C^B) = (C_H, C_H)$ is larger than or equal to that when $(C^A, C^B) = (C_L, C_H)$, the incumbent chooses C_H. This condition is given by

$$\gamma[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}] + (1-\gamma)F(C_H, C_H) \geq \gamma[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}] + (1-\gamma)F(C_L, C_H).$$

By rewriting this inequality, we obtain $(1-\gamma)[F(C_H, C_H) - F(C_L, C_H)] \geq 0$. This holds because of Lemma 2.

(i-2) When $\boldsymbol{\theta_A = 0}$:

In this case, the challenger chooses C_L.

If and only if the number of the voters who vote for the incumbent when $(C^A, C^B) = (C_H, C_L)$ is larger than or equal to that when $(C^A, C^B) = (C_L, C_H)$, the incumbent chooses C_H. This condition is

$$\gamma[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}] + (1-\gamma)F(C_H, C_L) \geq \gamma[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}] + (1-\gamma)F(C_L, C_L).$$

By rewriting this inequality, we obtain $(1-\gamma)[F(C_H, C_L) - F(C_L, C_L)] \geq 0$. This holds because of Lemma 2.

(ii) The incentive compatibility of challenger $\boldsymbol{B}$:

This is straightforwardly satisfied from the discussion in "only if" part.

(iii) The incentive compatibility of the media outlet:

This obviously holds from the discussion of footnote 15.

From (i)-(iii), if $\underline{\gamma} \leq \gamma \leq \bar{\gamma}$ holds, this specified equilibrium exists.

Finally, we obtain Proposition 1. Note that it is easily verified that $0 < \underline{\gamma} < \bar{\gamma} < 1$. ■

A.3 Proof of Lemma 3

Consider the incentive compatibility conditions of incumbent A and challenger B.

(i) The incentive compatibility of incumbent $\boldsymbol{A}$:

(i-1) When $\boldsymbol{\theta_A = g}$ and $\boldsymbol{p_{HH} > p^*}$(i.e., when the media outlet does not gather news):

In this case, the challenger chooses C_H.

If and only if the number of voters who vote for the incumbent when $(C^A, C^B) = (C_L, C_H)$ is larger than or equal to that when campaign allocation is $(C^A, C^B) = (C_H, C_H)$, the incumbent chooses C_H. Derive this condition.

When $(C^A, C^B) = (C_L, C_H)$, the sophisticated voters and the media outlet believe that $\theta_A = g$. As a result, in the election, the sophisticated voters believe that $\theta_A = g$ when $(C^A, C^B) = (C_L, C_H)$.

On the other hand, when $(C^A, C^B) = (C_H, C_H)$, the sophisticated voters and the media outlet believe that $\theta_A = g$ with probability p_{HH}. Since $p_{HH} > p^*$, the media outlet does not gather news. As a result, in the election, the sophisticated voters believe that $\theta_A = g$ with probability p_{HH} when $(C^A, C^B) = (C_L, C_H)$.

From this discussion, the condition is given by

$$\gamma[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}] + (1-\gamma)F(C_L, C_H)$$
$$\geq \gamma[\rho\Phi(\alpha d + (p_{HH} - 0.5)\beta g) + (1-\rho)\Phi(-\alpha d + (p_{HH} - 0.5)\beta g)]$$
$$+ (1-\gamma)F(C_H, C_H).$$

By rewriting this inequality, we have $\gamma \geq \gamma_1^+(p_{HH})$.

(i-2) When $\boldsymbol{\theta_A = g}$ and $\boldsymbol{p_{HH} \leq p^*}$(i.e., when the media outlet gathers news):

Only one difference from (i-1) is the belief formation when $(C^A, C^B) = (C_L, C_H)$. In this case, the sophisticated voters and the media outlet believe that $\theta_A = g$ with probability p_{HH}. Since $p_{HH} \leq p^*$, the media outlet gathers news and finds out the value of θ_A with probability δ. As a result, in the election, with probability $1-\delta$, the sophisticated voters believe that $\theta_A = g$ with probability p_{HH}. On the other hand, with probability δ, they believe that $\theta_A = g$.

Therefore, the incentive compatibility condition is given by

$$\gamma[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}] + (1-\gamma)F(C_L, C_H)$$
$$\geq \gamma[(1-\delta)\{\rho\Phi(\alpha d + (p_{HH} - 0.5)\beta g) + (1-\rho)\Phi(-\alpha d + (p_{HH} - 0.5)\beta g)\}$$
$$+ \delta\{\rho\Phi_{HH} + (1-\rho)\Phi_{LH}\}] + (1-\gamma)F(C_H, C_H).$$

By rewriting this inequality, we have $\gamma \geq \gamma_1^-(p_{HH})$.

(i-3) When $\boldsymbol{\theta_A = 0}$ and $\boldsymbol{p_{LL} > p^*}$:

In this case, the challenger chooses C_L.

If and only if the number of voters who vote for the incumbent when $(C^A, C^B) = (C_H, C_L)$ is larger than or equal to that when campaign allocation is $(C^A, C^B) = (C_L, C_L)$, the incumbent chooses C_H. Similarly in (i-1), this condition is given by

$$\gamma[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}] + (1-\gamma)F(C_H, C_L)$$
$$\geq \gamma[\rho\Phi(\alpha d + (p_{LL} - 0.5)\beta g) + (1-\rho)\Phi(-\alpha d + (p_{LL} - 0.5)\beta g)]$$
$$+ (1-\gamma)F(C_L, C_L).$$

By rewriting this inequality and using Lemma 2, we have $\gamma \leq \gamma_2^+(p_{LL})$.

(i-4) When $\boldsymbol{\theta_A = 0}$ and $\boldsymbol{p_{LL} \leq p^*}$:

Similarly in (i-2), the incumbent's incentive compatibility condition is given by

$$\gamma[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}] + (1-\gamma)F(C_H, C_L)$$
$$\geq \gamma\{(1-\delta)[\rho\Phi(\alpha d + (p_{LL} - 0.5)\beta g) + (1-\rho)\Phi(-\alpha d + (p_{LL} - 0.5)\beta g)]$$
$$+ \delta[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}]\} + (1-\gamma)F(C_L, C_L).$$

By rewriting this inequality and using Lemma 2, we have $\gamma \leq \gamma_2^-(p_{LL})$.

(ii) The incentive compatibility of challenger $\boldsymbol{B}$:

(ii-1) When $\boldsymbol{\theta_A = g}$ and $\boldsymbol{p_{LL} > p^*}$:

In this case, the incumbent chooses C_L.

If and only if the expected number of the voters who vote for the incumbent when $(C^A, C^B) = (C_L, C_H)$ is less than or equal to that when is $(C^A, C^B) = (C_L, C_L)$, the challenger chooses C_H. Similarly in (i-1), this condition is given by

$$\begin{aligned}\gamma[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}] + (1-\gamma)F(C_L, C_H) \\ \le \gamma[\rho\Phi(\alpha d + (p_{LL} - 0.5)\beta g) + (1-\rho)\Phi(-\alpha d + (p_{LL} - 0.5)\beta g)] \\ + (1-\gamma)F(C_L, C_L).\end{aligned}$$

By rewriting this inequality and using Lemma 2, we have $\gamma \le \gamma_1^+(p_{LL})$.

(ii-2) When $\boldsymbol{\theta_A = g}$ and $\boldsymbol{p_{LL} \le p^*}$:

Similarly in (i-2), the incumbent's incentive compatibility condition is given by

$$\begin{aligned}\gamma[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}] + (1-\gamma)F(C_L, C_H) \\ \le \gamma\{(1-\delta)[\rho\Phi(\alpha d + (p_{LL} - 0.5)\beta g) + (1-\rho)\Phi(-\alpha d + (p_{LL} - 0.5)\beta g)] \\ + \delta[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}]\} + (1-\gamma)F(C_L, C_L).\end{aligned}$$

By rewriting this inequality and using Lemma 2, we have $\gamma \le \gamma_1^-(p_{LL})$.

(ii-3) When $\boldsymbol{\theta_A = 0}$ and $\boldsymbol{p_{HH} > p^*}$:

In this case, the incumbent chooses C_H as campaign allocation.

If and only if the number of the voters who vote for the incumbent when $(C^A, C^B) = (C_H, C_L)$ is less than or equal to that when campaign allocation is $(C^A, C^B) = (C_H, C_H)$, the challenger chooses C_L.

Similarly in (i-1), this condition is given by

$$\begin{aligned}\gamma[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}] + (1-\gamma)F(C_H, C_L) \\ \le \gamma[\rho\Phi(\alpha d + (p_{HH} - 0.5)\beta g) + (1-\rho)\Phi(-\alpha d + (p_{HH} - 0.5)\beta g)] \\ + (1-\gamma)F(C_H, C_H).\end{aligned}$$

By rewriting this inequality, we have $\gamma \ge \gamma_2^+(p_{HH})$.

(ii-4) When $\boldsymbol{\theta_A = 0}$ and $\boldsymbol{p_{HH} \le p^*}$:

Similarly in (i-2), the incumbent's incentive compatibility condition is given by

$$\begin{aligned}\gamma[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}] + (1-\gamma)F(C_H, C_L) \\ \le \gamma\{(1-\delta)[\rho\Phi(\alpha d + (p_{HH} - 0.5)\beta g) + (1-\rho)\Phi(-\alpha d + (p_{HH} - 0.5)\beta g)] \\ + \delta[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}]\} + (1-\gamma)F(C_H, C_H)\end{aligned}$$

By rewriting this inequality, we have $\gamma \ge \gamma_2^-(p_{HH})$.

Lastly, the belief is obviously consistent and the mass media's strategy is obviously optimal by construction. Combining each condition derived in (i) and (ii), we complete the proof. ■

A.4 Proof of Lemma 4

For the first part, by the definition of the functions $\gamma_1^+(p)$ and $\gamma_1^-(p)$ ($\gamma_2^+(p)$ and $\gamma_2^-(p)$), the denominator of the $\gamma_1^+(p)$ and $\gamma_1^-(p)$ ($\gamma_2^+(p)$ and $\gamma_2^-(p)$) are decreasing (increasing) in p. Therefore we obtain the first part.

For the second part, by the definition of the functions γ_M and γ_m and monotonicity of the functions $\gamma_1^+(p)$ and $\gamma_1^-(p)$ ($\gamma_2^+(p)$ and $\gamma_2^-(p)$), this is straightforwardly obtained. ■

A.5 Proof of Proposition 2

From Lemma 3, there exists a separating equilibrium, in which (i) when the incumbent's ability is low, $(C^A, C^B) = (C_H, C_L)$, and (ii) when the incumbent's ability is high, if and only if there exist p_{HH} and p_{LL} such that $\gamma_M(p_{HH}) \leq \gamma \leq \gamma_m(p_{LL})$ holds.

From now on, we examine the condition under which there exist p_{HH} and p_{LL} such that this inequality holds.

(a) When $\hat{\boldsymbol{p}} > \boldsymbol{p}^*$.

a-1. When $\hat{\boldsymbol{p}} > \boldsymbol{p}^*$ and $\boldsymbol{\gamma_1^+(\hat{p}) < \gamma_1^-(p^*)}$

In this case, the lower bound of γ_M is $\min_{p_{HH}} \gamma_M(p_{HH}) = \gamma_1^+(\hat{p})$, and the upper bound of γ_m is $\max_{p_{LL}} \gamma_m(p_{LL}) = \gamma_1^-(p^*)$. Because of $\gamma_1^+(\hat{p}) < \gamma_1^-(p^*)$, if and only if $\gamma_1^+(\hat{p}) \leq \gamma \leq \gamma_1^-(p^*)$, there exists the equilibrium.

a-2. When $\hat{\boldsymbol{p}} > \boldsymbol{p}^*$ and $\boldsymbol{\gamma_1^+(\hat{p}) \geq \gamma_1^-(p^*)}$

In this case, the lower bound of γ_M is $\min_{p_{HH}} \gamma_M(p_{HH}) = \gamma_1^+(\hat{p})$, and the upper bound of γ_m is $\max_{p_{LL}} \gamma_m(p_{LL}) = \gamma_1^+(\hat{p})$. Therefore if and only if $\gamma = \gamma_1^+(\hat{p})$, there exists the equilibrium.

(b) When $\hat{\boldsymbol{p}} \leq \boldsymbol{p}^*$.

b-1. When $\hat{\boldsymbol{p}} \leq \boldsymbol{p}^*$ and $\boldsymbol{\gamma_1^-(\hat{p}) > \gamma_1^+(p^*)}$

In this case, the lower bound of γ_M is $\inf_{p_{HH}} \gamma_M(p_{HH}) = \gamma_1^+(p^*)$, and the upper bound of γ_m is $\max_{p_{LL}} \gamma_m(p_{LL}) = \gamma_1^-(\hat{p})$. Because $\gamma_1^-(\hat{p}) > \gamma_1^+(p^*)$ holds, if and only if $\gamma_1^-(\hat{p}) \geq \gamma > \gamma_1^+(p^*)$, there exists the equilibrium.

b-2. When $\hat{\boldsymbol{p}} \leq \boldsymbol{p}^*$ and $\boldsymbol{\gamma_1^-(\hat{p}) \leq \gamma_1^+(p^*)}$

In this case, the lower bound of γ_M is $\min_{p_{HH}} \gamma_M(p_{HH}) = \gamma_1^-(\hat{p})$, and the upper bound of γ_m is $\max_{p_{LL}} \gamma_m(p_{LL}) = \gamma_1^-(\hat{p})$. Therefore if and only if $\gamma = \gamma_1^-(\hat{p})$, there exists the equilibrium.

From (a) and (b), we obtain the proposition. ■

A.6 Proof of Lemma 6

Consider the incumbent's deviation incentive when $\theta_A = 0$. When $C^A = C_L$, the number of voters who vote for the incumbent is

$$\gamma[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}] + (1-\gamma)F(C_L, C_L).$$

When $C^A = C_H$, the lowest bound of the number of voters who vote for the incumbent is

$$\gamma[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}] + (1-\gamma)F(C_H, C_L).$$

Here, $F(C_H, C_L) > F(C_H, C_L)$. Thus, the incumbent has a strong incentive to deviate from $C^A = C_L$ to $C^A = C_L$. Therefore, such an equilibrium does not exist. ■

A.7 Proof of Lemma 7

Consider the challenger's deviation incentive when $\theta_A = g$. When $C^B = C_L$, the number of voters who vote for the incumbent is[23]

$$\gamma[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}] + (1-\gamma)F(C^A, C_L).$$

When $C^B = C_H$, the highest bound of the number of voters who vote for the incumbent is

$$\gamma[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}] + (1-\gamma)F(C^A, C_H).$$

Here, $F(C^A, C_L) > F(C^A, C_H)$. Thus, the challenger has a strong incentive to deviate from $C^B = C_L$ to $C^B = C_H$. Therefore, such an equilibrium does not exist. ■

A.8 Proof of Lemma 8

Consider the incumbent's deviation incentive when $\theta_A = 0$. When $C^A = C_H$, both the mass media and the sophisticated voters believe that $\theta_A = g$ with probability one. Then, the mass media does not gather news. As a result, the sophisticated voters believe that $\theta_A = g$ with probability one in the election. Thus, when $C^A = C_H$, the number of voters who vote for the incumbent is

$$\gamma[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}] + (1-\gamma)F(C_H, C^B).$$

When $C^A = C_L$, the number of voters who vote for the incumbent is

$$\gamma[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}(-\alpha d - 0.5\beta g)] + (1-\gamma)F(C_L, C^B).$$

Since $\Phi_{HH} > \Phi_{HL}, \Phi_{LH} > \Phi_{LL}, F(C_H, C^B) > F(C_L, C^B)$ hold, the incumbent has a strong incentive to deviate from $C^A = C_L$ to $C^A = C_H$. Therefore, such an equilibrium does not exist. ■

A.9 Proof of Lemma 9

First, consider the incumbent's incentive when $\theta_A = g$. When $C^A = C_L$, the number of voters

[23] Since this is on equilibrium path, the sophisticated voters believe that $\theta_A = g$ with probability one, provided that $C^B = C_L$. Thus, we obtain this number of voters.

who vote for the incumbent is

$$\gamma[\rho\Phi_{HH}+(1-\rho)\Phi_{LH}]+(1-\gamma)F(C_L,C^B).$$

When $C^A=C_H$, the number of voters who vote for the incumbent is

$$\gamma\{(1-\delta)[\rho\Phi_{HL}+(1-\rho)\Phi_{LL}]+\delta[\rho\Phi_{HH}+(1-\rho)\Phi_{LH}\}+(1-\gamma)F(C_H,C^B).$$

Thus, the incumbent chooses C_L when $\theta_A=g$ if and only if

$$\begin{aligned}(1-\delta)\gamma\{[\rho\Phi_{HH}+(1-\rho)\Phi_{LH}]-[\rho\Phi_{HL}+(1-\rho)\Phi_{LL}]\}\\ \geq(1-\gamma)[F(C_H,C^B)-F(C_L,C^B)].\end{aligned} \quad (1)$$

Second, consider the incumbent's incentive when $\theta_A=0$. When $C^A=C_L$, the number of voters who vote for the incumbent is

$$\gamma[\rho\Phi_{HH}+(1-\rho)\Phi_{LH}]+(1-\gamma)F(C_L,C^B).$$

(The reason is the same as in the proof of Lemma 8). When $C^A=C_H$, the number of voters who vote for the incumbent is

$$\gamma[\rho\Phi_{HL}+(1-\rho)\Phi_{LL}]+(1-\gamma)F(C_H,C^B).$$

Thus, the incumbent chooses C_H when $\theta_A=0$ if and only if

$$\begin{aligned}\gamma\{\rho\Phi_{HH}+(1-\rho)\Phi_{LH}-[\rho\Phi_{HL}+(1-\rho)\Phi_{LL}]\}\\ \leq(1-\gamma)[F(C_H,C^B)-F(C_L,C^B)].\end{aligned} \quad (2)$$

However, both inequalities (1) and (2) never hold at the same because the left-hand side of inequality (2) is strictly larger than that of inequality (1). Therefore, such an equilibrium does not exist. ■

A.10 Proof of Lemma 10

Consider the incumbent's deviation incentive when $\theta_A=0$. When $C^A=C_H$, the lowest number of voters who vote for the incumbent is

$$\gamma[\rho\Phi_{HL}+(1-\rho)\Phi_{LL}]+(1-\gamma)F(C_H,C_L).$$

When $C^A=C_L$, the number of voters who vote for the incumbent is

$$\gamma[\rho\Phi_{HL}+(1-\rho)\Phi_{LL}]+(1-\gamma)F(C_L,C_L).$$

Since $F(C_H,C_L)>F(C_L,C_L)$ holds, the incumbent has a strong incentive to deviate from $C^A=C_L$ to $C^A=C_H$. Therefore, such an equilibrium does not exist. ■

A.11 Proof of Lemma 11

Because of Lemma 4, $\gamma_1^+(p)$ and $\gamma_1^-(p)$ are increasing function in p. Therefore for any $p\in(0,1)$, $\gamma_1^+(p)>\gamma_1^+(0)=\underline{\gamma}$ and $\gamma_1^-(p)>\gamma_1^-(0)=\overline{\gamma}$.

Conversely, because of Lemma 4, $\gamma_2^+(p)$ and $\gamma_2^-(p)$ are decreasing function in p. Therefore for any $p\in(0,1)$, $\gamma_2^+(p)>\gamma_2^+(1)=\underline{\gamma}$ and $\gamma_2^-(p)>\gamma_2^-(1)=\overline{\gamma}$.■

A.12 Proof of Lemma 12

To begin with, prove the first part. $\gamma_1^+(\hat{p}) \leq \bar{\gamma}$ if and only if

$$(1-\delta)[\rho(\Phi_{HH}-\Phi_{HL})+(1-\rho)(\Phi_{LH}-\Phi_{LL})]$$
$$\leq \rho[\Phi_{HH}-\Phi(\alpha d+(\hat{p}-0.5)\beta g)]+(1-\rho)[\Phi_{LH}-\Phi(-\alpha d+(\hat{p}-0.5)\beta g)].$$

The left-hand side is decreasing with δ while the right-hand side is independent of δ. In addition, from Lemma 5, when $\delta = 0.5$, the inequality holds with equality. Therefore, we obtain the first part.

For the second part, $\gamma_1^+(p^*) \leq \bar{\gamma}$ if and only if

$$\rho\big(\Phi_{HH}-\Phi(\alpha d+(p^*-0.5)\beta g)\big)+(1-\rho)\big(\Phi_{LH}-\Phi(-\alpha d+(p^*-0.5)\beta g)\big)$$
$$\geq (1-\delta)[\rho(\Phi_{HH}-\Phi_{HL})+(1-\rho)(\Phi_{LH}-\Phi_{LL})].$$

The right-hand side is decreasing with δ, and the left-hand side is independent of δ. In addition, the left-hand side is always more than zero, and when $\delta = 1$, the right-hand side is zero i.e., when $\delta = 1$, the inequality above holds with a strong inequality. Thus, the second party is obtained. ■

A.13 Proof of Lemma 13

Suppose that such an equilibrium exists. Consider the following belief consistent with the equilibrium:

$$\pi\big(\theta_A = g|(C^A,C^B)\big) = \begin{cases} 0.5 \text{ if } (C^A,C^B)=(C_H,C_H) \\ 1 \text{ if } (C^A,C^B)=(C_H,C_L) \\ p_{LH} \text{ if } (C^A,C^B)=(C_L,C_H) \\ 1 \text{ if } (C^A,C^B)=(C_L,C_L) \end{cases},$$

where $p_{LH} \in (p^*, 0.5)$. Then, the media outlet never gathers news. Thus, the sophisticated voters never update this belief i.e., they use this belief in the election.

Incumbent A has no incentive to deviate from the equilibrium if and only if

$$\gamma[\rho\Phi(\alpha d)+(1-\rho)\Phi(-\alpha d)]+(1-\gamma)F(C_H,C_H)$$
$$\geq \gamma[\rho\Phi(\alpha d+(p_{LH}-0.5)\beta g)+(1-\rho)\Phi(-\alpha d+(p_{LH}-0.5)\beta g)]$$
$$+(1-\gamma)F(C_L,C_H).$$

This inequality always holds from $p_{LH} < 0.5$ and Lemma 2. Thus, the incumbent has no incentive to deviate from this equilibrium.

Next, challenger B has no incentive to deviate from the equilibrium if and only if

$$\gamma[\rho\Phi(\alpha d)+(1-\rho)\Phi(-\alpha d)]+(1-\gamma)F(C_H,C_H)$$
$$\leq \gamma[\rho\Phi_{HH}+(1-\rho)\Phi_{LH}]+(1-\gamma)F(C_H,C_L).$$

This inequality always holds from Lemma 2. Thus, the challenger has no incentive to deviate from this equilibrium.

Therefore, there is an equilibrium in which $(C^A,C^B)=(C_H,C_H)$. ■

A.14 Proof of Lemma 14

Step 1: $\boldsymbol{\Theta^A(C_L, C_H) = \{g, 0\}}$ or $\emptyset$.

To begin with, we prove that $\Theta^A(C_L, C_H) = \{g, 0\}$ or $\emptyset$. The equilibrium payoff of incumbent A is independent of θ^A. Thus, if the maximum payoff of incumbent A when $C^A = C_L$ is also independent of θ^A, $\Theta^A(C_L, C_H) = \{g, 0\}$ or $\emptyset$. Therefore, it suffices to prove that the maximum payoff of incumbent A when $C^A = C_L$ is independent of θ^A.

Consider the case where $\theta^A = g$. The maximum payoff is the payoff when the sophisticated voters and the media outlet believe that $\theta^A = g$. Thus, the maximum payoff is given by

$$\gamma[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}] + (1-\gamma)F(C_L, C_H).$$

Consider the case where $\theta^A = 0$. When the sophisticated voters and the media outlet believe that $\theta^A = g$, the media outlet does not gather news. As a result, the sophisticated voters believe that $\theta^A = g$ in the election. Thus, the maximum payoff is the payoff when the sophisticated voters and the media outlet believe that $\theta^A = g$. Hence, the maximum payoff is given by

$$\gamma[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}] + (1-\gamma)F(C_L, C_H).$$

Therefore, the maximum payoff of incumbent A when $C^A = C_L$ is independent of θ^A.

Step 2: $\boldsymbol{\Theta^B(C_H, C_L) = \{0\}}$ if and only if either 1 or 2 in the lemma holds.

Step 2-1 $\boldsymbol{0 \in \Theta^B(C_H, C_L)}$ if and only if $\boldsymbol{\gamma \geq \gamma_L}$.

Challenger B's maximum payoff is equivalent to incumbent A's minimum payoff/ number of obtained votes. From now on, we consider the latter instead of the former.

Suppose that $\theta^A = 0$. The smallest number of incumbent A's obtained votes when $C^B = C_L$ is that when the sophisticated voters and the mass media believe that $\theta^A = 0$. Thus, that is given by

$$\gamma[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}] + (1-\gamma)F(C_H, C_L). \tag{3}$$

On the other hand, the equilibrium number of incumbent A's votes is

$$\gamma[\rho\Phi(\alpha d) + (1-\rho)\Phi(-\alpha d)] + (1-\gamma)F(C_H, C_H). \tag{4}$$

Therefore, if and only if (3)≤(4), $0 \in \Theta^B(C_H, C_L)$. By rewriting this inequality, we have $\gamma \geq \gamma_L$.

Step 2-2 $\boldsymbol{g \notin \Theta^B(C_H, C_L)}$ if and only if (i) $\boldsymbol{I(\delta) > 0, \gamma < \gamma_H}$, and $\boldsymbol{\gamma \leq \gamma_H{}'}$, or (ii) $\boldsymbol{I(\delta) \leq 0}$ and $\boldsymbol{\gamma \leq \gamma_H{}'}$.

Suppose that $\theta_A = g$.

The number of voters who vote for incumbent A for when the sophisticated voters and the mass media believe that $\theta_A = 0$ is given by

$$(1-\delta)\gamma[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}] + \delta\gamma[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}] + (1-\gamma)F(C_H, C_L). \tag{5}$$

The number of voters who vote for incumbent A when the sophisticated voters and the mass media believe that $\theta_A = g$ with probability $p > p^*$ is given by

$$\gamma[\rho\Phi(\alpha d + (p-0.5)\beta g) + (1-\rho)\Phi(-\alpha d + (p-0.5)\beta g)] + (1-\gamma)F(C_H, C_L). \tag{6}$$

Here, (6) is increasing with p. Thus, there is no minimum of (6).

Therefore, if and only if (5)>(4) and $\inf_{p\in(p^*,1]}$(6)≥(4), $g \notin \Theta^B(C_H, C_L)$.

Case 1: $\boldsymbol{I(\delta) > 0}$. (5)>(4) is written as $\gamma < \gamma_H$, and $\inf_{p\in(p^*,1]}$(6)≥(4) is written as $\gamma \le \gamma_H{}'$.

Case 2: $\boldsymbol{I(\delta) \le 0}$. (5)>(4) always holds. And, $\inf_{p\in(p^*,1]}$(6)≥(4) is written as $\gamma \le \gamma_H{}'$.

From cases 1 and 2, we complete the proof of step 2-2.

From steps 2-1 and 2-2-, we complete the proof of step 2.

Step 3: Suppose that $\boldsymbol{\gamma \ne \gamma_L}$. Then, the sequential equilibrium in which $\boldsymbol{(C^A, C^B) = (C_H, C_H)}$ independently of $\boldsymbol{\theta^A}$ does not satisfy the intuitive criterion if and only if $\boldsymbol{\Theta^B(C_H, C_L) = \{0\}}$.

Step 3-1: "Only if" part

Prove the contrapositive. Suppose that $\Theta^B(C_H, C_L) \ne \{0\}$.

Case 1: $\boldsymbol{\Theta^B(C_H, C_L) = \{0, g\}}$ or $\boldsymbol{\emptyset}$. In this case, any belief π is allowed for $(C^A, C^B) = (C_H, C_L)$. In addition, from step 1, any belief π is allowed for $(C^A, C^B) = (C_L, C_H)$. Therefore, the equilibrium constructed in the proof of Lemma 13 satisfies the intuitive criterion.

Case 2: $\boldsymbol{\Theta^B(C_H, C_L) = \{g\}}$. In this case, only $\pi = 1$ is allowed for $(C^A, C^B) = (C_H, C_L)$. In addition, from step 1, any belief π is allowed for $(C^A, C^B) = (C_L, C_H)$. Therefore, the equilibrium constructed in the proof of Lemma 13 satisfies the intuitive criterion.

From cases 1 and 2, we complete the proof of contrapositive.

Step 3-2: "If" part

Suppose that $\Theta^B(C_H, C_L) = \{0\}$. Prove by contradiction. Suppose that a sequential equilibrium in which $(C^A, C^B) = (C_H, C_H)$ independently of θ^A satisfies the intuitive criterion. Only $\pi = 0$ is allowed for $(C^A, C^B) = (C_H, C_L)$. Thus, the number of voters who vote for incumbent A when $(C^A, C^B) = (C_H, C_L)$ is (3). Therefore, (3)≥(4) must hold to prevent challenger B's deviation.

However, $\Theta^B(C_H, C_L) = \{0\}$ and $\gamma \ne \gamma_L$ imply that $\gamma > \gamma_L$ from step 2-1. Thus, (3)<(4) holds. A contradiction.

Step 4: When $\boldsymbol{\gamma = \gamma_L}$, a sequential equilibrium in which $\boldsymbol{(C^A, C^B) = (C_H, C_H)}$ independently of $\boldsymbol{\theta^A}$ satisfies the intuitive criterion .

$\gamma = \gamma_L$ implies that $\gamma < \gamma_H$ if $I(\delta) > 0$, and $\gamma < \gamma_H{}'$. Thus, $\Theta^B(C_H, C_L) = \{0\}$. It means that only $\pi = 0$ is allowed for $(C^A, C^B) = (C_H, C_L)$. Thus, the number of voters who vote for incumbent A when $(C^A, C^B) = (C_H, C_L)$ is (3). Here, (3)=(4) holds. Therefore, challenger B has no deviation incentive.

In addition, from step 1, any belief π is allowed for $(C^A, C^B) = (C_L, C_H)$. Therefore, by setting π as in the proof of Lemma 13, incumbent A has no deviation incentive.

Therefore, there is a sequential equilibrium in which $(C^A, C^B) = (C_H, C_H)$ independently of θ^A, and which satisfies the intuitive criterion.

Combining each step, we complete the proof. Note that γ_L, $\gamma_H{}' \in (0,1)$, and $\gamma_L{}' > 0$ is easily obtained. ■

A.15 Proof of Lemma 15

Suppose that there is a sequential equilibrium in which $(C^A, C^B) = (C_L, C_L)$ independently of θ^A. Consider incumbent A's deviation incentive.

The equilibrium number of voters who vote for incumbent A is

$$\gamma[\rho\Phi(\alpha d) + (1-\rho)\Phi(-\alpha d)] + (1-\gamma)F(C_L, C_L). \tag{7}$$

If (7) is less than the smallest number of voters who vote for incumbent A when $C^A = C_H$, incumbent A has a strong deviation incentive.

Here, the number of voters who vote for incumbent A when the sophisticated voters and the mass media believe that $\theta_A = 0$ is given by (5). Also, the number of voters who vote for incumbent A when the sophisticated voters and the mass media believe that $\theta_A = g$ with probability $p > p^*$ is given by (6).

Thus, (7) is less than the smallest number of voters who vote for incumbent A when $C^A = C_H$ if and only if (5)>(7) and $\inf_{p\in[0,p^*)}$(6)≥(7). Since $F(C_H, C_H) = F(C_L, C_L)$, (7)=(4). Therefore, these conditions are equivalent to (5)>(4) and $\inf_{p\in[0,p^*)}$(6)≥(4). From the proof of Lemma 13, these condition hold when the condition in Lemma 14 is satisfied. Hence, incumbent A has a strong deviation incentive. ■

A.16 Proof of Theorem 2

Step 1: There is no pooling equilibria satisfying the intuitive criterion if and only if the condition in Lemma 14 is satisfied.

This is straightforwardly obtained from Lemmas 14 and 15.

Step 2: Negative campaign equilibrium [I] satisfies the intuitive criterion when the equilibrium exists.

Given unilateral deviation, only $(C^A, C^B) = (C_L, C_H)$ is observed as an off-equilibrium path when the sophisticated voters and the media outlet observes each candidate's campaign allocation. Thus, the restriction on the belief formation due to the intuitive criterion is only that for the case where $(C^A, C^B) = (C_L, C_H)$. Here, using the same logic in step 1 in the proof of Lemma 14, $\Theta^A(C_L, C_H) = \{g, 0\}$ or $\emptyset$. Thus, any π is allowed for $(C^A, C^B) = (C_L, C_H)$. Therefore, negative campaign equilibrium [I] satisfies the intuitive criterion when the equilibrium exists.

Step 3: Negative campaign equilibrium [II] satisfies the intuitive criterion when the equilibrium exists, the condition in Lemma 14 is satisfied, and $\gamma > \bar{\gamma}$.

Only $(C^A, C^B) = (C_L, C_L)$ and $(C^A, C^B) = (C_H, C_H)$ are observed as off-equilibrium paths when the sophisticated voters and the media outlet observes each candidate's campaign allocation. Thus, it is enough to consider these two cases.

Step 3-1: $\Theta^A(C_L, C_L) \cup \Theta^B(C_L, C_L) = \{0, g\}$.

To begin with, consider $\Theta^A(C_L, C_L)$. Challenger B chooses C_L only when $\theta_A = 0$. Thus, $\Theta^A(C_L, C_L) = \{0\}$ or $\emptyset$. Examine the condition under which $\Theta^A(C_L, C_L) = \{0\}$.

The equilibrium number of voters who vote for incumbent A is

$$\gamma[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}] + (1-\gamma)F(C_H, C_L). \tag{8}$$

On the other hand, the maximum number of voters who vote for incumbent A when $C^A = C_L$ is that when the sophisticated voters and the media outlet believe that $\theta_A = g$ i.e.,

$$\gamma[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}] + (1-\gamma)F(C_L, C_L). \tag{9}$$

Therefore, if and only if (9)≥(8), $\Theta^A(C_L, C_L) = \{0\}$. Here, this condition is rewritten as $\gamma \geq \gamma_L$. Since $\underline{\gamma} < \gamma_L$ holds, $\gamma \geq \underline{\gamma}$ is satisfied. Therefore, $\Theta^A(C_L, C_L) = \{0\}$.

Next, consider $\Theta^B(C_L, C_L)$. Incumbent A chooses C_L only when $\theta_A = g$. Thus, $\Theta^A(C_L, C_L) = \{g\}$ or $\emptyset$. Examine the condition under which $\Theta^B(C_L, C_L) = \{g\}$.

The equilibrium number of voters who vote for incumbent A is

$$\gamma[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}] + (1-\gamma)F(C_L, C_H). \tag{10}$$

The number of voters who vote for incumbent A when $C^B = C_L$, and the sophisticated voters and the mass media believe that $\theta_A = 0$ is given by

$$(1-\delta)\gamma[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}] + \delta\gamma[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}] + (1-\gamma)F(C_L, C_L). \tag{11}$$

Therefore, if (10)≥(11), $\Theta^B(C_L, C_L) = \{g\}$. Here, (10)≥(11) holds because $\gamma \geq \bar{\gamma}$. Therefore, $\Theta^B(C_L, C_L) = \{g\}$.

In summary, $\Theta^A(C_L, C_L) \cup \Theta^B(C_L, C_L) = \{0, g\}$.

Step 3-2: $\Theta^A(C_H, C_H) \cup \Theta^B(C_H, C_H) = \{0, g\}$.

To begin with, consider $\Theta^A(C_H, C_H)$. Challenger B chooses C_H only when $\theta_A = g$. Thus, $\Theta^A(C_H, C_H) = \{g\}$ or $\emptyset$. Examine the condition under which $\Theta^A(C_H, C_H) = \{g\}$.

The equilibrium number of voters who vote for incumbent A is (10).

On the other hand, the maximum number of voters who vote for incumbent A when $C^A = C_H$ is that when the sophisticated voters and the media outlet believe that $\theta_A = g$ i.e.,

$$\gamma[\rho\Phi_{HH} + (1-\rho)\Phi_{LH}] + (1-\gamma)F(C_H, C_H). \tag{12}$$

Since (12)>(10) holds, $\Theta^A(C_H, C_H) = \{g\}$.

Next, consider $\Theta^B(C_H, C_H)$. Incumbent A chooses C_H only when $\theta_A = 0$. Thus, $\Theta^B(C_H, C_H) = \{0\}$ or $\emptyset$. Examine the condition under which $\Theta^B(C_H, C_H) = \{0\}$.

The equilibrium number of voters who vote for incumbent A is (8).

On the other hand, the smallest number of voters who vote for incumbent A when $C^B = C_H$ is that when the sophisticated voters and the mass media believe that $\theta_A = 0$ i.e,

$$\gamma[\rho\Phi_{HL} + (1-\rho)\Phi_{LL}] + (1-\gamma)F(C_H, C_H). \tag{13}$$

Since (13)<(8) holds, $\Theta^B(C_H, C_H) = \{0\}$.

In summary, $\Theta^A(C_H, C_H) \cup \Theta^B(C_H, C_H) = \{0, g\}$.

From steps 3-1 and 3-2, any π is allowed for $(C^A, C^B) = (C_L, C_L)$ and $(C^A, C^B) = (C_H, C_H)$. Therefore, negative campaign equilibrium [II] satisfies the intuitive criterion.

From steps 1-3, if and only if the conditions in Theorem 1 and the condition in Lemma 14 are satisfied, (i) at least one separating equilibrium in which the challenger chooses C_L if and only if $\theta_A = 0$, satisfy the intuitive criterion, and (ii) all the other equilibria (pooling equilibria) do not satisfy the intuitive criterion. Moreover, $\gamma_L < \gamma_1^-(p^*)$ if $p^* < \hat{p}$, and $\gamma_L < \gamma_1^-(\hat{p})$ if $p^* \geq \hat{p}$ from Lemma 16. As a result, we have the condition in the theorem. ■