

# Oligopoly Theory (5)

## First-Mover and Second-Mover Advantage

### Aim of this lecture

- (1) To understand the Stackelberg Model
- (2) To understand the relationship among first mover and second mover advantage, strategic substitutes and complements, and the stability condition.

# Outline of the Fifth Lecture

5-1 Stackelberg Model

5-2 Strategic Substitutes and First-Mover Advantage

5-3 Strategic Complements and Second-Mover Advantage

5-4 Incomplete Information and Second-Mover Advantage

5-5 Price leadership

# Stackelberg Duopoly

Firm 1 and firm 2 compete in a homogeneous product market.

Firm 1 chooses its output  $Y_1 \in [0, \infty)$ . After observing  $Y_1$ , firm 2 chooses its output  $Y_2 \in [0, \infty)$ .

Each firm maximizes its own profit  $\Pi_i$ .

$\Pi_i = P(Y)Y_i - C_i(Y_i)$ ,  $P$ : Inverse demand function,

$Y$ : Total output,  $Y_i$ : Firm  $i$ 's output,  $C_i$ : Firm  $i$ 's cost function

# Stackelberg Equilibrium

Stackelberg Equilibrium ~ the subgame perfect Nash equilibrium.

We solve the game by backward induction.

First, we consider the behavior of firm 2.

$$Y_2 = R_2(Y_1)$$

Second, we consider the behavior of firm 1

$$\Pi_1 = P(Y_1 + R_2(Y_1))Y_1 - C_1(Y_1)$$

I assume that it is concave.

# Stackelberg Equilibrium

Question: Is it possible that Stackelberg leader's profit is strictly smaller than the one at the Cournot equilibrium?

Is it possible that

$$P(Y_1^L + R_2(Y_1^L))Y_1^L - C_1(Y_1^L) < P(Y_1^C + Y_2^C)Y_1^C - C_1(Y_1^C)?$$

# Stackelberg Equilibrium

Suppose that  $R_2' < 0$ . (Strategic Substitutes)

The first order condition for firm 1 is

$$P + P'(1 + R_2') Y_1 - C_1' = 0$$

When  $Y_1 = Y_1^C$ ,  $P + P'Y_1 - C_1' = 0$ , so

$P + P'(1 + R_2') Y_1 - C_1'$  must be (positive, negative, zero).

# Stackelberg Equilibrium

Suppose that  $R_2' > 0$ . (Strategic Complements)

$$P + P'(1 + R_2') Y_1 - C_1' = 0$$

When  $Y_1 = Y_1^C$ ,  $P + P'Y_1 - C_1' = 0$ , so

$P + P'(1 + R_2') Y_1 - C_1'$  must be (positive, negative, zero) if  $R_2' > 0$ .

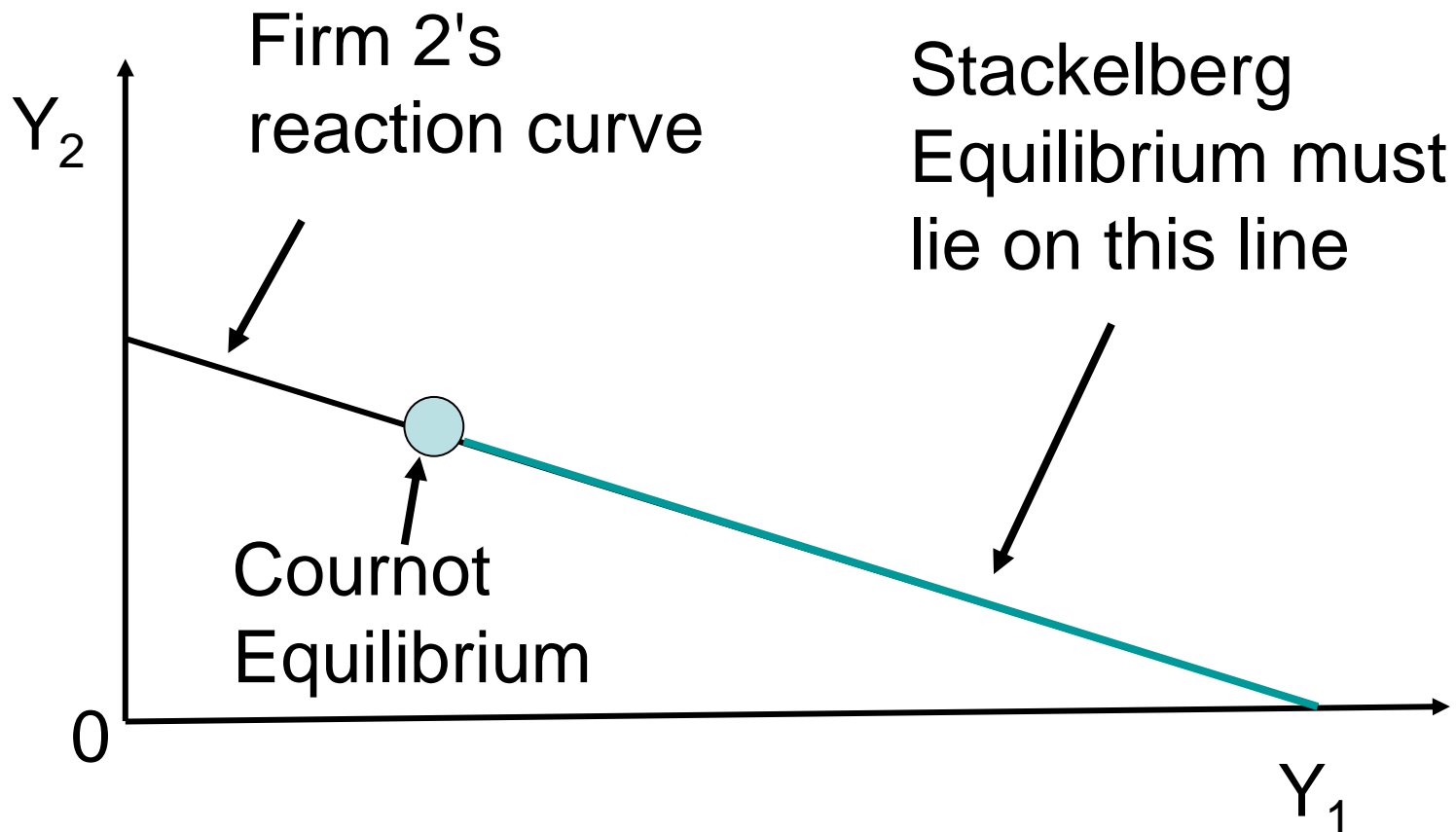
# First-Mover Advantage

Stackelberg Leader's profit is strictly larger than the Cournot counterpart if  $R' \neq 0$ .

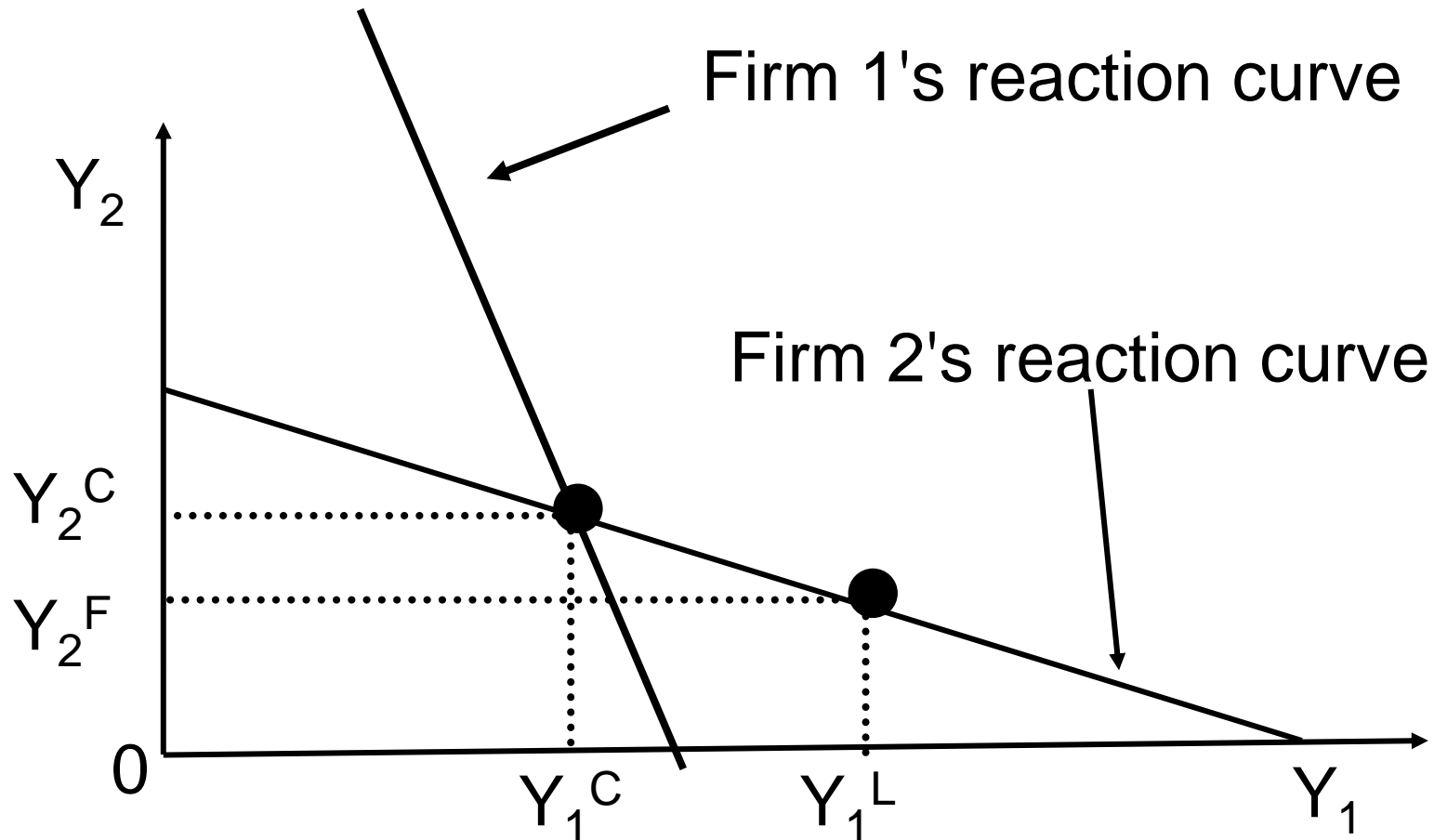
Question: Suppose that  $R_2' < 0$  (Strategic Substitutes). Is Stackelberg Follower's profit larger or smaller than the Cournot counterpart?



# Stackelberg Equilibrium



# Stackelberg Equilibrium



the superscript F denotes the outcome of the Stackelberg Follower

# First-Mover Disadvantage under Incomplete Information

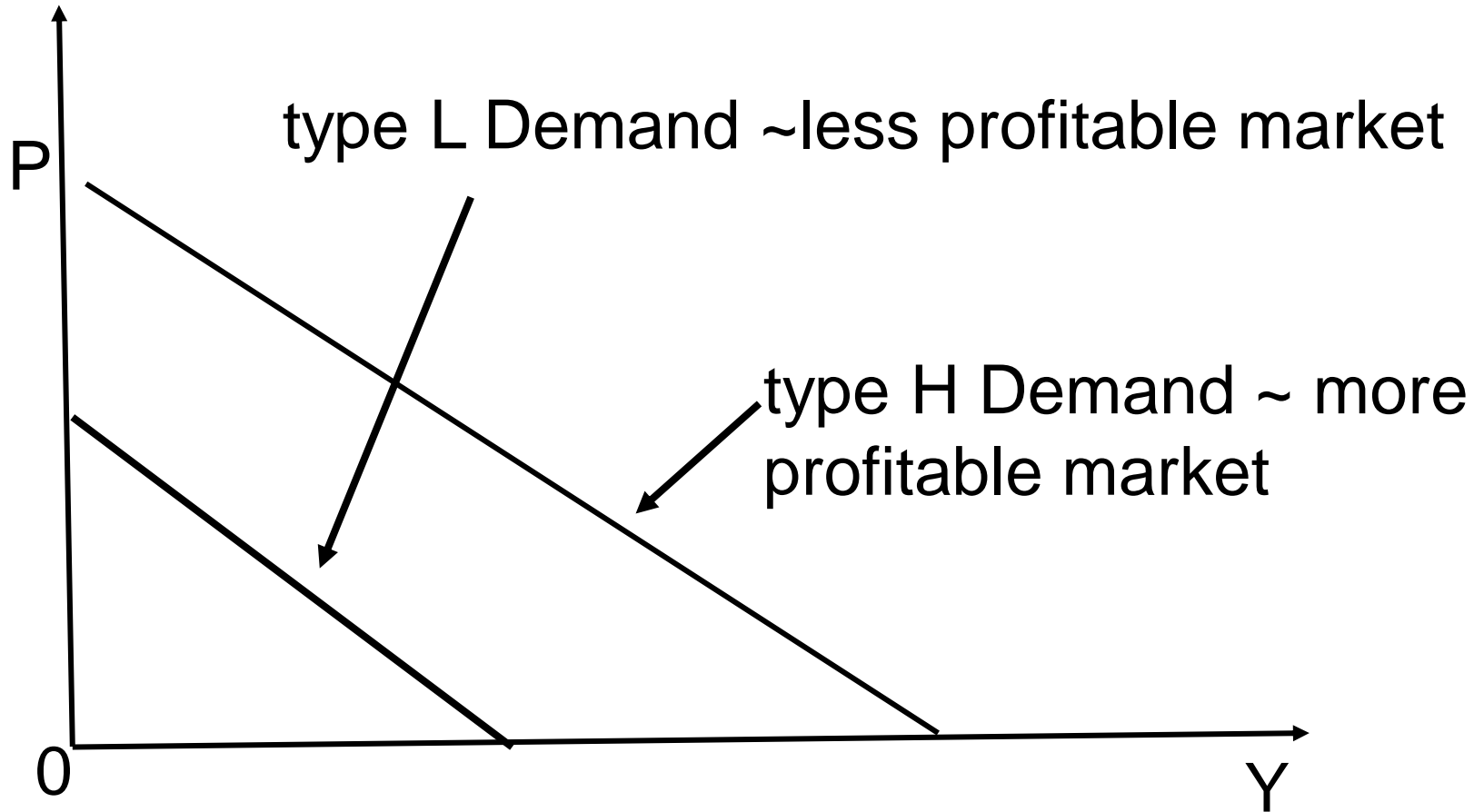
We consider the case of strategic substitute.

Consider the following situation. The first-mover knows whether the demand is large or small. The second-mover does not know it.

The second-mover may obtain the information on the demand condition from the output of the First-Mover. → The output of the first-mover serves as a signal.

Consider a separating equilibrium.

# Demand condition



# signaling

Let  $Y(H)$  be the Stackelberg Leader's output when the information is complete and firm 1 is type H.

Let  $Y(L)$  be the Stackelberg Leader's output when the information is complete and firm 1 is type L.

Naturally,  $Y(H) > Y(L)$ .

Consider the incomplete information game.

If the second-mover thinks that the rival is type L, it chooses a smaller output.

Thus, type H has an incentive for making the second-mover misunderstand that the demand is small and it may choose  $Y_1 = Y(L)$  rather than  $Y(H)$ .

# signaling

If even type H leader has an incentive to choose  $Y_1 = Y(L)$ , then the rational second-mover thinks that the demand may H.

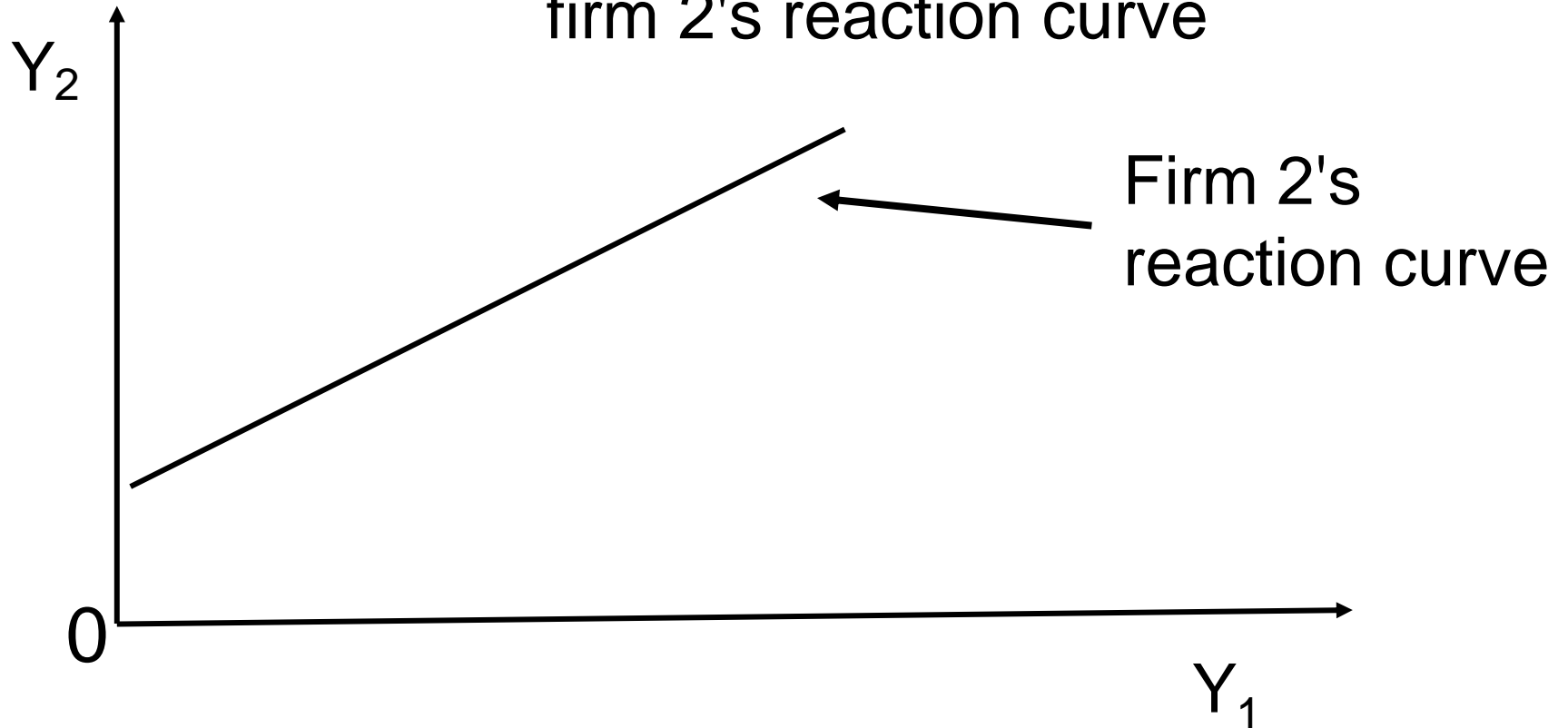
To avoid this situation and to show its true type, type L has an incentive to reduce its output and to make firm 1 to be sure that the demand is small.

As a result,  $Y_1 < Y(L)$  for type L.

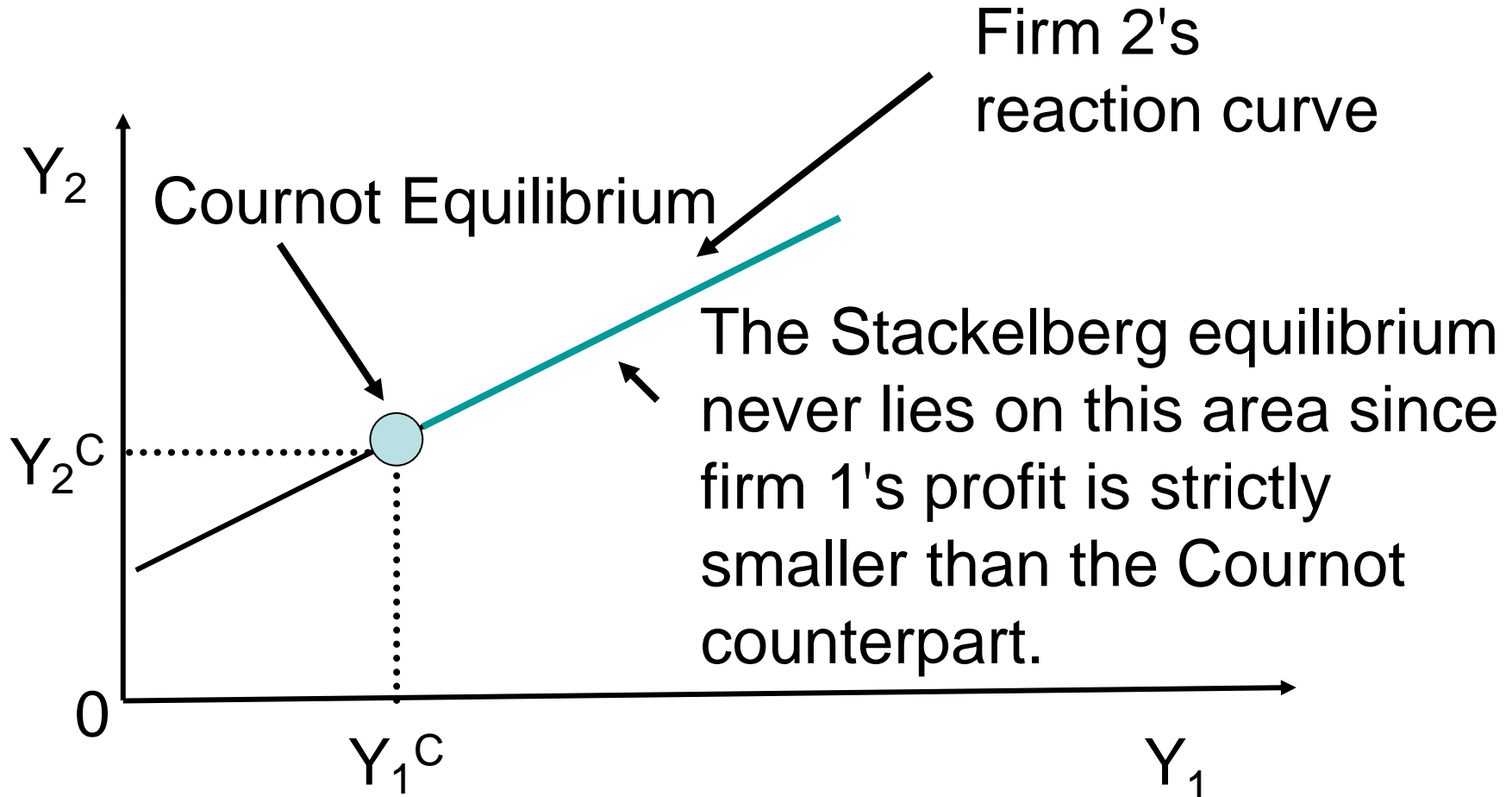
→ If this effect is sufficiently large, the expected profit of the first-mover can be smaller than that of the second-mover. → first-mover disadvantage

# strategic complements

Firm 1 can choose any point on firm 2's reaction curve

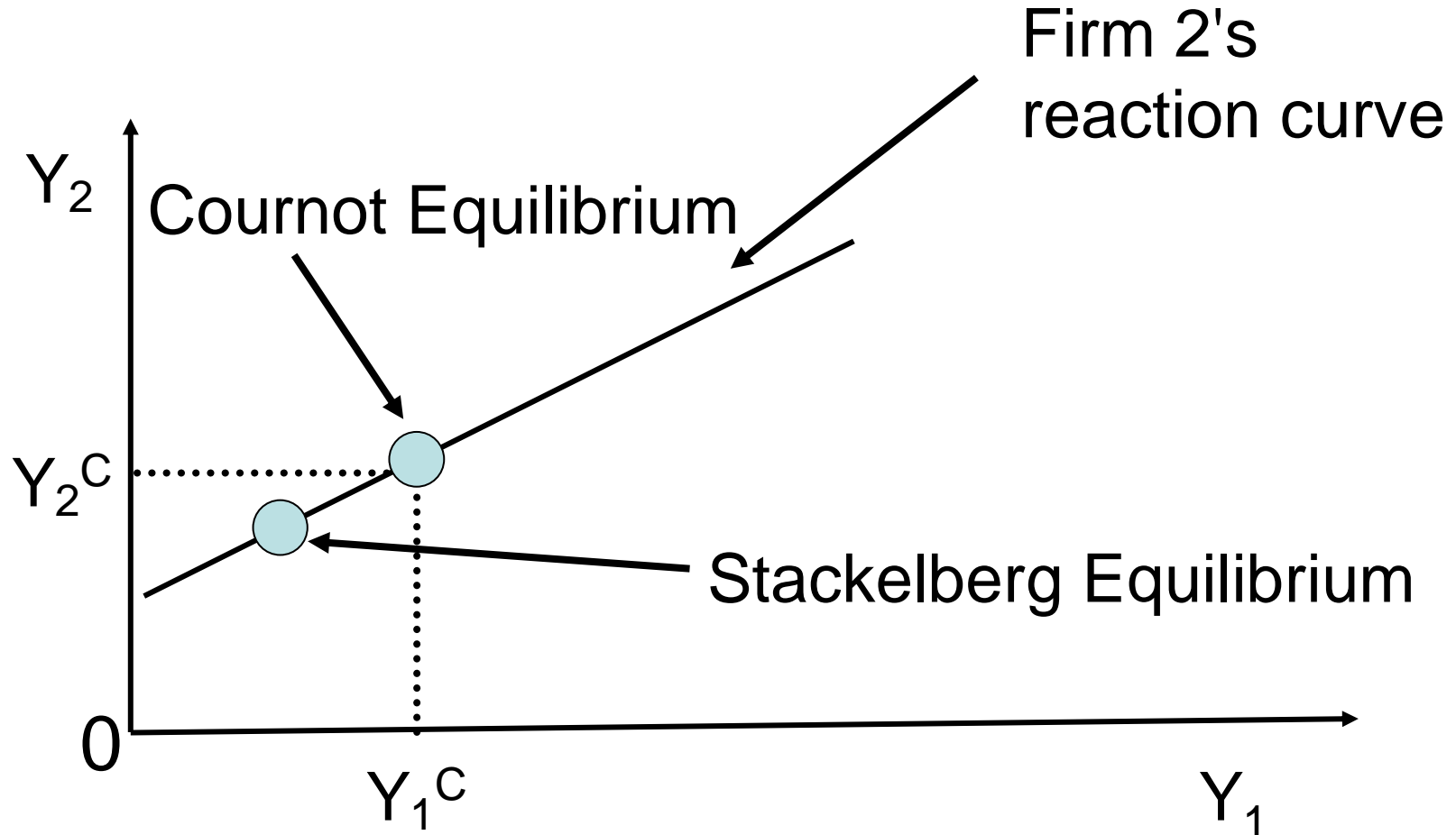


# Strategic Complements

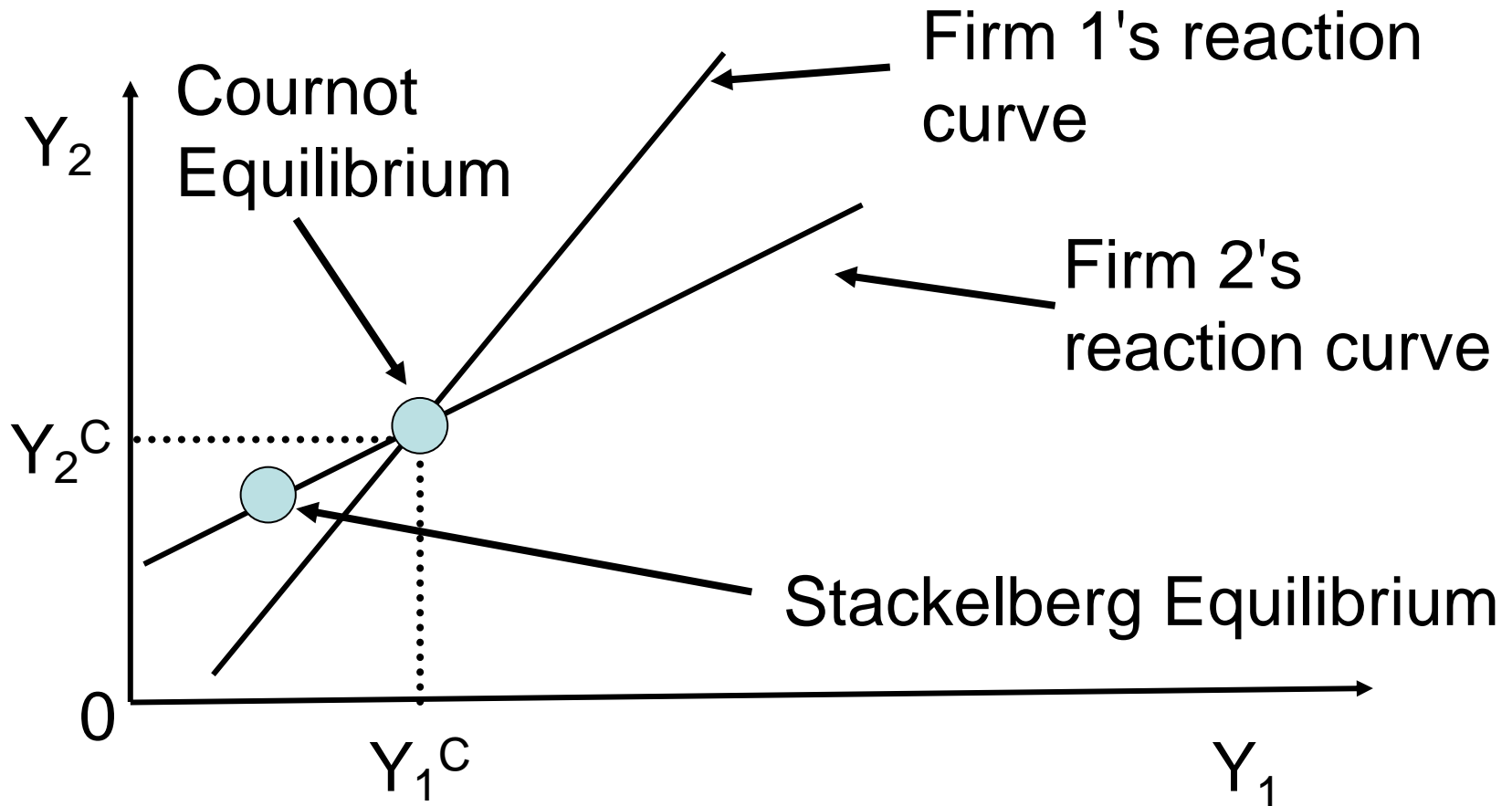




# Strategic Complements



# Strategic Complements

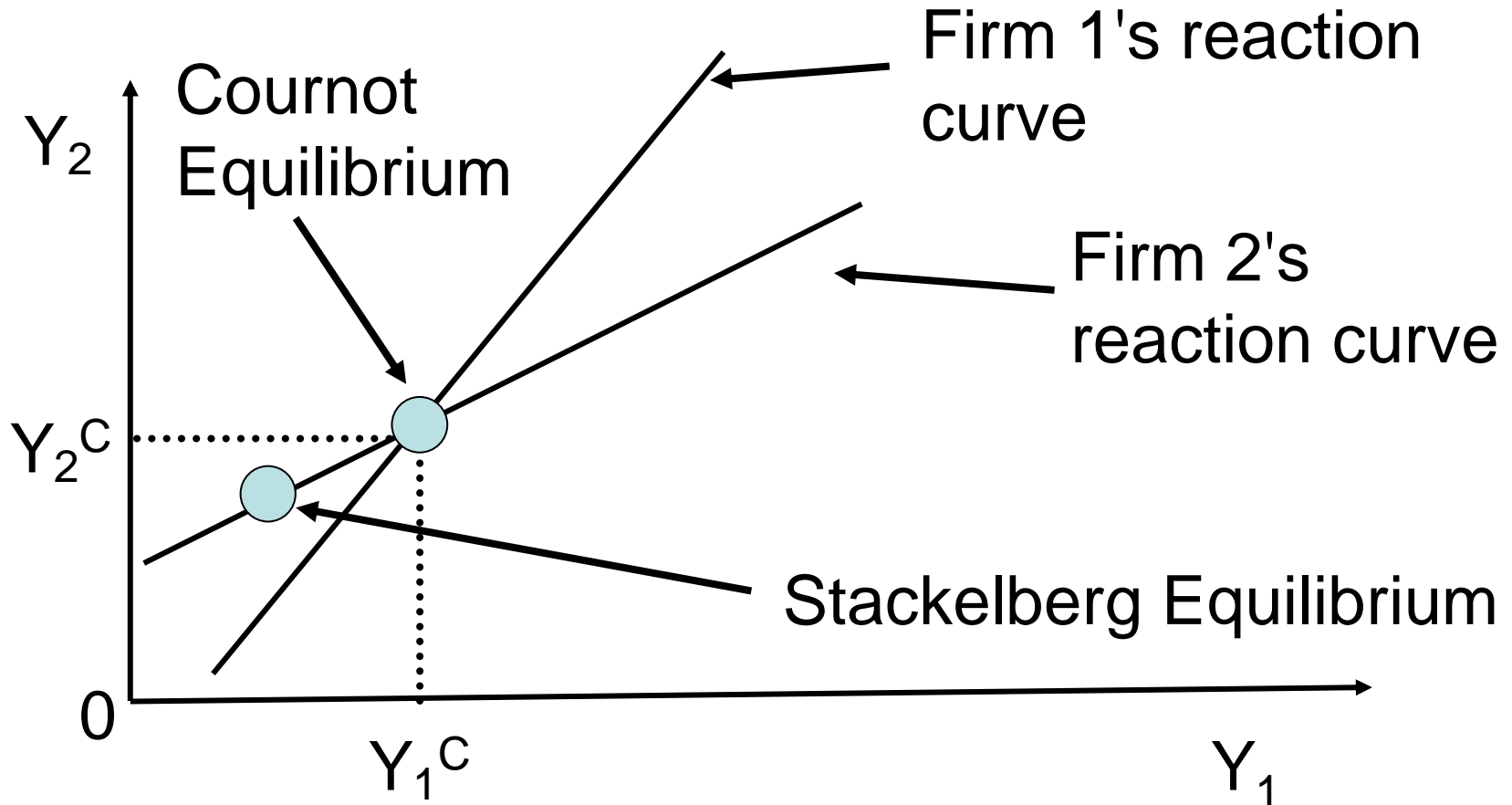


# Strategic Complements

Stackelberg Leader's profit is strictly larger than the Cournot counterpart if  $R' \neq 0$ .

Question: Suppose that  $R' > 0$  (Strategic Complements). Stackelberg Follower's profit is larger or smaller than the Cournot counterpart?

# Strategic Complements



# Second-Mover Advantage

Suppose that the two firms are symmetric except for the role of the game (either leader or the follower). Usually, Follower's profit is larger than the Leader's since

(1)  $|R_i'| < 1$  (stability condition)

$$\rightarrow Y_2^C - R_2(Y_1^L) < Y_1^C - Y_1^L$$

Firm 1 contributes more significantly for the reduction of total output.

(2) Firm 2 chooses the optimal output given  $Y_1^L$ , while  $Y_1 \neq R_1(Y_2^F)$ .

# Stackelberg Oligopoly

- (1) Firm 1 produces and then other firms produce
- (2) Firm 1 produces, then firm 2 produces, then firm 3 produces,....., and finally firm N produces.  
(Generalized Stackelberg Model) ~ Anderson and Engers (1992,1994)
- (3) Firm 1, firm 2, ..., firm m produces independently, and then firm m+1, firm m+2, ... firm N produce  
(Multiple Leadership)

# The Model

Players: identical  $m$  ( $\in [0, N]$ ) leaders,  
identical  $N - m$  followers.

Payoff: Its own profits

First, leaders choose their output independently.

After observing the leaders' outputs followers  
choose their outputs.

The market opens at the end of the game.

# Assumptions

**Assumption 1**  $P(X)$  is twice differentiable and  $P'(X) < 0$  for all  $X$  such that  $P(X) > 0$ .

**Assumption 2**  $C(x)$  is twice differentiable and  $C'(x) > 0$ ,  $C''(x) \geq 0$  for all  $x \geq 0$ .

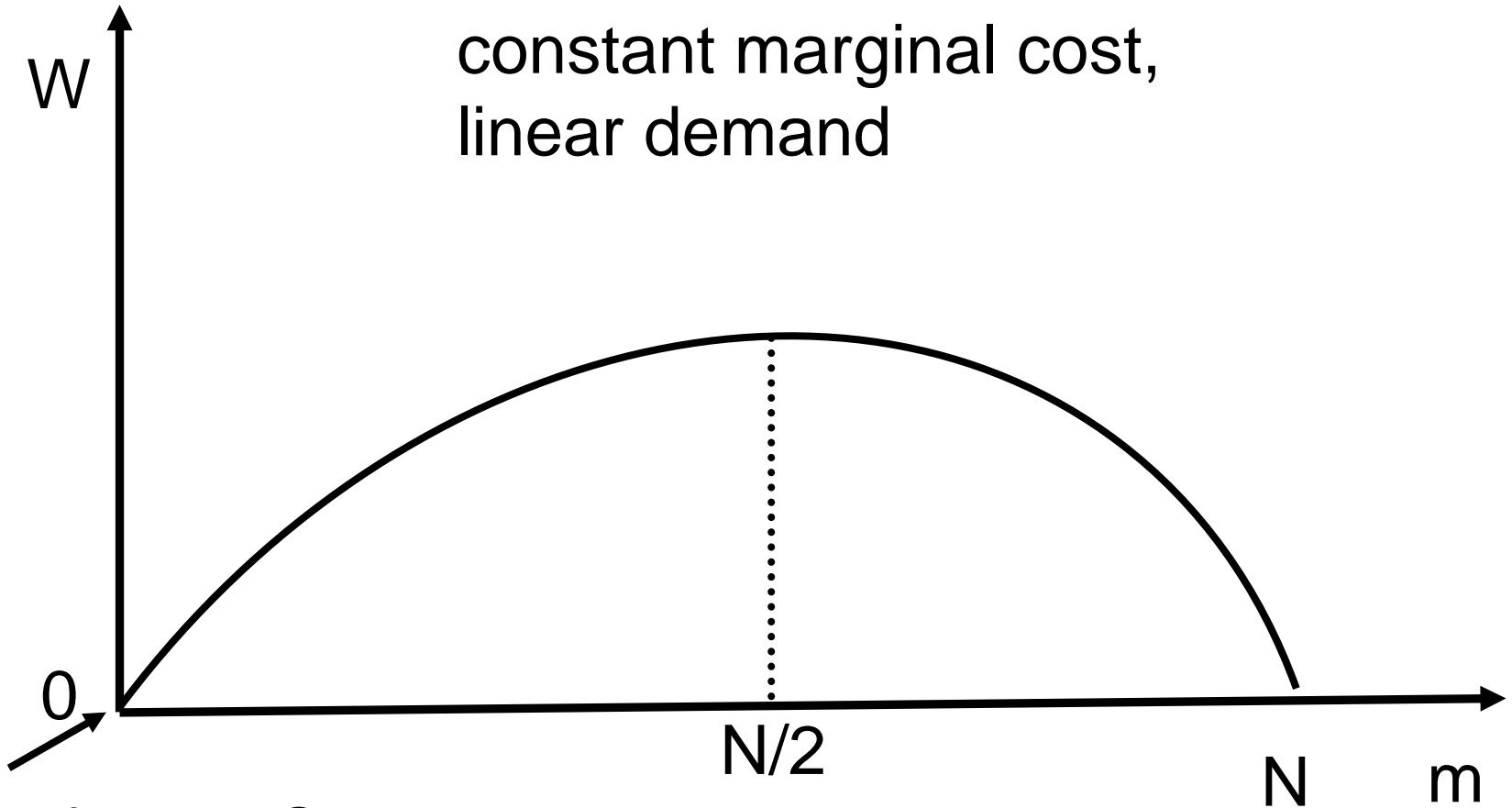
**Assumption 3 (strategic substitutes)**  $P''(X)x + P'(X) < 0$  for all  $X$  such that  $P(X) > 0$  and  $x \in (0, X)$ .

**Assumption 4** The model has the unique equilibrium for all  $m \in [0, N]$  and  $N > 0$ . The equilibrium is symmetric and all firms produce positive outputs.



# Daughety (1990, AER)

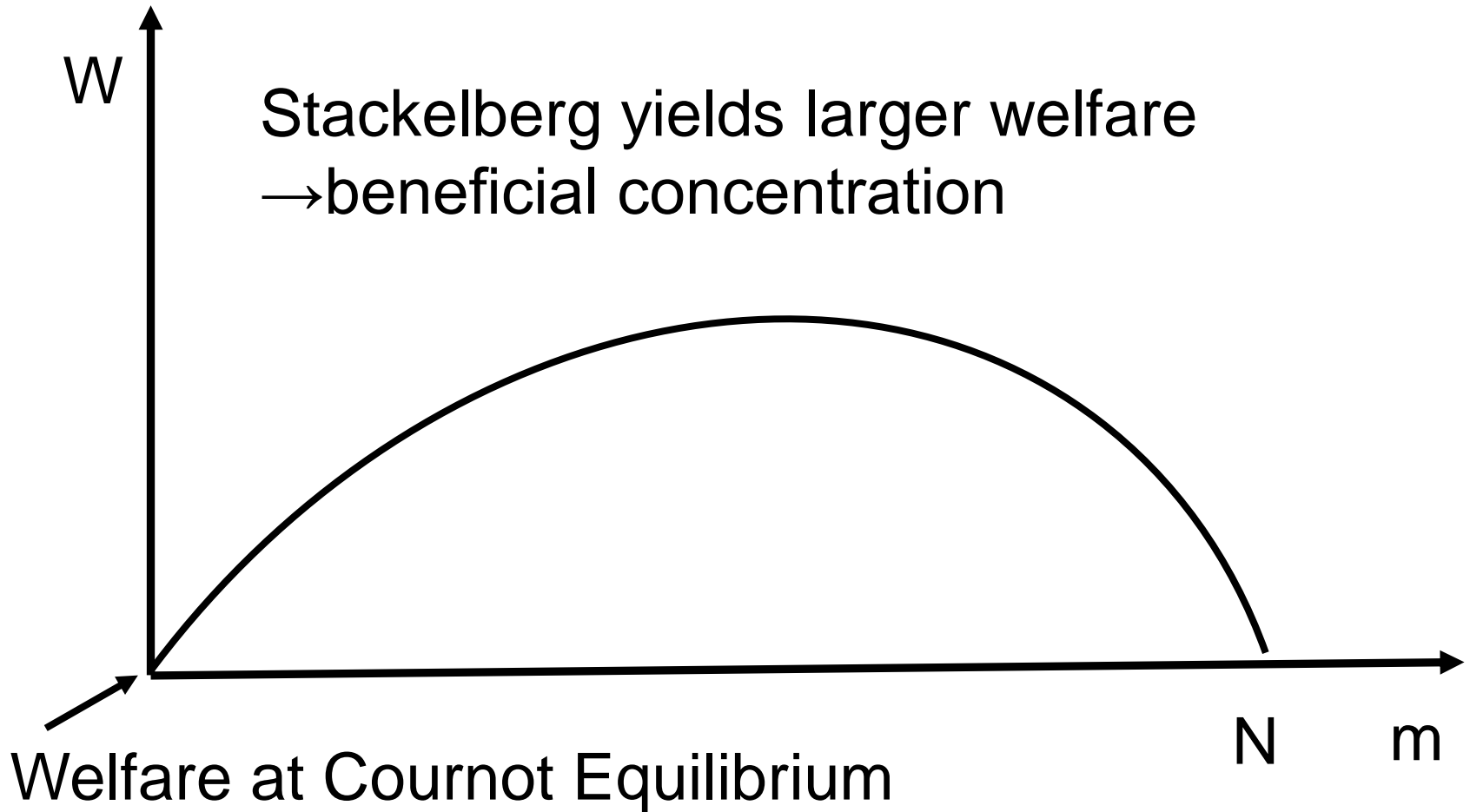
constant marginal cost,  
linear demand



Welfare at Cournot Equilibrium

Oligopoly Theory

# constant marginal cost



# constant marginal cost

Suppose that Assumptions 1-4 are satisfied.

If  $C''(x) = 0$  for all  $x \geq 0$ , then  $W^*(m) > W^*(0)$  for all  $m \in (0, N)$ .

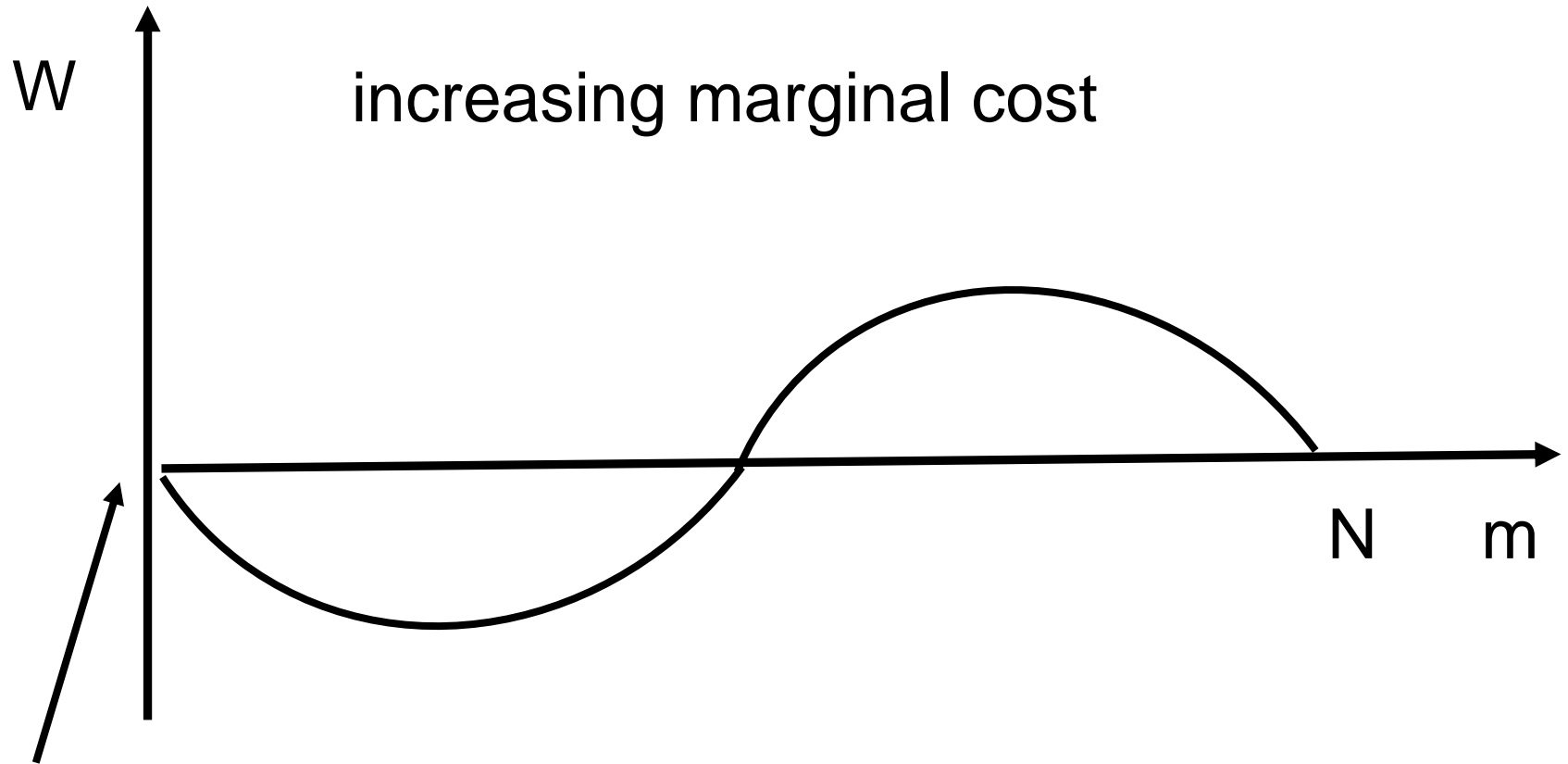
If marginal cost is constant, beneficial concentration always takes place.

→ This is because Stackelberg model yields larger total output than the Cournot.

(generalization of Daughety (1990) for general demand)

# increasing marginal cost

## Ino and Matsumura (2016, BETA)



Welfare at Cournot Equilibrium

Oligopoly Theory

# Increasing Marginal Cost

Suppose that Assumptions 1-4 are satisfied.

Then, (i)  $W^*(m)$  at  $m = 0$  can be either negative or positive and (ii)  $W^*(m)$  at  $m = N$  is always negative.

(i) Introducing small number of leaders into the Cournot model can be either beneficial or harmful for welfare.

(ii) Introducing small number of followers into the Cournot model is always beneficial for welfare.

# Linear-Quadratic Costs

Suppose that  $P = a - X$  and  $C(x) = cx + kx^2$ . If  $k > 0$ , there exists  $N' > 0$  such that  $W'(m)$  at  $m = 0$  is negative for all  $N > N'$ .

For any quadratic cost functions, there are cases where leadership is harmful.

Leadership becomes harmful more likely when the number of follower is large.

# Why can introducing a small number of leaders into the Cournot model be harmful ?

Consider the Stackelberg model with one leader (firm 1). Then firm 1 becomes a followers (Cournot).

→ Production substitutions from firm 1 to the other firms. This production substitution improves production efficiency when marginal cost is increasing and can dominate the positive effect of increasing CS.

~ This welfare-improving production substitution effect is strong when the number of followers is large.

# Why is introducing a small number of followers into the Cournot model always beneficial?

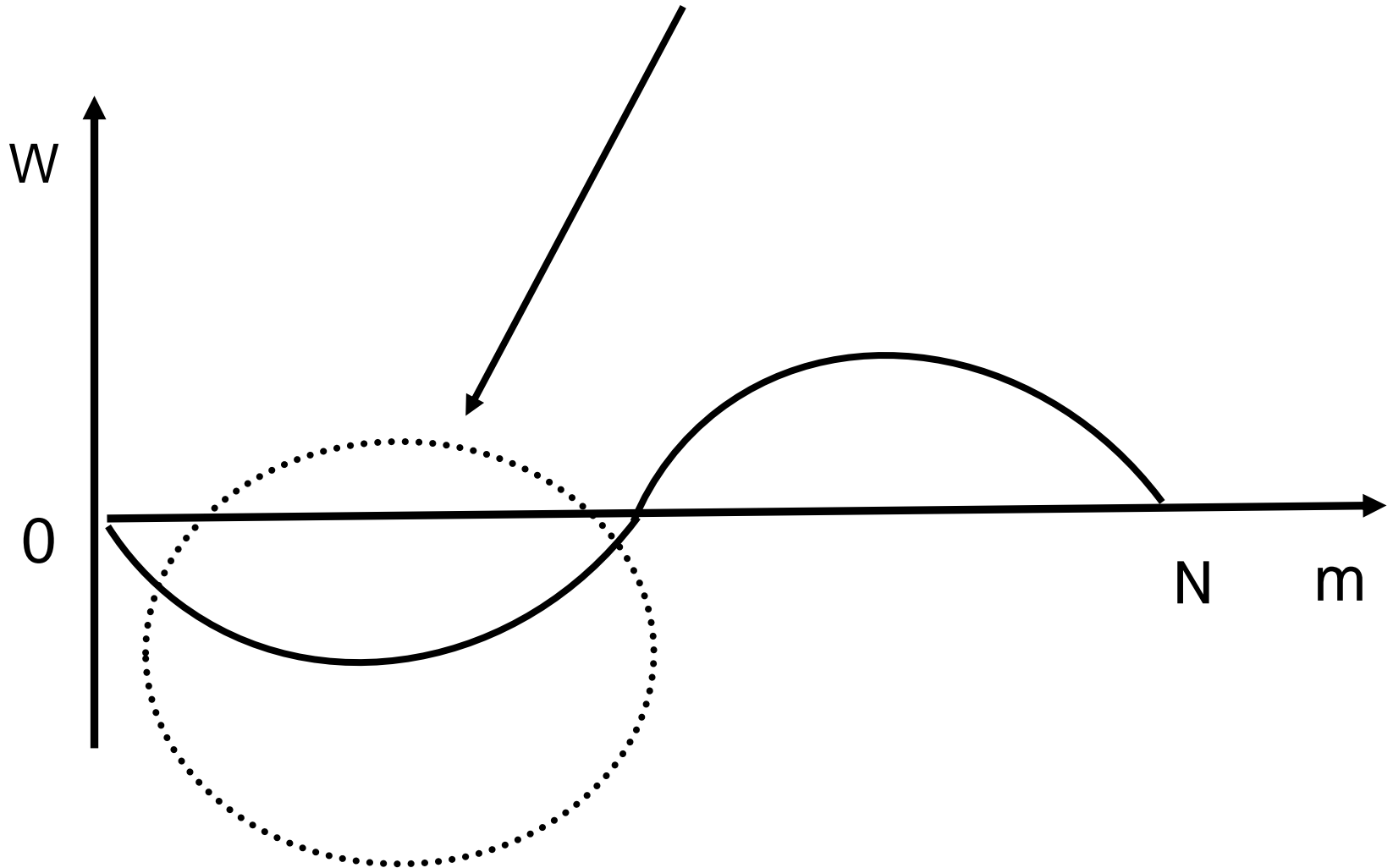
Consider the Stackelberg model with one follower (firm  $n$ ). Then firm 1 becomes a leader (Cournot).

→ Production substitutions from firm 1, 2...,  $N - 1$  to firm  $N$ . This production substitution improves production efficiency when marginal cost is increasing, but this effect is negligible because  $\lim_{m \rightarrow N} x^L = \lim_{m \rightarrow N} x^F = x^C$  (Cournot output).

~ When the number of followers is small, the difference of output level between each leader and follower is negligible.



# Why convex?



# Why convex ?

Consider the Stackelberg model with  $m$  leaders. Then firm  $m+1$  becomes a leader.

→ Production substitutions from firm  $m+2$ , firm  $m+3$ , ... firm  $N$  (firm 1, firm 2, ..., firm  $m$ ) to firm  $m+1$ . This production substitution worsens (improves) production efficiency.

The former (latter) effect is weaker (stronger) when  $m$  is large.

~ An increase of the number of the leaders more likely improve welfare when  $m$  is large.

# price leadership

The leader announces the price change first, and then other firms follow this price change.

Some researchers suspect that this is a collusive pricing, implicit cartel.

However, if we regard this as a price version Stackelberg, it is natural that a higher price of the leader induces a higher price of the follower (strategic complements)

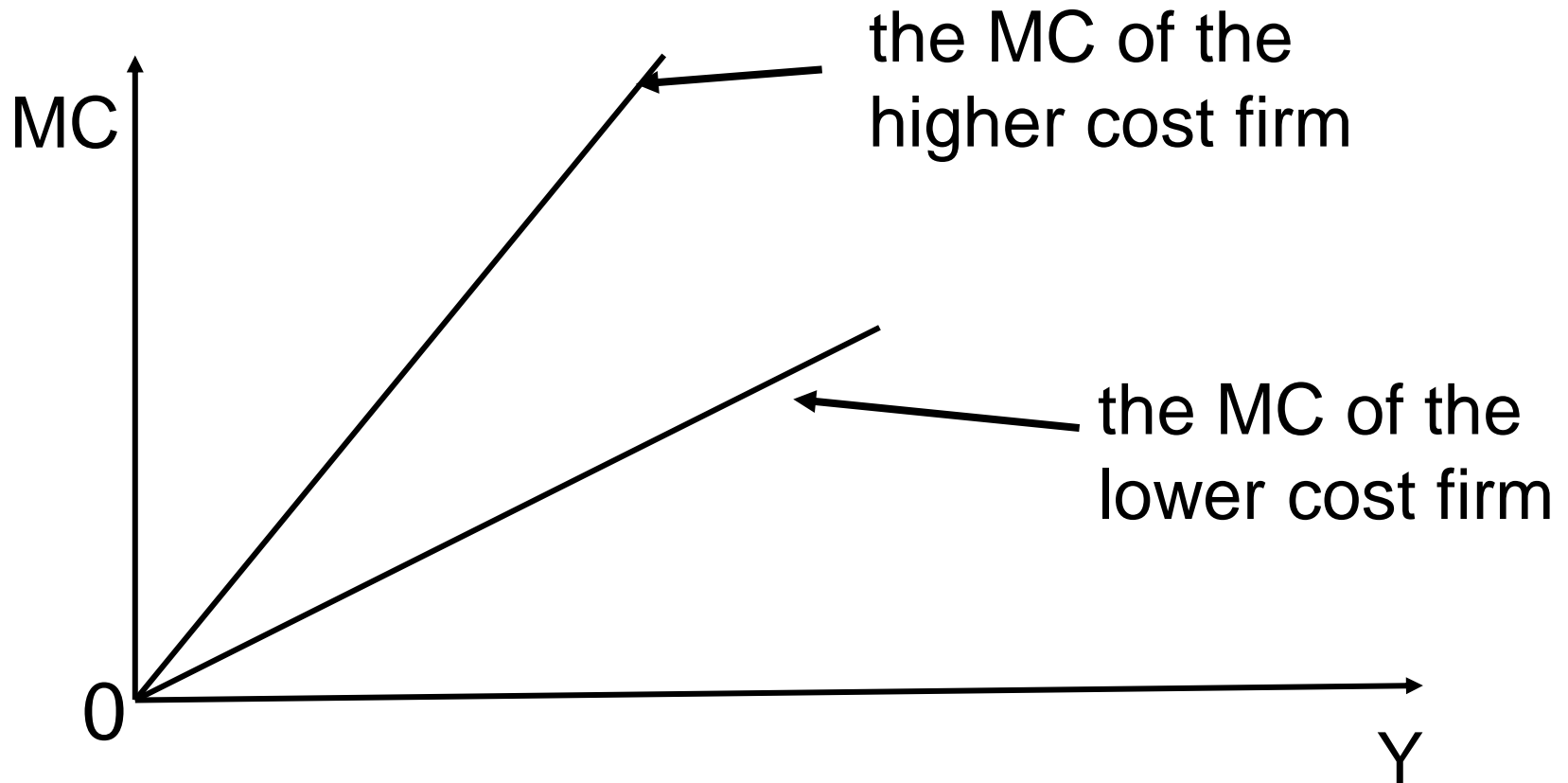
# price leadership (Ono, 1978, Economica)

Homogeneous product market, no supply obligation, duopoly, increasing marginal cost, price-setting.

One firm chooses the price and then the other firm chooses its price after observing the price of the rival. (Stackelberg)

He compares the equilibrium payoffs when the firm with higher cost is the leader to those when the firm with lower cost is the leader.

# Asymmetric Costs



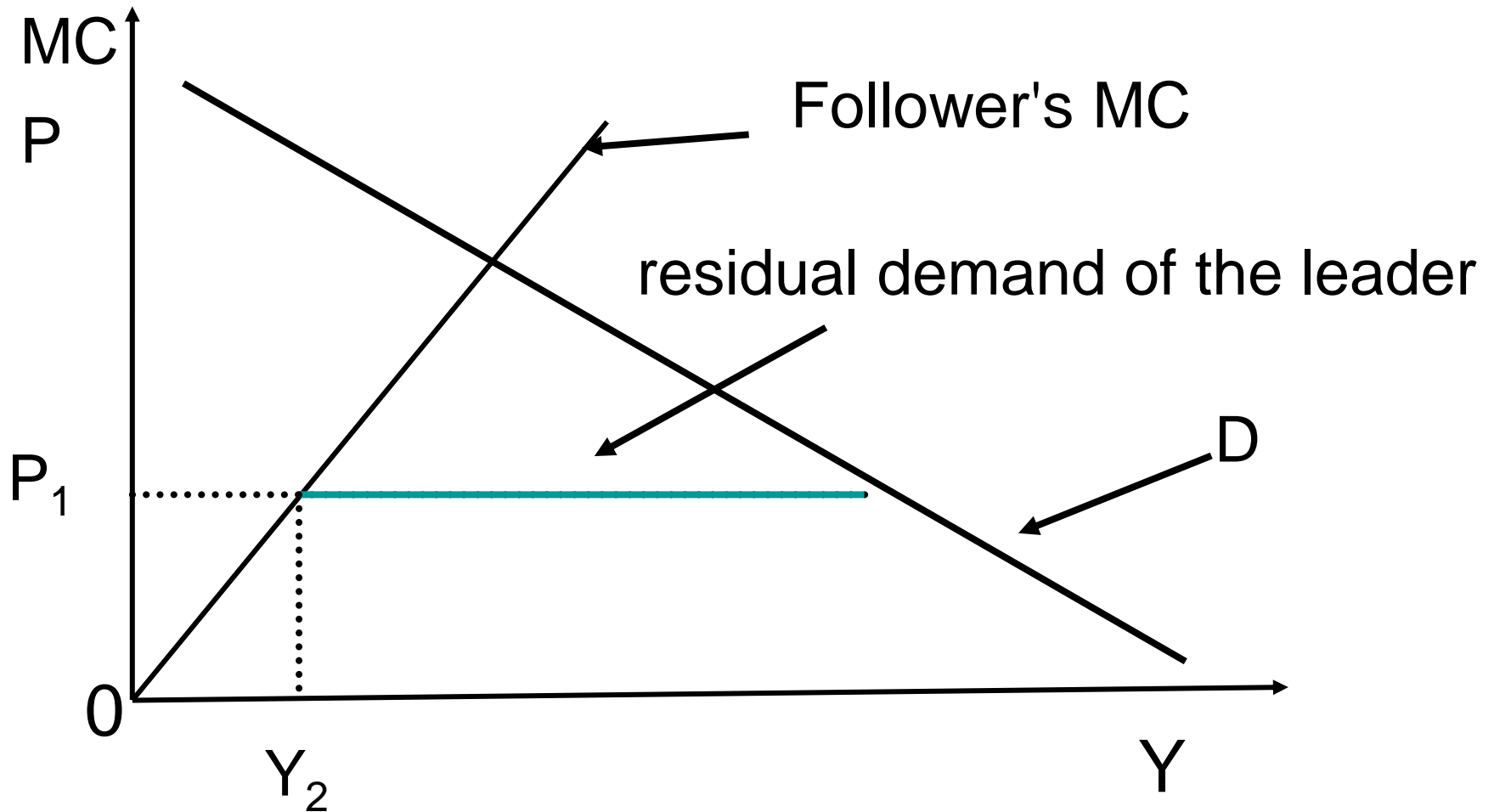
# Follower's pricing

- (1) Suppose that the leader's price is higher than the monopoly price of the follower. Then, the follower names its monopoly price and obtains the whole market.
- (2) Suppose that the leader's price is lower than the monopoly price of the follower. Then,
  - (a) names a higher price than the leader and obtains the residual demand, or
  - (b) the follower names the price slightly lower than the rival's and obtains the whole market. (price undercutting)

# Firm 1's pricing

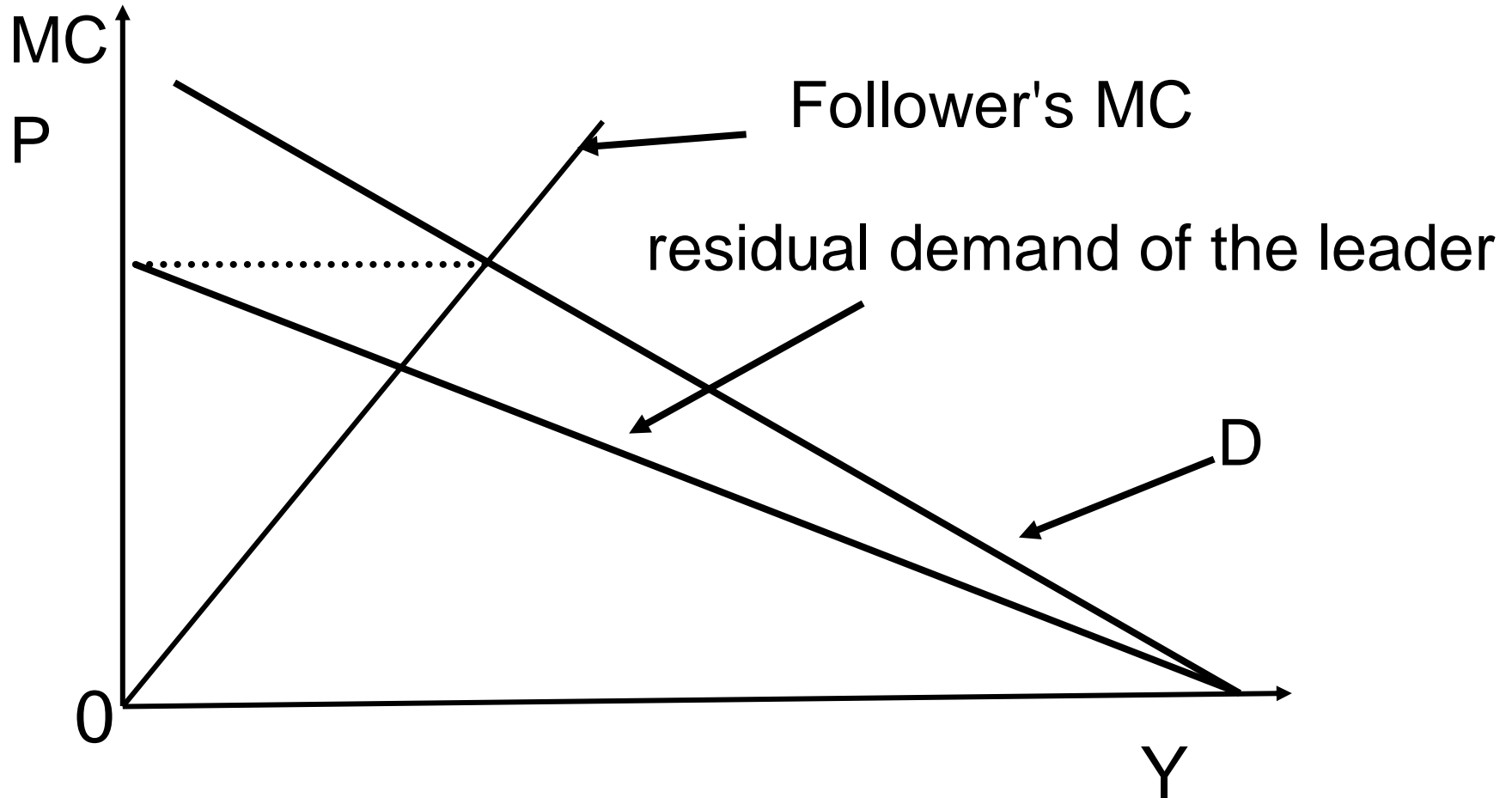
Ono (1978) assume that the follower undercut the leader's price. Predicting this behavior of the follower, the leader chooses its price.

# Residual demand

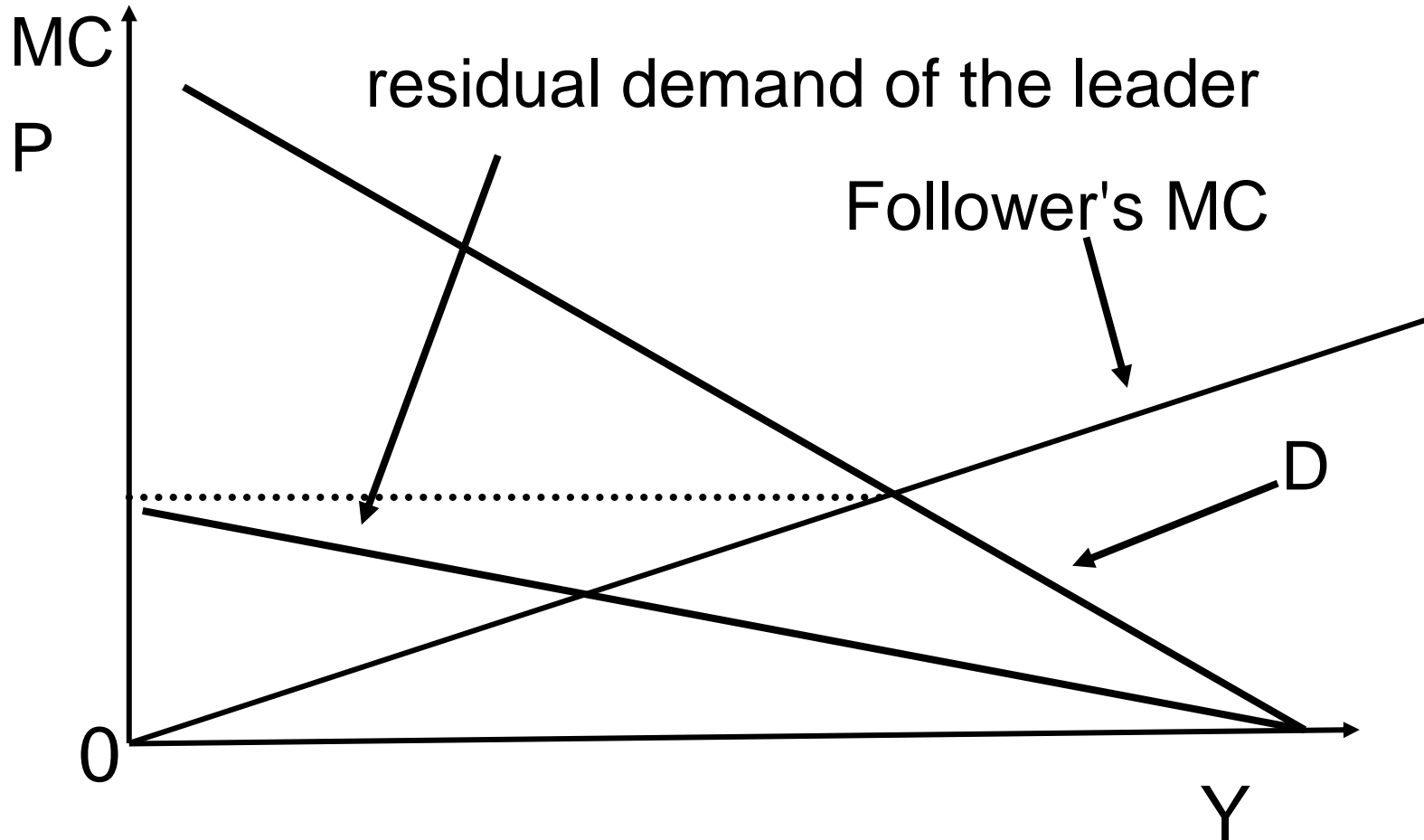




# Residual demand



# Residual Demand



# price leadership

Suppose that the firm with lower cost becomes the follower. → It produces a lot as a price taker

→ Predicting this aggressive behavior, the firm with higher cost names a low price.

Suppose that the firm with higher cost becomes the follower. → It does not produce a lot as a price taker

→ Predicting this less aggressive behavior, the firm with higher cost names a high price. ~ **beneficial for both firms.**

**He concludes that the lower cost firm takes price leadership if the cost difference between two firms is large.**

# Contribution of Ono (1978)

- (1) pioneering work on Timing Game → 6th lecture
- (2) pioneering work on Price Leadership.  
~ the lower cost firm becomes the leader
- (3) Mutual Beneficial Price Leadership can appear when the cost difference between two firms are sufficiently large.

# Subsequent Works

- Ono (1982, *Economica*) Oligopoly Version
- Denekere and Kovenock (1992, *RES*)

## ~Capacity Constraint

→ The firm with more capacity becomes the leader.

- Amir and Stepanova (2006, *GEB*) ~ differentiated product market

→ The firm with lower cost firm becomes the leader and it is mutually beneficial if cost difference is large.

- Ishibashi (2007, *IJIO*)

## ~Capacity Constraint + repeated game

→ The firm with more capacity becomes the leader.

# Problems in Ono (1978)

(1) Is mutually beneficial leadership is always realized in equilibrium ?

- He did not formulate the Timing Game.

(a) Is the outcome where the lower cost firm becomes the leader always an equilibrium ?

(b) Is it a unique equilibrium?

(c) If not, the equilibrium with lower cost firm's leadership is robust ?

~ No game theoretic foundation

**I discuss this point in the 6th lecture.**

# Problems in Ono (1978)

(2) Is price undercutting always best reply?

The answer is NO. Undercutting is not always the best reply.

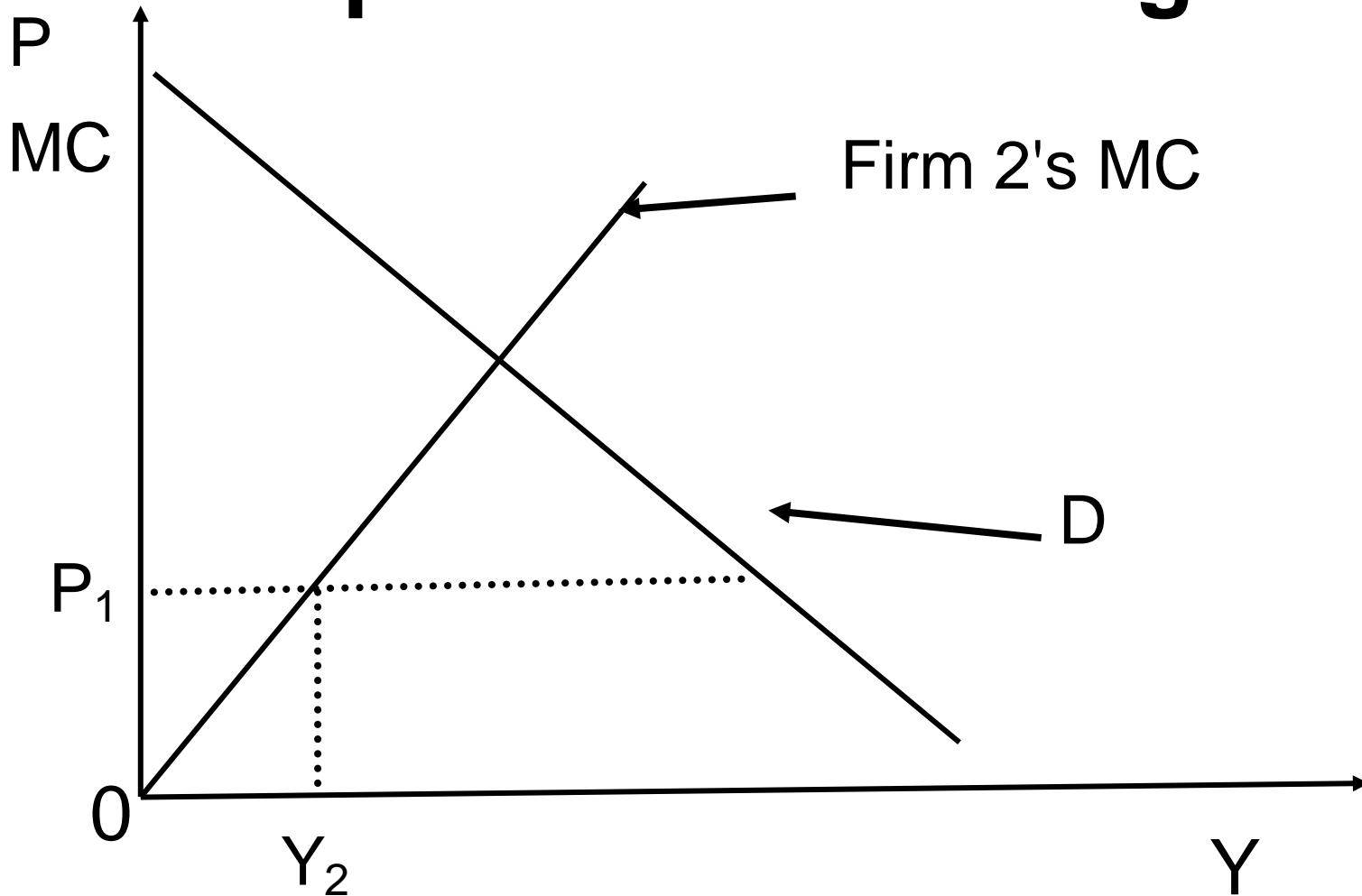
# Dastidar (2004, EER)

Consider a Stackelberg duopoly with common increasing marginal cost in a homogeneous product market.

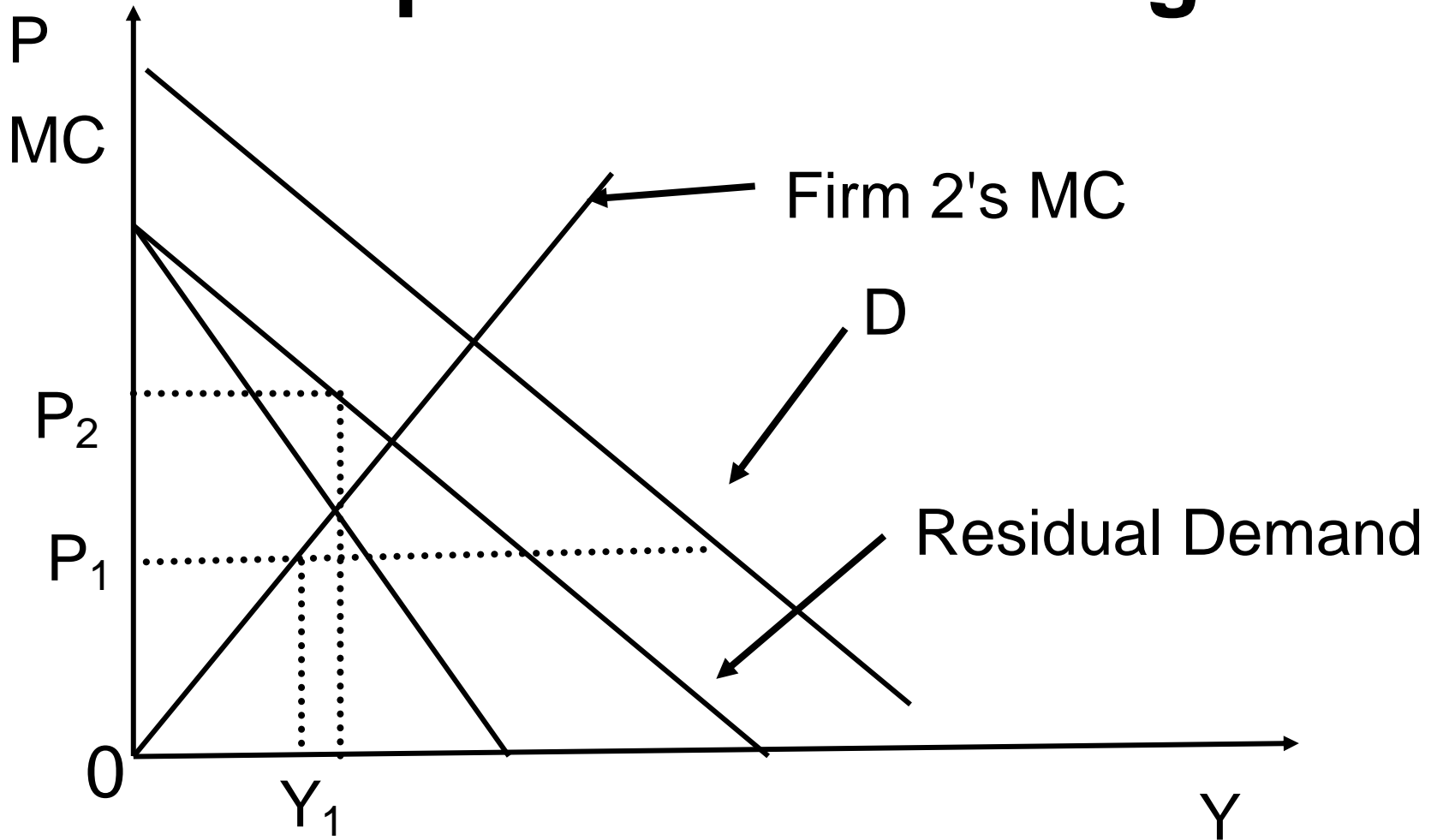
Firm 1 names the price and after observing the price firm 2 names the price.



# price-undercutting



# no price-undercutting



# price-undercutting vs non-undercutting

An increase in the price of the leader makes the undercutting strategy more profitable and non-undercutting strategy less profitable.

→ There exists  $p^*$  such that the follower takes non-undercutting strategy if and only if  $p \leq p^*$ .

In equilibrium, firm 1 names  $P_1 = p^*$ , firm 2 takes non-undercutting strategy, and two firms obtain the same profits.

Two prices appear in the homogeneous product market. The leader engages in marginal cost-pricing, while the follower does not.

# Problems in Ono (1978)

(3) The assumption of price-undercutting is innocuous or harmful?

The answer is `not innocuous'. This assumption changes the results

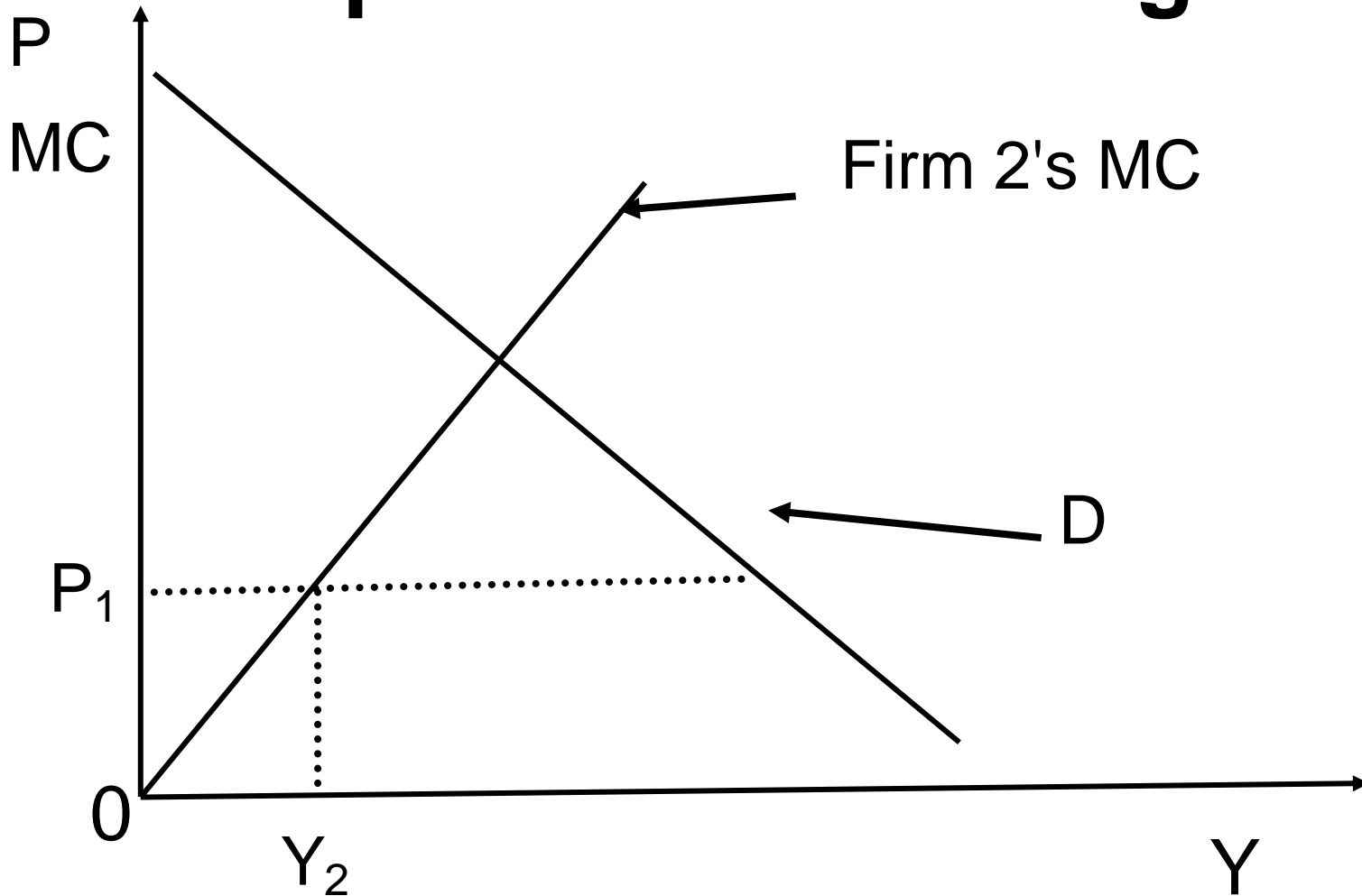
In equilibrium, the follower does not undertake the price.

- (i) The price leadership by the higher price firm is mutually beneficial even when the cost difference is small.
- (ii) It is a unique equilibrium, or it is the risk-dominant equilibrium.

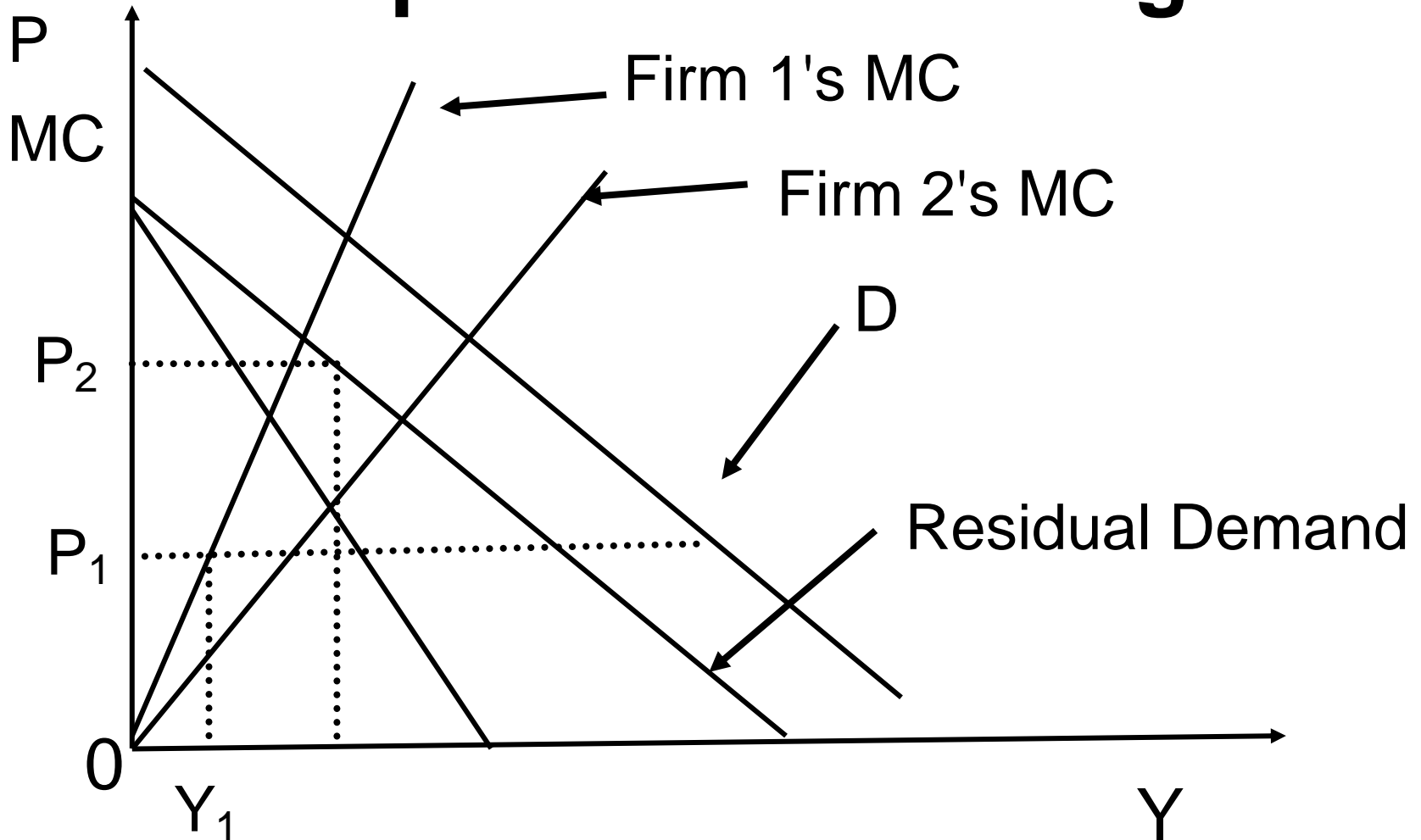
# Hirata and Matsumura (2011, JoE)

- (i) The price leadership by the higher price firm is mutually beneficial even when the cost difference is small.
- (ii) It is a unique equilibrium, or it is the risk-dominant equilibrium (I will explain the concept of risk dominance in the 6th lecture).

# price-undercutting



# no price-undercutting



# Intuition behind the results

Suppose that the leader has a higher cost. ~ It is easy to induce the follower to take non-undercutting strategy (taking a residual demand).

It can name a relatively higher price, and it is mutually beneficial.