

# Oligopoly Theory (4)

## Market Structure and Competitiveness

### Aim of this lecture

- (1) To understand the concept of HHI.
- (2) To understand the relationship between the relative profit maximization and the market competition.

# Outline of the fourth lecture

4-1 HHI and competitiveness

4-2 The number of the firms and competitiveness

4-3 Demand elasticity and competitiveness

4-4 Market definition

4-5 Conjectural variation and market conduct

4-6 Relative profit maximization approach and  
common ownership

# HHI (Herfindahl-Hirschman Index )

$$\text{HHI} = \sum_{i=1}^n (\text{firm } i\text{'s market share})^2$$

A higher HHI → Higher market concentration

This index is used in the contexts of anti-trust legislation and regulations for network industries.

Usually, the percent representation is used.

$$\text{Monopoly } (100)^2 = 10000$$

$$\text{Symmetric duopoly } (50)^2 + (50)^2 = 5000$$

In the lecture we use

$$\text{Monopoly } (1)^2 = 1,$$

$$\text{Symmetric duopoly } (0.5)^2 + (0.5)^2 = 0.5$$

# HHI (Herfindahl-Hirschman Index )

(1) An increase of the number of the firms  
(increases, decreases) HHI

Consider an  $n$ -firm, symmetric oligopoly.

Question:

(a) Derive HHI.

(b) HHI is (increasing, decreasing) in  $n$ .

# HHI (Herfindahl-Hirschman Index )

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(increases, decreases) HHI

Consider an n-firm, symmetric oligopoly.

Question:

(a) Derive HHI.

# HHI (Herfindahl-Hirschman Index )

(1) An increase of the number of the firms decreases HHI

n-firm, symmetric oligopoly,

Question: HHI is (increasing, decreasing) in n.

# Asymmetry among firms and HHI

(2) An increase of asymmetries among the firms  
(increases, decreases) HHI

Question: Consider a duopoly. The share of firm 1  
is  $b > 1/2$ .

(a) Derive HHI

(b) HHI is (increasing, decreasing) in  $b$ .

# Asymmetry among firms and HHI

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(b) HHI is (increasing, decreasing) in  $b$ .

# The relationship between the number of firms and competition

The larger the number of firms is, the tougher the market competition is ~ The world of Cournot Limit Theorem

Is it always true?

The inverse causality may be true. ~ The tougher competition yields a smaller number of firms.

A larger number of firms can survive under less severe competition.

Only a small number of firm can survive under very severe competition. ~ free-entry-exit markets.

# The number of firms and economic welfare

The larger the number of the firms is, the closer the equilibrium outcome to the Walrasian. ~ Cournot Limit Theorem

Walrasian outcome is the first best (Pareto efficient) ~  
The first fundamental theorem of welfare economics

Then, it is natural to think that the larger the number of the firms is, the more efficient the outcome is.

However, it is not true in free entry markets with positive entry cost. →9th Lecture.

# The asymmetry among firms and economic welfare

Consider a duopoly.

An increase of the asymmetry among the firms increases HHI.

# The asymmetry between firms and economic welfare

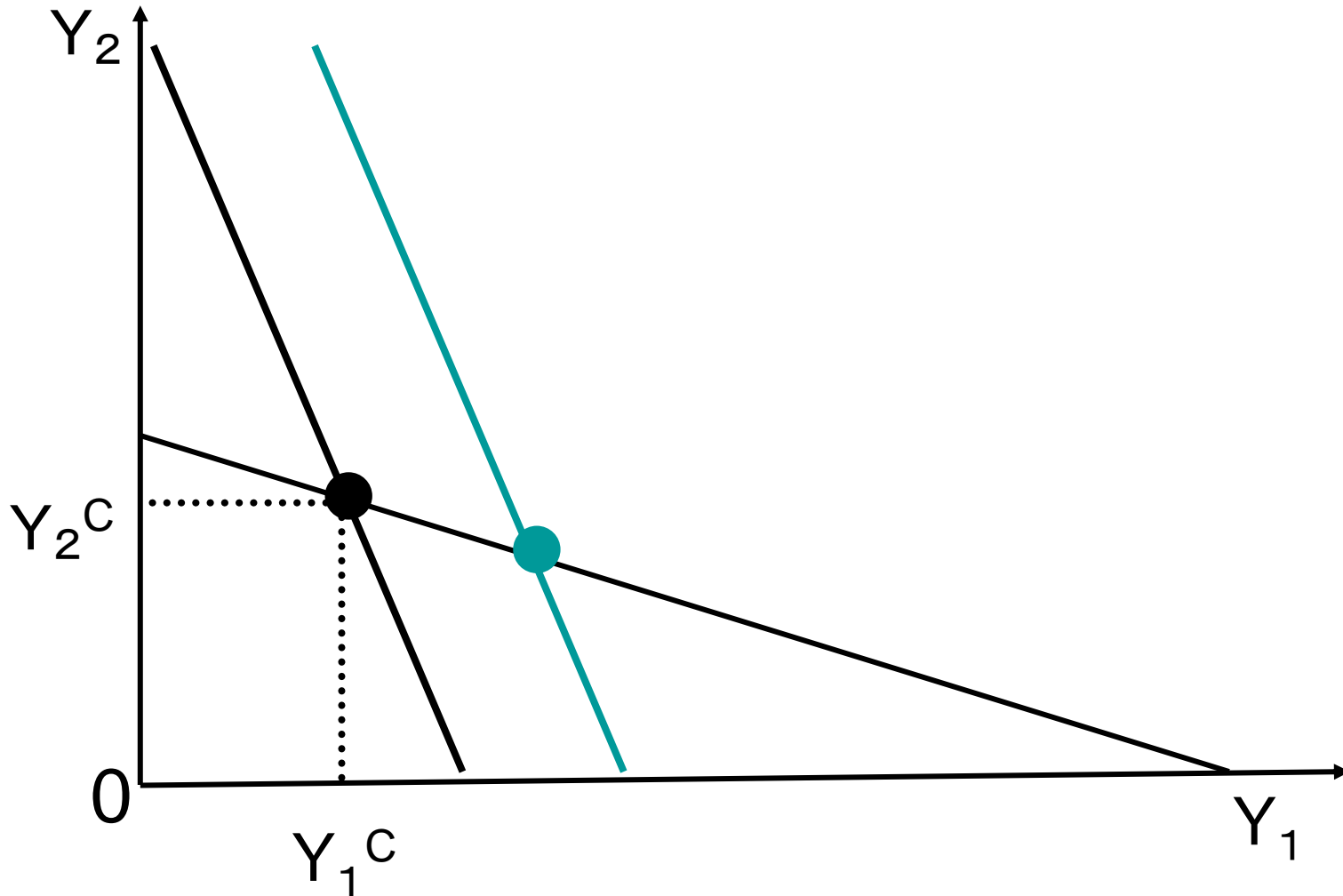
Consider the following example.

Duopoly, constant marginal costs, Cournot competition, homogeneous product markets, decreasing demand function, strategic substitutes, continuous reaction function, Cournot equilibrium is unique and stable.

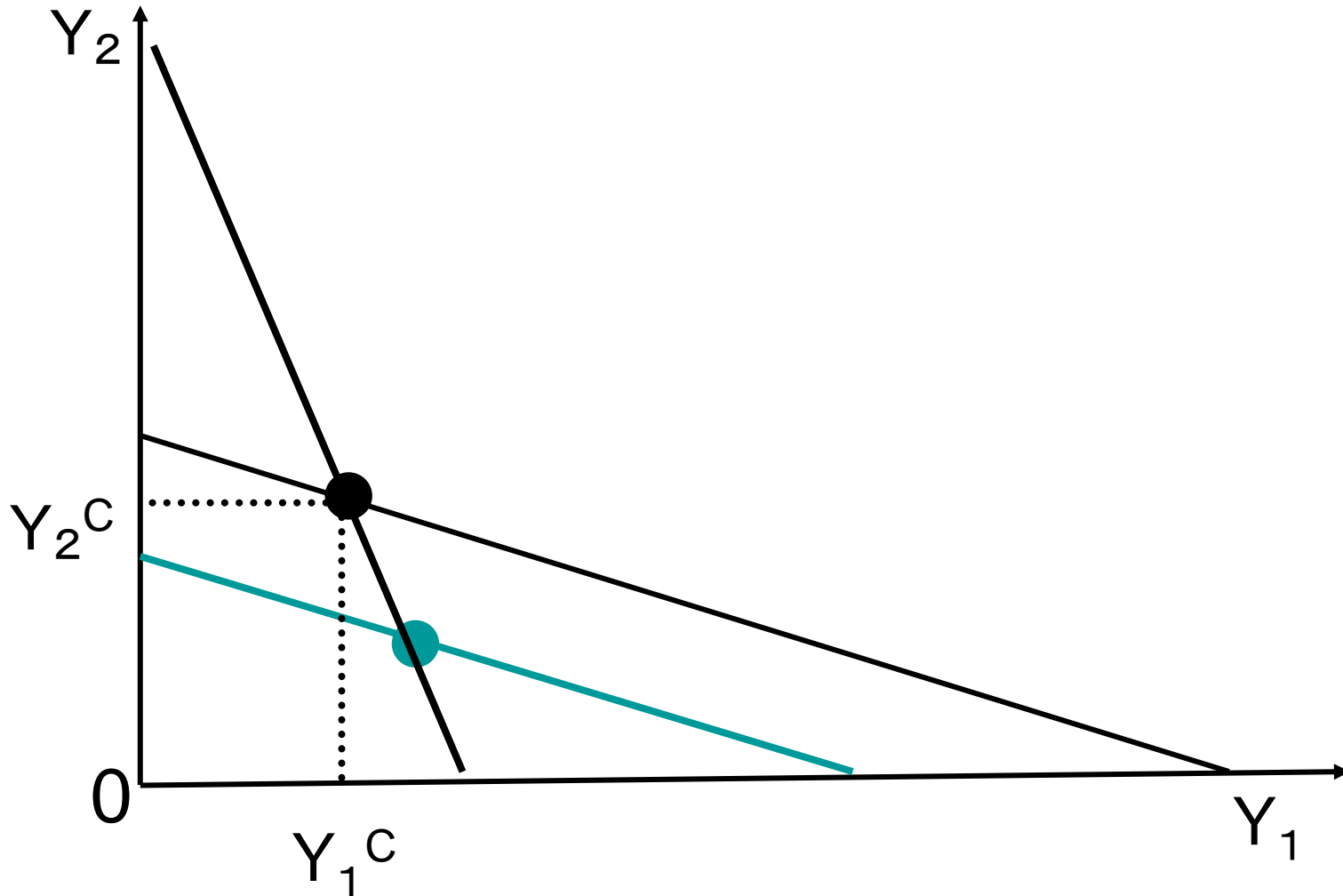
Firm 1's marginal cost is  $c - d$ , firm 2's is  $c + d$ , where  $c > d > 0$ .

**Question: An increase in  $d$  (increases, decreases) the output of firm 1 at the Cournot equilibrium.**

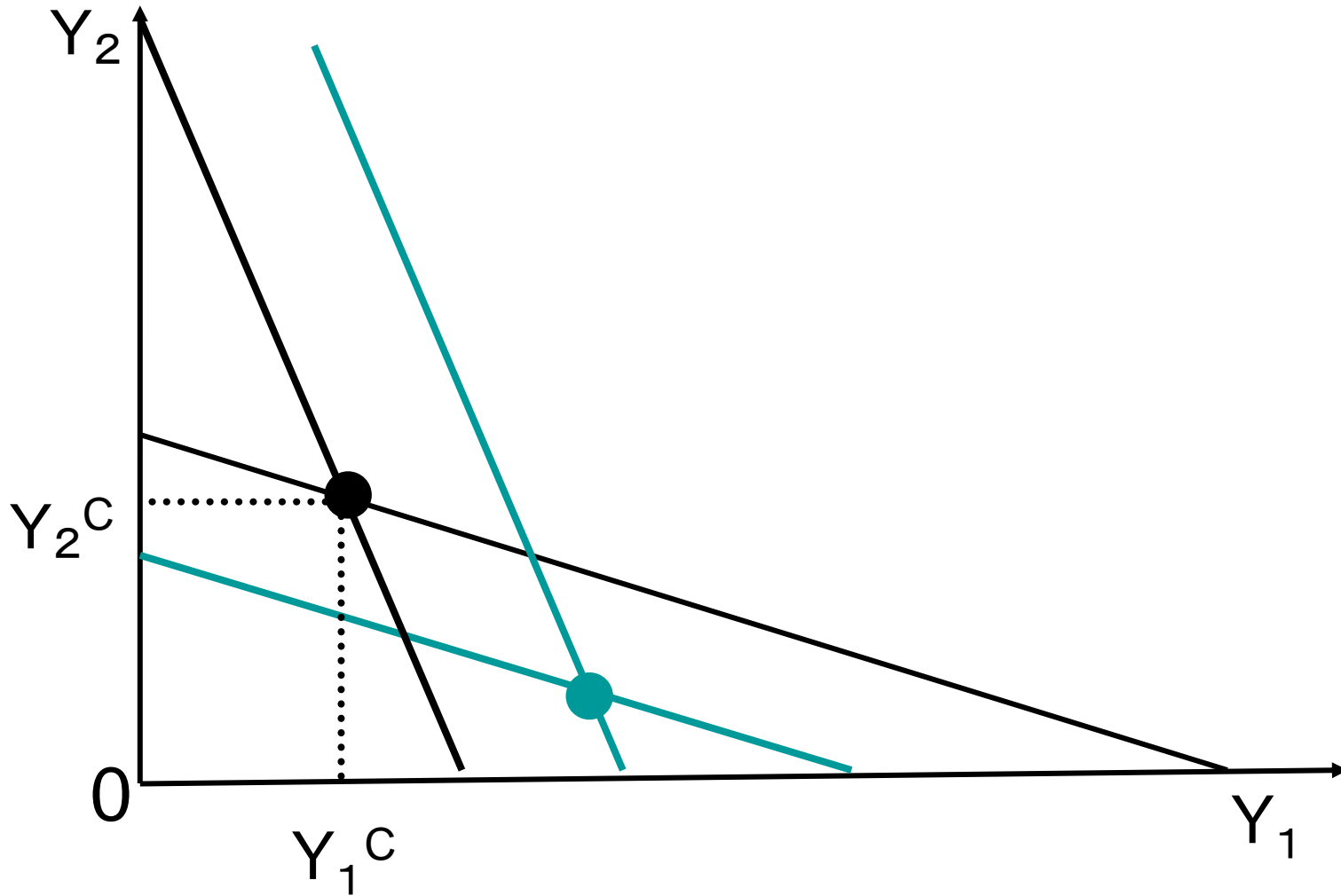
# A decrease of its own cost



# An increase of its rival's cost



# Total effect





# The asymmetry between firms and economic welfare

The first-order condition for profit-maximizing

$$P + P'Y_1 = c - d, \quad P + P'Y_2 = c + d$$

$\Rightarrow 2P + P'Y = 2c \sim d$  does not appear

$\rightarrow Y$  (and thus CS) does not depend on  $d$ .

Question:

Total production cost,  $(c - d)Y_1 + (c + d)Y_2$  is  
(increasing, decreasing) in  $d$ .

# Conjectural variation

conjectural variation

Each firm considers that one unit increase of its output increases the  $r$ -unit increase of total output (i.e., it considers that if it increases the output by one unit, then the rivals increase their output by  $r - 1$  unit).

The model is Cournot if  $r = 1$ .

Conjectural Variation Model ~The general model including Cournot as a special case.

However, is the model with  $r \neq 1$  reasonable?

# The solution of the conjectural variation model

Consider a symmetric duopoly in a homogeneous product market with constant marginal cost.

The first-order condition of firm 1 is

$$P + P'Y_1 = c$$

We assume that the second-order condition is satisfied.

We can derive the equilibrium in the model from

$$P + P'Y_1 = c, \quad P + P'Y_2 = c$$

# Conjectural variation model

Suppose that  $r \neq 1$ .

An increase of firm 1's output affects the firm 2's output. → The output of firm 2 must be determined after the decision of firm 1.

An increase of firm 2's output affects the firm 1's output. → The output of firm 1 must be determined after the decision of firm 2.

⇒ It is impossible that both are true.

Inconsistent model as a one-shot static model.

# Why do we still use the conjectural variation model?

(1) an unmodeled dynamic model

~ the model considers the dynamic interaction between firms

→ OK. But we should formulate a dynamic model formally instead of using an unmodeled model.

(2) 'r' represents a degree of market competition.

# The solution

The first-order condition in the CV Model,  $P + P'r Y_1 = c$

The first-order condition in the Cournot Model

$$P + P'Y_1 = c \quad (r = 1)$$

The first-order condition in the Walrasian (or Bertrand)

$$P = c \quad (r = 0)$$

The first-order condition for the joint profit Maximization (Collusion),  $P + P'(Y_1 + Y_2) = c \quad (r = 2)$

The models with different competition structure are included as special cases.

The more  $r$  is, the less severe the competition is.

# Relative profit maximization and perfect competition

Suppose that each firm maximizes the difference between its own profit and the rival's  $\sim U_1 = \pi_1 - \pi_2$   
→ more aggressive behavior

The first-order condition

$$P + P'Y_1 - C_1' - P'Y_2 = 0$$

→ at the symmetric equilibrium ( $Y_1 = Y_2$ ),  $P = MC$ .

(price taking)

⇒ In the context of quantity-setting competition, only two firms yield Walrasian.

This is evolutionary stable (Vega-Redondo, 1997)

# Relative profit maximization approach and competition

Consider a symmetric duopoly in a homogeneous product market. Consider a quantity-setting competition. Suppose that  $U_1 = \pi_1 - \alpha\pi_2$ ,  $\alpha \in [-1, 1]$ . The first-order condition is  $P + P'Y_1 - C_1' - \alpha P'Y_2 = 0$ .

$\alpha = 1$  ~ perfect competition,  $\alpha = 0$  ~ Cournot,

$\alpha = -1$  ~ Collusion

This model can describe from perfectly competitive market to the collusive market by the single model.

We can discuss the competition which is less severe than Bertrand and more severe than the Cournot.

Matsumura and Matsushima (2012), Matsumura et al. (2013), Matsumura and Okamura (2015)



# Advantage of relative profit maximization approach

Consistent model.

We can use a standard Nash equilibrium in the model without inconsistency.

More importantly, **I believe** that the model is quite realistic.

# Rationale for relative profit maximization approach

- (1) CEO Market, Management Market
- (2) evolutionary approach
- (3) envy, altruism
- (4) strategic commitment
- (5) political science (election, each candidate tries to obtain the votes larger than the rival, not absolute volume of votes).
- (6) status

# Exercise (2)

Consider a symmetric quantity-setting duopoly.

The payoff of firm 1 is given by  $U_1 = \pi_1 - \alpha\pi_2$  and the payoff of firm 2 is given by  $U_2 = \pi_2 - \alpha\pi_1$ ,  $\alpha \in [-1, 1]$

The demand is given by  $P = A - Y$ , where  $A$  is a positive constant. The marginal cost of each firm is  $c$ , where  $c$  is a positive constant and  $A > c$ .

# Exercise (2)

- (a) Derive the reaction function of firm 1.
- (b) Derive  $|R_1'|$ . Make sure that it has downward sloping (strategic substitutes) for  $\alpha < 1$ . Make sure that the stability condition is satisfied for  $\alpha \in (-1, 1]$ .
- (c) Derive the output of firm 1 at the equilibrium.
- (d) The output of firm 1 at the equilibrium is (increasing, decreasing) in  $\alpha$ .
- (e) Derive the equilibrium price.

# Exercise (2a)

Consider a symmetric quantity-setting duopoly.

The payoff of firm 1 is given by  $U_1 = \pi_1 - \alpha\pi_2$  and the payoff of firm 2 is given by  $U_2 = \pi_2 - \alpha\pi_1$ ,  $\alpha \in [-1, 1]$

The demand is given by  $P = A - Y$ , where  $A$  is a positive constant. The marginal cost of each firm is  $c$ , where  $c$  is a positive constant and  $A > c$ .

**Question: (a) Derive the reaction function of firm 1.**

# Exercise (2b)

Consider a symmetric quantity-setting duopoly.

The payoff of firm 1 is given by  $U_1 = \pi_1 - \alpha\pi_2$  and the payoff of firm 2 is given by  $U_2 = \pi_2 - \alpha\pi_1$ ,  $\alpha \in [-1, 1]$

The demand is given by  $P = A - Y$ , where  $A$  is a positive constant. The marginal cost of each firm is  $c$ , where  $c$  is a positive constant and  $A > c$ .

Question: (b) Derive  $|R_1|$ . Make sure that it has downward sloping (strategic substitutes) for  $\alpha < 1$ . Make sure that the stability condition is satisfied for  $\alpha \in (-1, 1]$ .

# Exercise (2c)

Consider a symmetric quantity-setting duopoly.

The payoff of firm 1 is given by  $U_1 = \pi_1 - \alpha\pi_2$  and the payoff of firm 2 is given by  $U_2 = \pi_2 - \alpha\pi_1$ ,  $\alpha \in [-1, 1]$

The demand is given by  $P = A - Y$ , where  $A$  is a positive constant. The marginal cost of each firm is  $c$ , where  $c$  is a positive constant and  $A > c$ .

**Question:(c) Derive the output of firm 1 at the equilibrium.**

# Exercise (2d)

Consider a symmetric quantity-setting duopoly.

The payoff of firm 1 is given by  $U_1 = \pi_1 - \alpha\pi_2$  and the payoff of firm 2 is given by  $U_2 = \pi_2 - \alpha\pi_1$ ,  $\alpha \in [-1, 1]$

The demand is given by  $P = A - Y$ , where  $A$  is a positive constant. The marginal cost of each firm is  $c$ , where  $c$  is a positive constant and  $A > c$ .

Question:(d) The output of firm 1 at the equilibrium is (increasing, decreasing) in  $\alpha$ .



# Exercise (2e)

Consider a symmetric quantity-setting duopoly.

The payoff of firm 1 is given by  $U_1 = \pi_1 - \alpha\pi_2$  and the payoff of firm 2 is given by  $U_2 = \pi_2 - \alpha\pi_1$ ,  $\alpha \in [-1, 1]$

The demand is given by  $P = A - Y$ , where  $A$  is a positive constant. The marginal cost of each firm is  $c$ , where  $c$  is a positive constant and  $A > c$ .

**Question: (e) Derive the equilibrium price.**