Oligopoly Theory (4)
Price Competition and Endogenous Competition Structure

Aim of This Lecture

(1) To understand the difference between price and quantity competition.

(2) To understand the basic property of Bertrand Model.

(3) To understand the idea of endogenous price-quantity competition

Outline of the Fourth Lecture

4-1 Bertrand Model with Constant Marginal Costs
4-2 Rationing Rule
4-3 Bertrand Equilibrium and Perfect Competition
4-4 Quantity-Setting vs Price-Setting
4-5 Bertrand Model with Increasing Marginal Costs

Duopoly

Suppose that there are two or more firms in the market

～The price depends on both its own output and the rivals' outputs.

～The output depends on both its own price and the rivals' prices.

⇒The competition structure depends on whether firms choose their outputs or prices.

Quantity competition Model (The second lecture)

Price competition Model (The fourth lecture)

Which model should we use?(The fourth lecture)

Bertrand Duopoly

Firm 1 and firm 2 compete in a homogeneous product market.

Each firm i independently chooses its price $P_{i.}$

Each firm maximizes its own profit $\Pi_{i.}$

$\Pi_i = P_iY_i - c_iY_i$ (constant marginal cost)

Y_i: Firm i's output, c_i: Firm i's marginal cost

If firm 1 is the monopolist, its profit is $(P_1 - c_1)\,D(P_1)$. I assume that it is concave. Let P_1^M be the monopoly price.

Bertrand duopoly model (integer constraint version)

constant marginal costs, integer values

$c_1 \leqq c_2 < P_1^M$ (if $c_2 \geqq P_1^M$, firm 1 becomes the monopolist, and we need not discuss oligopoly market)

Each firm independently chooses its margin over its cost (names its price)

$P_1 \in \{c_1+\varepsilon, c_1+2\varepsilon, c_1+3\varepsilon, \ldots\}$

$P_2 \in \{c_2+\varepsilon, c_2+2\varepsilon, c_2+3\varepsilon, \ldots\}$

We do not allow non-positive margin. Naming the price lower than its cost is a weakly dominated strategy.

rationing rule

If $P_1 < P_2$, only firm 1 supplies $D(P_1)$.

If $P_1 > P_2$, only firm 2 supplies $D(P_2)$.

If $P_1 = P_2$, each firm supplies $D(P_1)/2$.

$D(P)$ is decreasing in P.

Bertrand duopoly model (integer constraint version)

constant marginal costs, integer values

$c_1 \leqq c_2 < P_1M$

Each firm independently chooses its margin over its cost (names its price)

$P_1 \in \{c_1+\varepsilon, c_1+2\varepsilon, c_1+3\varepsilon, \ldots\}$

$P_2 \in \{c_2+\varepsilon, c_2+2\varepsilon, c_2+3\varepsilon, \ldots\}$

Question: Suppose that $c_1 < c_2$. Derive the pure strategy Nash equilibrium.

Bertrand duopoly model (integer constraint version)

constant marginal costs, integer values

$c_1 \leqq c_2 < P_1^M$

Each firm independently chooses its margin over its cost (names its price)

$P_1 \in \{c_1+\varepsilon, c_1+2\varepsilon, c_1+3\varepsilon, \ldots\}$

$P_2 \in \{c_2+\varepsilon, c_2+2\varepsilon, c_2+3\varepsilon, \ldots\}$

Question: Suppose that $c_1 < c_2$. Suppose that $P_2 = c_2 + 3\varepsilon < P_1^M$. Derive the best reply of firm 1.

Bertrand duopoly model (integer constraint version)

constant marginal costs, integer values

$c_1 \leqq c_2 < P_1^M$

Each firm independently chooses its margin over its cost (names its price)

$P_1 \in \{c_1+\varepsilon, c_1+2\varepsilon, c_1+3\varepsilon, \ldots\}$

$P_2 \in \{c_2+\varepsilon, c_2+2\varepsilon, c_2+3\varepsilon, \ldots\}$

Question: Suppose that $c_1 < c_2$, $\varepsilon = 1$, and $c_2 + 2\varepsilon < P_1^M$. Suppose that $P_1 = c_2 + 2$. Derive the best reply of firm 2.

Bertrand duopoly model (integer constraint version)

constant marginal costs, integer values

$c_1 \leqq c_2 < P_1^M$

Each firm independently chooses its margin over its cost (names its price)

$P_1 \in \{c_1+\varepsilon, c_1+2\varepsilon, c_1+3\varepsilon,\ldots\}$

$P_2 \in \{c_2+\varepsilon, c_2+2\varepsilon, c_2+3\varepsilon,\ldots\}$

Question: Suppose that $c_1 < c_2$, $\varepsilon = 1$, and $c_2 + 2\varepsilon < P_1^M$.

Suppose that $P_2 = c_2 + 1$. Derive the best reply of firm 1.

Bertrand duopoly model (integer constraint version)

constant marginal costs, integer values

$c_1 \leqq c_2 < P_1^M$

Each firm independently chooses its margin over its cost (names its price)

$P_1 \in \{c_1+\varepsilon, c_1+2\varepsilon, c_1+3\varepsilon, \ldots\}$

$P_2 \in \{c_2+\varepsilon, c_2+2\varepsilon, c_2+3\varepsilon, \ldots\}$

Question Suppose that $c_1 < c_2$. Derive the pure strategy Nash equilibrium.

Properties of Bertrand Model with Cost Asymmetry

The lowest cost firm monopolizes the market.

The equilibrium price is equal to the marginal cost of the second lowest cost firm.

The equilibrium price converges to the marginal cost of the supplier when the cost difference converges to zero.

→ The competition between only two firms yields the same equilibrium price under the perfect competition. (Bertrand Paradox)

Why is $P_2 \leqq c_2$ assumed?

The strategy $P_2 \leqq c_2$ is weakly dominated by the strategy $P_2 = c_2 + \varepsilon$. Thus, it is not plausible.

But for the completeness of the analysis, I dare drop this assumption for a moment.

Non-Positive Margin

Suppose that the price -cost margin can be non-positive.

$P_1 \in \{c_1, c_1+\varepsilon, c_1-\varepsilon, c_1+2\varepsilon, c_1-2\varepsilon , c_1+3\varepsilon,\ldots\}$

$P_2 \in \{c_2, c_2+\varepsilon, c_2-\varepsilon, c_2+2\varepsilon, c_2-2\varepsilon , c_2+3\varepsilon,\ldots\}$

Question: Suppose that $c_2 = 100$, $c_1 = 90$, $\varepsilon = 1$, and the monopoly price of firm 1 is higher than 100. Describe the set of Nash equilibrium prices.

Non-Positive Margin

Suppose that the price -cost margin can be non-positive.

$P_1 \in \{c_1, c_1+\varepsilon, c_1-\varepsilon, c_1+2\varepsilon, c_1-2\varepsilon, c_1+3\varepsilon, \ldots\}$

$P_2 \in \{c_2, c_2+\varepsilon, c_2-\varepsilon, c_2+2\varepsilon, c_2-2\varepsilon, c_2+3\varepsilon, \ldots\}$

Question: Suppose that $c_2 = 100$, $c_1 = 90$, $\varepsilon = 1$, and the monopoly price of firm 1 is higher than 100. Does $(P_1, P_2) = (100, 101)$ constitutes an equilibrium?

Non-Positive Margin

Suppose that the price -cost margin can be non-positive.

$P_1 \in \{c_1, c_1+\varepsilon, c_1-\varepsilon, c_1+2\varepsilon, c_1-2\varepsilon, c_1+3\varepsilon,...\}$

$P_2 \in \{c_2, c_2+\varepsilon, c_2-\varepsilon, c_2+2\varepsilon, c_2-2\varepsilon, c_2+3\varepsilon,...\}$

Question: Suppose that $c_2 = 100$, $c_1 = 90$, $\varepsilon = 1$, and the monopoly price of firm 1 is higher than 100. Suppose that $P_2 = 100$. Derive the best reply of firm 1.

Non-Positive Margin

Suppose that the price -cost margin can be non-positive.

$P_1 \in \{c_1, c_1+\varepsilon, c_1-\varepsilon, c_1+2\varepsilon, c_1-2\varepsilon , c_1+3\varepsilon,\ldots\}$

$P_2 \in \{c_2, c_2+\varepsilon, c_2-\varepsilon, c_2+2\varepsilon, c_2-2\varepsilon , c_2+3\varepsilon,\ldots\}$

Question: Suppose that $c_2 = 100$, $c_1 = 90$, $\varepsilon = 1$, and the monopoly price of firm 1 is higher than 100. Suppose that $P_1 = 99$. Derive the best reply of firm 2.

non-positive margin: multiple equilibrium

$P_1 \in \{c_1, c_1+\varepsilon, c_1-\varepsilon, c_1+2\varepsilon, c_1-2\varepsilon , c_1+3\varepsilon,\ldots\}$
$P_2 \in \{c_2, c_2+\varepsilon, c_2-\varepsilon, c_2+2\varepsilon, c_2-2\varepsilon , c_2+3\varepsilon,\ldots\}$

$c_2 = 100$, $c_1 = 90$, $\varepsilon = 1$.

Answer: $(P_1, P_2) = (100, 101)$, $(P_1, P_2) = (99,100)$
$(P_1, P_2) = (98,99)$,$(P_1, P_2) = (97,98)$, $(P_1, P_2) = (96,97)$
$(P_1, P_2) = (95,96)$,$(P_1, P_2) = (94,95)$,$(P_1, P_2) = (93,94)$
$(P_1, P_2) = (92,93)$,$(P_1, P_2) = (91,92)$

Multiple equilibria but except for the first one is implausible because they are supported by weakly dominated strategies.

Symmetric Bertrand duopoly model (integer constraint version)

constant marginal costs, integer values

$c_1 \leqq c_2 < P_1 M$

Each firm independently chooses its margin over its cost (names its price)

$P_1 \in \{c_1+\varepsilon, c_1+2\varepsilon, c_1+3\varepsilon, \ldots\}$

$P_2 \in \{c_2+\varepsilon, c_2+2\varepsilon, c_2+3\varepsilon, \ldots\}$

Question: Suppose that $c_1 = c_2$. Derive the pure strategy Nash equilibrium.

Symmetric Bertrand duopoly model (integer constraint version)

Each firm independently chooses its margin over its cost (names its price)

$P_1 \in \{c_1+\varepsilon, c_1+2\varepsilon, c_1+3\varepsilon,\ldots\}$

$P_2 \in \{c_2+\varepsilon, c_2+2\varepsilon, c_2+3\varepsilon,\ldots\}$

Question Suppose that $c_1 = c_2$. Derive the pure strategy Nash equilibrium.

Answer: $P_1 = P_2 = c_2 + \varepsilon$

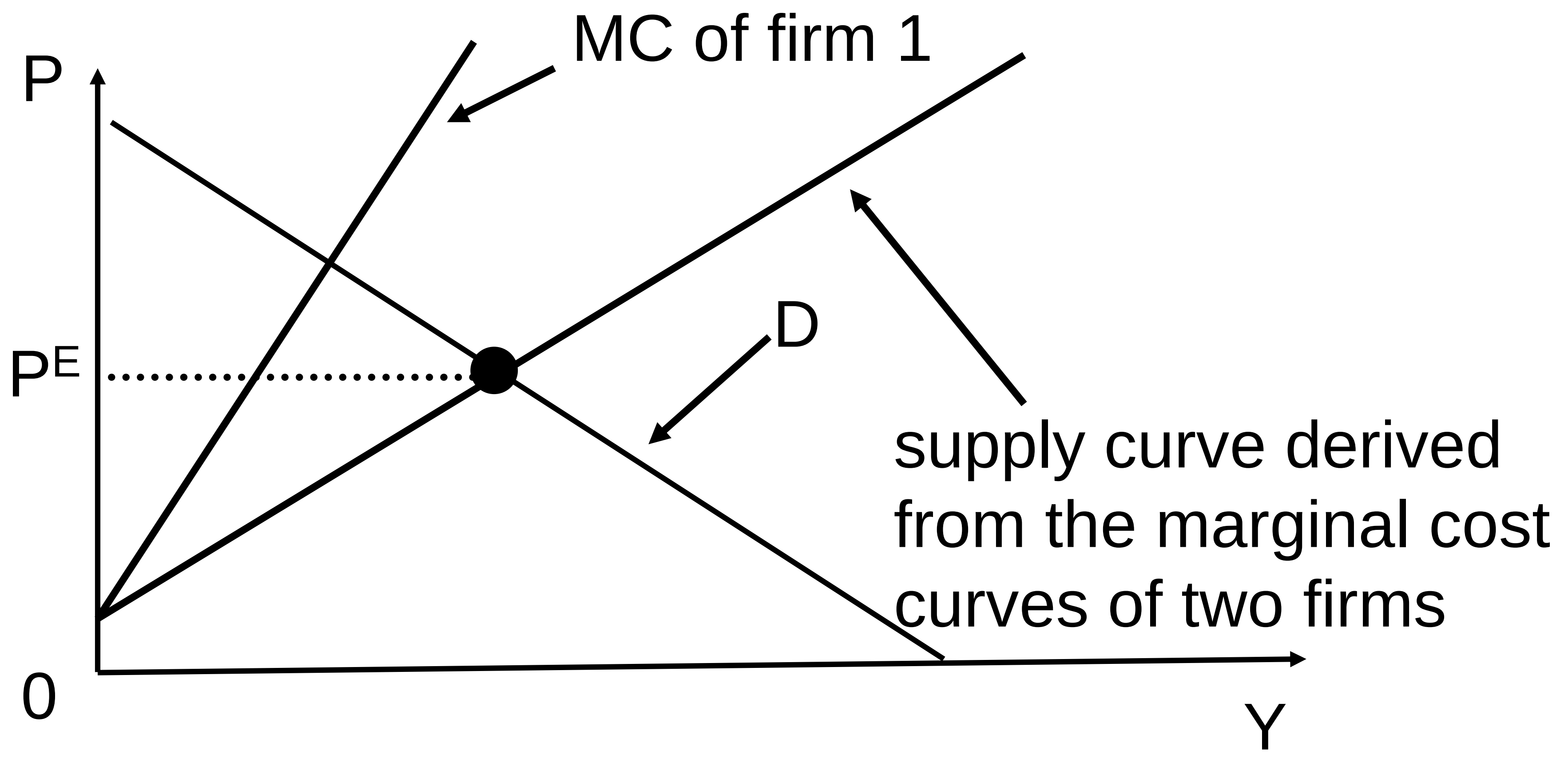
~ Bertrand Paradox

increasing marginal cost

Henceforth we assume that ε is sufficiently small and neglect it.

P = marginal cost ~($P = MC+\varepsilon$)

Bertrand Equilibrium with Increasing Marginal Costs



Bertrand Equilibrium with Increasing Marginal costs

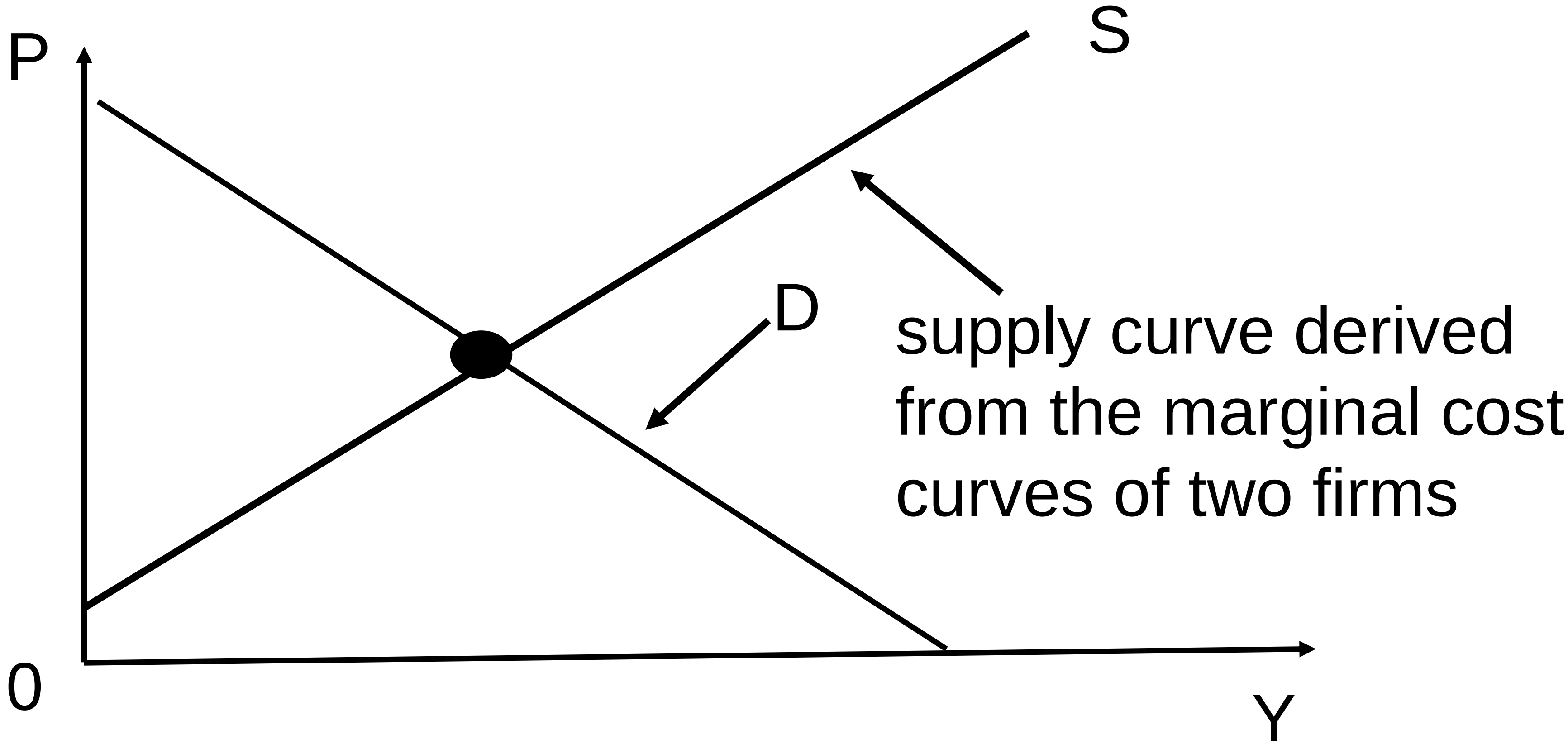
In the equilibrium both firms name $P = P^E$ and obtain the demand $D(P^E)/2$.

Suppose that firm 1 raises its price. → The profit is zero, so it has no incentive for raising its price.

Suppose that firm 1 reduces its price. →It obtains the demand $D(P_1)$. Because $P^E = c_1'(D(PE)/2)$, the profit is maximized given the price. Because c' is increasing,

$P^E D(P^E)/2 - c_1(D(P^E)/2) > P_1 D(P_1) - c_1(D(P_1))$.

Bertrand Equilibrium with Increasing Marginal costs



Continuum Equilibrium

Both higher and lower prices than the perfectly competitive price can be equilibrium prices.

Define P^H by $P^H D(P^H)/2 - c_1(D(P^H)/2) = P^H D(P^H) - c_1(D(P^H))$.

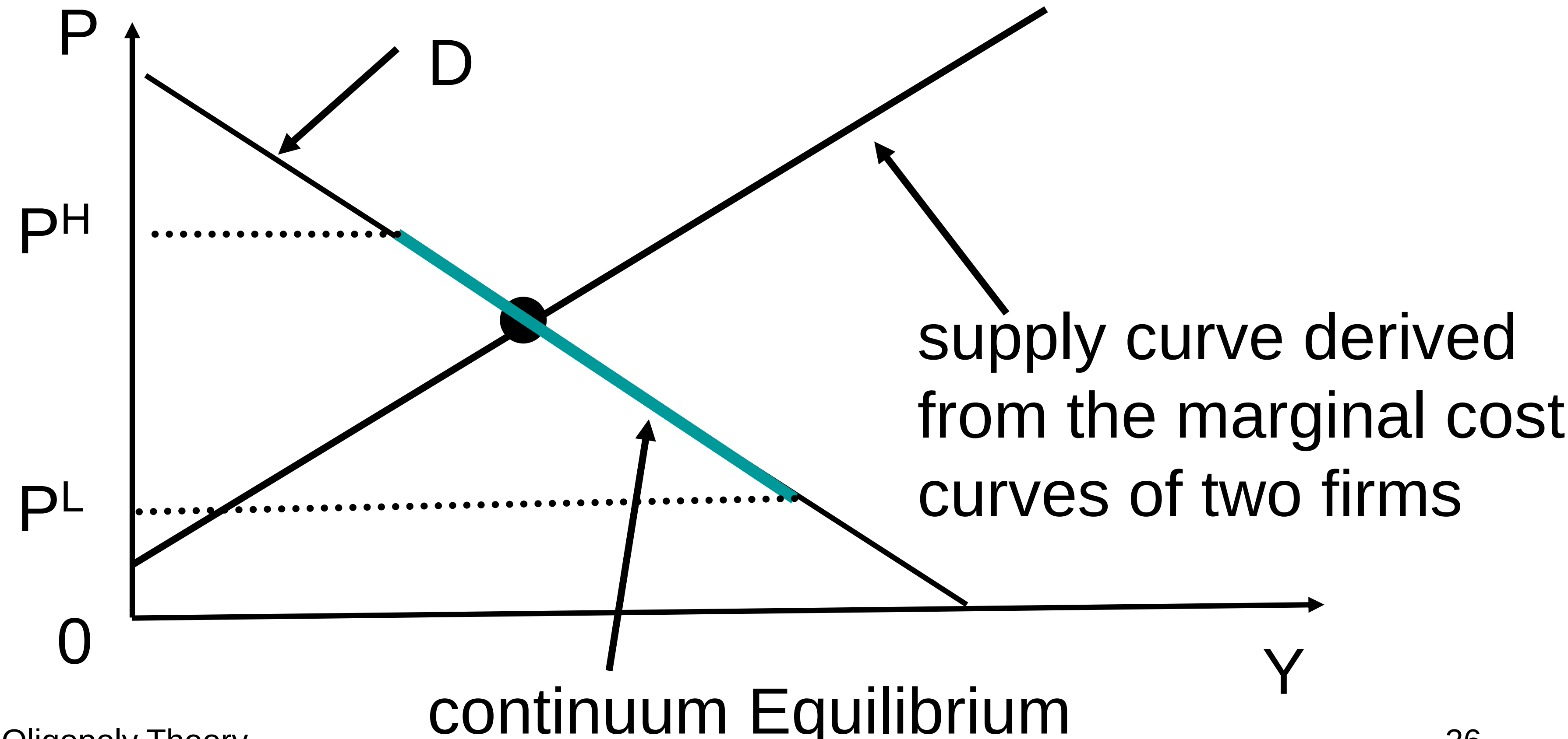
If $P_1 > P^H$, then $P_1 D(P_1)/2 - c_1(D(P_1)/2) < P_1 D(P_1) - c_1(D(P_1))$.

Define P^L by $P^L D(P^L)/2 - c_1(D(P^L)/2) = 0$.

If $P_1 > P^L$, then $P_1 D(P_1)/2 - c_1(D(P_1)/2) < 0$.

Any price $P \in (P^L, P^H)$ can be an equilibrium price.

Bertrand Equilibrium with Increasing Marginal costs



Uniqueness of Bertrand Equilibria

Hirata and Matsumura (2010)

Does this result (indeterminacy of equilibria) depend on the assumption of homogeneous product?

$p_1 = a - q_1 - bq_2$ $\quad$ $p_2 = a - q_2 - bq_1$ $\quad$ $b \in (-1,1]$

$b > 0$ supplementary products

$b = 1$ homogeneous product

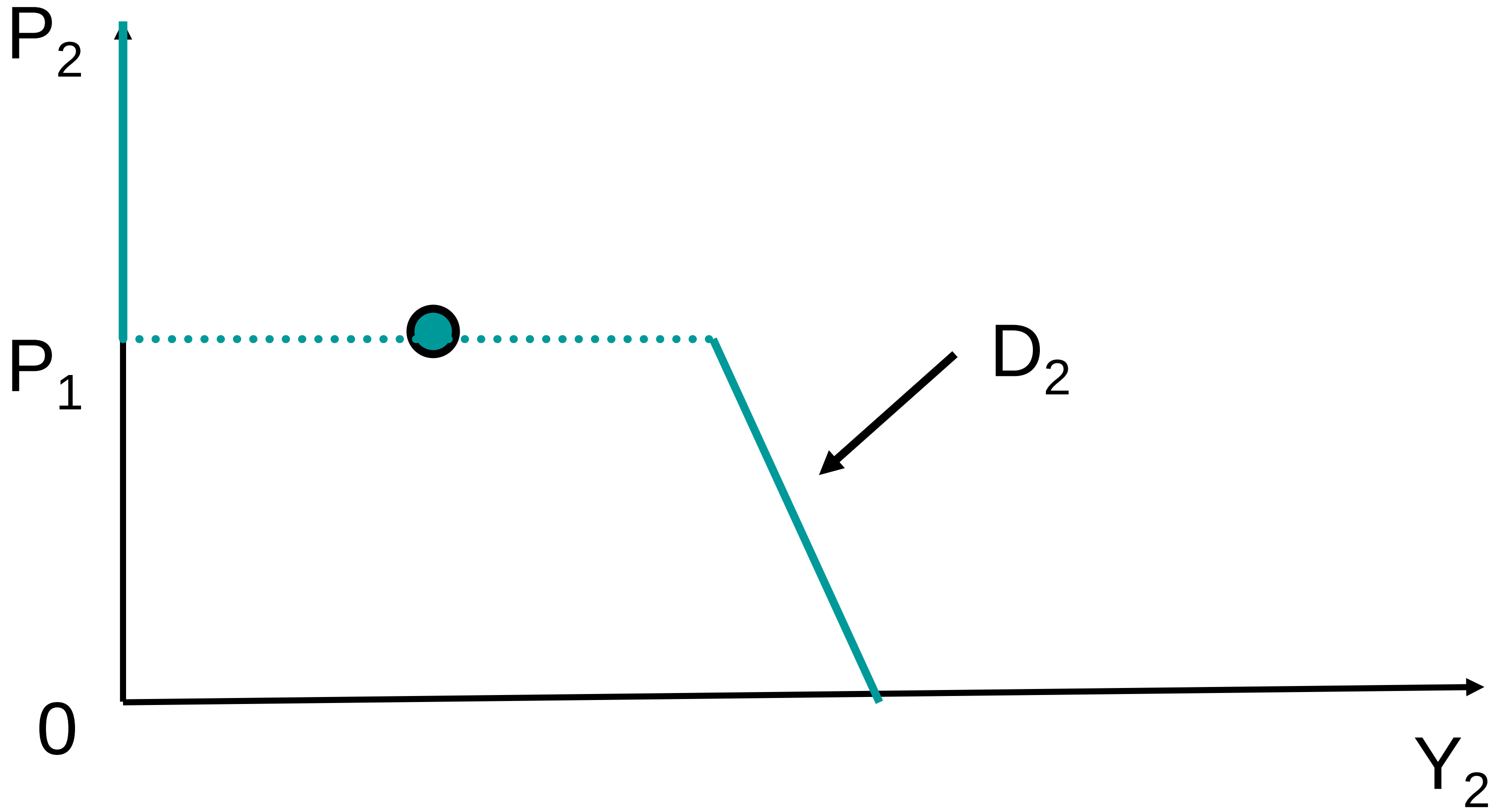
b represents the degree of product differentiation.

If $b = 1$, a continuum of equilibria exists.

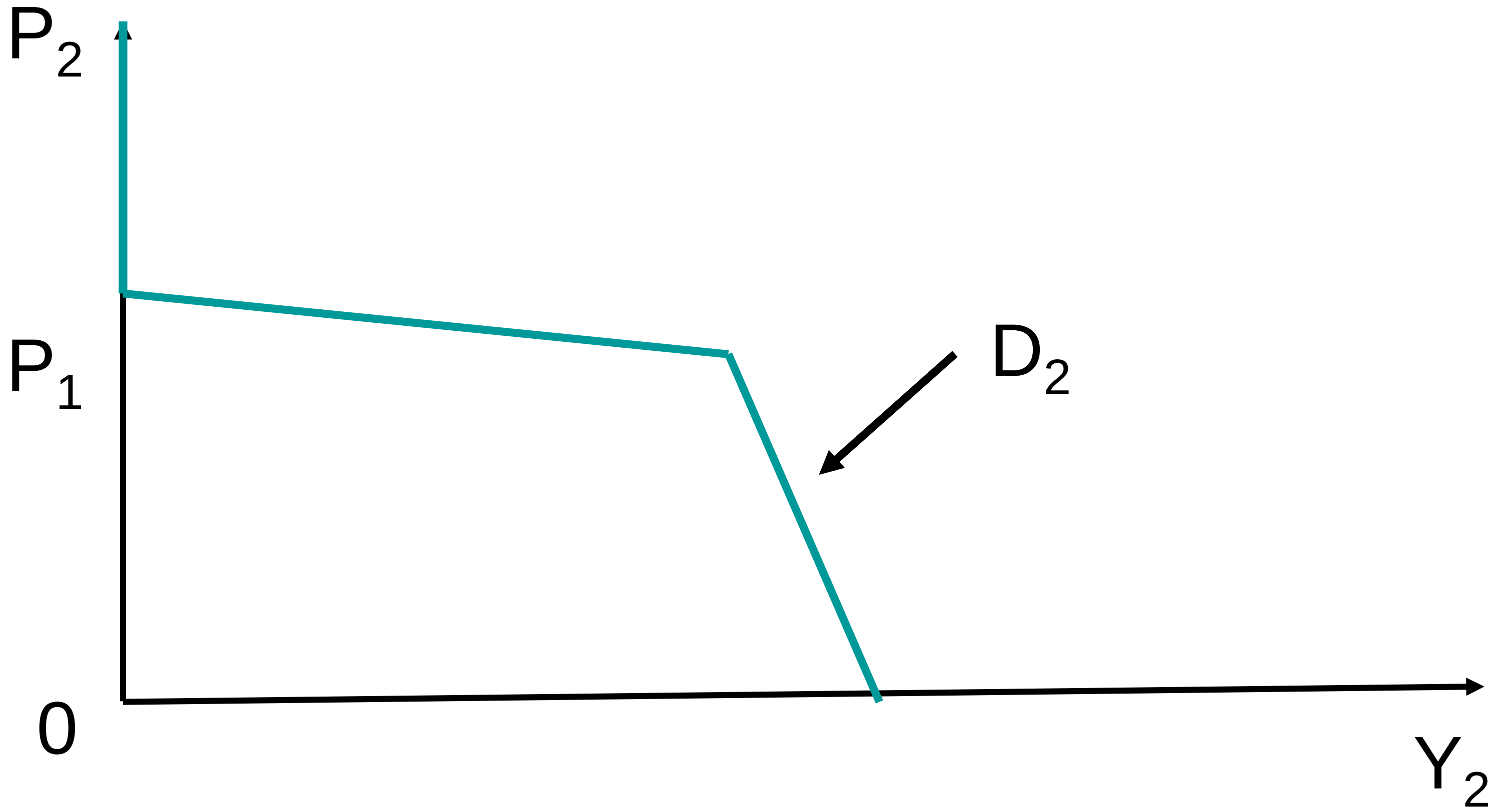
If $b \in (0,1)$, the equilibrium is unique and it converges to Walrasian as $b \to 1$.

It is also true under more general demand functions.

Homogeneous Product Market



Differentiated Product Market



supply obligation

If $P_1 < P_2$, only firm 1 supplies $D(P_1)$.

If $P_1 > P_2$, only firm 2 supplies $D(P_2)$.

If $P_1 = P_2$, each firm supplies $D(P_1)/2$.

This implies that the firms cannot choose their outputs.

The firm must meet the demand = supply obligation.

Such markets exist, (telecommunication, electric power distribution, gas distribution, water power,...)

However, it is not a plausible model formulation in many industries.

rationing rule revisited

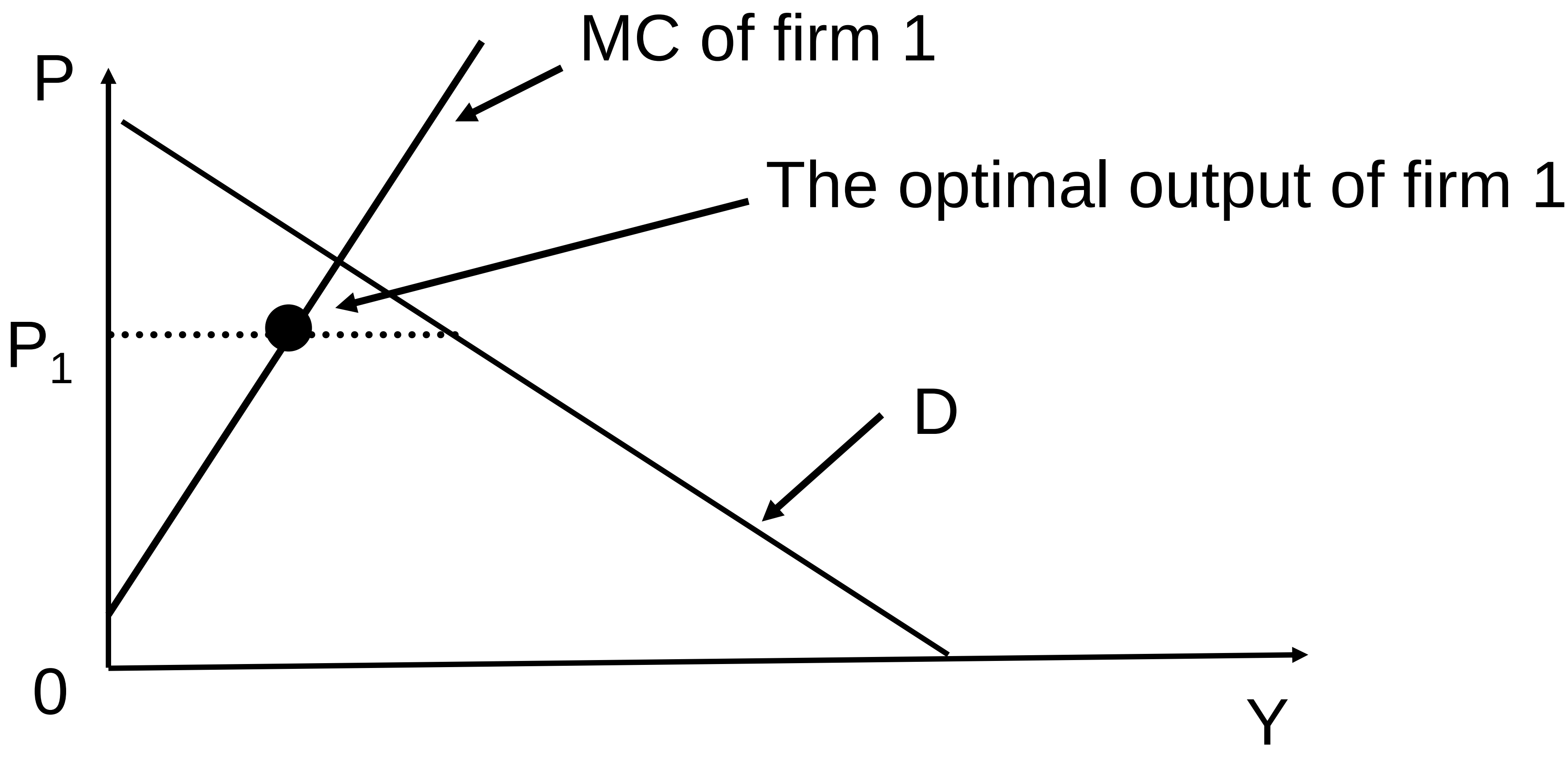
If $P_1 < P_2$, only firm 1 supplies $D(P_1)$.

If $P_1 > P_2$, only firm 2 supplies $D(P_2)$.

If $P_1 = P_2$, each firm supplies $D(P_1)/2$.

There is no problem if the marginal cost is constant because each firm has no incentive to restrict its output, but this assumption is problematic when marginal cost is increasing.

Bertrand Equilibrium with Increasing Marginal Costs



rationing rule

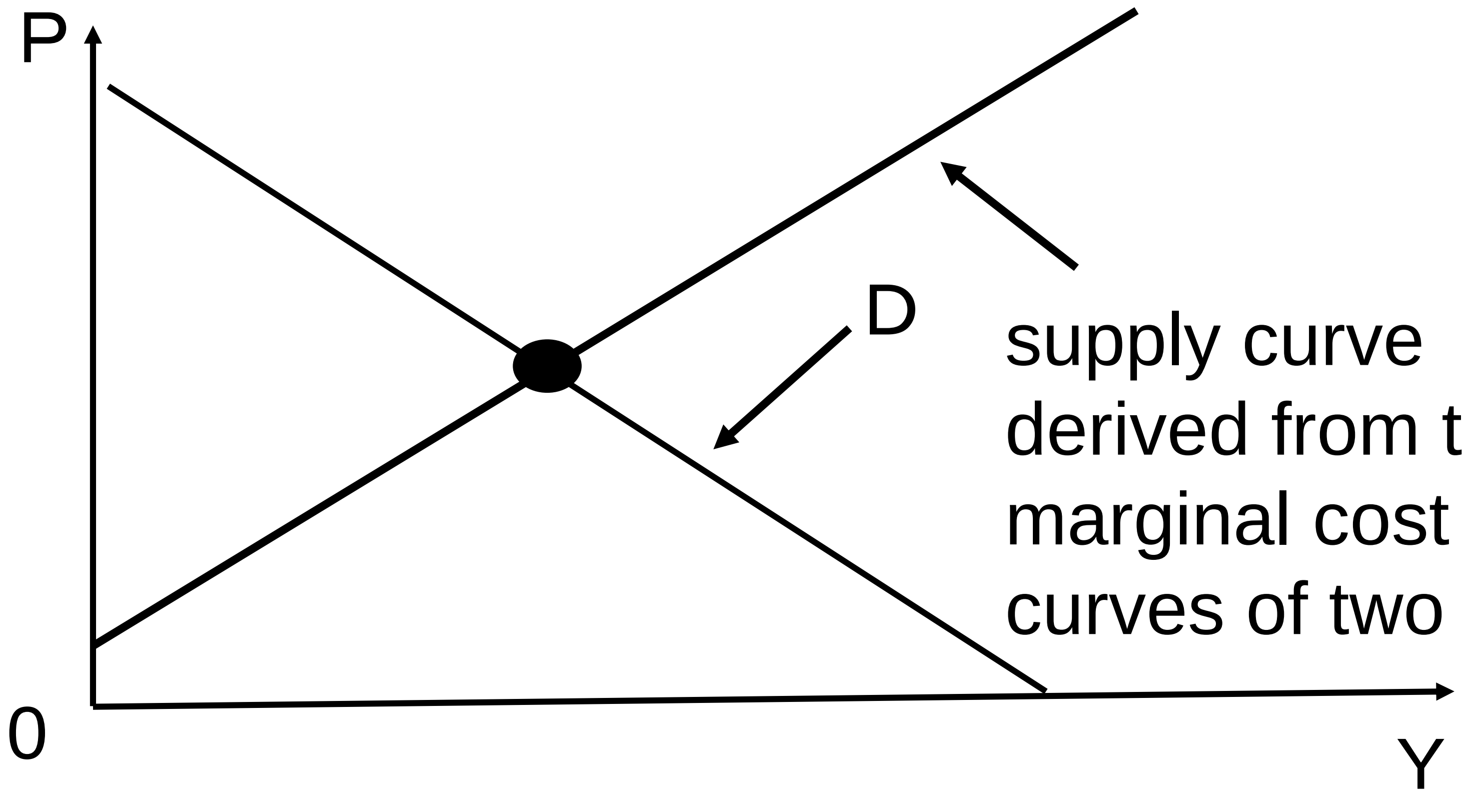
$P_1 < P_2 \rightarrow D_1 = D(P_1)$, $D_2 = \max\{D(P_2) - Y_1, 0\}$

$P_1 > P_2 \rightarrow D_2 = D(P_2)$, $D_1 = \max\{D(P_1) - Y_2, 0\}$

$P_1 = P_2 \rightarrow D_1 = D(P_1)/2 + \max\{D(P_2)/2 - Y_2, 0\}$

Suppose that firm 1 names a lower price. It can choose its output Y_1, which is not larger than $D_1 = D(P_1)$, and then firm 2 can choose its output Y_2, which is not larger than the remaining demand $D_2 = D_2 = \max\{D(P_2) - Y_1, 0\}$.

Pure Strategy Symmetric Bertrand Equilibrium



supply curve derived from the marginal cost curves of two firms

Bertrand Equilibrium with Increasing Marginal Costs

Suppose that $P_1 = P_2 = MC_1 = MC_2$ at a pure strategy equilibrium. →We derive a contradiction

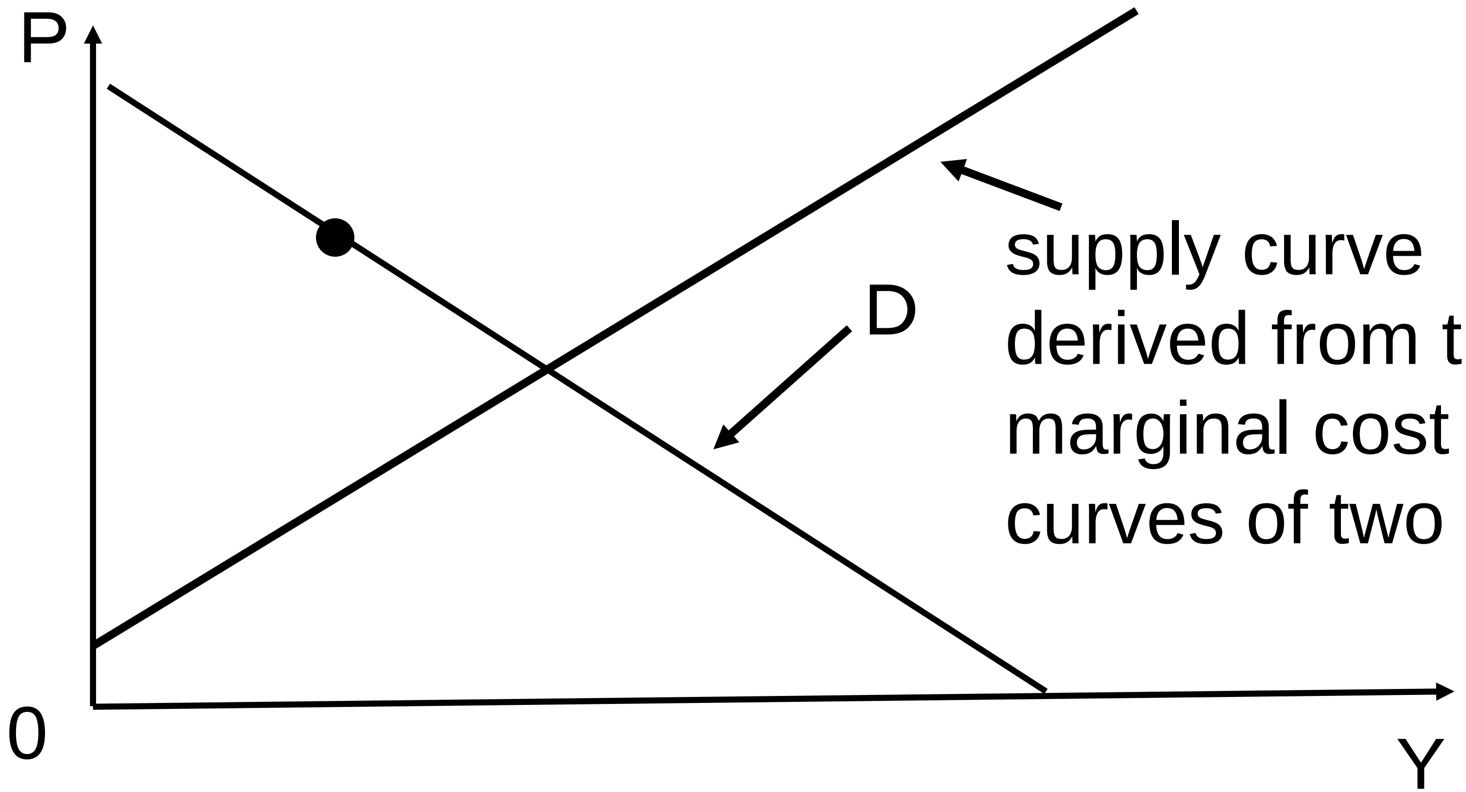
Suppose that firm 1 deviates from the strategy above and raises its price.

→Firm 2 has no incentive to increase its output because its output before the deviation of firm 1 is best given P_2.

→Given Y_2, firm 1 obtains the residual demand.

→Because $P_1 = MC_1 > MR_1$ before the deviation, a slight increase of P_1 must increase the profit of firm 1, a contradiction.

Pure Strategy Symmetric Bertrand Equilibrium



supply curve derived from the marginal cost curves of two firms

pure strategy symmetric Bertrand Equilibrium

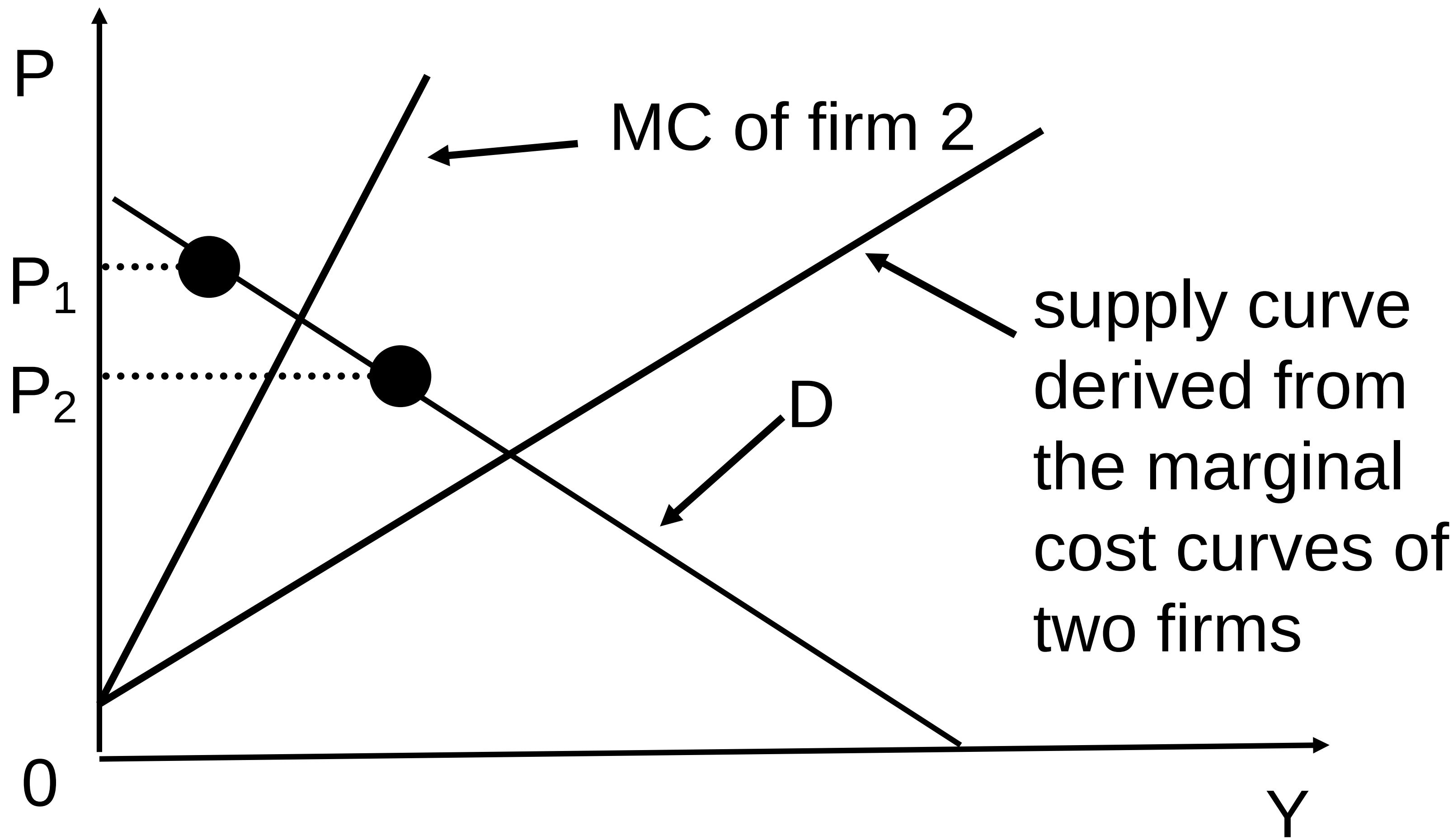
Suppose that $P_1 = P_2 > MC_1 = MC_2$ at a pure strategy equilibrium. →We derive a contradiction

Suppose that firm 1 deviates from the strategy above and reduces its price slightly.

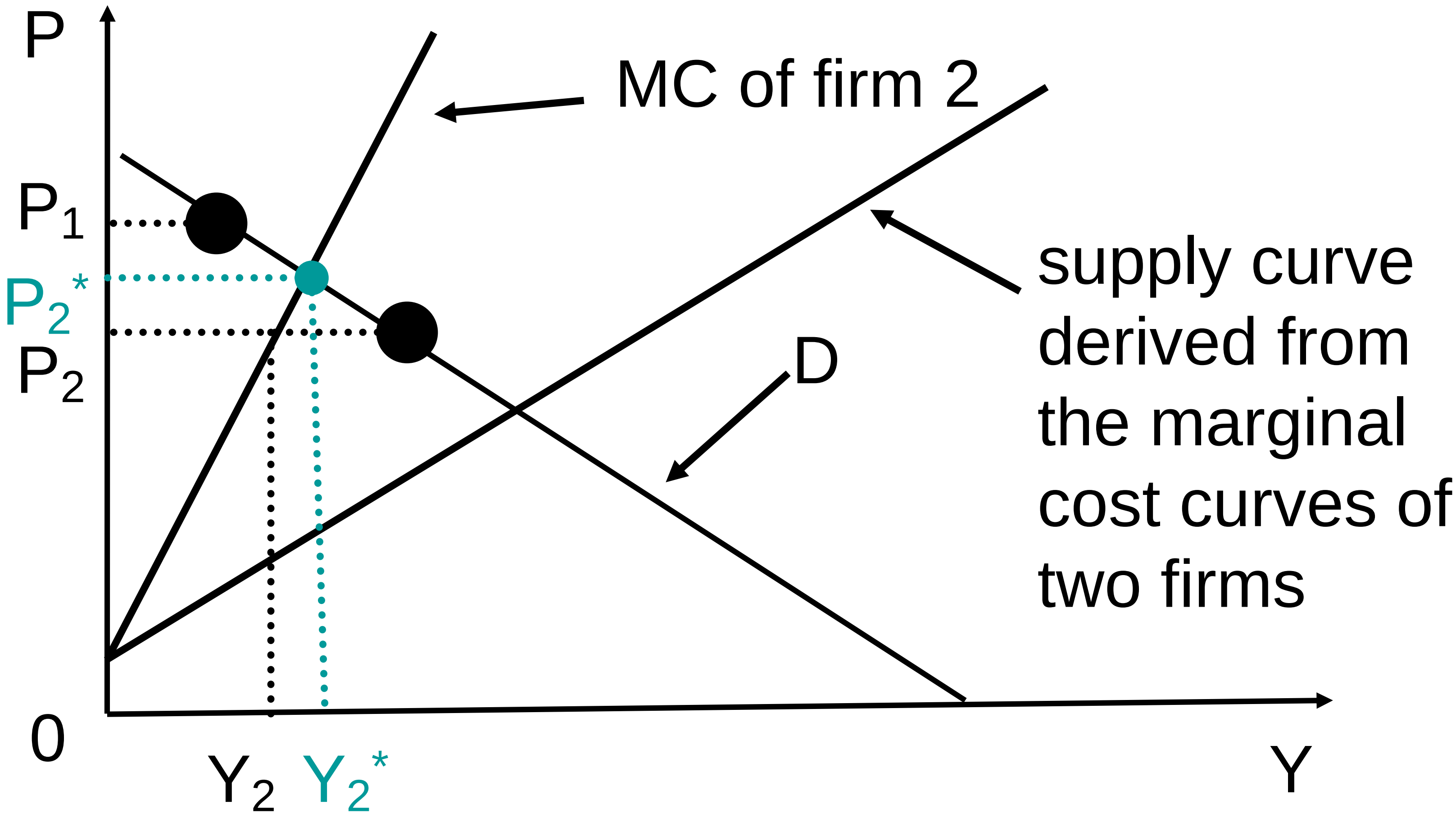
→Firm 1 can increase its demand (demand elasticity is infinite). Because $P_1 > MC_1$, the deviation increases the profit of firm 1, a contradiction.

⇒No symmetric Bertrand equilibrium exists.

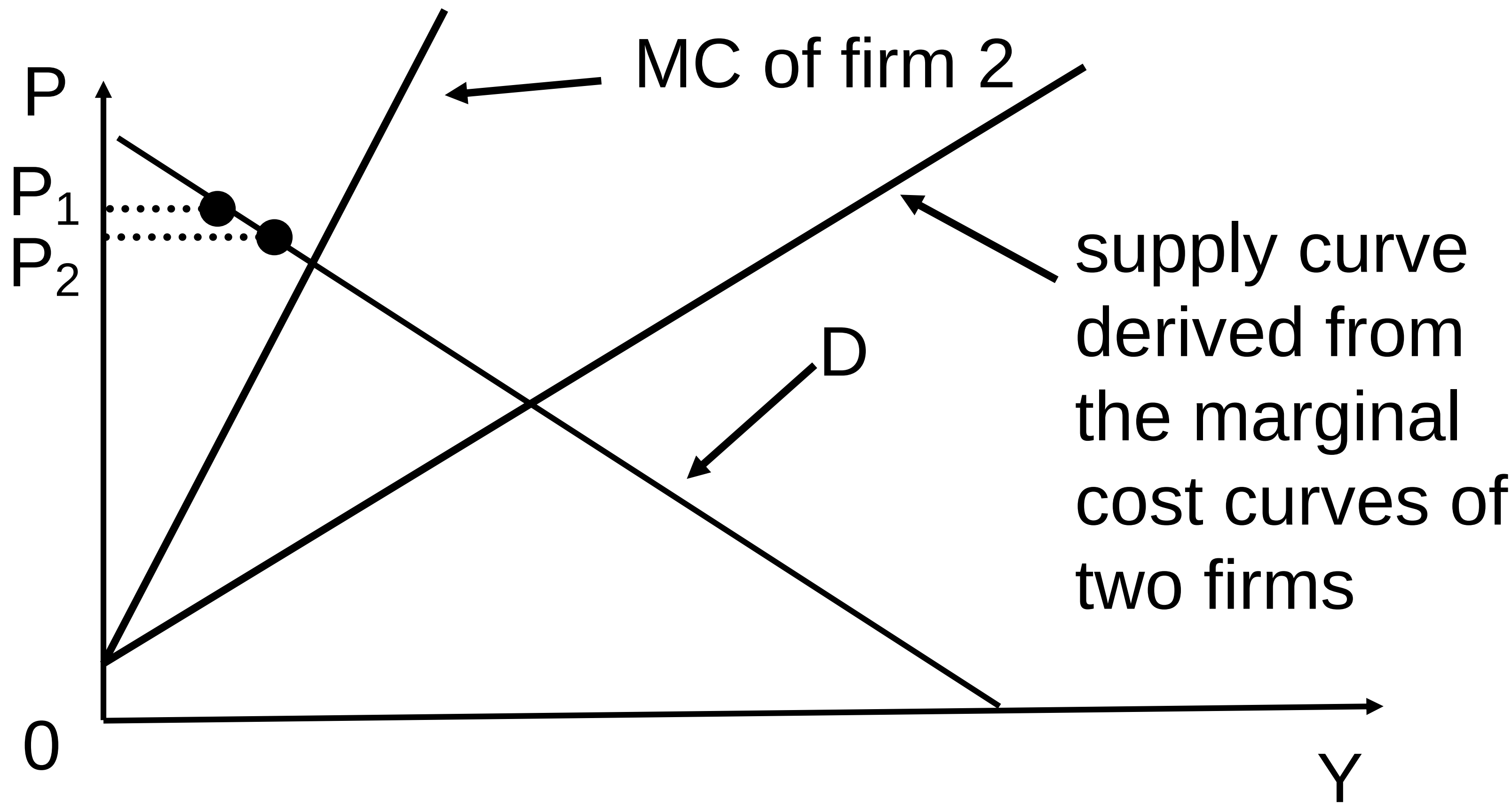
pure strategy asymmetric Bertrand Equilibrium



The deviation increases the profit of firm 2, a contradiction



pure strategy asymmetric Bertrand Equilibrium



The deviation of firm 1 increases the profit of firm 1, a contradiction

The profit of firm 1 is zero, and it has an incentive to name the price slightly lower than the rival's

⇒Neither symmetric nor asymmetric pure strategy Bertrand equilibrium exists.

Edgeworth Cycle

consider the symmetric Bertrand duopoly. consider the following capacity constraint.

Marginal cost of firm i is c if $Y_i \leqq K$ and ∞ otherwise.

If K is sufficiently large, the equilibrium outcome is same as the Bertrand model with constant marginal cost.

If K is sufficiently small, then the equilibrium price is derived from $2K = D(P)$. Firms just produce the upper limit output K.

Otherwise →No pure strategy equilibrium

(a similar problem under increasing marginal cost case appears) ~ a special case of increasing marginal cost.

Product Differentiation

It is rare that firms produce homogeneous product in oligopoly markets.

Even if firm 1 names higher price than firm 2, firm 1's demand does not become zero. This is because products are differentiated.

The degree of product differentiation is given exogenously→A model is presented today

The degree of product differentiation is endogenously determined.→I will present models when spatial competition models are discussed.

A linear demand with product differentiation

Firm 1's demand $P_1 = a - Y_1 - bY_2$

Firm 2's demand $P_2 = a - Y_2 - bY_1$

$b \in [0,1]$

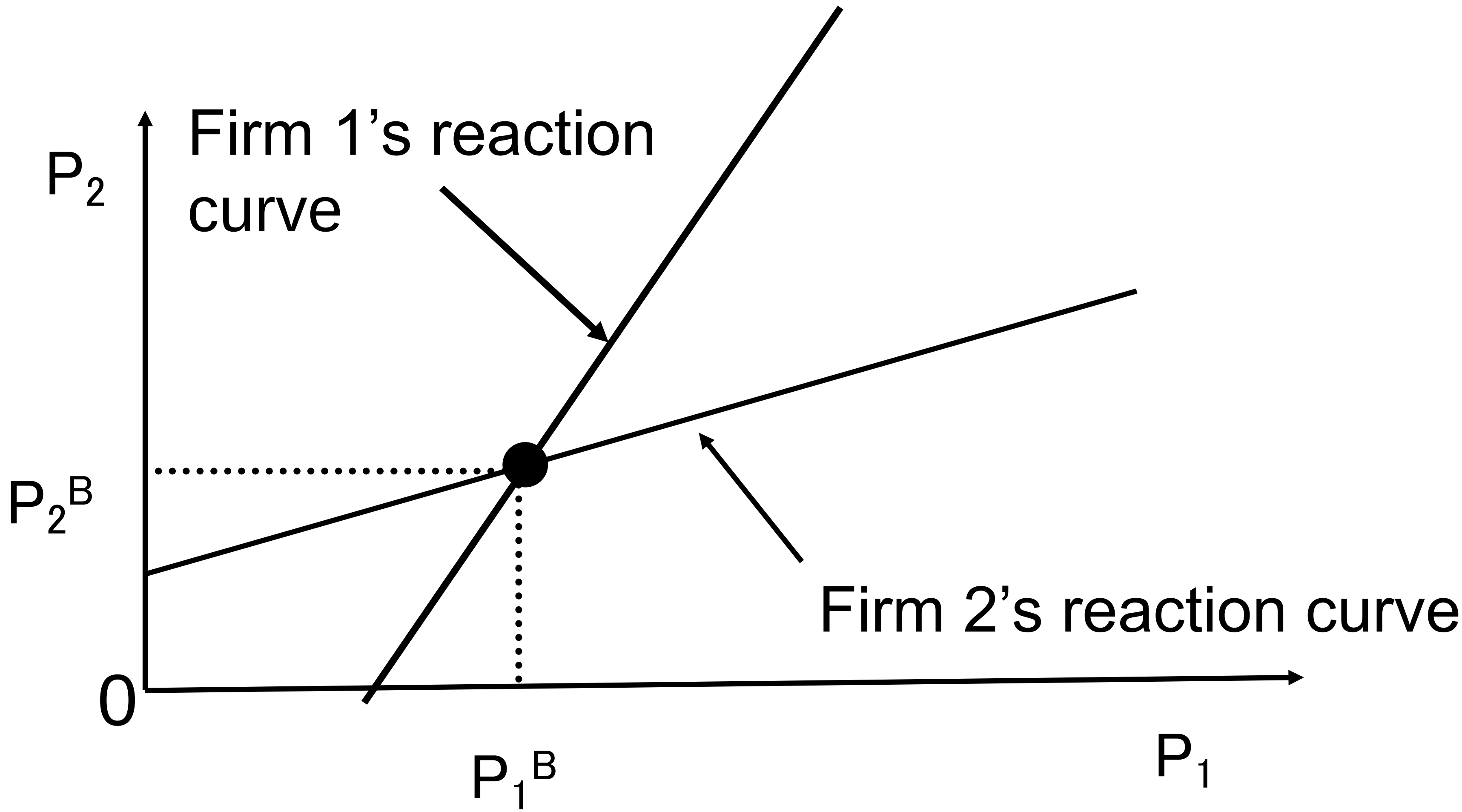
$b = 1 \rightarrow$ homogeneous product

$b = 0 \rightarrow$ no rivalry

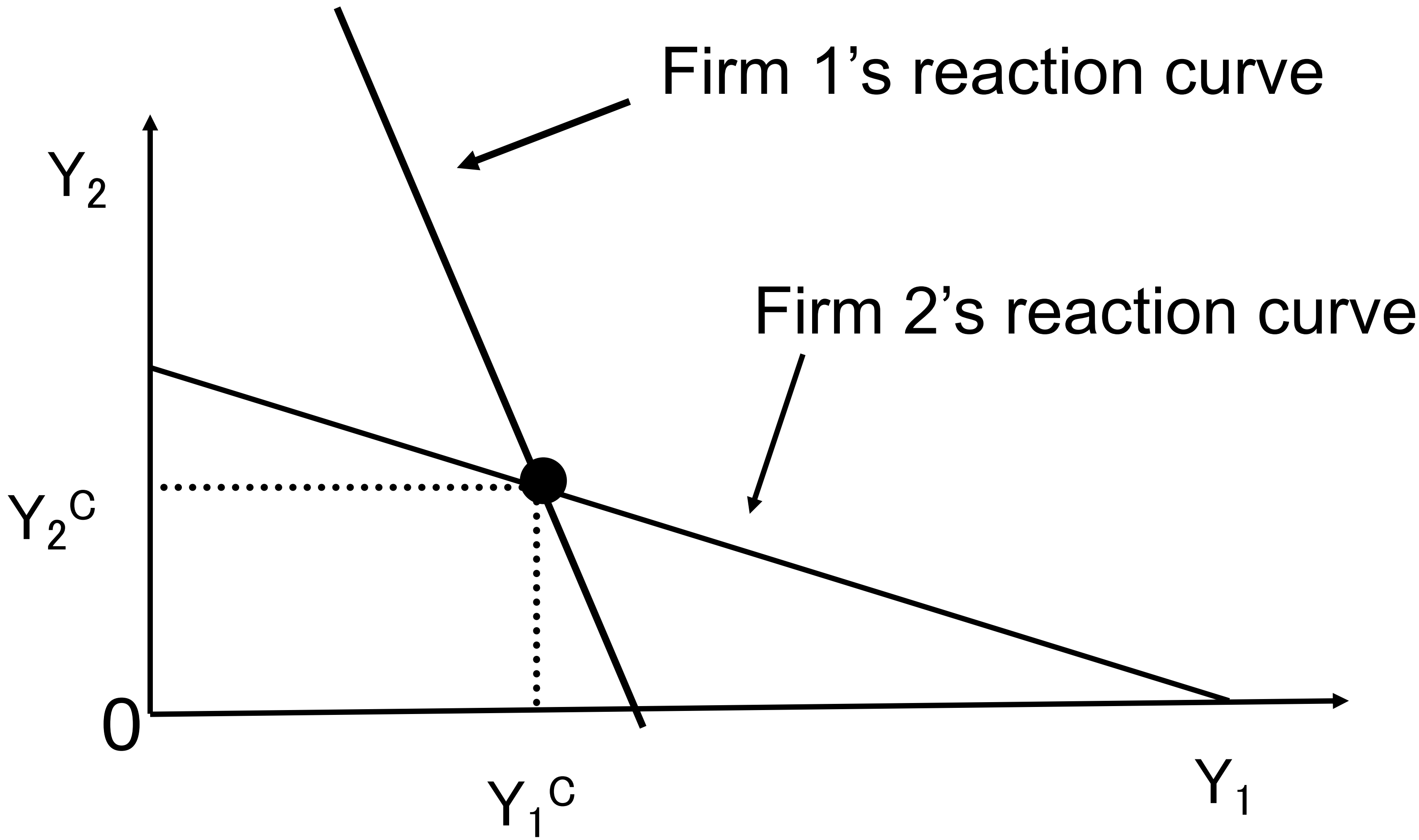
A smaller in b implies the larger degree of product differentiation.

We can use this demand system under both Cournot and Bertrand competition.

Bertrand Equilibrium



Cournot Equilibrium



Bertrand or Cournot?

Consider a symmetric duopoly in a homogeneous product market. Suppose that the marginal cost is constant and it is not too small.

In the first stage, firms choose their output independently. In the second stage, after observing the outputs, firms choose their prices independently.

→Cournot outcome ~ Kreps and Scheinkman (1983), and this result depends on the rationing rule.

See also Friedman (1988)

quantity-setting or price-setting

Cournot and Bertrand yield different results.

Which model should we use ? Which model is more realistic?

→It depends on the market structure

Quantity-setting model is more plausible when quantity change is less flexible than the price change

(e.g., it takes more time, and/or more cost, to change the quantity than the price)

Examples of inflexibility of price-setting

・mail-order retailers that send catalogues to consumers incur substantial costs when they change price.

・Regulated market such as Japanese telecommunication markets where the firms must announce the prices and cannot change them frequently.

See Eaton and Lipsey (1989), Friedman (1983,1988). and Matsushima and Matsumura (2003).

an example of inflexible quantity choice

If additional capacity investments and/or additional employment of workers are required to increase its output, it must take long time and large cost to adjust the production. →Quantity-setting model is more plausible.

I think that many of manufacturing industries, such as steel and automobile industries are good examples of this. However, if the firms have idle capacity and can increase its output very quickly, the price-setting model may be plausible.

quantity-setting or price-setting

（2）Each firm can choose whether it chooses price contract or quantity contract.

In the first stage, each firm independently choose whether it chooses price contract or quantity contract. After observing the rival's choice, each firm chooses either price or quantity, depending on its first stage choice.

This model is formulated by (Singh and Vives,1984)

quantity-setting or price-setting

Suppose that products are substitutes. In equilibrium, both firms choose quantity contracts. (Singh and Vives,1984)→choosing price contract increases the demand elasticity of the rival's and it accelerates competition.

Bertrand is reasonable only when firms cannot choose quantity contracts.