Free Entry under Common Ownership*

Susumu Sato† and Toshihiro Matsumura‡

April 30, 2020

Abstract

This study investigates the equilibrium and welfare properties of free entry under common ownership. We formulate a model in which incumbents under common ownership choose whether to enter a new market. Using a circular-market model, we find that an increase in common ownership reduces entries, which may or may not improve welfare. Welfare has an inverted-U shaped relationship with the degree of common ownership, which implies that there is a strictly positive optimal degree of common ownership. However, if firms have common ownership only after the entry, common ownership always harms welfare. We also show that our results are robust under Cournot competition.

JEL classification L13, L22

Keywords Overlapping ownership; Free entry, Insufficient entry, Excessive entry, Circular markets

---

*This work was supported by JSPS KAKENHI (18K01500,18J10212,19H01494). We are responsible for any remaining errors.

†Graduate School of Economics, The University of Tokyo, 7-3-1, Hongo, Bunkyo-ku, Tokyo 113-0033, Japan. Email:susumusato@live.jp

‡Institute of Social Science, The University of Tokyo, 7-3-1, Hongo, Bunkyo-ku, Tokyo 113-0033, Japan. Phone:(81)-3-5841-4932. Fax:(81)-3-5841-4905. Email:matsumur@iss.u-tokyo.ac.jp

Highlights

Our model examines firms with common ownership before their market entry.

Common ownership reduces entries, which may or may not improve welfare.

The optimal degree of common ownership is strictly positive but not large.

Common ownership is harmful if firms have common ownership only after their entry.

1 Introduction

The same set of institutional investors often owns many listed firms (Azar et al., 2018). Partial ownership by common owners in the same industries may internalize industry-wide externalities and improve welfare.[1] However, common ownership reduces firms' incentives to compete in product markets and may be harmful for welfare (Azar et al., 2018). Common ownership has become a central issue in recent debates on antitrust policies because the degree of common ownership has grown substantially in recent years. Some empirical studies show that common ownership has a substantial effect on the strategic behavior of firms held by institutional stockholders.[2]

In some markets, common ownership affects firms' entry decisions. For example, Newham et al. (2018) document that an increase in common ownership decreases the likelihood of the entry of generic medications in pharmaceutical markets. However, the body of theoretical literature on welfare and the policy implications of common ownership in free-entry markets is quite small.

In this study, we consider the welfare impact of common ownership in free-entry markets. To consider the welfare impacts of common ownership, we must first understand whether the presence of common ownership mitigates or exacerbates excessive entry in free-entry markets (Mankiw and Whinston, 1986). On one hand, after the entry, common ownership tends to make firms less aggressive and thus, increases firms' profits, which increases incentives for entry. Common ownership may, on the other hand, make firms internalize the business-stealing effects at the time of entry, which then reduces the incentives for entry. When the latter effect dominates the former effect, the presence of common ownership may mitigate excessive entry and thereby improve welfare.

We formulate a model in which incumbent firms under common ownership choose whether to enter a new market. Using a circular-market model developed by Vickrey (1964) and popularized by Salop (1979), we investigate how the degree of common ownership affects the equilibrium and welfare

[1] López and Vives (2019) point out that common ownership internalizes a spillover effect of R&D and may accelerate welfare-improving R&D.

[2] See Backus et al. (2019) for an example of a rise in common ownership in the US, and Schmalz (2018) for a review of empirical studies that suggest links between common ownership and firms' behavior. For antitrust concerns, see Elhauge (2016).

properties in free-entry markets. We find that an increase in common ownership always reduces entries, which may or may not improve welfare. This means that both excessive and insufficient entries can emerge, which is in sharp contrast to the excess entry result without common ownership. Moreover, we find an inverted-U shaped relationship between the degree of common ownership and welfare. This result implies that a small degree of common ownership improves welfare, whereas a large degree of common ownership reduces welfare. The optimal degree of common ownership is strictly positive but not large.

However, if there is no common ownership before the entry and firms are under common ownership only after the entry, an increase in common ownership increases entry, which is harmful for welfare, because the number of entering firms is always excessive in this case. In other words, the welfare-maximizing degree of common ownership is zero.

We adopt the standard circular-market model of Vickrey (1964) and Salop (1979) because of tractability. In the circular-market framework, industry demand is inelastic, and equilibrium outputs do not depend on common ownership. This property greatly simplifies the analysis, and we obtain clear results. However, if the demand is elastic, common ownership reduces the equilibrium outputs, which yields additional welfare loss. Therefore, our analysis may overestimate the welfare-improving effect of common ownership. To check the robustness of the results, we analyze the incentive to enter a new market with Cournot competition under elastic demand. With linear differentiated and constant-elasticicity demand functions, we find that (i) common ownership reduces the incentive to enter a new market, and (ii) when the degree of common ownership is large (small), entry incentives can be insufficient (excessive). These findings suggest that our main result—that an increase in common ownership may or may not improve welfare—is robust, because the entry incentive is reduced by common ownership and it may or may not be excessive for welfare.

The remainder of the paper is organized as follows. Section 2 formulates a free-entry model of common ownership. Section 3 presents an analysis of a circular-market model and shows our main results. Section 4 discusses the result of the model without common ownership before entry.

Section 5 and Appendix B examine Cournot competition with linear demand and constant-elasticity demand, respectively. Section 6 concludes. All proofs of propositions are relegated to Appendix A.

2 Model

There are $N \in \mathbb{N}$ established firms under common ownership that are potential entrants in a new market. Among the potential entrants, n firms enter the market and compete in prices.[3] Following the recent theoretical literature on common ownership (e.g., López and Vives, 2019), we assume that each firm i has the following post-entry objective function

$$\psi_i = \pi_i(p) + \lambda \sum_{j \neq i} \pi_j(p),$$

where

$$\pi_i(p) := d_i(p)(p_i - c) - F$$

is the product-market profit of firm i given a price profile $p := (p_j)_{j=1,\ldots,N}$, c is the constant marginal cost of production, F is the entry cost, and λ is the degree of common ownership.

Assuming a symmetric demand system and symmetric equilibrium in a product market, we obtain the equilibrium price $p^S(n, \lambda)$ and profit $\pi^S(n, \lambda)$ as functions of n and λ, where the superscript S denotes the short-run equilibrium (given the number of firms). We assume that π^S is decreasing in n.

Each firm enters the market whenever ψ_i increases as a result of its entry. Then, the number of firms in free-entry equilibrium is given by

$$\psi^E(n, \lambda) = \psi^O(n-1, \lambda), \tag{1}$$

[3]The model describes the following situation. There are incumbents under common ownership. A new market emerges in which incumbents are potential new entrants. For example, pharmaceutical companies under common ownership consider whether to enter a new immune checkpoint drug market with R&D expenditure. Another example is the Japanese gas market, which was liberalized in 2017, after which some electricity and oil companies under common ownership entered the new market.

where

$$\psi^E(n,\lambda) := \pi^S(n,\lambda) + \lambda(n-1)\pi^S(n,\lambda)$$

and

$$\psi^O(n,\lambda) := \lambda n \pi^S(n,\lambda)$$

are the value of objective functions when a firm enters the market and when it does not. Let $n^*(\lambda)$ be the solution to equation (1). By arranging equation (1), we have

$$\pi^S(n^*,\lambda) = \lambda(n^*-1)\left\{\pi^S(n^*-1,\lambda) - \pi^S(n^*,\lambda)\right\}.$$

Assuming $n^* > 1$, we obtain $\pi^S(n^*,\lambda) > 0$.

3 Circular market

In this section, we present the welfare analysis of the equilibrium number of firms using the circular-market model of Vickery (1964) and Salop (1979). Consumers are located uniformly on a circle with a perimeter equal to 1 and density is unitary around the circle. Firms are located around the circle. Consumers buy one unit of the good at the lowest cost (product price + transportation cost). Transportation cost is proportional to the distance and the unit transportation cost is $t > 0$. We assume that the willingness to pay for the product is so high that all consumers buy the products. Moreover, in this section, we restrict our attention to the case with $\lambda < 1/2$.[4]

First, we consider the price competition stage. Suppose that firm i chooses price p_i and all other firms choose p^S. Each firm has only two real competitors, that is, the two firms on either side of it.[5] A consumer located at the distance of $x \in (0, 1/n)$ from firm i is indifferent about purchasing

[4]In our circular-market model this section, the number of entering firms is at most one regardless of F if $\lambda \geq 1/2$; thus, competition among firms never emerges in equilibrium. In this case, the number of entering firms is never excessive, and an increase in λ never improves welfare. See also footnote 9.

[5]If $n = 2$, the firm competes with the same rival on each side, and the following analysis applies to this case as well. However, the following analysis does not apply when $n = 1$, because the monopolist has no competitor and obtains profit greater than $\pi^S(1)$ in (2).

from firm i or purchasing from its closest neighbor if $p_i + tx = p^S + t(1/n - x)$. Each firm i faces demand of

$$d_i(p_i, p^S) = 2x = \frac{1}{n} - \frac{p_i - p^S}{t}.$$

Firm i maximizes $(p_i - c)d_i + 2\lambda(p^S - c)(1/n - d_i/2)$. Note that firm i's pricing affects the profit of only its two neighboring firms. The first-order condition is

$$d_i - \frac{p_i - c}{t} + \lambda \frac{p^S - c}{t} = 0.$$

The second-order condition is satisfied.

Substituting $p_i = p^S$ into the above first-order condition, we obtain

$$p^S(n, \lambda) = c + \frac{t}{n(1-\lambda)}.$$

Given the number of firms n, the equilibrium profit is

$$\pi^S(n, \lambda) = (p^S(n, \lambda) - c)d_i(p^S(n, \lambda), p^S(n, \lambda)) - F = \frac{1}{n^2}\frac{t}{1-\lambda} - F. \tag{2}$$

We obtain the number of firms that maximizes welfare by minimizing the following sum of transportation and entry costs:

$$K(n) := 2n \int_0^{1/2n} tx dx + nF = \frac{t}{4n} + nF,$$

which leads to the socially optimal number of firms

$$n^O = \frac{1}{2}\sqrt{\frac{t}{F}}.$$

We assume that $\sqrt{t/F}/2 > 1$ so that $n^O > 1$.

By arranging the free-entry condition (1), we obtain the following condition:[6]

$$G(n,\lambda,t,F) := \frac{t}{1-\lambda}\frac{1}{n^2}\left\{1-\lambda\frac{2n-1}{n-1}\right\} - F = 0,$$

or equivalently,

$$H(n,\lambda,F/t) := n-1-\lambda(2n-1) - \frac{(1-\lambda)F}{t}n^2(n-1) = 0.$$

H is a cubic function of n and equation $H=0$ has at most three solutions. Figure 1 illustrates the shape of $H(n,\lambda,F/t)$.[7]

One of three possible solutions is negative, and thus, it is not equilibrium. Two are positive whenever they exist and the largest solution is the unique stable equilibrium. Positive solutions exist and the larger solution exceeds one unless F/t is too large.[8] We denote the unique stable equilibrium number of firms as $n^*(\lambda, F/t)$.[9] We denote the unique stable equilibrium number of firms as $n^*(\lambda, F/t)$.

[6] $H >\ (=,<)\ 0$ if $\psi^E(n,\lambda) >\ (=,<)\ \psi^O(n,\lambda)$.

[7] We set $\lambda = 0.3, F = 0.05$, and $t = 3$.

[8] The local maximum of H is given by the first-order condition

$$3n^2 - 2n - \frac{t}{(1-\lambda)F}(1-2\lambda) = 0.$$

Thus, the local maximum of H is attained at

$$\hat{n}(\lambda,t,F) = \frac{1+\sqrt{1+3\frac{t}{(1-\lambda)F}(1-2\lambda)}}{3}.$$

Because $\lambda < 1/2$, we find that $\hat{n}$ is decreasing in F/t, in $F/t \to 0$, and then in $\hat{n} \to \infty$. Therefore, there exists $\hat{\beta}$ such that $\hat{n} > 1$ if and only if $F/t < \hat{\beta}$.

By the envelope theorem, we have

$$\frac{dH(\hat{n}(\lambda,F/t),\lambda,F/t)}{d(F/t)} = -(1-\lambda)\hat{n}^2(\hat{n}-1) < 0,$$

as long as $\hat{n} > 1$. $F/t \to 0$, then $\hat{n} \to \infty$ and $\hat{n} \to \infty$, and then $dH/d(F/t) \to -\infty$. Therefore, together with the fact that $H(1,\lambda) < 0$, there exists $\hat{\alpha} < \hat{\beta}$ such that $\hat{n} > 1$ and $H(\hat{n},\lambda,F/t) > 0$ for $F/t < \hat{\alpha}$. Finally, if $H(\hat{n},\lambda,F/t) > 0$, there are two positive solutions to $H(n,\lambda) = 0$. In summary, n^* exists and $n^* > 1$ holds if F/t is not too large.

[9] If $H(n,\lambda,F/t)$ has no positive solution, and then, the equilibrium number of the firms is one (if the monopolist obtains positive profits) or zero (even the monopolist cannot obtain positive profits). If $\lambda \geq 1/2$, then $H(n,\lambda,F/t)$ has no positive solution, and thus, either the monopoly or no entry emerges in equilibrium.

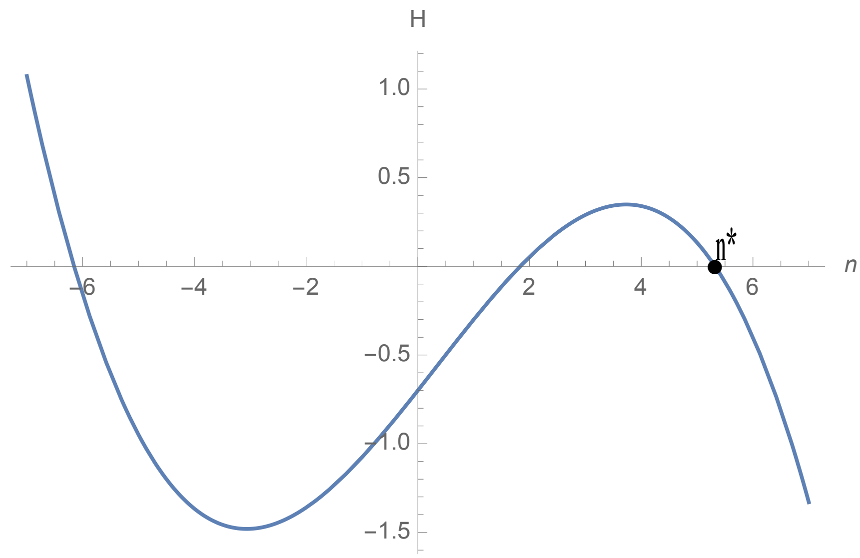

Figure 1: The stable equilibrium

Calculations show that $\partial H/\partial\lambda < 0$ and $\partial H/\partial(F/t) < 0$ at $n = n^*(\lambda, F/t)$. These results imply the following proposition.

Proposition 1 *$n^*(\lambda, F/t)$ decreases with λ and F/t.*

Proof See Appendix A.

Proposition 1 is consistent with Newham et al.'s (2018) findings that the presence of common ownership reduces the incentive for entry in pharmaceutical markets. Common ownership internalizes the business-stealing effects at the entry stage, which reduces the incentives for entry. Common ownership also mitigates competition in the product market and thus raises the profits of entering firms. Although this increases the entry incentive, it also increases the rival's profits, which reduces the entry incentive. These two effects are canceled out, and thus, the entry-enhancing effect due to lesser competition in the product market is weak. Therefore, the entry-enhancing effect of common ownership at the product market stage is dominated by the entry-restricting effect of common ownership at the entry stage.

We now discuss the welfare implications. Evaluating $G(n, \lambda, t, F)$ at $n = n^O$, we can check whether $n^* > n^O$ (excess entry) or $n^* < n^O$ (insufficient entry); the latter holds if and only if

$$\Gamma(n^O, \lambda, F) := G(n^O, \lambda, t, F) = \frac{F}{1-\lambda}\left\{3 - \lambda\frac{7n^O - 3}{n^O - 1}\right\} < 0,$$

where we use $n^O = \sqrt{t/F}/2$.

Figure 2 illustrates the range for excess and insufficient entries.[10]

Because $\partial\Gamma/\partial\lambda < 0$, we find that $\Gamma(n^O, \lambda, F) < 0$ (insufficient entry) more likely holds when λ is greater. Thus, we obtain the following proposition.

Proposition 2 *(i) The equilibrium welfare decreases with λ if and only if $n^*(\lambda, F/t) < n^O$ (insufficient entry). (ii) There exists $\bar{\lambda} \in (0, 3/7)$ such that $n^*(\lambda, F/t) < n^O$ holds (i.e., insufficient entry emerges) if and only if $\lambda > \bar{\lambda}$.*

Proof See Appendix A.

When $\lambda = 0$, the entry is excessive for welfare owing to the business-stealing effects (Vickrey, 1964; Salop, 1979; Mankiw and Whinston, 1986). Until λ hits the critical value $\bar{\lambda}$, entry is excessive. Because an increase in common ownership reduces the equilibrium entry, it improves welfare. However, once λ exceeds $\bar{\lambda}$, insufficient entry emerges because the business-stealing effects are internalized, and further increases in common ownership reduce welfare. Thus, welfare has an inverted-U shape relationship with the degree of common ownership. Moreover, our result shows that there is a strictly positive socially optimal degree of common ownership, and it is smaller than 3/7.[11]

[10] We set $t/F = 1,000$.

[11] If we deviate from the standard setting of the circular-market model, the number of entering firms can be insufficient even without common ownership. For example, Matsumura and Okamura (2006) show that the number of entering firms can be insufficient if the transportation cost function is strictly concave with respect to the distance. For discussions of excessive and insufficient entry in the circular-market model, see also Anderson et al. (1992), Matsumura (2002), and Gu and Wenzel (2009). If the case of insufficient entry without common ownership, common ownership may be less likely to improve welfare.

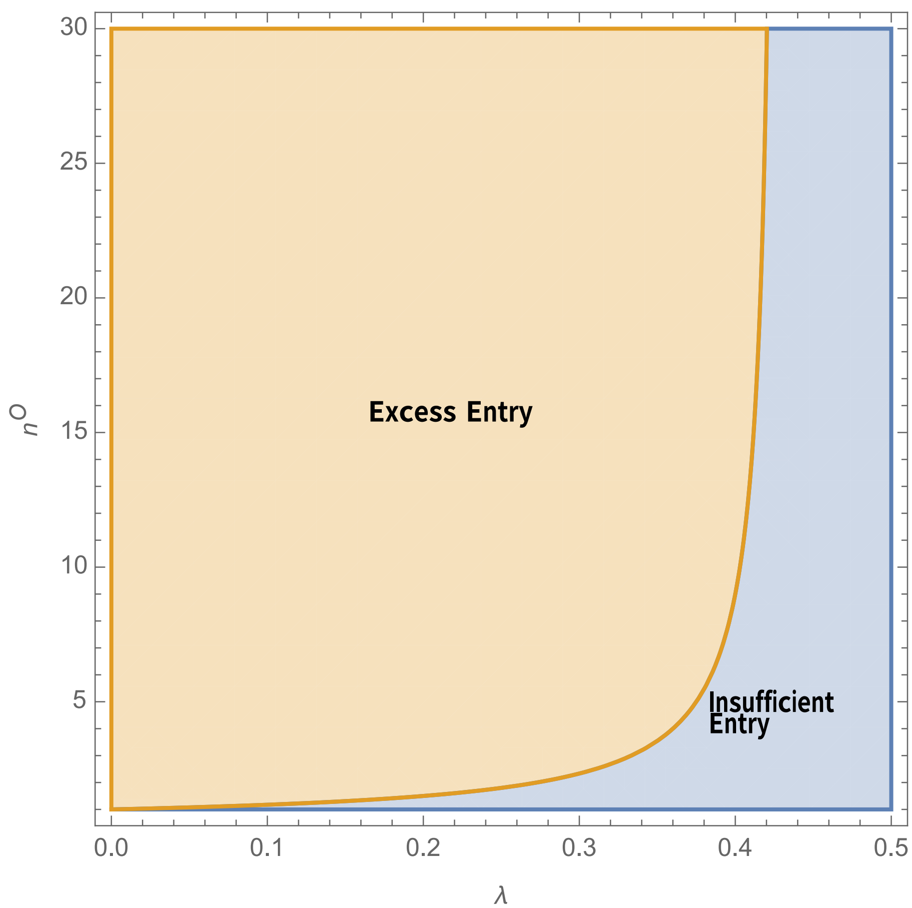

Figure 2: Range for insufficient entry

4 The case without common ownership before entry

We now discuss an alternative model in which entrants do not have common ownership before the entry but have common ownership with entering firms after the entry. In this case, $\psi^O(n,\lambda)=0$, and thus, the number of firms in free-entry equilibrium is given by

$$\pi^S(n,\lambda)=0. \tag{3}$$

Let n^{**} be the solution to equation (3). Because $\pi^S(n^*,\lambda)>0$ and $\pi^S(n^{**},\lambda)=0$, we obtain $n^*<n^{**}$.

Using the circular-market model, we obtain the following equilibrium number of firms n^{**}:

$$n^{**}(\lambda,F/t)=\sqrt{\frac{t}{(1-\lambda)F}}>\frac{1}{2}\sqrt{\frac{t}{F}}=n^O.$$

Thus, for any $\lambda\in[0,1)$, $n^{**}(\lambda,F/t)$ is excessive for welfare. Because $n^{**}(\lambda,F/t)$ is increasing in λ, common ownership exacerbates excessive entry. Therefore, an increase in λ is always harmful for welfare.

Figure 3 illustrates the relationship among n^O, n^*, and n^{**}.[12] In general, $n^*=n^{**}$ if $\lambda=0$, and $n^*<n^{**}$ otherwise. In the circular-market model, $n^{**}>n^O$ for any λ and $n^*<n^O$ ($n^*>n^O$) when λ is large (small).

5 Monopoly or duopoly

In our main setting of the circular market, demand is assumed to be inelastic, but this may be a restrictive assumption. The decrease in competition due to common ownership reduces post-entry outputs and thus, raises prices. If the demand is elastic, this yields additional welfare loss. However, the analysis of free entry under common ownership with general elastic demand functions turns out

[12]in the numerical example, $t/F=2,000$.

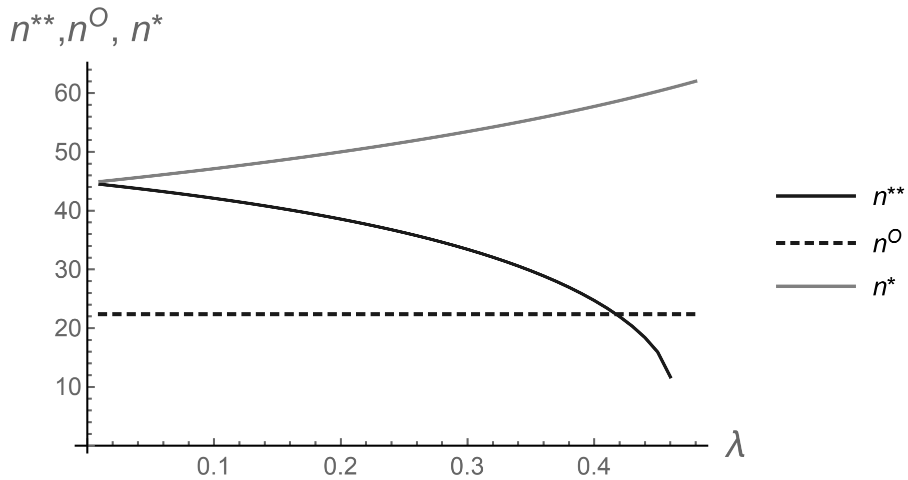

Figure 3: Comparison of n^O, n^*, and n^{**}

to be intractable.

Furthermore, in the previous sections, we do not consider an integer constraint for the number of entering firms. The number of entering firms may be insufficient even without common ownership under the integer constraint (Matsumura, 2000), which may affect our results on welfare implications of common ownership. However, the general analysis under the integer constraint is also intractable.

Therefore, we present an alternative analysis. Instead of a general discussion, we investigate an entry incentive in a duopoly model in differentiated product markets under quantity competition.[13]

One firm (firm 1) has already entered the new market and another firm (firm 2) under common ownership chooses whether to enter this new market. We adopt a standard duopoly model in differentiated product markets with a linear demand system (Singh and Vives, 1984). The quasi-

[13]Gu and Wenzel (2009, 2012) and Gu et al. (2016) investigate a circular-market model with an elastic demand function. However, introducing common ownership into their models makes the analysis intractable and we fail to derive clear-cut results.

linear utility function of the representative consumer is

$$U(q_0, q_1, y) = \alpha(q_0 + q_1) - \frac{1}{2}(q_1^2 + 2\delta q_1 q_2 + q_2^2) + y, \tag{4}$$

where q_i $(i = 1, 2)$ is the consumption of the good produced by firm i, y is the consumption of an outside good that is competitively provided (with a unitary price), α $(> c)$ is a positive constant, and $\delta \in (0, 1)$ represents the degree of product differentiation: a smaller δ indicates a larger degree of product differentiation. For a large enough income, the above utility function yields the following demand functions (Vives, 1999)

$$p_i = \alpha - q_i - \delta q_j \ (i, j = 1, 2, \ i \neq j).$$

The post-entry gross profit and welfare functions are given by

$$\begin{aligned} \pi_i(q_i, q_j) &= (\alpha - q_i - \delta q_j - c)q_i, && (5) \\ W(q_1, q_2) &= \alpha(q_0 + q_1) - \frac{1}{2}(q_1^2 + 2\delta q_1 q_2 + q_2^2) - c(q_1 + q_2). && (6) \end{aligned}$$

We compare the monopoly and duopoly equilibria and investigate firm 2's incentive to enter the market and the welfare gain from the entry.

We first analyze the monopoly subgame in which $q_2 = 0$. In this case, the incumbent maximizes $\pi_1(q_1, 0)$ with respect to q_1, which leads to the monopoly output $q^M = (\alpha - c)/2$. The profit and welfare under monopoly are given by

$$\pi^M = \frac{(\alpha - c)^2}{4},$$
$$W^M = \frac{3(\alpha - c)^2}{8}.$$

Next, we consider the duopoly subgame. Each firm i maximizes $\psi_i = \pi_i + \lambda \pi_j$ $(i, j = 1, 2, \ i \neq j)$, which is characterized by the first-order condition

$$\alpha - q_i - \delta q_j - c - q_i - \lambda\delta q_j = 0.$$

In the equilibrium, we obtain the duopoly output of each firm

$$q^D = \frac{\alpha - c}{2 + (1+\lambda)\delta}.$$

Substituting $q_1 = q_2 = q^D$ into (5) and (6), we obtain the profit and welfare under duopoly:

$$\pi^D(\delta, \lambda) = \frac{(1+\lambda\delta)(\alpha - c)^2}{\{2 + (1+\lambda)\delta\}^2},$$
$$W^D(\delta, \lambda) = \frac{\{3 + (1+2\lambda)\delta\}(\alpha - c)^2}{\{2 + (1+\lambda)\delta\}^2}.$$

Finally, we compare the incentive to enter the market and the welfare gain from the entry. Suppose that the entry incurs a fixed cost $F > 0$. Then, firm 2 is willing to enter the market if and only if

$$\Delta\pi(\delta, \lambda) := \pi^D - \lambda(\pi^M - \pi^D) \geq F.$$

The entry improves welfare if and only if

$$\Delta W(\delta, \lambda) := W^D - W^M \geq F.$$

If $\Delta W > \Delta\pi$, the private incentive for entering the market is too small (insufficient entry). Conversely, if $\Delta W < \Delta\pi$, it is too large (excess entry).

The following proposition suggests that an increase in common ownership reduces the entry incentive and increases the range for the insufficient entry.

Proposition 3 *(i) Both ΔW and $\Delta\pi$ decreases with λ. (ii) For any $\lambda \in [0, 1]$, there exists $\hat{\delta}(\lambda)$ such that $\Delta W > \Delta\pi$ (insufficient entry) if and only if $\delta < \hat{\delta}(\lambda)$. (iii) $\hat{\delta}(0) = 2/3$, $\hat{\delta}(1) = 1$, and*

$\hat{\delta}(\lambda) \in (\hat{\delta}(0), \hat{(}1))$ *for all* $\lambda \in (0,1)$.

Proof See Appendix A.

According to Proposition 3(i), an increase in common ownership discourages firm 2's entry, which is consistent with our main results presented in Section 3. According to Proposition 3(ii, iii), insufficient entry emerges unless the degree of product differentiation is small. In the insufficient entry case, an increase in common ownership has two welfare-reducing effects. The first is to raise the price–cost margin in the duopoly market (when $\Delta\pi \geq F$). The second is to change the competition structure from duopoly to monopoly. When the entry incentive is insufficient (i.e., $\delta < \hat{\delta}$), this change harms welfare. Therefore, an increase in common ownership is harmful for welfare. However, when the degree of product differentiation is small (i.e., δ is close to 1), the entry incentive can be excessive, and an increase in common ownership that changes the competition structure (change from duopoly to monopoly) may improve welfare, because common ownership reduces the entry incentive. This result is consistent with our main results.

Figure 4 suggests that $\hat{\delta}(\lambda)$ is increasing in λ. This implies that the greater the common ownership, the more likely insufficient entry is (and thus, the more likely is the welfare-reducing effect of common ownership).

The results that common ownership reduces entry, the entry incentive may be excessive (insufficient) when λ is small (large), and an increase in λ may improve (reduce) welfare when λ is small (large) are not specific to our linear demand formulation. In Appendix B, we analyze a constant-elasticity demand system and show that the entry incentive decreases with common ownership. Moreover, we show that the entry incentive is excessive when $\lambda = 0$ and can be insufficient when λ is close to one as long as the demand elasticity is large. These results imply that an increase in λ improves (may reduce) welfare when λ is small (large). In this sense, our main results are not specific to our linear demand formulation.

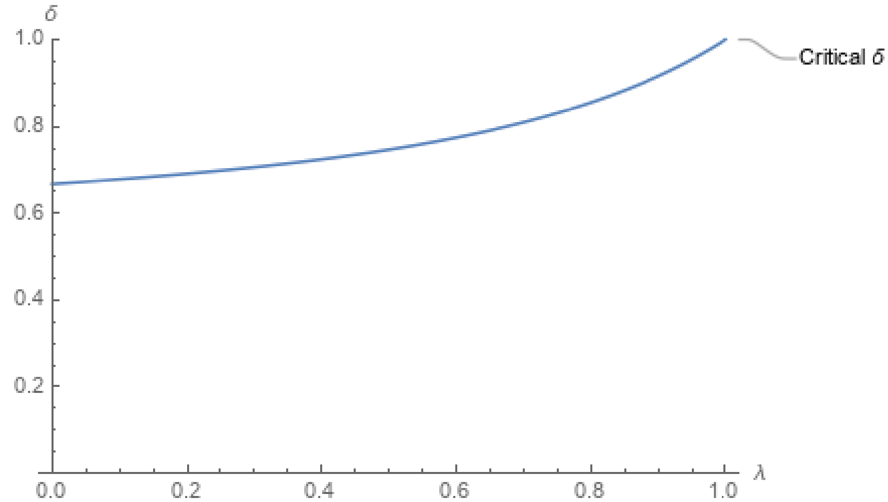

Figure 4: The relationship between $\hat{\delta}$ and λ

6 Concluding remarks

In this study, we investigate the effect of common ownership on the entry incentive and welfare consequences. First, using a circular-market model, we show that whether or not firms have common ownership before entry is crucial. If firms have common ownership before entry, common ownership reduces the number of firms. This may or may not improve welfare, because the equilibrium number of firms may or may not be insufficient. Meanwhile, if firms do not have common ownership before entry, common ownership increases the number of firms, which is harmful for welfare. This is because the number of entering firms is always excessive in this case.

Second, comparing monopoly and duopoly equilibrium in differentiated product markets under Cournot competition, we find that common ownership before entry reduces the entry incentive for a potential entrant, and the entry incentive is insufficient for welfare when the degree of common ownership or the degree of product differentiation is large. In this case, an increase in common ownership harms welfare. However, when the degree of common ownership and the degree of product differentiation are small, entry incentives can be excessive, and an increase in common ownership may improve welfare.

In this study, we consider a situation in which common owners, such as financial institutions, own the firms symmetrically. However, financial institutions, such as active funds, may invest in firms asymmetrically. Extending our analysis to this case remains for future research.

Appendix A: Proofs of Propositions

Proof of Proposition 1

The implicit function theorem implies that

$$\frac{dn^*}{d\lambda} = -\frac{\partial G/\partial\lambda}{\partial G/\partial n}\bigg|_{n=n^*(\lambda)}, \quad \text{and } \frac{dn^*}{d(F/t)} = -\frac{\partial G/\partial(F/t)}{\partial G/\partial n}\bigg|_{n=n^*(\lambda)}.$$

We obtain

$$\frac{\partial G}{\partial\lambda} = -\frac{t}{(1-\lambda)^2 n(n-1)} < 0$$

and $\text{sign}(\partial G/\partial(F/t)) = \text{sign}(\partial H/\partial(F/t)) < 0$ at $n = n^*$. Because n^* is the largest solution to a cubic equation with negative coefficient on n^3, $\text{sign}(\partial G/\partial n) = \text{sign}(\partial H/\partial n) < 0$ at $n = n^*$. These imply Proposition 1. Q.E.D.

Proof of Proposition 2

Proposition 2-(i) simply holds owning to the concavity of welfare function. $\Gamma(n^O, \lambda, F) < 0$ if and only if

$$\lambda > \frac{3(n^O-1)}{7n^O-3} =: \bar{\lambda}.$$

Let $\gamma(n^O) := 3(n^O-1)/(7n^O-3)$. Because $n^O = \sqrt{t/F}/2 > 1$, $\gamma(1) = 0$, $\gamma'(n^O) > 0$, and $\lim_{n^O\to\infty}\gamma(n^O) = 3/7$, we obtain $\bar{\lambda} = \gamma(n^O) \in (0, 3/7)$ for any $n^O > 1$. Q.E.D.

Proof of Proposition 3

First, we show Proposition 3(i). Because $W^D(\delta,\lambda)$ decreases with λ and W^M is independent of λ, $\Delta W(\delta,\lambda)$ decreases with λ. We then show that $\Delta\pi(\delta,\lambda)$ decreases with λ. To this end, note that

$$\Delta\pi(\delta,\lambda) = \frac{(\alpha-c)^2}{\{2+(1+\lambda)\delta\}^2}(1+\lambda)(1+\lambda\beta) - \frac{\lambda}{4}(\alpha-c)^2.$$

We obtain

$$\frac{\partial\Delta\pi}{\partial\lambda} = \frac{\xi(\lambda)}{\{2+(1+\lambda)\delta\}^3}, \tag{7}$$

where

$$\xi(\lambda) := \{2 + (1+\lambda)\delta\}(1 + \beta + \lambda\delta) - 2\delta(1+\lambda)(1+\lambda\delta) - \frac{\{2 + (1+\lambda)\delta\}^2}{4}.$$

Because the denominator of (7) is positive, we show that the numerator of (7) is negative.

Because $\xi'''(\lambda) = -(3/2)\beta^3 < 0$, we find that ξ'' is decreasing. Because $\xi''(0) = -\beta^2 - (3/2)\beta^3 < 0$, we find that $\xi'' < 0$, so ξ' is decreasing. Because $\xi'(0) = -2\beta^2 - (3/4)\beta^3 < 0$, we find that $\xi' < 0$, and thus, ξ is decreasing. Because $\xi(0) = -2\beta - \beta^2/2 - \beta^3/4 < 0$, we find that $\xi(\lambda) < 0$ for all $\lambda \in [0,1]$. This implies Proposition 3(i).

Next, we show Proposition 3(ii). We find that $\Delta W - \Delta\pi > 0$ if and only if

$$\Omega(\delta, \lambda) := 2 - \lambda + (1 + \lambda - \lambda^2)\delta - \frac{3 - 2\lambda}{8}\{2 + (1+\lambda)\delta\}^2 > 0.$$

Because $\Omega(0, \lambda) = 1/2 > 0$, $\Omega(1, \lambda) = (\lambda - 1)(2\lambda^2 + 3\lambda + 1)/8 \leq 0$, and Ω is strictly concave with respect to δ, we find that there exists unique $\hat{\delta}(\lambda) \in (0, 1]$ such that $\Omega(\delta, \lambda) > 0$ if and only if $\delta < \hat{\delta}(\lambda)$, where $\Omega(\hat{\delta}, \lambda) = 0$. This implies Proposition 3(ii)

Finally, we show Proposition 3(iii). Solving $\Omega(\hat{\delta}, 0) = 0$, we obtain $\hat{\delta}(0) = 2/3$. Solving $\Omega(\hat{\delta}, 1) = 0$, we obtain $\hat{\delta}(1) = 1$. We now show that $\Omega(\hat{\delta}(0), \lambda) > 0$ and $\Omega(\hat{\delta}(1), \lambda) < 0$ for all $\lambda \in (0, 1)$. This implies that $\hat{\delta}(\lambda) \in (\hat{\delta}(0), \hat{\delta}(1))$ for all $\lambda \in (0, 1)$ because Ω is continuous and $\hat{\delta}(\lambda)$ is uniquely determined.

Because $\Omega(2/3, \lambda)$ is zero when $\lambda = 0$ and $\Omega(2/3, \lambda)$ is increasing in λ, $\Omega(2/3, \lambda) > 0$ for $\lambda \in (0, 1]$. $\Omega(1, \lambda) = (\lambda - 1)(2\lambda^2 + 3\lambda + 1)/8 < 0$ for all $\lambda \in [0, 1)$. Q.E.D.

Appendix B: Cournot competition with constant-elasticity demand

Setting Consider a homogeneous-product duopoly with a constant-elasticity demand function. The inverse demand function is

$$p(Q) = Q^{-\frac{1}{\eta}},$$

where $\eta > 1$ is the price elasticity of the demand. As in Section 5, we consider firms competing in quantities. The post-entry gross profit function is given by

$$\pi_i = \pi(q_i, Q) = (p(Q) - c)q_i,$$

where $Q = q_1 + q_2$. Taking first-order conditions for (symmetric) equilibrium in monopoly and duopoly[14], we obtain the equilibrium prices, quantities, and profits:

$$p^M = \frac{\eta}{\eta - 1}c, \quad q^M = \left(\frac{\eta}{\eta - 1}c\right)^{-\eta}, \pi^M = \frac{(\eta - 1)^{\eta - 1}}{\eta^\eta}c^{1-\eta},$$

$$p^D = \frac{\eta}{\eta - \frac{1+\lambda}{2}}c, \quad q^D = \frac{1}{2}\left(\frac{\eta}{\eta - \frac{1+\lambda}{2}}c\right)^{-\eta}, \pi_D = \frac{1}{2}\frac{\left(\eta - \frac{1+\lambda}{2}\right)^{\eta-1}}{\eta^\eta}c^{1-\eta}.$$

Entry incentive We obtain the entry incentive of firm 2 by the following equation

$$\Delta\pi(\lambda) = \pi_D - \lambda(\pi_M - \pi_D) = \frac{c^{1-\eta}}{\eta^\eta}\left\{\frac{1+\lambda}{2}\left(\eta - \frac{1+\lambda}{2}\right)^{\eta-1} - \lambda(\eta-1)^{\eta-1}\right\}.$$

Differentiating $\Delta\pi(\lambda)$, we obtain

$$\Delta\pi'(\lambda) = \frac{c^{1-\eta}}{\eta^\eta}\left\{\frac{1}{2}\left(\eta - \frac{1+\lambda}{2}\right)^{\eta-1} - (\eta-1)^{\eta-1} - \frac{1+\lambda}{4}(\eta-1)\left(\eta - \frac{1+\lambda}{2}\right)^{\eta-2}\right\} < 0,$$

where we use the fact that the function $a(\eta - a)^{\eta-1}$ is increasing in a as long as $a \leq 1$ to show that

$$\frac{1}{2}\left(\eta - \frac{1+\lambda}{2}\right)^{\eta-1} - (\eta-1)^{\eta-1} < \frac{1+\lambda}{2}\left(\eta - \frac{1+\lambda}{2}\right)^{\eta-1} - (\eta-1)^{\eta-1} < 0.$$

Thus, the entry incentive $\Delta\pi$ decreases with common ownership λ.

[14]The second-order conditions are satisfied.

Welfare gain from entry Next, we show that $\Delta\pi(\lambda) > \Delta W(\lambda)$ holds around $\lambda = 0$, and that $\Delta\pi(\lambda) < \Delta W(\lambda)$ holds around $\lambda = 1$ when η is sufficiently large.

The quasi-linear utility function $U(Q) + y$ that gives the inverse demand function $p(Q) = Q^{-1/\eta}$ is

$$U(Q) = \frac{\eta}{\eta - 1} Q^{\frac{\eta-1}{\eta}}.$$

This gives the post-entry gross market surplus as a function of total output Q:

$$\tilde{W}(Q) = U(Q) - cQ = \frac{\eta}{\eta - 1} Q^{\frac{\eta}{\eta-1}} - cQ.$$

Then, we have

$$W^M = \tilde{W}(Q^M) = \frac{c^{1-\eta}}{\eta^\eta}(\eta - 1)^\eta \left\{ \left(\frac{\eta}{\eta - 1} \right)^2 - 1 \right\},$$

$$W^D(\lambda) = \tilde{W}(2q^D) = \frac{c^{1-\eta}}{\eta^\eta} \left(\eta - \frac{1+\lambda}{2} \right)^\eta \left(\frac{\eta^2}{(\eta - 1)\left(\eta - \frac{1+\lambda}{2}\right)} - 1 \right).$$

Therefore, we have

$$\Delta W = W^D(\lambda) - W^M.$$

At $\lambda = 0$, we have

$$\Delta\pi(0) = \frac{c^{1-\eta}}{\eta^\eta} \frac{1}{2} \left(\eta - \frac{1}{2} \right)^{\eta-1}$$

$$\Delta W(0) = \frac{c^{1-\eta}}{\eta^\eta} \left[\left(\eta - \frac{1}{2} \right)^{\eta-1} \left(\frac{\eta^2}{\eta - 1} - \eta + \frac{1}{2} \right) - (\eta - 1)^\eta \left\{ \left(\frac{\eta}{\eta - 1} \right)^2 - 1 \right\} \right]$$

$$\Delta\pi(0) - \Delta W(0) = \frac{c^{1-\eta}}{\eta^\eta(\eta - 1)} \left[(\eta - 1)^{\eta-1} (2\eta - 1) - \left(\eta - \frac{1}{2} \right)^{\eta-1} \right].$$

This is positive if and only if

$$2 > \frac{\left(\eta - \frac{1}{2}\right)^{\eta-2}}{(\eta - 1)^{\eta-1}}. \tag{8}$$

Taking the logarithm of both sides, condition (8) holds if and only if

$$\Phi(\eta) := \log 2 - (\eta - 2)\log\left(\eta - \frac{1}{2}\right) + (\eta - 1)\log(\eta - 1) > 0.$$

We show that $\Phi(\eta) > 0$ for all $\eta > 1$ and thus $\Delta\pi(0) - \Delta W(0) > 0$. To show this, we take the following steps.

1. We show that $\Phi''(\eta) > 0$ for $\eta > 1$, that is, $\Phi(\eta)$ is convex. This is shown by observing that $\Phi(\eta)$ has the first derivative

$$\Phi'(\eta) = -\log(2\eta - 1) + \log 2 + \log(\eta - 1) + \frac{3}{2\eta - 1} \tag{9}$$

   and the second derivative

$$\Phi''(\eta) = \frac{2(1\eta^2 + 2\eta - 3)}{(2\eta - 1)^2(\eta - 1)} > 0$$

   for $\eta > 1$.

2. $\lim_{\eta\to 1} \Phi'(\eta) < 0$ and $\Phi'(2) > 0$, which implies that there is unique $\hat{\eta} \in (1, 2)$ such that $\Phi'(\hat{\eta}) = 0$ holds. From equation (9), we have

$$\lim_{\eta\to 1} \Phi'(\eta) = -\infty$$

   and

$$\Phi'(2) = -\log 3 + \log 2 + 1 = \log e - \log\left(\frac{3}{2}\right) > 0.$$

3. We show that $\Phi(\hat{\eta}) > 0$, which implies that $\Phi(\eta) > \Phi(\hat{\eta}) > 0$ for all $\eta > 1$. At $\hat{\eta}$, we have

$$(\hat{\eta} - 1)\log(\hat{\eta} - 1) = (\hat{\eta} - 1)\log(2\hat{\eta} - 1) - (\hat{\eta} - 1)\log 2 - \frac{3(\hat{\eta} - 1)}{2\hat{\eta} - 1}.$$

Substituting this into $\Phi(\eta)$, we have

$$\Phi(\hat{\eta}) = \log(2\hat{\eta} - 1) - \frac{3(\hat{\eta} - 1)}{2\hat{\eta} - 1}.$$

Define

$$\phi(\eta) =: \log(2\eta - 1) - \frac{3(\eta - 1)}{2\eta - 1}.$$

We have

$$\phi'(\eta) = \frac{2(\eta - 2)}{(2\eta - 1)^2} < 0$$

for $\eta \in (1, 2)$, which implies that $\phi(\hat{\eta}) > \phi(2)$ since $\hat{\eta} < 2$. Moreover, we have $\phi(2) = \log 3 - \log e > 0$. Thus, we have

$$\Phi(\hat{\eta}) = \phi(\hat{\eta}) > \phi(2) > 0.$$

The convexity of $\Phi(\eta)$ implies that $\hat{\eta}$ attains the minimum of $\Phi(\eta)$ in $\eta > 1$. Therefore, we finally have that $\Phi(\eta) \geq \Phi(\hat{\eta}) > 0$ for all $\eta > 1$.

At $\lambda = 1$, we have $\Delta\pi(1) = \Delta W = 0$. Moreover, we have

$$\Delta\pi'(1) = -\frac{c^{1-\eta}}{\eta^\eta} \frac{3(\eta - 1)^{\eta-1}}{4},$$

and

$$\Delta W'(1) = -\frac{c^{1-\eta}}{\eta^\eta} \frac{(\eta - 1)^{\eta-1}}{2} \frac{\eta}{\eta - 1}.$$

Therefore, in the neighborhood of $\lambda = 1$, we have

$$\Delta\pi(\lambda) - \Delta W(\lambda) \simeq -(1 - \lambda) \left\{ \Delta\pi'(1) - \Delta W'(1) \right\} = -(1 - \lambda) \frac{c^{1-\eta}}{\eta^\eta} \frac{(\eta - 1)^{\eta-2}}{4} (\eta - 3),$$

which is negative if and only if $\eta > 3$. Thus, the entry incentive is insufficient for $\lambda \simeq 1$ if $\eta > 3$.

References

Anderson, S. P., de Palma, A., Thisse, J.-F., 1992. *Discrete Choice Theory of Product Differentiation*, MIT Press, Cambridge, Massachusetts.

Azar, J., Schmalz, M. C., Tecu, I., 2018. Anti-competitive effects of common ownership. *Journal of Finance* 73(4), 1513–1565.

Backus, M., Conlon, C., Sinkinson, M., 2019. Common ownership in America: 1980–2017. NBER Working Paper No.25454

Elhauge, E., 2016. Horizontal shareholding. *Harvard Law Review* 109, 1267–1317.

Gu, Y., Rasch, A., Wenzel, T., 2016. Price-sensitive demand and market entry. *Papers in Regional Science*, 95(4), 865–875.

Gu, Y., Wenzel, T., 2009. A note on the excess entry theorem in spatial models with elastic demand. *International Journal of Industrial Organization* 27(5), 567–571.

Gu, Y., Wenzel, T., 2012. Price-dependent demand in spatial models. *B.E. Journal of Economic Analysis & Policy*, 12(1), Article 6.

López, Á. L., Vives, X., 2019. Overlapping ownership, R&D spillovers, and antitrust policy. *Journal of Political Economy* 127(5), 2394–2437.

Mankiw N. G., Whinston M. D., 1986. Free entry and social inefficiency. *RAND Journal of Economics* 17(1), 48–58.

Matsumura, T., 2000. Entry regulation and social welfare with an integer problem. *Journal of Economics* 71(1), 47–58.

Matsumura, T., Okamura, M. 2006. A note on the excess entry theorem in spatial markets. *International Journal of Industrial Organization* 24(5), 1071–1076.

Newham M., Seldeslachts, J., Banal-Estañol, A., 2018. Common ownership and market entry: Evidence from the pharmaceutical industry. Barcelona GSE Working Paper: 1042

Salop, S. C., 1979. Monopolistic competition with outside goods. *Bell Journal of Economics* 10(1), 141–156.

Schmalz, M. C., 2018. Common-ownership concentration and corporate conduct. *Annual Review of Financial Economics* 10, 413–448.

Singh, N., Vives, X., 1984. Price and quantity competition in a differentiated duopoly. *RAND Journal of Economics* 15(4), 546–554.

Vickrey, W. S., 1964. *Microstatics*, Harcourt, Brace and World, New York.

Vives, X., 1999. *Oligopoly Pricing: Old Idea and New Tools*, The MIT Press, Cambridge, MA, London, England.