Mixed Duopoly, Product Differentiation, and Competition

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Abstract

We examine the relationship between the equilibrium price and the degree of product differentiation in a mixed duopoly in which a welfare-maximizing public enterprise competes against a profit-maximizing private firm. Existing works on private economy show that increased product differentiation mitigates price competition. We find that in a mixed economy, increased product differentiation can accelerate competition when the demand is elastic. The private firm chooses to locate itself too close to the public firm (and thus makes the resulting degree of product differentiation too low) for social welfare, which never appears in the private economy. Finally, we discuss the welfare implications of public and private leadership and of privatization of the public firm.

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Key words: product differentiation, location-price model, elastic demand, mixed market

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1 Introduction

In many developed, developing, and transitional economies, state-owned public firms are quite active, and many of them compete with private firms. Studies of mixed oligopolies involving both private and public enterprises have become increasingly popular. In addition, in the wake of the recent financial crisis, many private enterprises facing financial problems have been nationalized, either fully or partially. The studies of mixed oligopolies involving both state-owned public enterprises and private enterprises have therefore attracted the interest of researchers.

In mixed oligopolies, public firms rarely manufacture products that are identical to those manufactured by private firms; thus, the products in such markets are quite differentiated. Competition between public and private firms influences the product positioning of firms and therefore affects both consumer welfare and the social surplus. Cremer et al. (1991) adopted a standard location-price model on the Hotelling line and showed that in a mixed duopoly, the equilibrium location (and hence, the degree of product differentiation) is optimal for social welfare. Matsumura and Matsushima (2004) showed that their result is robust because the location is optimal under any cost difference between public and private firms. Lu and Poddar (2007) considered partial privatization in this context and showed that the equilibrium location depends on the degree of privatization.

However, these papers, along with the many other papers on location price with mill pricing, assumed that the total demand is inelastic; that is, each consumer consumes one unit of the product.

The assumption of inelastic demand can be realistic in the case of certain durable goods, such as

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1 The analysis of mixed oligopolies dates back to Merrill and Schneider (1966). Recently, the literature on this topic has become richer and more diverse. See Ohnishi (2010), Tomaru and Saito (2010), and Tomaru et al. (2011) for recent developments in this field.

2 For recent studies adopting mill-pricing models, see Inoue et al. (2009), Lu (2006), Nilssen and Sørgard (2002), and Ogawa and Sanjo (2007). The delivered-pricing model is another model employed extensively in this field. See Heywood and Ye (2009) and Matsushima and Matsumura (2003, 2006). For studies of other approaches to differentiated product markets, see Anderson et al. (1997), Fujiwara (2007), Matsumura et al. (2009), Matsumura and Shimizu (2010), and Ogawa (2006).

3 For the discussion of partial privatization, see Börs (1991), Fershtman (1990), and Matsumura (1998).
as automobiles, houses, and commuter transportation services. However, in many markets, such as leisure transport services, overnight delivery, energy, and financial markets, all of which are typical examples of mixed markets, this assumption seems less plausible. Thus, it is important to investigate whether the well-known results in this field depend on this assumption.

We drop the inelastic demand assumption of the standard location-price model and investigate the welfare implication of equilibrium location. First, we show that in a mixed duopoly, decreased product differentiation may relax competition, resulting in higher prices when the rival is a public enterprise and the demand is elastic. Second, we show that the location of a private firm is too close to that of a public firm for social welfare; that is, the equilibrium degree of product differentiation is too small. As mentioned above, Cremer et al. (1991) investigated a mixed duopoly in which a private firm competes against a public firm in the location-price model under inelastic total demand (demand for the two firms). In their model, as in the standard model of a private duopoly, reduced product differentiation always accelerates competition, resulting in lower prices. However, this effect depends on the assumption of inelastic demand. Under elastic total demand, the competition-accelerating effect of reduced product differentiation is weaker and can relax competition for certain ranges of locations. Thus, the private firm has a weaker incentive for product differentiation under elastic demand. As a result, the equilibrium degree of product differentiation is too small for social welfare; this is in contrast to the result known from the literature on mixed markets. We also investigate the case of partial privatization. We show that this result holds true if the degree of privatization is small, but the opposite result holds if the degree of privatization is large.

Third, we discuss two sequential location choice models. Matsumura and Matsushima (2003) investigated this problem in the inelastic demand case. They showed that if the public firm is the leader and chooses its location before the private firm (public leadership), the outcome is the same as in the simultaneous location choice case. They also showed that if the private firm is the leader and

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4 We adopt the constant elasticity demand function discussed by Anderson and de Palma (2000) and Gu and Wenzel (2009) in a spatial context.
chooses its location before the public firm (private leadership), the degree of product differentiation is lower and it reduces welfare. We re-examine this problem under the elastic demand assumption and find that under public leadership, the public firm chooses a position closer to the centre, resulting in further reduction of product differentiation. Despite the lower degree of product differentiation, it improves welfare. Thus, under elastic demand, public leadership does matter. We also find that under public leadership, the private firm chooses a position closer to the centre, again resulting in a degree of product differentiation lower than in the simultaneous location choice case. This reduces welfare because even in the simultaneous move case, the location of the private firm is too close to the centre.

Finally, we investigate the effect of privatization of the public firm. In a private duopoly with inelastic total demand, firms have strong incentives to locate far from their rivals in order to relax price competition; this results in maximal product differentiation. We show that this finding holds true under elastic total demand. Thus, privatization reduces welfare.

2 The Model

We formulate a duopoly model with a linear city à la Hotelling. Consumers are uniformly distributed with density 1 along the interval [0, 1]. Let $x_i \in [0, 1]$ be the location of firm $i$ ($i = 1, 2$). A consumer living at $x \in [0, 1]$ incurs a transport cost of $t(x_i - x)^2$ when he/she purchases the product from firm $i$. The consumers have the following indirect utility function:

$$V(p) = \bar{v} - v(p) = \bar{v} - \frac{p^{1-\alpha}}{1-\alpha},$$

where $\bar{v}$ and $\alpha$ are positive constants and $\bar{v}$ is sufficiently large. To ensure the existence of a pure strategy equilibrium, we assume that $\alpha$ is not too large. We require $\alpha \leq 4/13$; otherwise, no pure strategy equilibrium exists in the location choice stage.\(^5\) Firms 1 and 2 produce the same physical

\(^5\) As we discuss later, a larger $\alpha$ reduces the equilibrium distance between the locations of the two firms. When $\alpha = 4/13$, $(x_1, x_2) = (1/2, 11/18)$ in equilibrium, and the equilibrium distance between the two firms is never smaller than this value in pure strategy equilibria.
product. The common marginal cost is constant and normalized to zero.\footnote{This assumption is standard in the Hotelling line model, for both mixed and private markets. See, among others, d’Aspremont et al., (1979), Cremer et al., (1991), Sanjo (2009), and Inoue et al., (2009). The cost difference between firms often yields some problems, such as the non-existence of a pure strategy equilibrium. See, for example, Matsumura and Matsushima (2009, 2011). We do not consider the entry-exit decisions of the firms.}

The utility of the consumer located at $x$ is given as follows:

$$u(x) = \begin{cases} V(p_1) - t(x_1 - x)^2 & \text{if bought from firm 1,} \\ V(p_2) - t(x_2 - x)^2 & \text{if bought from firm 2.} \end{cases}$$

(1)

The functional form of the indirect utility implies that the conditional demand of each consumer is

$$-V'(p) = v'(p) = p^{-\alpha},$$

where $\alpha$ represents the elasticity of the demand as

$$-\frac{v''(p)p}{v'(p)} = \alpha,$$

and the model collapses to a model with inelastic demand when $\alpha = 0$.

For a consumer living at $\bar{x}(p_1, p_2, x_1, x_2)$, where

$$V(p_1) - t(x_1 - \bar{x})^2 = V(p_2) - t(x_2 - \bar{x})^2,$$

(2)

the utility remains the same regardless of the firm chosen. Henceforth, we assume that $x_1 < x_2$. We can rewrite (2) as follows:

$$\bar{x} = \frac{x_1 + x_2}{2} - \frac{v(p_1) - v(p_2)}{2t(x_2 - x_1)} = \frac{x_1 + x_2}{2} - \frac{p_1^{1-\alpha} - p_2^{1-\alpha}}{2(1-\alpha)t(x_2 - x_1)}.$$

(3)

Firm $i$’s demand $D_i$ ($i = 1, 2$) is $D_1 = p_1^{-\alpha}\bar{x}$, $D_2 = p_2^{-\alpha}(1 - \bar{x})$.\footnote{We implicitly assume that $\bar{x} \in (0, 1)$. If we do not make this assumption, then $D_1 = p_1^{-\alpha}\max\{0, \min\{\bar{x}, 1\}\}$ and $D_2 = p_2^{-\alpha}(1 - \max\{0, \min\{\bar{x}, 1\}\})$. We can show that without this implicit assumption, our results hold and $\bar{x} \in (0, 1)$ in equilibrium. This assumption is made only for simplicity.}

Each firm $i$ simultaneously chooses its price $p_i \geq 0$. Firm 1’s payoff is its profit, while firm 2’s payoff is the social surplus (the sum of consumer surplus and the firms’ profits). Firm 1’s profit is...
\( \Pi_1 = p_1 D_1 = p_1^{1-\alpha} \bar{x} \). The social surplus is

\[
W = \Pi_1 + \Pi_2 + V(p_1) \bar{x} + V(p_2)(1 - \bar{x}) - \int_{0}^{\bar{x}} t(x_1 - x)^2 dx - \int_{\bar{x}}^{1} t(x_2 - x)^2 dx
\]

\[
= p_1^{1-\alpha} \bar{x} + p_2^{1-\alpha}(1 - \bar{x}) + \bar{v} - \frac{p_1^{1-\alpha}}{1 - \alpha} \bar{x} - \frac{p_2^{1-\alpha}}{1 - \alpha}(1 - \bar{x}) - \int_{0}^{\bar{x}} t(x_1 - x)^2 dx - \int_{\bar{x}}^{1} t(x_2 - x)^2 dx.
\]

3 Analysis

In this section, we assume that the firms’ locations are given exogenously. In the next section, we analyse the endogenous location choice.

We consider the price competition given \( x_1 \) and \( x_2 \). The first-order conditions for firms 1 and 2 are, respectively, given as

\[
(1 - \alpha)p_1^{\alpha - 1}\left(\bar{x} - \frac{p_1^{1-\alpha}}{1 - \alpha} \frac{1}{2t(x_2 - x_1)}\right) = 0 \iff \bar{x} - \frac{v(p_1)}{2t(x_2 - x_1)} = 0 \tag{4}
\]

\[
p_2^{-\alpha}\left(\alpha(1 - \bar{x}) + (p_2^{1-\alpha} - p_1^{1-\alpha}) \frac{1}{2t(x_2 - x_1)}\right) = 0 \iff \alpha(1 - \bar{x}) - (1 - \alpha) \frac{v(p_1) - v(p_2)}{2t(x_2 - x_1)} = 0. \tag{5}
\]

The second-order conditions are satisfied unless \( \alpha \) is too large.

Let the superscript ‘PE’ denote the equilibrium outcome (price equilibrium) given the locations. In the next section, we use the superscript ‘E’ to denote the equilibrium outcome in the location-price model.

From (3), (4), and (5), we have the following equilibrium outcomes as long as the solution is

\[ \frac{dW}{dp_2} = -p_2^{-\alpha}\{\alpha(1 - \bar{x}) - (1 - \alpha) \frac{v(p_1) - v(p_2)}{2t(x_2 - x_1)}\} = p_2^{-\alpha}\{-\alpha + (1 - \alpha) \frac{x_1 + x_2}{2} - (1 - 2\alpha)\bar{x}\}, \]

where we apply (3) for the second equality. Because \( \partial \bar{x}/\partial p_2 > 0 \), the solution minimizes the objective unless \( \alpha < 1/2 \). Note that we assume \( \alpha \leq 4/13 < 1/2 \).
interior (i.e. $p_1^{PE}, p_2^{PE} > 0$ in equilibrium).³

$$
\bar{x}^{PE} = \frac{1 - \alpha}{1 - 2\alpha} x_1 + \frac{x_2}{2} - \frac{\alpha}{1 - 2\alpha},
$$

(6)

$$
v(p_1^{PE}) = 2t(x_2 - x_1) \left( \frac{1 - \alpha}{1 - 2\alpha} x_1 + \frac{x_2}{2} - \frac{\alpha}{1 - 2\alpha} \right),
$$

(7)

$$
v(p_2^{PE}) = 2t(x_2 - x_1) \left( \frac{1 - \alpha}{1 - 2\alpha} x_1 + \frac{x_2}{2} - \frac{2\alpha}{1 - 2\alpha} \right).
$$

(8)

From (7), (8), and the definition of $v(p)$, we obtain the equilibrium prices.

**Proposition 1** Suppose that the solution is interior ($p_1^{PE}, p_2^{PE} > 0$). (i) $p_1^{PE}$ is increasing in $x_1$ if and only if $x_1 < \alpha / (1 - \alpha)$. (ii) $p_2^{PE}$ is increasing in $x_1$ if and only if $x_1 < 2\alpha$.

**Proof** Taking the log of (7) and (8) and differentiating each with respect to $x_1$, we obtain

$$
\frac{\partial p_1^{PE}}{\partial x_1} v'(p_1^{PE}) = -\frac{1}{x_2 - x_1} + \frac{1}{x_1 + x_2 - \frac{\alpha}{1 - \alpha}},
$$

(9)

$$
\frac{\partial p_2^{PE}}{\partial x_1} v'(p_2^{PE}) = -\frac{1}{x_2 - x_1} + \frac{1}{x_1 + x_2 - 4\alpha}.
$$

(10)

Because $v$ and $v'$ are positive, $\partial p_1^{PE} / \partial x_1$ is positive if and only if (9) is positive and $\partial p_2^{PE} / \partial x_1$ is positive if and only if (10) is positive, implying (i) and (ii), respectively. **Q.E.D.**

Proposition 1 states that when the demand is inelastic ($\alpha = 0$), reduced product differentiation never raises the prices. It also states that when the demand is elastic ($\alpha > 0$), reduced product differentiation can relax price competition. We now provide the underlying intuition.

When the total demand is inelastic, a high price does not yield a welfare loss. The only source of distortion is inefficient transportation. Thus, firm 2 chooses its price to economize the transportation cost, resulting in the two firms’ prices being equal ($p_1 = p_2$) regardless of their locations. An increase

³ The solution is interior if and only if $x_1 + x_2 > 4\alpha$. If $x_1 + x_2 \leq 4\alpha$, then $p_2^{SE}$ is the corner solution 0 and the equilibrium price system becomes

$$
\bar{x}^{PE} = \frac{x_1 + x_2}{4}, \quad v(p_1^{PE}) = 2t(x_2 - x_1) \bar{x}^{PE} = \frac{t(x_2 - x_1)(x_1 + x_2)}{2}, \quad v(p_2^{SE}) = 0.
$$

In this case, Proposition 1 does not hold because $p_1^{PE}$ is decreasing in $x_1$ and $p_2^{PE}$ is independent of $x_1$. 

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in $x_1$ increases the demand elasticity of firm 1 and increases its incentive for price discounting, thus accelerating competition.

When the total demand is elastic, a high price by firm 2 yields a welfare loss. Thus, firm 2 has a stronger incentive to discount prices. Conversely, a too-low price by firm 2 yields inefficient transportation. This trade-off of price discounting determines the price of firm 2. An increase in $x_1$ raises the demand elasticity of firm 1 and thus strengthens the second effect. Thus, an increase in $x_1$ can raise $p_2$ (and so $p_1$) through strategic interaction.

If firm 1 can choose $x_1$, it never chooses $x_1 \leq 2\alpha$. An increase in $x_1$ increases the market share of firm 1 (Hotelling effect). If $x_1 \leq 2\alpha$, firm 1 can increase its profit by increasing $x_1$, a contradiction. However, even when $x_1 > 2\alpha$, the price raising effect of an increase in $x_1$ is weaker when the total demand is elastic than when it is inelastic. The basic principle behind this fact is the same as discussed in the previous paragraphs.

4 Location Choice

Next, we consider the choice of location. Consider the following two-stage location-price model. First, firms independently choose their locations. Second, after observing the locations, firms face Bertrand competition.

Let the superscript ‘E’ denote the equilibrium outcome in the two-stage game. The second-stage competition has already been analysed in the previous section. For the location choice, the first-order condition of firm 1 is

$$\frac{\partial \Pi^E_{1}}{\partial x_1} = (p^E_{1})^{1-\alpha} \left( \frac{1}{2} + \frac{v(p^E_{2}) - v(p^E_{1})}{2t(x_2 - x_1)^2} - \frac{v(p^E_{1})}{2t(x_2 - x_1)} \right)$$

$$= (p^E_{1})^{1-\alpha} \left( \frac{1}{x_2 - x_1} \left( \frac{1}{1 - 2\alpha} \right) \left( \frac{x_2}{2} \right) + \frac{\alpha}{1 - \alpha} \right) = 0. \quad (11)$$

The second-order condition is satisfied unless $\alpha$ is too large. From (11), we have the following reaction
function of firm 1 in the first stage:

$$\hat{x}_1^*(x_2) = \frac{1}{3} x_2 + \frac{2}{3} \frac{\alpha}{1 - \alpha}. \tag{12}$$

The first-order condition for firm 2 is

$$\frac{\partial W_{PE}}{\partial x_2} = \left(2t\bar{x} + 2t(x_2 - x_1)\frac{1}{2(1 - 2\alpha)} \right) \left(-\bar{x} + (1 - \alpha)v(p_{PE}^2)\frac{1}{2t(x_2 - x_1)} \right) + \left(\frac{1}{2} + \frac{v(p_{PE}^1) - v(p_{PE}^2)}{2t(x_2 - x_1)^2} \right) \left(1 - \alpha\right) \left(v(p_{PE}^1) - v(p_{PE}^2) \right) + t(1 - x_2)^2 - t(\bar{x} - x_2)^2 = 0. \tag{13}$$

The second-order condition is satisfied unless $\alpha$ is too large. Substituting (12) into (13) and rearranging using (6), we have

$$\frac{\partial W_{PE}}{\partial x_2} = 2t(1 - 2\alpha) \left(\bar{x}^2 - \frac{3}{2} \frac{1 - \alpha}{2(1 - 2\alpha)} + \frac{1}{2} \right) = 0. \tag{14}$$

From (14), we have

$$\bar{x}^E = \frac{3}{4} \frac{1 - \alpha}{1 - 2\alpha} - \sqrt{\left(\frac{3}{4} \frac{1 - \alpha}{1 - 2\alpha}\right)^2 - \frac{1}{2}}. \tag{15}$$

Substituting (12) and (15) into (6), we have

$$(x_1^E, x_2^E) = \left(\frac{3}{8} - \frac{1}{2} \sqrt{\frac{9}{16} - \frac{1}{2} \left(\frac{1 - 2\alpha}{1 - \alpha}\right)^2} + \frac{\alpha}{1 - \alpha}, \frac{9}{8} - \frac{3}{2} \sqrt{\frac{9}{16} - \frac{1}{2} \left(\frac{1 - 2\alpha}{1 - \alpha}\right)^2} + \frac{\alpha}{1 - \alpha}\right). \tag{16}$$

As expected,

$$\lim_{\alpha \to 0} (x_1^E, \bar{x}^E, x_2^E) = (1/4, 1/2, 3/4),$$

which is the equilibrium location in the standard model with inelastic demand.

We now discuss the welfare implication of the equilibrium location. First, consider the case in which the government can choose the firms’ locations and prices. It is obvious that the first best is achieved and that $x_1 = 1/4$ and $x_2 = 3/4$. The first-best location pattern is equal to the equilibrium one if and only if $\alpha = 0$. Second, consider the case in which the government can choose only the firms’ locations. Because firm 2 maximizes welfare, it is obvious that $\partial W_{PE}/\partial x_2 = 0$ at the equilibrium.
given $x_1$ and the second-stage competition. We consider the welfare implication of firm 1’s location.

**Proposition 2** Suppose that the government does not intervene in the pricing policy. Then,

$$\left. \frac{\partial W^P}{\partial x_1} \right|_{(x_1,x_2)=(x_1^E,x_2^E)} < 0.$$

**Proof**

$$\frac{\partial W^P}{\partial x_1} = 2t\alpha \left( \frac{1 - \alpha}{1 - 2\alpha} x_1 - \alpha \left( \frac{2t(x_2 - x_1)\alpha}{1 - 2\alpha} \left( 1 - \frac{x_1 + x_2}{2} \right) \right) \left( \frac{1}{2} - \frac{\alpha}{1 - 2\alpha} \frac{1 - x_1 + x_2}{x_2 - x_1} \right) \right) + t \left( \frac{1 - \alpha}{1 - 2\alpha} \frac{x_1 + x_2}{2} - \frac{\alpha}{1 - 2\alpha} x_1 \right)^2 - t(0 - x_1)^2. \quad (17)$$

Substituting $(x_1, x_2) = (\hat{x}_1^*(x_2), x_2)$ into (17), we have

$$\frac{\partial W^P}{\partial x_1} = 2t \frac{\alpha^2}{1 - \alpha} \left( \frac{1}{2} \hat{x} - 1 \right); \quad (18)$$

this is negative because $\hat{x} \leq 1$. Note that $x_1^E = \hat{x}_1^*(x_2^E)$. Q.E.D.

Proposition 2 states that a decrease in $x_1$ improves welfare. This implies that in equilibrium, the private firm chooses a location that is too close to the public firm for social surplus (thereby making the degree of product differentiation too small). As stated in the discussion preceding Proposition 1, firm 1 has a strong incentive to increase $x_1$ for a strategic purpose when demand is elastic. Cremer et al. (1991) and Matsumura and Matsushima (2004) showed that under inelastic total demand, the location pattern is efficient for social welfare in a mixed duopoly. Under elastic demand, firm 1 has a stronger incentive to increase $x_1$, thus yielding Proposition 2.

**5 Partial privatization**

In this section, we adopt the partial privatization approach. The public firm cares about its own profits and welfare. We consider the following payoff of the public firm:

$$\theta \Pi_2 + W, \quad \theta \geq 0, \quad (19)$$
where $\theta$ indicates the degree of privatization.

Then, the price equilibrium outcomes are generalized as

$$\bar{x}_{PE} = \frac{(1 - \alpha + \theta(1 - \alpha)) \frac{x_1 + x_2}{2} - (\alpha - \theta(1 - \alpha))}{1 - 2\alpha + 3\theta(1 - \alpha)},$$

(20)

$$v(p^1_{PE}) = 2t(x_2 - x_1)\bar{x}_{PE},$$

(21)

$$v(p^2_{PE}) = 2t(x_2 - x_1)\left(2\bar{x}_{PE} - \frac{x_1 + x_2}{2}\right),$$

(22)

and the equilibrium outcomes of the two-stage game are generalized as

$$\bar{x}^E = \frac{1}{4} \frac{3(1 - \alpha)(1 + 2\theta)}{1 - 2\alpha + 2\theta(1 - \alpha)} - \sqrt{\left(\frac{1}{4} \frac{3(1 - \alpha)(1 + 2\theta)}{1 - 2\alpha + 2\theta(1 - \alpha)}\right)^2 - \frac{1}{2}},$$

(23)

$$x_1^E = \frac{1}{2} \frac{1 - 2\alpha + 3\theta(1 - \alpha)}{1 - \alpha + \theta(1 - \alpha)} \bar{x}^E + \frac{\alpha - \theta(1 - \alpha)}{1 - \alpha + \theta(1 - \alpha)},$$

(24)

$$x_2^E = \frac{3}{2} \frac{1 - 2\alpha + 3\theta(1 - \alpha)}{1 - \alpha + \theta(1 - \alpha)} \bar{x}^E + \frac{\alpha - \theta(1 - \alpha)}{1 - \alpha + \theta(1 - \alpha)}.$$

(25)

Now the welfare implication of firm 1’s location is unclear:

$$\frac{\partial W^{PE}}{\partial x_1} \bigg|_{(x_1, x_2) = (x_1^E, x_2^E)} = 2t \frac{\alpha^2 + \theta(1 - \alpha)(1 - 2\alpha)}{(1 - \alpha)(1 + \theta)} \left(2\bar{x}^E - \frac{\alpha(\alpha - \theta(1 - \alpha))}{\alpha^2 + \theta(1 - \alpha)(1 - 2\alpha)}\right).$$

(26)

The sign of (26) depends on $\theta$. If $\theta$ is sufficiently close to 0, the sign is negative.\footnote{A sufficient condition for this is $\alpha^2 > \theta(1 - \alpha)$.} Thus, we obtain a result similar to Proposition 2 for small $\theta$. However, the sign is positive for large $\theta$. Thus, if the degree of privatization is large, $x_1$ is too small for social welfare. As we discuss in section 6, full privatization yields maximal differentiation; that is, $x_1 = 0$ and obviously $x_1$ is too small for social welfare under full privatization. Even under partial privatization, $x_1$ can be too small for social welfare.\footnote{A sufficient condition for this is $\alpha < \theta(1 - \alpha)$.}

### 6 Sequential location choice

In this section, we discuss two sequential location choice models. In mixed oligopolies, it is often observed that the public firms are incumbents and compete against new private entrants. Such
public firms (public telecom and telecommunication companies, national airlines, public electric power corporations, and national Yahata steel in Japan) can assume leadership for product positioning. Thus, we investigate the effect of public leadership. We also investigate private leadership. Private leadership often yields higher welfare (Pal (1998), Ino and Matsumura (2010)). For theoretical completeness, we discuss this case as well.

6.1 Leadership by the public firm

The public firm and the private firm are often an incumbent and a new entrant, respectively. Thus, the former may be able to take the lead in product positioning (location choice).

In this section, we consider a situation in which the public firm chooses its location first, followed by the private firm. Let the superscript ‘L’ (public leadership) denote the equilibrium location of this game. As discussed in the previous section, \(x^E_1\) is too large for social welfare if the firms choose their locations simultaneously. Thus, the public firm (firm 2) has an incentive to manipulate \(x_2\) to reduce \(x_1\). Because firm 1’s best reply (the optimal location of firm 1 given \(x_2\)) is increasing in \(x_2\), firm 2 has an incentive to reduce \(x_2\) for a strategic purpose. This leads to the following proposition.

**Proposition 3** (i) \(x^L_1 < x^E_1\) and \(x^L_2 < x^E_2\). (ii) \(x^E_2 - x^E_1 > x^L_2 - x^L_1\). (iii) \(\pi^L_1 < \pi^E_1\).

**Proof** From (6) and (12), we have

\[
x^E_1 = \frac{1}{2} \frac{1 - 2\alpha}{1 - \alpha} x^E + \frac{\alpha}{1 - \alpha}, \quad x^E_2 = \frac{3}{2} \frac{1 - 2\alpha}{1 - \alpha} x^E + \frac{\alpha}{1 - \alpha}.
\]  

(27)

Similarly, we obtain

\[
x^L_1 = \frac{1}{2} \frac{1 - 2\alpha}{1 - \alpha} x^L + \frac{\alpha}{1 - \alpha}, \quad x^L_2 = \frac{3}{2} \frac{1 - 2\alpha}{1 - \alpha} x^L + \frac{\alpha}{1 - \alpha}.
\]  

(28)

\[\text{For the discussion on this point, see Ino and Matsumura (2010). For the pioneering work on endogenous timing in a mixed oligopoly, see Pal (1998). See also Tomaru and Kiyono (2010).} \]
The first-order condition for firm 2 is
\[
\frac{\partial W}{\partial x_2}(\hat{x}_1^*, x_2) = \frac{\partial W}{\partial x_2}(\hat{x}_1^*, x_2) + \hat{x}_1^* \frac{\partial W}{\partial x_1}(\hat{x}_1^*, x_2)
\]
\[
= 2t(1 - 2\alpha) \left( \frac{1}{2} \frac{1}{1 - 2\alpha} \hat{x}(\hat{x}_1^*, x_2) + \frac{1}{2} \right) + \frac{1}{3} 2t \frac{\alpha^2}{1 - \alpha} \left( \frac{1}{2} \hat{x}(\hat{x}_1^*, x_2) - 1 \right)
\]
\[
= 0.
\]
(29)

This yields
\[
\bar{x}_L = \frac{3}{4} \frac{1 - \alpha}{1 - 2\alpha} - \frac{1}{12} \frac{\alpha^2}{(1 - \alpha)(1 - 2\alpha)}
\]
\[
- \sqrt{\left( \frac{3}{4} \frac{1 - \alpha}{1 - 2\alpha} - \frac{1}{12} \frac{\alpha^2}{(1 - \alpha)(1 - 2\alpha)} \right)^2 - \left( \frac{1}{2} - \frac{1}{3} \frac{\alpha^2}{(1 - \alpha)(1 - 2\alpha)} \right)^2} < \bar{x}_E.
\]
(30)

Thus, we have
\[
x_1^E - x_1^L = \frac{3}{2} \frac{1 - 2\alpha}{1 - \alpha} (\bar{x}_E - \bar{x}_L) > 0,
\]
(31)
\[
x_2^E - x_2^L = \frac{3}{2} \frac{1 - 2\alpha}{1 - \alpha} (\bar{x}_E - \bar{x}_L) > 0,
\]
(32)
\[
(x_2^E - x_1^E) - (x_2^L - x_1^L) = \left( \frac{3}{2} \frac{1 - 2\alpha}{1 - \alpha} (\bar{x}_E - \bar{x}_L) > 0.\right.
\]
(33)

Finally, because
\[
\Pi_1 = (1 - \alpha)v(p_1)\bar{x},
\]
(34)
from (6), (7), (30), and (33), we have
\[
\Pi_1^E - \Pi_1^L = (1 - \alpha)2t((x_2^E - x_1^E)(\bar{x}_E)^2 - (x_2^L - x_1^L)(\bar{x}_L)^2) > 0.
\]
(35)

Q.E.D.

By definition, the leadership of firm 2 improves firm 2’s payoff (welfare). However, it reduces the degree of product differentiation between firms. Because \(x_1\) is too large for social welfare, firm 2 has an incentive to reduce \(x_1\). Thus, firm 2 chooses a smaller \(x_2\) under public leadership than in the simultaneous location choice case. This strategic location choice reduces the resulting degree of product differentiation and simultaneously improves welfare.
6.2 Leadership by the private firm

Let the superscript ‘1L’ (firm 1’s leadership, private leadership) denote the game in which the private firm locates first.

**Proposition 4**

(i) \( x^{1L}_1 > x^E_1 \) and \( x^{1L}_2 > x^E_2 \) and (ii) \( x^{1L}_2 - x^{1L}_1 < x^E_2 - x^E_1 \).

**Proof** Let \( \hat{x}_2(x_1) \) denote the best response location of firm 2 to the location of firm 1. Then, by 

\[
\frac{\partial W^{PE}}{\partial x_2} = t \left( \left(1 - (1 - \alpha)\frac{\hat{x}_2 + x_1}{2}\right) - (1 - \alpha)(x_2 - x_1) \right) \left(1 - (1 - \alpha)\frac{\hat{x}_2 + x_1}{2}\right) - \alpha(2 - 3\alpha) \frac{1}{1 - 2\alpha} 
\]

for \( x_2 = \hat{x}_2(x_1) \),

\[ \Pi^{PE}_1 = (1 - \alpha)2t(x_2 - x_1)(\hat{x}^{PE})^2 \]

\[ = 2t \left(1 - \alpha - (1 - 2\alpha)\hat{x}^{PE}\right) - \frac{\alpha(2 - 3\alpha)}{1 - \alpha - (1 - 2\alpha)\hat{x}^{PE}}(\hat{x}^{PE})^2. \]  

(37)

By 

\[
\frac{d\Pi^{PE}_1(x_1, \hat{x}_2)}{dx_1} = \frac{\partial \Pi^{PE}_1}{\partial x_1}(x_1, \hat{x}_2) + \hat{x}_2 \frac{\partial \Pi^{PE}_1}{\partial x_2}(x_1, \hat{x}_2) = \hat{x}_2 \frac{\partial \Pi^{PE}_1}{\partial x_2}(x_1, \hat{x}_2) > 0
\]

(38)

for \( x_1 = x^E_1 \),

\[ x^{1L}_1 > x^E_1 \].

(39)

from which the remaining properties follow. **Q.E.D.**

Under private leadership, firm 1 chooses a larger \( x_1 \) for a strategic purpose than in the simultaneous location choice case. An increase in \( x_1 \) increases \( x_2 \) and increases the profits of firm 1. Thus, \( x^{1L}_1 \) is larger than \( x^E_1 \). In the simultaneous location choice case, \( x_1 \) is too large for welfare, and private leadership further increases \( x_1 \). Thus, private leadership reduces welfare. This result stands in stark contrast to the results for a mixed oligopoly under Cournot competition.\(^{13}\)

\(^{13}\) For discussions on the advantage of private leadership under Cournot competition, see Pal (1998) and Ino and Matsumura (2010).
7 The effect of privatization

We discuss the effect of privatization of firm 2. Suppose that both firms are private. Let the superscript ‘P’ denote the equilibrium outcomes in the private duopoly.

**Proposition 5**  
(i) $x_1^P = 0$ and $x_2^P = 1$.  
(ii) $W^P < W^E < W^L$.

**Proof** By (3) and (34), this case is strategically equivalent to the case with $\alpha = 0$, whose equilibrium location is known to be $(0, 1)$. Thus, $(x_1^P, x_2^P) = (0, 1)$.

$W^P < W^E$ follows because

$$W^P = W^{PSE}(0, 1) < W^{PSE}(x_1^E, 1 - x_1^E) \leq W^{SE}(x_1^E, 1 - x_1^E) \leq W^{SE}(x_1^E, x_2^E) = W^E,$$

where $W^{PSE}(x_1^E, 1 - x_1^E)$ is the second-stage social surplus given the locations of firms in the private oligopoly.

Finally, by Proposition 2, $W^E < W^L$ follows straightforwardly. Q.E.D.

The privatization leads to maximal differentiation and reduces welfare. However, we must emphasize that the second result may depend on the assumption that the public firm is as efficient as the private firm. If privatization improves the production efficiency of the public firm, privatization can improve welfare.

8 Concluding remarks

In this paper, we investigate a location-price model in a mixed duopoly. We introduce elastic demand into the standard model and find that in the price-competition stage, reduced product differentiation can relax price competition. We also show that the resulting degree of product differentiation is too small for social welfare. As mentioned in the Introduction, many papers have explored a mixed oligopoly with inelastic demand and have derived many important policy implications. However, the policy and welfare implications of the results that hold only under inelastic demand are limited.
Determining whether the above results depend on the assumption of inelastic demand is essential to assessing their robustness and applicability. This issue remains to be addressed in future research.

In this paper, we assume that there is no cost difference between public and private firms. In the literature on mixed oligopolies, cost difference often plays an important role. Incorporating cost differences in this model is a difficult task but worth discussing in future research.\textsuperscript{14} Another possible extension is to incorporate foreign competition. In the literature on mixed oligopoly, it is well known that the nationality of private firms affects the optimal behaviour of the public firm.\textsuperscript{15} This extension also remains an issue for future research.

\textsuperscript{14} In recent works, Matsumura and Ogawa (2010), who assume cost difference between two firms, and Bárcena-Ruíz and Garzón (2010), who assume identical cost, present contrasting results in an endogenous timing game under Cournot competition.

References


