Endogenous Public and Private Leadership with Diverging Social and Private Marginal Costs*

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Abstract

We investigate endogenous timing in a mixed duopoly with price competition and different social versus private marginal costs. We find that any equilibrium timing patterns—Bertrand, Stackelberg with private leadership, Stackelberg with public leadership, and multiple Stackelberg equilibria—emerge. When the foreign ownership share in a private firm is less than 50%, public leadership is more likely to emerge than private leadership. Conversely, private leadership can emerge in a unique equilibrium when the foreign ownership share in a private firm is large. These results may explain recent policy changes in public financial institutions in Japan. We also find there is a nonmonotonic relationship between the welfare advantage of public and private leadership and the difference between social and private marginal costs for a private firm. A nonmonotonic relationship does not emerge in profit ranking. Similar results are obtained under quantity competition, although some properties are different.

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1 Introduction

In Japan, during the post-war reconstruction and high-growth period from 1945 to the 1970s, state-owned public enterprises—especially public financial institutions—played a leading role in the Japanese economy. It was widely believed that lending by public financial institutions (e.g., the Development Bank of Japan) had a pump-priming effect on lending by private banks and Japan Post was the world's largest bank in the 1970s and 1980s, occupying a dominant position for raising money from households.[1] Since the 1980s, some public enterprises have undergone major reforms. For example, three major nonfinancial, state-owned public enterprises—Japan Railway, Japan Tobacco Incorporated, and Nippon Telegraph and Telephone Corporation—were privatized. In addition, the Japanese flag carrier (Japan Airline) was privatized in 1987. However, the government continued to hold shares in major public financial institutions, which played dominant roles in Japanese financial markets.

The Koizumi Cabinet (April 2001–September 2006) disrupted this state-centered scenario, by declaring that public financial institutions should play a secondary role to private firms, with private firms leading the market. As a result, there was significant downscaling of major public institutions. Once again, however, public institutions have begun to lead in Japanese markets (Matsumura and Ogawa, 2017b). Newly established public financial institutions, such as the Industrial Revitalization Corporation of Japan, the Enterprise Turnaround Initiative Corporation of Japan, the Regional Economy Vitalization Corporation of Japan, and the Private Finance Initiative Promotion Corporation of Japan, currently play leading roles in financial markets. The Nikkei newspaper, refers to this phenomenon as "Kiko capitalism" (state institution capitalism) (Nikkei, November 22, 2011). This type of capitalism is still expanding under the current Abe Cabinet (Nikkei, October 8, 2013). For example, the government has tried to establish new public financial institutions, such as the Japan Investment Corporation.[2]

The topic of whether public or private firms should lead markets and how this affects market

[1]See Horiuchi and Sui (1993). It has been observed that, globally, the public sector plays an important role in lending markets. See Bose *et al.* (2014).

[2]This drive for new institutions is not limited to Japan. For example, the Korea Development Bank plays an important role in financing Korean industry and recently supplied funding to rescue Hanjin Heavy Industries (Nikkei, February 13, 2019). In Indonesia, the state-owned bank, Bank Mandiri, has expanded by acquiring the privately owned Permata Bank (NNA Asia, April 10, 2019).

equilibrium has been actively discussed in the literature on mixed oligopolies.[3] Pal (1998) adopted the observable delay game formulated by Hamilton and Slutsky (1990), and investigated the endogenous roles in which public and private firms compete in mixed oligopolies. He showed that public firms should be the followers in welfare and also become the followers in equilibrium. The literature on endogenous roles in mixed oligopolies is rich and diverse. Tomaru and Kiyono (2010) showed that both private and public leadership outcomes emerge under general demand and increasing marginal costs. Matsumura (2003) introduced foreign competition, showing that public firms should be the leaders and that they become the leaders in equilibrium. Lu (2006) extended this analysis to an oligopoly case. Nakamura and Inoue (2007) introduced managerial delegation and showed that public firms become the followers. Matsumura and Ogawa (2010) adopted Matsumura's (1998) partial privatization approach and showed that under partial privatization, private leadership is a unique equilibrium or is risk dominant unless the degree of privatization is large.[4] Capuano and De Feo (2010) introduced the shadow cost of public funds and showed that private leadership equilibrium is robust. Tomaru and Saito (2010) considered a subsidized, mixed duopoly and showed that private leadership emerges under an optimal subsidy policy. Matsumura and Ogawa (2017b) introduced product differentiation and showed that public leadership can be risk dominant, although it is worse for welfare than private leadership is.[5]

Most studies on endogenous roles in mixed oligopolies have investigated quantity competition. Until the 1990s, it was difficult to raise funds. In this situation—based on Kreps and Scheinkman (1983) and Friedman (1988)—it would be reasonable to use quantity competition models to analyze the Japanese financial market. Since March 2001, however, Japanese financial markets have been loosened considerably in response to quantitative monetary-easing policy, and similar situations have prevailed globally. Thus, price competition models may be more appropriate for analyzing the role of public institutions in current financial markets. Moreover, in mixed oligopolies—as Matsumura and Ogawa (2012), Haraguchi and Matsumura (2014), and Din and Sun (2016) have shown— price competition is more natural than

[3]For recent developments in mixed oligopolies, see Cato and Matsumura (2019), Dong and Bárcena-Ruiz (2017), Sato and Matsumura (2019a,b), Pi *et al.* (2018), Shuai (2017), and works cited therein.

[4]On the concept of risk dominance, see Harsanyi and Selten (1988).

[5]For the importance of sequential-move games in mixed oligopolies, see Gelves and Heywood (2013), Heywood and Ye (2009a), Ino and Matsumura (2010), Pi *et al.* (2018), and Wang and Lee (2013).

quantity competition in endogenous competition structure models.[6] Thus, it is important to investigate endogenous roles in the price competition model.

The literature on endogenous timing with price competition in mixed oligopolies is relatively sparse. Bárcena-Ruiz (2007) investigated price competition and showed that Bertrand equilibrium emerges in a mixed duopoly as a unique equilibrium. Din and Sun (2016) showed that his result holds even when the competition structure is endogenized.[7]

In this study, we extend the model of Bárcena-Ruiz (2007) in two directions. First, we introduce a foreign-ownership share in a private firm.[8] The other direction is to allow private marginal cost to differ from social marginal cost, which appears in various important situations. The social marginal cost is larger than the private marginal cost if there is a negative externality of production, such as pollution. The same is true if a production subsidy is introduced. The social marginal cost is smaller than the private marginal cost if a licensing royalty exists. The same holds if a vertical relationship exists, and there is a double marginalization problem.[9] Thus, our model formulation incorporates many important issues from such fields as industrial organization, public economics, and environmental economics. In particular, the externalities that yield divergent social and private costs are important for the analysis of mixed oligopolies, because these externalities may serve as the rationale for the existence of public financial institutions.

We find that any distribution of roles, simultaneous-move equilibrium, unique equilibrium with public leadership, unique equilibrium with private leadership, and multiple equilibria with public and

[6] Matsumura and Ogawa (2012), Haraguchi and Matsumura (2014), and Din and Sun (2016) used an endogenous competition structure model formulated by Singh and Vives (1984). For more on the topic of welfare and profit ranking over price and quantity competition in mixed duopolies in a simultaneous-move game, see Ghosh and Mitra (2010, 2014), and in a sequential-move game, see Hirose and Matsumura (2019). See Haraguchi and Matsumura (2016) for an oligopoly version.

[7] In this study, similar to Bárcena-Ruiz (2007) and Din and Sun (2016), we assume that a public firm is a welfare maximizer, whereas a private firm is a profit maximizer. Bárcena-Ruiz and Sedano (2011), Matsumura and Ogawa (2014), and Naya (2015) discussed different payoff functions and showed that sequential-move outcomes can emerge in equilibrium.

[8] The literature on mixed oligopolies has shown that foreign ownership in private firms often matters. For pioneering works on foreign competition in mixed oligopolies, see Corneo and Jeanne (1994), Fjell and Pal (1996), and Pal and White (1998). Foreign ownership is important in the context of public policies in mixed oligopolies. See also Bárcena-Ruiz and Garzón (2005a, b), Heywood and Ye (2009b), Lee *et al.* (2013), Lin and Matsumura (2012), and Wang and Lee (2013).

[9] For discussions on tax-subsidy policy in mixed oligopolies, see Mujumdar and Pal (1998). For discussions on licensing in mixed oligopolies, see Ye (2012) and Kim *et al.* (2018). For discussions on vertical relationship in mixed oligopolies, see Matsumura and Matsushima (2012), Chang and Ryu (2015), and Wu *et al.* (2016).

private leadership emerge in equilibrium. The distribution depends on the foreign ownership share in the private firm, the difference between social and private marginal costs, and the degree of product differentiation. Our results suggest that both the Koizumi and Abe Cabinets' policies could be regarded as reasonable. Moreover, we find that public leadership is more likely to emerge in equilibrium when the foreign ownership share in private firms is small, which is in sharp contrast to the result for quantity competition. Our results may explain the policy shift from the Koizumi to Abe Cabinets, as the presence of foreign financial institutions in the Japanese banking industry has become weaker recently.[10]

Next, we investigate welfare and profit ranking, comparing public and private leadership. We find that public leadership is better than private leadership for social welfare when the difference between social and private marginal costs is small. Private leadership becomes better than public leadership for social welfare when the cost difference reaches a threshold value. However, as the cost difference becomes larger and reaches another threshold value, public leadership again becomes better than private leadership for social welfare. In other words, there is a nonmonotonic relationship between the advantage of public leadership and the difference between social and private marginal costs.

This nonmonotonic relationship does not emerge in the ranking of a private firm's profit. Private (public) leadership yields greater profit for the private firm than public (private) leadership when the difference between social and private marginal costs is small (large).

From these results, we find that public leadership and private leadership can be payoff dominant and risk dominant to private leadership and public leadership, respectively, depending on the difference between social and private marginal costs, the degree of product differentiation, and the foreign ownership share in the private firm.[11] Our results highlight the importance of these three factors for the equilibrium role of public firms and welfare implications.

Matsumura and Ogawa (2017a) and Lee and Xu (2018) are most closely related to this study. They introduced an environmental problem and showed that the degree of negative externality affects

[10] For example, City Bank exited from the Japanese investment banking market in 2009 and from the retail banking market in 2016. Standard Chartered Bank and HSBC exited from the Japanese private banking market in 2012, and the Royal Bank of Scotland exited in 2017. In 2006, Ripplewood Holdings, which was a dominant stockholder in Shinsei Bank, withdrew its investment.

[11] Matsumura and Ogawa (2009) showed that if one outcome is payoff dominant, this outcome is either the unique equilibrium or the risk-dominant equilibrium in the observable delay game.

equilibrium roles in mixed duopolies. Matsumura and Ogawa (2017a) showed that quantity (price) competition yields a simultaneous-move (sequential-move) outcome under a significant negative externality. Lee and Xu (2018) examined an endogenous timing game in product differentiated duopolies under price competition and showed that when environmental externalities are significant, public leadership yields greater welfare than private leadership does, and that public leadership is more robust than private leadership as an equilibrium outcome. Lee and Xu (2018) also found that that privatization can result in a public leader becoming a private leader, but this worsens welfare. However, they assumed that the private firm is domestic and focused on the symmetric negative externality case only.[12]

The rest of this paper is organized as follows. Section 2 presents the model. Sections 3 investigates fixed timing games. Section 4 shows the equilibrium timing of the observable delay game. Section 5 compares the welfare and profit of private firms. Section 6 discusses a model with quantity competition. Section 7 concludes.

2 The Model

The quasi-linear utility function of a representative consumer, $U(q_0, q_1) = a(q_0 + q_1) - (q_0^2 + 2bq_0q_1 + q_1^2)/2 - (p_0q_0 + p_1q_1)$, provides the following demand function. Firms 0 and 1 produce differentiated commodities, and the demand function is given by $q_i = (a(1-b) - p_i + bp_j)/(1-b^2)$ $(i = 0, 1,\ i \neq j)$, where p_i and q_i are firm i's price and quantity, respectively, a is a positive constant, and $b \in (0, 1)$ represents the degree of product differentiation. A smaller b implies larger product differentiation. The marginal production costs are constant. Let c_i and s_i denote firm i's private and social marginal costs, respectively. We assume that $a > s_0 > c_1$ for analytical simplicity. Let $\theta \in [0, 1]$ be the foreign ownership share in firm 1.

Firm 0 is a domestic state-owned public firm, and its payoff is the total social surplus SW. SW is the sum of profit of firm 0, $(p_0 - c_0)q_0$, the domestic profit share in firm 1, $(1-\theta)(p_1 - c_1)q_1$, consumer surplus, $U(q_0, q_1)$, and the social gain of externality $(c_0 - s_0)q_0 + (c_1 - s_1)q_1$. Then, the total social

[12] For an example of asymmetric diverging social and private marginal costs among firms in mixed oligopolies, see Haraguchi and Matsumura (2020).

surplus is given by

$$SW = (p_0 - s_0)q_0 + (p_1 - s_1)q_1 + \left[a(q_0 + q_1) - \frac{q_0^2 + 2bq_0q_1 + q_1^2}{2} - p_0q_0 - p_1q_1 - \theta(p_1 - c_1)q_1 \right] \ (:= V_0).$$

Firm 1 is a private firm and its payoff is its own profit, $\pi_1 = (p_1 - c_1)q_1$ $(:= V_1)$. The firms choose prices p_0 and p_1, and then q_0 and q_1 are obtained from the demand functions.

Note that $(p_0 - s_0)q_0 + (p_1 - s_1)q_1$ is not producer surplus. If the cost difference between social and private marginal costs is due to unit production tax, then $(p_0 - s_0)q_0 + (p_1 - s_1)q_1$ represents producer surplus plus tax revenue. If the cost difference is due to negative externality, then $(p_0 - s_0)q_0 + (p_1 - s_1)q_1$ represents producer surplus minus loss of negative externality. If the cost difference is due to royalty or price-cost margin of inputs, then $(p_0 - s_0)q_0 + (p_1 - s_1)q_1$ represents firms' and the licenser's profits or firms' and input suppliers' profits.[13] Our analysis applies to cases in which there are multiple sources of the divergence of social and private costs. Therefore, $s_0 - c_0$ and $s_1 - c_1$ may have opposite signs.

The game runs as follows. In the first stage, each firm i $(i = 0, 1)$ independently chooses whether to move early $(t_i = 1)$ or late $(t_i = 2)$. In the second stage, if both firms make the same choice, $t_0 = t_1$, each firm i $(i = 0, 1)$ independently chooses p_i (Bertrand). If firm i chooses $t_i = 1$ and firm $j (\neq i)$ chooses $t_j = 2$, firm i choses p_i and then firm j chooses p_j after observing p_i (Stackelberg). See Table 1 for the payoff matrix of the observable delay game in our environment, where V_i^F (res. V_i^L) denotes firm i's equilibrium payoff in the sequential-move game when it is the follower (res. leader), and V_i^B denotes each firm's equilibrium payoff in the simultaneous-move game (Bertrand). We solve this game by backward induction and the equilibrium concept is the subgame perfect Nash equilibrium.

Table 1: Payoff matrix of the observable delay game.

0\1	$t_1 = 1$	$t_1 = 2$
$t_0 = 1$	(V_0^B, V_1^B)	(V_0^L, V_1^F)
$t_0 = 2$	(V_0^F, V_1^L)	(V_0^B, V_1^B)

[13] If the price-cost margin of input suppliers is m and foreign ownership share in input suppliers is θ', then the social marginal cost is $c_0 - \theta' m$.

3 Three Fixed Timing Games

In this section, we discuss the second stage game given t_0 and t_1. Let $\Delta_i := s_i - c_i$. If there is a negative (positive) externality of the production of firm i, Δ_i is positive (negative). If there is a production subsidy (production tax), Δ_i is positive (negative). If firm i pays a licensing royalty and the licenser is a domestic investor, Δ_i is negative. We assume that the following three games have interior solutions.

3.1 Bertrand ($t_0 = t_1 = 1$ or $t_0 = t_1 = 2$)

First, we consider the simultaneous-move game (Bertrand competition). Each firm maximizes its payoff V_i with respect to p_i. The first-order conditions are

$$\begin{aligned}\frac{\partial V_0}{\partial p_0} &= \frac{-p_0 + s_0 - b(p_1 - s_1) - b\theta(p_1 - c_1)}{1 - b^2} = 0,\\ \frac{\partial V_1}{\partial p_1} &= \frac{a(1-b) + c_1 + bp_0 - 2p_1}{1 - b^2} = 0.\end{aligned}$$

The second-order conditions are satisfied. From the first-order conditions, we obtain the following reaction functions for firms 0 and 1, respectively:

$$\begin{aligned}R_0(p_1) &= s_0 - b\Delta_1 + b(1-\theta)(p_1 - c_1),\\ R_1(p_0) &= \frac{a(1-b) + c_1 + bp_0}{2}.\end{aligned}$$

Given p_1, firm 0's optimal price does not depend on c_0. Because firm 0 cares about welfare, only social marginal cost s_0 matters. For this reason, Δ_0 does not appear in firm 0's reaction function. However, given p_1, firm 0's optimal price depends on Δ_1. The higher Δ_1 is, the higher the likelihood that firm 1's profit-maximizing price yields an excessive output level for welfare. Thus, firm 0 has a greater incentive to reduce firm 1's output when Δ_1 is larger. For this reason, given p_1, firm 0's optimal price is decreasing in Δ_1.

Given p_0, firm 1's optimal price does not depend on s_1, because firm 1 cares only about its own profit.

These reaction functions lead to the following equilibrium prices:

$$p_0^B = \frac{b(1-\theta)(a(1-b)-c_1)+2s_0-2b\Delta_1}{2-b^2(1-\theta)}, \tag{1}$$

$$p_1^B = \frac{a(1-b)+bs_0+(1-b^2(1-\theta))c_1-b^2\Delta_1}{2-b^2(1-\theta)}. \tag{2}$$

The resulting equilibrium outputs are, respectively,

$$q_0^B = \frac{(2-b^2)(a(1-b)-s_0+bs_1)+b(1-b^2)(a-c_1)\theta}{(1-b^2)(2-b^2(1-\theta))}, \tag{3}$$

$$q_1^B = \frac{a(1-b)+bs_0-c_1-b^2\Delta_1}{(1-b^2)(2-b^2(1-\theta))}. \tag{4}$$

The resulting welfare and firm 1's profit are, respectively,

$$V_0^B = \frac{X_1}{2(1-b^2)(2-b^2(1-\theta))^2}, \tag{5}$$

$$V_1^B = \frac{(a(1-b)+bs_0-c_1+b^2\Delta_1)^2}{(1-b^2)(2-b^2(1-\theta))^2}, \tag{6}$$

where X_1 and the other coefficients $X_i (i = 1, 2, ..., 16)$ that appear throughout the paper are reported in Appendix A.

3.2 Stackelberg with Public Leadership ($t_0 = 1, t_1 = 2$)

Second, we consider a sequential-move game in which firm 1 chooses $p_1 = R_1(p_0)$, and firm 0 maximizes its payoff, $V_0(p_0, R_1(p_0))$. We obtain

$$p_0^L = \frac{b(1-2\theta)(a(1-b)-c_1)+2(2-b^2)s_0-2b\Delta_1}{4-3b^2+2b^2\theta}, \tag{7}$$

$$p_1^F = \frac{a(1-b)(2-b^2)+2(1-b^2(1-\theta))c_1+b(2-b^2)s_0-b^2\Delta_1}{4-3b^2+2b^2\theta}. \tag{8}$$

The resulting equilibrium outputs are, respectively,

$$q_0^L = \frac{a(1-b)(4+b-3b^2-b^3)-(2-b^2)^2s_0+b(3-2b^2)c_1+b(2-b^2)\Delta_1+2b(1-b^2)(a-c_1)\theta}{(1-b^2)(4-3b^2+2b^2\theta)},$$

$$q_1^F = \frac{(2-b^2)(a(1-b)+bs_0-c_1)-b^2\Delta_1}{(1-b^2)(4-3b^2+2b^2\theta)}.$$

The resulting welfare and firm 1's profit are, respectively,

$$V_0^L = \frac{X_2}{2(1-b^2)(4-3b^2+2b^2\theta)}, \tag{9}$$

$$V_1^F = \frac{((2-b^2)((1-b)a+bs_0-c_1)-b^2\Delta_1)^2}{(1-b^2)(4-3b^2+2b^2\theta)^2}. \tag{10}$$

3.3 Stackelberg with Private Leadership ($t_0 = 2, t_1 = 1$)

Third, we consider a sequential-move game in which firm 0 chooses $p_0 = R_0(p_1)$, and firm 1 maximizes its payoff, $V_1(R_0(p_1), p_1)$. We obtain

$$p_0^F = \frac{(1-\theta)b(a(1-b) - c_1 - bs_0 + b^2\Delta_1) + 2s_0 - 2b\Delta_1}{2(1-b^2(1-\theta))}, \quad (11)$$

$$p_1^L = \frac{a(1-b) + bs_0 + (1-2b^2(1-\theta))c_1 - b^2\Delta_1}{2(1-b^2(1-\theta))}. \quad (12)$$

The resulting equilibrium outputs are, respectively,

$$q_0^F = \frac{2(1-b^2)(a(1-b) - s_0 + bs_1) + b\theta(a(1-b)(1+2b) - bs_0 - (1-2b^2)c_1 + b^2\Delta_1)}{2(1-b^2)(1-b^2(1-\theta))},$$

$$q_1^L = \frac{a(1-b) + bs_0 - c_1 - b^2\Delta_1}{2(1-b^2)}.$$

The resulting welfare and firm 1's profit are, respectively,

$$V_0^F = \frac{X_3}{8(1-b^2)(1-b^2(1-b^2(1-\theta))^2}, \quad (13)$$

$$V_1^L = \frac{(a(1-b) + bs_0 - c_1 - b^2\Delta_1)^2}{4(1-b^2)(1-b^2(1-\theta))}. \quad (14)$$

4 Equilibrium Role

In this section, we discuss the first stage choices and show the equilibrium outcome in the observable delay game. Before presenting our main results (Propositions 1 and 3), we present three supplementary results on the comparison of the equilibrium prices among three fixed timing games (Lemmas 1–3). These lemmas are helpful for understanding the intuition behind our main results.

First, we present a result highlighting the strategic behavior of the leaders.

Lemma 1 *(i) There exists $\tilde{\Delta} > 0$ such that $p_0^B > p_0^L$ and $p_1^B > p_1^F$ if and only if $\Delta_1 < \tilde{\Delta}$. (ii) $\tilde{\Delta}$ is increasing in θ. (iii) $p_1^B \leq p_1^L$ and thus, $p_0^B \leq p_0^F$ regardless of Δ_1, and the equality holds if and only if $\theta = 1$.*

Proof See Appendix B.

First, we explain the intuition behind Lemma 1(i,ii). Suppose that Δ_1 is small. Because of the price-making behavior of firm 1, its resulting output level is too low (firm 1's price is too high) for welfare, and thus, firm 0 has an incentive to lower firm 1's price. As the leader, firm 0 chooses a lower price than p_0^B because firm 1's reaction curve is upward sloping (strategic complement). Therefore, $p_0^L < p_0^B$ when Δ_1 is small.

Suppose that Δ_1 is large. Because firm 1 chooses its price without considering its high social cost, firm 1's resulting output level is too high (its price is too low) for welfare, and thus, firm 0 has an incentive to raise firm 1's price. As a leader, firm 0 chooses a higher price than p_0^B. Therefore, $p_0^L > p_0^B$ when Δ_1 is large.

Because firm 1's output is more likely to be excessive for domestic welfare when firm 1 has higher foreign ownership (i.e., θ is larger), $p_0^L < p_0^B$ is more likely to hold when θ is larger (Lemma 1(ii)).

Second, we explain the intuition behind Lemma 1(iii). Firm 1's profit increases with firm 0's price, and firm 0's reaction curve is upward sloping (strategic complement) unless $\theta = 1$ (and firm 0's optimal price is independent of p_1 if $\theta = 1$). Thus, firm 1 chooses a higher price than p_1^B to raise firm 0's price unless $\theta = 1$. Therefore, $p_1^L > p_1^B$, regardless of Δ_1. Because the strategy of firm 0 is strategic complement, $p_0^F > p_0^B$ holds, unless $\theta = 1$.

Next, we present another supplementary result highlighting how Δ_1 affects firms' equilibrium prices among the three games.

Lemma 2 *(i)* $0 > \partial p_0^L/\partial \Delta_1 > \partial p_0^B/\partial \Delta_1$ *and* $0 > \partial p_0^L/\partial \Delta_1 > \partial p_0^F/\partial \Delta_1$. *In other words, firm 0's equilibrium prices are decreasing in the cost difference between public and private marginal cost in the private firm and an increase in the cost difference reduces firm 0's price under the public leadership less significantly than it reduces firm 0's price under Bertrand competition and private leadership. (ii)* $0 > \partial p_1^F/\partial \Delta_1 > \partial p_1^B/\partial \Delta_1$ *and* $0 > \partial p_1^F/\partial \Delta_1 > \partial p_1^L/\partial \Delta_1$. *In other words, firm 1's equilibrium prices are decreasing in the cost difference and an increase in the cost difference reduces firm 1's price under public leadership less significantly than it reduces firm 1's price under Bertrand competition and private leadership.*

Proof See Appendix B.

We explain the intuition behind Lemma 2. Lemma 2 states that p_0^B, p_0^L, and p_0^F are decreasing in Δ_1. As Δ_1 increases, the output level of firm 1 is more likely to be excessive for welfare. Thus, firm 0 has a greater incentive to reduce q_1 as Δ_1 increases. Given p_1, a decrease in p_0 reduces q_1. Therefore, firm 0 chooses a smaller p_0 as Δ_1 increases in all three fixed timing games.

Firm 0 has a greater incentive to raise p_1 to reduce q_1 as Δ_1 increases. Thus, as the leader, firm 0 has a greater incentive to raise p_0^L when Δ_1 is larger. This partially cancels out the abovementioned price-reducing effect of Δ_1. Such an effect does not exist when firm 0 is the follower or when two firms move simultaneously. Therefore, an increase in Δ_1 reduces p_0^L less significantly than it reduces p_0^B and p_0^F.

Finally, we compare firm 0's price under public leadership as opposed to private leadership.

Lemma 3 *(i) There exists $\bar{\Delta}$ ($\geq \tilde{\Delta}$) such that $p_0^L < p_0^F$ if and only if $\Delta_1 < \bar{\Delta}$ and $\bar{\Delta} = \tilde{\Delta}$ if and only if $\theta = 1$. In other words, firm 0's price is higher under public leadership than under private leadership only if Δ_1 is large. (ii) There exists $\breve{\Delta}$ ($\geq \tilde{\Delta}$) such that $p_1^L > p_1^F$ if and only if $\Delta_1 < \breve{\Delta}$ and $\breve{\Delta} = \tilde{\Delta}$ if and only if $\theta = 1$. In other words, firm 1's price is higher under public leadership than under private leadership only if Δ_1 is large. (iii) $\tilde{\Delta} \leq \breve{\Delta}$ and the equality holds if and only if $\theta = 1$. In other words, $p_1^L < p_1^F$ holds only if $p_0^L > p_0^F$ holds.*

Proof See Appendix B.

Regardless of Δ_1, $p_0^F \geq p_0^B$ (Lemma 1(iii)). When $\Delta_1 < \tilde{\Delta}$, $p_0^L < p_0^B$. Thus, when Δ_1 is small, $p_0^L < p_0^F$. Lemma 2(i) states that p_0^F decreases more significantly than p_0^L as Δ_1 increases. Thus, $p_0^L > p_0^F$ holds when Δ_1 is large. These yield Lemma 3(i).

Regardless of Δ_1, $p_1^L \geq p_1^B$ (Lemma 1(iii)). When $\Delta_1 < \tilde{\Delta}$, $p_1^F < p_0^B$. Thus, when Δ_1 is small, $p_1^L > p_1^F$. Lemma 2(ii) states that p_1^L decreases more significantly than p_1^F as Δ_1 increases. Thus, $p_1^L < p_1^F$ holds when Δ_1 is large. These yield Lemma 3(ii).

We now explain the intuition. As a leader, firm 0 has a stronger strategic incentive to raise p_1 from p_1^B in order to reduce negative externality due to firm 1's production, when Δ_1 is larger. Thus, $p_0^L - p_0^B$ is larger when Δ_1 is larger. Because of the strategic complementarity, $p_1^F - p_1^B$ is larger when Δ_1 is larger.

p_0^B decreases as Δ_1 increases because firm 0 has a stronger incentive to reduce q_1. p_1^B also decreases as Δ_1 increases because of the strategic complementarity. The lower price levels result in larger q_0 and

q_1. As a leader, firm 1 has a weaker strategic incentive to raise p_1 from p_1^B because it leads to larger profit distortion due to the larger q_1. Thus, $p_1^L - p_1^B$ is smaller when Δ_1 is larger. Because of the strategic complementarity, $p_0^F - p_0^B$ is smaller when Δ_1 is larger. Under these conditions, $p_0^L > p_0^F$ and $p_1^L < p_1^F$ hold when Δ_1 is large.

When Δ_1 is zero, $p_0^L < p_0^F$ and $p_1^L > p_1^F$ hold, and the signs of the inequalities are reversed when Δ_1 is large. Δ_1 directly affects firm 0's payoff, and it affects firm 1's payoff indirectly, only through the strategic interaction between firms 0 and 1. Thus, a change of Δ_1 affects firm 0's behavior more significantly than firm 1's. As a result, the reverse of the inequality is more likely to take place in p_0 than in p_1.

Figure 1-1(1-2) illustrates the relationship between the equilibrium price of firm 0 (firm 1) and Δ_1.

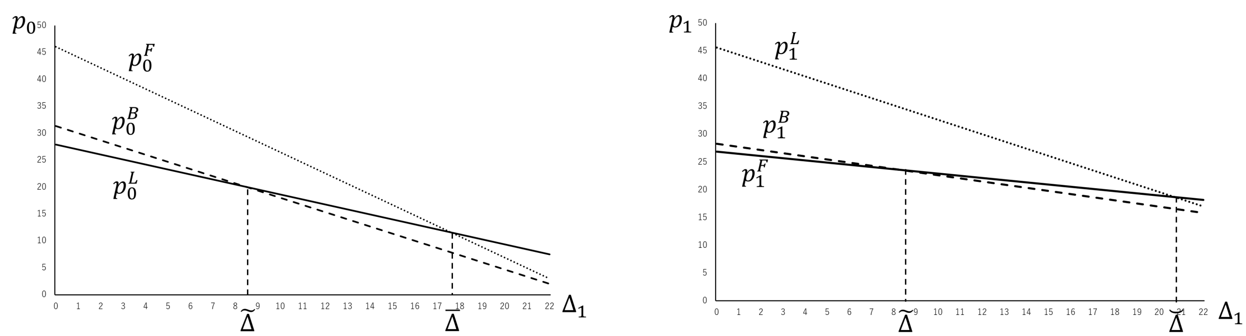

Figure 1-1 Figure 1-2

Figure 1: Numerical examples of equilibrium prices of public firm and private firm where $a = 100$, $s_0 = 20$, $c_1 = 15$, $b = 0.85$, and $\theta = 0$.

We now present one of our main results. Proposition 1 describes the equilibrium roles in the observable delay game.

Proposition 1 *Suppose that $\theta < 1$. (i) There exists $\Delta_a > 0$ such that in equilibrium $t_0 = t_1 = 1$ ($t_0 \neq t_1$) if $\Delta_1 < \Delta_a$ ($\Delta_1 > \Delta_a$). In other words, Bertrand equilibrium appears if and only if the cost difference between social and private marginal costs in the private firm is small. (ii) There exists $\Delta_b (\geq \Delta_a)$ such that a unique Stackelberg equilibrium exists (two Stackelberg equilibria exist) if $\Delta_1 \in (\Delta_a, \Delta_b)$ (if $\Delta_1 \geq \Delta_b$). In other words, a unique Stackelberg equilibrium exists if the cost difference lies in the* *middle range, whereas two Stackelberg equilibria exist if the cost difference is large. (iii) Suppose that* $\Delta_1 \in (\Delta_a, \Delta_b)$*. There exists* $\theta_a < 1/2$ *such that in equilibrium* $(t_0, t_1) = (1, 2)$ *(*$(t_0, t_1) = (2, 1)$*) if* $\theta < \theta_a$ *(*$\theta > \theta_a$*), and* $\Delta_a = \Delta_b$ *if* $\theta = \theta_a$*. In other words, when a unique Stackelberg equilibrium exists, the equilibrium is public leadership if the foreign share in the private firm is small, and the public leadership equilibrium is never the unique equilibrium if the foreign ownership share exceeds 50%.*

Proof See Appendix B.

Proposition 1(i) states that whether Bertrand or Stackelberg emerges in equilibrium depends on the difference between the social and private marginal costs in the private firm. If there is no difference or if the private marginal cost exceeds the social marginal cost (i.e., the cost difference Δ_1 is negative), then Bertrand emerges regardless of the foreign ownership share in the private firm or the degree of product differentiation. However, if the cost difference Δ_1 exceeds the threshold value, then Stackelberg emerges. This implies that if there are negative externalities of production or if the private firm's production is subsidized, Stackelberg can emerge in equilibrium. Proposition 1(iii) states that two Stackelberg equilibria (public leadership and private leadership equilibria) emerge if the cost difference Δ_1 exceeds another threshold value. Proposition 1(iii) states that public leadership is more likely to become a unique equilibrium than private leadership when the foreign ownership share in the private firm is lower. This result may explain the recent revival of public leadership by Japanese public financial institutions, as discussed in the Introduction.

We now explain the intuition behind Proposition 1. Suppose that Δ_1 is small. Lemma 1(i) states that $p_0^L < p_0^B$. Given $t_0 = 1$, firm 1 has an incentive to prevent firm 0's leadership by choosing $t_1 = 1$. Therefore, the public leadership equilibrium does not emerge. Lemma 1(iii) states that $p_1^L > p_1^B$. As we explain in the intuitive explanation for Lemma 1(i), firm 0 prefers firm 1 to name a lower price when Δ_1 is small. Thus, given $t_1 = 1$, firm 0 has an incentive to prevent firm 1's leadership by choosing $t_0 = 1$. Therefore, the private leadership equilibrium does not emerge. Because neither public nor private leadership emerges in equilibrium, the only equilibrium outcome is Bertrand.

Suppose that Δ_1 is large. Lemma 1(i) states that $p_0^L > p_0^B$. Given $t_0 = 1$, firm 1 chooses $t_1 = 2$ to raise p_0. Therefore, the public leadership equilibrium emerges. As we explain in the intuitive explanation for Lemma 1(i), firm 0 prefers a higher p_1 when Δ_1 is large. Thus, given $t_1 = 1$, firm 0 chooses $t_0 = 2$ to raise p_1. Therefore, the private leadership equilibrium emerges.

Suppose that Δ_1 is intermediate and θ is small. Given $t_1 = 1$, choosing $t_0 = 2$ raises p_1. Although a marginal increase in p_1 from p_1^B improves welfare, p_1^L can be too high for welfare, and it is possible that p_1^B is better for welfare than p_1^L. In this case, private leadership fails to result in equilibrium, and the unique equilibrium is public leadership.

Suppose that Δ_1 is intermediate and that θ is large. Firm 1's output level is more likely to be too high (price level is too low) for welfare when θ is larger because the higher market share of firm 1 increases the outflow of the surplus to foreign investors. Thus, it is less likely that p_1^B $(< p_1^L)$ is better than p_1^L for welfare. Therefore, the private leadership equilibrium is more likely to survive. This effect can be so strong that the private leadership equilibrium exists even when the condition for the existence of the public leadership equilibrium is not satisfied. For this reason, the unique equilibrium can involve private leadership when θ is large.

We now present a result when $\theta = 1$ (i.e., when the private firm is completely foreign owned), which is not covered by our main result (Proposition 1).

Proposition 2 *Suppose that $\theta = 1$. (i) $V_0^B = V_0^F$ and $V_1^L = V_1^B$ (i.e., the public leadership game yields the same outcome as Bertrand game). Moreover, $V_0^L = V_0^B = V_0^F$ and $V_1^L = V_1^B = V_1^F$ when $\Delta_1 = \tilde{\Delta}$ (i.e., the three games may yield the same equilibrium outcomes). (ii) Bertrand is an equilibrium outcome if $\Delta_1 \leq \tilde{\Delta}$. (iii) Public leadership is an equilibrium outcome if $\Delta_1 \geq \tilde{\Delta}$. (iv) Private leadership is an equilibrium outcome regardless of Δ_1.*

Proof See Appendix B.

From (1), we find that the public firm's optimal price does not depend on p_1 when $\theta = 1$. When $\theta < 0$, firm 0's reaction curve is upward sloping. A higher p_1 reduces q_1, which is suboptimal for welfare. To mitigate this welfare loss, firm 0 chooses higher p_0 and raises q_1 when p_1 is higher. However, firm 1's reaction curve becomes less steep when the foreign ownership share in firm 1 is larger. When p_1 is higher, the outflow of the private firm's profit to foreign owners is more significant. To restrict this profit outflow, firm 0 has a stronger incentive to raise its own output. This effect partially cancels out the above effect, and these two effects are completely canceled out when $\theta = 1$. This is why firm 0's reaction function becomes flat when $\theta = 1$. Because the public firm's reaction curve is flat, as the leader, the private firm cannot affect p_0 and thus, chooses the same price as in the Bertrand case. This leads to Proposition 2(i).

Proposition 2(ii) states that Bertrand equilibrium emerges when Δ_1 is small; whereas the public leadership equilibrium emerges when Δ_1 is large. These results are the same as those for Proposition 1(ii). However, Proposition 2(iv) states that the private leadership equilibrium emerges regardless of Δ_1, which is in sharp contrast to Proposition 1(ii). Thus, some readers might perceive that there is a discontinuity with respect to θ at the point where $\theta = 1$.

We consider that the discrepancy between Propositions 1 and 2 is smaller than what it appears at first glance. In the private leadership equilibrium, both firms adopt weakly dominated strategies when $\Delta_1 < \tilde{\Delta}$. When $\Delta_1 < \tilde{\Delta}$, both Bertrand and the private leadership equilibria exist. However, Bertrand equilibrium is risk dominant. Thus, from the viewpoint of the standard discussion of equilibrium selection (Harsanyi and Selten, 1988), it is natural to focus on Bertrand equilibrium, not the private leadership equilibrium, when Δ_1 is small, and this result is similar to that of Proposition 1.

The result shows a possible risk of using the model with pure foreign private firms in mixed oligopolies. The result that the private leadership equilibrium always exists holds only when there is no domestic investor in the private firm.

5 Welfare and Profit Ranking

We now present another main result (Proposition 3), which concerns welfare and profit ranking in public and private leadership. Let

$$\begin{aligned}
\Delta_c &:= \frac{(a(1-b)+bs_0-c_1)[(1-b^2)(6-5b^2-2\theta(2-5b^2))-b^2\theta^2(4-7b^2+2b^2\theta))-X_4]}{(4-3b^4)(1-b^2)+8b^4(1-b^2)\theta(1-\theta)+b^4\theta^2(4-b^2-2b^2\theta)}, \\
\Delta_d &:= \frac{(a(1-b)+bs_0-c_1)[5b^4-12b^2+8+8b^2\theta(1-b^2)+4b^4\theta^2-X_5]}{b^2(4b^4\theta^2+12b^2\theta(1-b^2)+9b^4-20b^2+12)}. \\
\Delta_e &:= \frac{(a(1-b)+bs_0-c_1)[(1-b^2)(6-5b^2-2\theta(2-5b^2))-b^2\theta^2(4-7b^2+2b^2\theta))+X_4]}{(4-3b^4)(1-b^2)+8b^4(1-b^2)\theta(1-\theta)+b^4\theta^2(4-b^2-2b^2\theta)}.
\end{aligned}$$

Before presenting our main result, we present a supplemental and technical result on Δ_c, Δ_d, and Δ_e, which is indispensable for our main result, Proposition 3.

Lemma 4 *Suppose that* $\theta < 1$. *(i)* $\Delta_c, \Delta_d, \Delta_e > 0$. *(ii)* $\Delta_c < \Delta_e$.

Proof See Appendix B

We now present our main result.

Proposition 3 *Suppose that* $\theta < 1$. *(i)* $V_0^L < V_0^F$ *if and only if* $\Delta_1 \in (\Delta_c, \Delta_e)$ *(i.e., public leadership yields greater welfare than private leadership if the cost difference between social and private marginal costs in the private firm is small or large). (ii)* $V_1^L > V_1^F$ *if and only if* $\Delta_1 < \Delta_d$ *(i.e., private leadership yields greater profit for the private firm than public leadership if the cost difference is small).*

Proof See Appendix B.

Suppose that $\theta < 1$. Public leadership is better than private leadership for welfare ($V_0^L > V_0^F$) when the difference between social and private marginal costs Δ_1 is small ($\Delta_1 < \Delta_c$). Private leadership becomes better than public leadership for social welfare when the cost difference reaches a threshold value (Δ_c). However, as the cost difference becomes larger and reaches yet another threshold value (Δ_e), public leadership again becomes better than private leadership for welfare. In other words, there is a nonmonotonic relationship between the advantage of public leadership and the cost difference between social and private marginal costs (Proposition 3(i)).

A similar nonmonotonic relationship does not emerge in the ranking of the private firm's profit (Proposition 3(ii)). Private leadership yields greater profit for firm 1 than public leadership ($V_1^L > V_1^F$) when the cost difference between social and private marginal costs is small ($\Delta_1 < \Delta_d$). Public leadership becomes better for the private firm than private leadership when the cost difference exceeds a threshold value (Δ_d).

Moreover, Proposition 3 implies that public leadership is payoff dominant to private leadership when $\Delta_1 > \max\{\Delta_d, \Delta_e\}$. In addition, if $\Delta_c < \Delta_d$, private leadership can be payoff dominant to public leadership (private leadership is payoff dominant to public leadership when $\Delta_1 \in (\Delta_c, \min\{\Delta_d, \Delta_e\})$). Although we fail to prove that $\Delta_c \leq \Delta_d$ always holds, we numerically show that the inequality $\Delta_c < \Delta_d$

holds for a wide range of parameter values.[14]

As Matsumura and Ogawa (2009) showed, payoff dominance implies risk dominance in the observable delay game. Therefore, our results show that both private and public leadership can be risk dominant and payoff dominant. This implies that both types of leadership can be robust, depending on the cost difference between social and private marginal costs. These results are in sharp contrast to those of Capuano and De Feo (2010) and Matsumura and Ogawa (2010), and they may explain the recent fluctuations in the Japanese government's policy regarding public financial institutions, as discussed in the Introduction.

We now explain the intuition behind the results on welfare ranking in Proposition 3. As the leader, firm 0 can choose $p_0 = p_0^B$. This implies that $V_0^L \geq V_0^B$, regardless of Δ_1 and θ. As the leader, firm 1 chooses $p_1 = p_1^L > p_1^B$. We have already explained that p_1^L is too high for welfare when Δ_1 is small. Thus, $V_0^F < V_0^B$. Therefore, $V_0^L > V_0^F$ when Δ_1 is small.

As Δ_1 reaches a threshold value, p_1^L becomes optimal for welfare. Thereafter, p_1^L becomes too low for welfare as Δ_1 increases. As the leader, firm 1 chooses $p_1 = p_1^L > p_1^B$, the welfare advantage of private leadership increases, and eventually $V_0^L < V_0^F$ holds. Note that $p_0^L \neq R(p_1^F)$ and $p_0^F = R(p_1^L)$. In other words, the public firm chooses the optimal price with p_1 as the follower, but it does not choose the optimal price with p_1 as the leader. Therefore, private leadership advantage appears as long as p_1^L is close to the optimal price for welfare.

When Δ_1 increases further, the difference between welfare optimal p_1 and p_1^L becomes larger, which worsens the welfare performance of private leadership. Therefore, the advantage of public leadership again emerges when Δ_1 is very large.

We then explain the intuition behind the results on profit ranking in Proposition 3. As the leader, firm 1 can choose $p_1 = p_1^B$. This implies that $V_1^L \geq V_1^B$ regardless of Δ_1 and θ. Moreover, because $p_1^L \neq p_1^B$, $V_1^L > V_1^B$ holds. As we show, $p_0^L < p_0^B$ and thus $V_1^F < V_1^B$, when Δ_1 is small. Therefore, $V_1^L > V_1^F$ when Δ_1 is small.

As Δ_1 reaches a threshold value, $p_0^L = p_0^B$ (and thus $V_1^F = V_1^B$) holds, and thereafter, $p_0^L > p_0^B$

[14]For example, we fix $a = 100$, $s_0 = 10$, $c_1 = 5$, and set $b = 0.1, 0.4, 0.6$, and 0.9. In these cases, $\Delta_c < \Delta_d$ holds for any $\theta \in [0, 1)$. We do not find a numerical example in which the inequality $\Delta_c > \Delta_d$ holds.

(and thus $V_1^F > V_1^B$) holds as Δ_1 increases. The profit advantage of public leadership increases as Δ_1 increases, and eventually $V_1^L < V_1^F$ holds. Note that $p_1^L \neq R(p_0^F)$ and $p_1^F = R(p_0^L)$. In other words, the private firm chooses the optimal price given p_0 as the follower, but it does not choose the optimal price given p_0 as the leader. Therefore, public leadership advantage emerges.

We now present a result when $\theta = 1$, which is not covered by our main result (Proposition 3). Before presenting Proposition 4, we present a supplementary result (Lemma 5), which is helpful for understanding Proposition 4.

Lemma 5 *If $\theta = 1$, then $\Delta_c = \Delta_d = \Delta_e = \tilde{\Delta}$.*

Proof See Appendix B.

Proposition 4 *Suppose that $\theta = 1$. (i) $V_0^L \geq V_0^F$ and the equality holds if and only if $\Delta_1 = \Delta_d$ (i.e., private leadership never yields greater welfare than public leadership). (ii) $V_1^L > V_1^F$ if and only if $\Delta_1 < \Delta_d$ (i.e., the private leadership yields greater profit for the private firm than the public leadership when the cost difference is small).*

Proof See Appendix B.

Proposition 4(ii) states that private leadership yields greater profits for the private firm than public leadership when Δ_1 is small, which is the same result as in Proposition 3(ii).

Proposition 4(i) states that public leadership always yields greater welfare than private leadership, in contrast to Proposition 3. Proposition 4 is a degenerated version of Proposition 3. Because all Δ_c, Δ_d, and Δ_e converge to the same value when $\theta \to 1$ (Lemma 5), the range of Δ_1 for the welfare advantage of private leadership over public leadership, (Δ_c, Δ_e), disappears as $\theta \to 1$ (Figure 2). This result again shows a possible risk of using the model with pure foreign private firms in mixed oligopolies. The result that public leadership always yields greater welfare holds only when there is no domestic investor in the private firm.

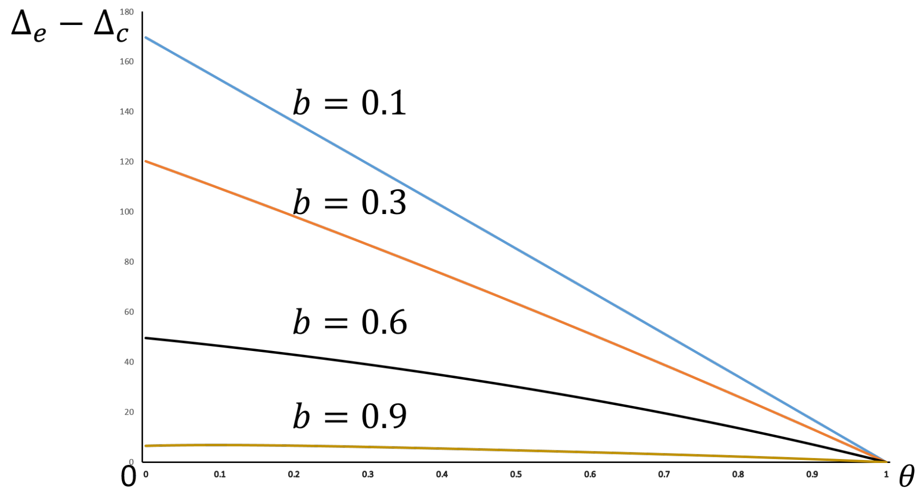

Figure 2: Numerical examples of $\Delta_e - \Delta_c$ where $a = 100$, $s_0 = 10$, and $c_1 = 5$.

The intuition behind Proposition 4(ii) is as follows. As stated in the previous section, firm 0's optimal price does not depend on p_1. Therefore, Bertrand equilibrium and private leadership equilibrium yield the same prices, and thus, the same profits and welfare. Because as the leader, firm 0 can choose p_0^B (and then firm 1 chooses p_1^B), the welfare in the public leadership equilibrium is never smaller than that in the private leadership equilibrium.

Proposition 4(i) states that profit ranking and welfare ranking depend on the sign of $p_0^L - p_0^B$. Remember that $p_0^L < (>, =) p_0^B$ if $\Delta_1 < (>, =) \tilde{\Delta}$. Because $V_0^L = V_0^B$ holds only when $p_0^L = p_0^B$, and $V_0^B = V_0^F$ always holds when $\theta = 1$, welfare ranking depends only on whether $p_0^L = p_0^B$. This leads to $\Delta_c = \Delta_e = \tilde{\Delta}$. Because $V_1^F > (<, =) V_1^B$ holds only when $p_0^L > (<, =) p_0^B$ and $V_1^L = V_1^B$, profit ranking depends only on the sign of $p_0^L - p_0^B$. This leads to $\Delta_d = \tilde{\Delta}$. Remember that Δ_d is a threshold value determining the sign of $V_1^L - V_1^F$ and that $\tilde{\Delta}$ is a threshold value determining the sign of $p_0^L - p_0^F$.

6 Quantity Competition

In this section, we briefly discuss quantity competition for a robustness check. The firms choose their outputs. The inverse demand function is given by $p_i = a - q_i - bq_j$ $(i = 0, 1,\ i \neq j)$. All other settings are the same as in the previous sections. The detailed calculation process and proofs of propositions are presented in Appendix C. First, we describe the equilibrium roles in the observable delay game in quantity competition.

Proposition 5 *Suppose that $\theta < 1$. (i) There exists Δ_f such that in equilibrium $t_0 = t_1 = 1$ ($t_0 \neq t_1$) if $\Delta_1 > \Delta_f$ ($\Delta_1 < \Delta_f$). In other words, Cournot equilibrium appears if and only if the cost difference between social and private marginal costs in the private firm is large. (ii) There exists $0 < \Delta_g (\leq \Delta_f)$ such that a unique Stackelberg equilibrium exists (two Stackelberg equilibria exist) if $\Delta_1 \in (\Delta_g, \Delta_f)$ (if $\Delta_1 \leq \Delta_g$). In other words, a unique Stackelberg equilibrium exists if the cost difference in the private firm lies in the middle range, whereas two Stackelberg equilibria exist if cost difference in private firm is small. (iii) Suppose that $\Delta_1 \in (\Delta_g, \Delta_f)$. There exists $\theta_a < 1/2$ such that in equilibrium $(t_0, t_1) = (1, 2)$ ($(t_0, t_1) = (2, 1)$) if $\theta < \theta_a$ ($\theta > \theta_a$), and $\Delta_g = \Delta_f$ if $\theta = \theta_a$. In other words, when a unique Stackelberg equilibrium exists, the equilibrium is public leadership if the foreign share in the private firm is small, and public leadership is never the unique equilibrium if the foreign ownership exceeds 50%.*

Proof See Appendix C.

We present a result when $\theta = 1$, which is not covered by Proposition 5.

Proposition 6 *Suppose that $\theta = 1$. (i) There exists $\dot{\Delta}$ such that Cournot (Public leadership) is an equilibrium outcome if $\Delta_1 \geq \dot{\Delta}$ (if $\Delta_1 \leq \dot{\Delta}$). (ii) Private leadership is an equilibrium outcome regardless of Δ_1.*

Proof See Appendix C.

Propositions 5 and 6 are similar to Propositions 1 and 2. Any equilibrium timing patterns– Cournot, Stackelberg with private leadership, Stackelberg with public leadership, and multiple Stackelberg equilibria– emerge, and public leadership is an equilibrium when the foreign ownership share in private firms is small. However, one important difference exists. Under price competition, Bertrand emerges in equilibrium when the difference between social and private marginal costs is small, whereas under quantity competition, Cournot emerges in equilibrium when the difference between social and private marginal costs is large.

We now describe welfare and profit ranking in public and private leadership. Let

$$\begin{aligned}
\Delta_h &:= -\frac{(a(1-b)+bs_0-c_1)[(1-2\theta)+(1-\theta)\sqrt{4-3b^2+2b^2\theta}]}{2(1-b^2(1-\theta))}, \\
\Delta_i &:= -\frac{(a(1-b)+bs_0-c_1)[-4(1-b^2(1-\theta))+(4-3b^2+2b^2\theta)\sqrt{1-b^2(1-\theta)}]}{2(1-b^2(1-\theta))}, \\
\Delta_j &:= -\frac{(a(1-b)+bs_0-c_1)[(1-2\theta)-(1-\theta)\sqrt{4-3b^2+2b^2\theta}]}{2(1-b^2(1-\theta))}.
\end{aligned}$$

Proposition 7 *Suppose that* $\theta < 1$. *(i)* $\Delta_h < \Delta_j$. *(ii)* $V_0^L > V_0^F$ *if and only if* $\Delta_1 \in (\Delta_h, \Delta_j)$ *(i.e., private leadership yields greater welfare than public leadership if the cost difference between social and marginal cost in the private firm is large or small). (iii)* $V_1^L < V_1^F$ *if and only if* $\Delta_1 < \Delta_i$ *(i.e., public leadership yields grater profit for the private firm if the cost difference is small).*

Proof See Appendix C.

We present a result when $\theta = 1$, which is not covered by Proposition 7.

Proposition 8 *Suppose that* $\theta = 1$ *(i)*$\Delta_h = \Delta_i = \Delta_j$. *(ii)* $V_0^L \geq V_0^F$ *and the equality holds if and only if* $\Delta_1 = \Delta_i$ *(i.e., private leadership never yields greater welfare than public leadership). (iii)* $V_1^L < V_1^F$ *if and only if* $\Delta_1 < \Delta_i$ *(i.e., public leadership yields greater profit for the private firm than private leadership if the cost difference is small).*

Proof See Appendix C.

Proposition 7 and 8 are similar to Propositions 3 and 4. There is a nonmonotonic relationship between the welfare advantage of public and private leadership and the difference between social and private marginal costs for a private firm. A nonmonotonic relationship does not emerge in profit ranking. However, two important differences exist. Under price (quantity) competition, public leadership is better (worse) for welfare than private leadership is when the difference between social and private marginal costs is either close to zero or quite large. Moreover, under price (quantity) competition, private leadership yields greater profit for the private firm than public leadership if Δ_1 is small (large).

7 Concluding Remarks

In this study, we investigate endogenous timing in a mixed duopoly with price competition when social marginal costs are allowed to differ from private marginal costs. We find that any equilibrium timing patterns—Bertrand, Stackelberg with private leadership, Stackelberg with public leadership, and multiple Stackelberg equilibria—emerge. When the foreign ownership share in a private firm is less than 50%, public leadership is more likely to emerge than private leadership. Conversely, private leadership can emerge in a unique equilibrium when the foreign ownership share in a private firm is large. These results may explain recent policy changes in public financial institutions in Japan. We also find there is a nonmonotonic relationship between the welfare advantage of public and private leadership and the difference between the social and private marginal costs for a private firm. A nonmonotonic relationship does not emerge in profit ranking.

We also investigate quantity competition and find the following similar results. Any equilibrium timing pattern can emerge depending on cost parameter and foreign ownership share in the private firm. Moreover, there is a nonmonotonic relationship between the welfare advantage of public and private leadership and the difference between social and private marginal costs for a private firm, and such a nonmonotonic relationship does not emerge in profit ranking.

However, the equilibrium and payoff ranking have the opposite direction under quantity competition. Under price (quantity) competition, the simultaneous-move equilibrium appears if and only if the cost difference between the social and private marginal costs of the private firm is small (large). Under price competition (quantity), public (private) leadership is better for welfare than private (public) leadership if the cost difference is small or large. Moreover, under price (quantity) competition, private leadership yields greater (smaller) profit for firm 1 than public leadership if the cost difference is small.

We also show some results that indicate a possible risk of using the model with pure foreign private firms in mixed oligopolies. For example, we show that under price competition, public leadership yields greater welfare than private leadership regardless of cost and demand parameters when the private firm is completely owned by foreign investors, but this result does not hold if the domestic ownership share is positive. In the literature on mixed oligopolies, the assumption that the private firm is completely owned by foreign investors is often adopted, but a robustness check for this assumption is important in mixed oligopolies.

In this study, we assume that the private and social costs are exogenous, but this assumption may be restrictive. The cost difference between private and social marginal costs depends on public policies such as tax/subsidy policies, licensing strategies of firms, and R&D investments by both public and private sectors. Future research should extend our analysis in this direction.

Moreover, we consider a single market model. As Haraguchi *et al.* (2018) pointed out, public firms face competitive pressure from neighboring markets, and thus, extension of our analysis to a multi-market model presents an opportunity for future research.[15]

In this study, we also consider a duopoly model. Extending our analysis to n-firm oligopolies is beyond the scope of the current study, and this is left for future research, although it would be quite a difficult assignment.

[15] For discussions on optimal privatization policy in multi-market models, see Bárcena-Ruiz and Garzón (2017), and Dong *et al.* (2018).

Appendix A

$$
\begin{aligned}
X_1 &:= -(1-b^2)(a-c_1)b^2(a(1-2b)+2bs_0-c_1)\theta^2 - [2b^2(1-b^2)(a(1-b)+bs_0-c_1)\Delta_1 \\
&- 2b^3(1-b^2)\Delta_0^2 + 4b(1-b^2)(a(1+b-b^2)-bc_0-(1-b^2)c_1)\Delta_0 + 2(1-b^2)^2c_1^2 \\
&- 4(1-b^2)^2(a(1-b)+bc_0)c_1 - 2b^2(1-b^2)c_0^2 + 4a(1-b^2)(1+b-b^2)c_0 \\
&+ 2a^2(1-b^2)(1-2b-2b^2+2b^3)]\theta + b^4\Delta_1^2 - 2(2-2b^2+b^4)(a(1-b)+bs_0-c_1)\Delta_1 \\
&+ (2b^4-5b^2+4)\Delta_0^2 - 2((b^5-3b^3+3b)c_1-(2b^4-5b^2+4)c_0-(b^5-2b^4-3b^3+5b^2+3b-4)a)\Delta_0 \\
&+ (b^4-3b^2+3)c_1^2 - 2(3-3b^2+b^4)(a(1-b)+bc_0)c_1 + (2b^4-5b^2+4)c_0^2 \\
&- 2a(1-b)(b^4-b^3-4b^2+b+4)c_0 + a^2(1-b)(2b^4-b^3-7b^2+b+7), \\
X_2 &:= -[4b(1-b^2)(a-c_1)\Delta_0 + 2(1-b^2)c_1^2 - 4(1-b^2)(a(1-b)+bc_0)c_1 + 4ab(1-b^2)c_0 \\
&+ 2a^2(1-b^2)(1-2b)]\theta + b^2\Delta_1^2 - 2(2-b^2)(a(1-b)+bs_0-c_1)\Delta_1 + (2-b^2)^2\Delta_0^2 \\
&+ 2((2b^3-3b)c_1+(2-b^2)^2c_0-(b^4+2b^3-4b^2-3b+4)a)\Delta_0 - (2b^2-3)c_1^2 \\
&- 2(3-2b^2)(a(1-b)+bc_0)c_1 + (2-b^2)^2c_0^2 - 2a(1-b)(4+b-3b^3-b^3)c_0 \\
&+ a^2(1-b)(7+b-5b^2-b^3), \\
X_3 &:= [b^6\Delta_1^2 - 2b^4(a(1-b)+bs_0-c_1)\Delta_1 + b^4\Delta_0^2 - 2b^3((4b^2-3)c_1-bc_0+(3+b-4b^2)a)\Delta_0 \\
&+ b^2(4b^2-3)c_1^2 + 2b^2(3-4b^2)(a(1-b)+bc_0)c_1 + b^4c_0^2 - 2ab^3(1-b)(3+4b)c_0 \\
&+ a^2b^2(1-b)(8b^2+3b-3)]\theta^2 + [2b^4(1-b^2)\Delta_1^2 - 8b^2(1-b^2)(a(1-b)+bs_0-c_1)\Delta_1 \\
&+ 6b^2(1-b^2)\Delta_0^2 - 4b(1-b^2)((4b^2-1)c_1-3bc_0+(1+3b-4b^2)a)\Delta_0 - 2(1-b^2)(1-4b^2)c_1^2 \\
&+ 4(1-b^2)(1-4b^2)(a(1-b)+bc_0)c_1 + 6b^2(1-b^2)c_0^2 - 4ab(1-b)^2(1+b)(1+4b)c_0 \\
&- 2a^2(1-b^2)(1+b)(1-b-8b^2)]\theta + b^4(b^2-1)\Delta_1^2 - 2(1-b^2)(2-3b^2)(a(1-b)+bs_0-c_1)\Delta_1 \\
&+ (1-b^2)(4-5b^2)\Delta_0^2 + 2(1-b^2)((4b^3-3b)c_1-(5b^2-4)c_0-(4b^3-5b^2-3b+4)a)\Delta_0 \\
&- (1-b^2)(3-4b^2)c_1^2 - 2(1-b^2)(3-4b^2)(a(1-b)+bc_0)c_1 + (1-b^2)(4-5b^2)c_0^2 \\
&- 2a(1-b)^2(1+b)(4+b-4b^2)c_0 + a^2(1-b)^3(1+b)(7+8b),
\end{aligned}
$$

$$
\begin{aligned}
X_4 &:= 2(1-b^2)(1-\theta)(1-b^2(1-\theta))\sqrt{4-3b^2+2b^2\theta},\\
X_5 &:= 2(1-b^2)(4-3b^2+2b^2\theta)\sqrt{1-b^2(1-\theta)},\\
X_6 &:= (1-b^2)(4-3b^2)(a(1-b)+bs_0-c_1)+b^2(1-b^2)\theta(5(a(1-b)+bs_0-c_1)-3(4-b^2)\Delta_1)\\
&+ b^2\theta^2((2+b^2)(a(1-b)+bs_0-c_1)-3b^2(2-b^2)\Delta_1)+b^4\theta^3(a(1-b)+bs_0-c_1-b^2\Delta_1),\\
X_7 &:= [b^4-7b^2+8+4b^2\theta-b^4\theta]((1-b)a+bs_0-c_1)-(6b^2-4b^4+3b^4\theta)\Delta_1,\\
X_8 &:= (-2b^6\theta^3+b^4(7b^2-4)\theta^2+8b^4(1-b^2)\theta+3b^6-3b^4-4b^2+4)\Delta_1^2+2(2b^4\theta^3-b^2(7b^2-4)\theta^2\\
&+ 2(1-b^2)(2-5b^2)\theta-5b^4+11b^2-6)(a(1-b)+bs_0-c_1)\Delta_1-(2b^2\theta^3+b^2(1-4b^2)\theta^2\\
&+ 4(1-b^2)(1-2b^2)\theta-(1-b^2)(5-4b^2))(a(1-b)+bs_0-c_1)^2,\\
X_9 &:= b^2(4b^4\theta^2+12b^2(1-b^2)\theta+9b^4-20b^2+12)\Delta_1^2-2(4b^4\theta^2+8b^2(1-b^2)\theta+5b^4-12b^2+8)(a(1-b)\\
&+ bs_0-c_1)\Delta_1+(4b^2\theta^2+4b^2\theta(1-b^2)+4b^2-11b^2+8)(a(1-b)+bs_0-c_1)^2,\\
X_{10} &:= a^2(7-6b-2b^2(1-b))-((4-b^2)(2a-c_0)-6ab-b^2c_0)c_0-(3-b^2)(2a-c_1-2b(a-c_0))c_1\\
&- (2(4-b^2)(a-c_0)-2b(3-b^2)(a-c_1)-(4-b^2)\Delta_0)\Delta_0-2(2-b^2)(a-c_1-b(a-s_0))\Delta_1\\
&- 2\theta(a^2(1-2b-2b^2+2b^3)+b(2a(1-b^2)+b(2a-c_0))c_0-(1-b^2)(2a-c_1-2b(a-c_0))c_1\\
&+ b(2(a-c_1)(1-b^2)+b(2a-c_1-\Delta_0))\Delta_0+b^2(a-c_1-b(a-s_0))\Delta_1)\\
&- b^2\theta^2(a-c_1)(a-c_1-2b(a-s_0)),\\
X_{11} &:= (7-6b)a^2-2c_0(a(4-3b)-2c_0)-3c_1(2a-c_1-2b(a-c_0))-2\Delta_0(4(a-c_0)-3b(a-c_1)-2\Delta_0)\\
&- \Delta_1(4(a-c_0-b(a-s_0))+b^2\Delta_1)-2(a-c_1)(a-c_1-2b(a-s_0))\theta,\\
X_{12} &:= (1-b^2)(a^2(1-b)(7+b)-c_0(4(2a-c_0)-6ab-b^2(2a-c_0))-3c_1(2a-c_1-2b(a-c_0))\\
&- \Delta_0((4-b^2)(2a-2c_0-\Delta_0)-6b(a-c_1))-4\Delta_1(a-c_1-b(a-s_0)))\\
&- 2\theta(2b^2\Delta_1(a-c_1-b(a-s_0))+b\Delta_0(2(1-3b^2)(a-c_1)+b(3-b^2)(2a-2c_0-\Delta_0))\\
&+ bc_0(2a(1-3b^2)+b(2a-c_0)(3-b^2))-c_1(1-3b^2)(2a-c_1-2b(a-c_0))+a^2(1-b^2)(1-b-7b^2-b^3))\\
&+ b^2\theta^2(a^2(b^2+6b-3)+bc_0(b(c_0-2a)-6a)+3c_1(2a-c_1-2b(a-c_0))\\
&- b\Delta_0(6(a-c_1)+b(2a-2c_0-\Delta_0))),
\end{aligned}
$$

$$
\begin{aligned}
X_{13} &:= (-(1-b^2)(4-b^2) - b^2(7-2b^2)\theta + b^2(2-3b^2)\theta^2 + b^4\theta^3)(a - c_1 - b(a - s_0)) \\
&+ (4(1-b^2)(2-b^2) + 4b^2(3-2b^2)\theta + 4b^4\theta^2)\Delta_1, \\
X_{14} &:= (4 - 2b^2(1-\theta) - b^2)(a - c_1 - b(a - s_0)) + (2 - b^2(1-\theta))(2(a - c_1 - b(a - s_0)) - b^2\Delta_1), \\
X_{15} &:= (3(1-b^2) - 4(1-2b^2)\theta - 7b^2\theta^2 + 2b^2\theta^3)(a(1-b) + bs_0 - c_1)^2 \\
&- 4(1 - b^2 - (2-3b^2)\theta + 2b^2\theta^2)(a(1-b) + bs_0 - c_1)\Delta_1 - 4((1-b^2)^2 + 2b^2\theta + b^4\theta^2)\Delta_1^2, \\
X_{16} &:= (8 - 9b^2 + 12b^2\theta - 4b^2\theta^2)(a(1-b) + bs_0 - c_1)^2 - 16(1 - b^2(1-\theta))(a(1-b) + bs_0 - c_1)\Delta_1 \\
&+ 4b^2(1 - b^2(1-\theta))\Delta_1^2.
\end{aligned}
$$

Appendix B

Proof of Lemma 1

From (1) and (7), we obtain

$$\begin{aligned} p_0^B - p_0^L &= \frac{2b((1-b^2(1-\theta))(a(1-b)+bs_0-c_1)-(2(1-b^2)+b^2\theta)\Delta_1)}{(2-b^2(1-\theta))(4-3b^2+2b^2\theta)} > 0 \\ &\Leftrightarrow \Delta_1 < \tilde{\Delta} := \frac{(1-b^2(1-\theta))(a(1-b)+bs_0-c_1)}{2(1-b^2)+b^2\theta}. \end{aligned}$$

Because we suppose that $a > s_0 > c_1$, we obtain $a(1-b)+bs_0-c_1 = (a-c_1)-b(a-s_0) > 0$. This implies $\tilde{\Delta} > 0$. From (2) and (8), we obtain

$$\begin{aligned} p_1^B - p_1^F &= \frac{b^2((1-b^2(1-\theta))(a(1-b)+bs_0-c_1)-(2(1-b^2)+b^2\theta)\Delta_1)}{(2-b^2(1-\theta))(4-3b^2+2b^2\theta)} > 0 \\ &\Leftrightarrow \Delta_1 < \frac{(1-b^2(1-\theta))(a(1-b)+bs_0-c_1)}{2(1-b^2)+b^2\theta} = \tilde{\Delta}. \end{aligned}$$

These results imply Lemma 1(i).

Differentiating $\tilde{\Delta}$ with respect to θ yields

$$\frac{\partial\tilde{\Delta}}{\partial\theta} = \frac{b^2(1-b^2)(a(1-b)+bs_0-c_1)}{(2(1-b^2)+b^2\theta)^2} > 0.$$

This implies Lemma 1(ii).

Because we assume an interior solution in the price competition stage, from (4), we obtain $a(1-b)+bs_0-c_1-b^2\Delta_1 > 0$. From (2) and (12), we obtain

$$p_1^B - p_1^L = -\frac{(1-\theta)b^2(a(1-b)+bs_0-c_1-b^2\Delta_1)}{2(1-b^2(1-\theta))(2-b^2(1-\theta))} \leq 0, \tag{15}$$

and the equality in (15) holds if and only if $\theta = 1$. From (1) and (11), we obtain

$$p_0^B - p_0^F = -\frac{(1-\theta)^2b^3(a(1-b)+bs_0-c_1-b^2\Delta_1)}{2(1-b^2(1-\theta))(2-b^2(1-\theta))} \leq 0, \tag{16}$$

and the equality in (16) holds if and only if $\theta = 1$. These imply Lemma 1(iii). ■

Proof of Lemma 2

From (1), (7) and (11), we obtain

$$\frac{\partial p_0^B}{\partial \Delta_1} = -\frac{2b}{2-b^2(1-\theta)} < 0, \tag{17}$$

$$\frac{\partial p_0^L}{\partial \Delta_1} = -\frac{2b}{4-3b^2+2b^2\theta} < 0, \tag{18}$$

$$\frac{\partial p_0^F}{\partial \Delta_1} = -\frac{b(2-b^2(1-\theta))}{2(1-b^2(1-\theta))} < 0. \tag{19}$$

From (17)-(19), we obtain

$$\frac{\partial p_0^B}{\partial \Delta_1} - \frac{\partial p_0^L}{\partial \Delta_1} = -\frac{2b(2-b^2(2-\theta))}{(2-b^2(1-\theta))(4-3b^2+2b^2\theta)} < 0,$$

$$\frac{\partial p_0^F}{\partial \Delta_1} - \frac{\partial p_0^L}{\partial \Delta_1} = -\frac{b(3(1-b^2)^2+2b^2\theta(2(1-b^2)+b^2\theta)+1-b^4\theta)}{2(1-b^2(1-\theta))(4-3b^2+2b^2\theta)} < 0.$$

These results imply Lemma 2 (i).

From (2), (12) and (8), we obtain

$$\frac{\partial p_1^B}{\partial \Delta_1} = -\frac{b^2}{2-b^2(1-\theta)} < 0, \tag{20}$$

$$\frac{\partial p_1^L}{\partial \Delta_1} = -\frac{b^2}{2(1-b^2(1-\theta))} < 0, \tag{21}$$

$$\frac{\partial p_1^F}{\partial \Delta_1} = -\frac{b^2}{4-3b^2+2b^2\theta} < 0. \tag{22}$$

From (20)-(22), we obtain

$$\frac{\partial p_1^B}{\partial \Delta_1} - \frac{\partial p_1^F}{\partial \Delta_1} = -\frac{b^2(2-b^2(2-\theta))}{(2-b^2(1-\theta))(4-3b^2+2b^2\theta)} < 0,$$

$$\frac{\partial p_1^L}{\partial \Delta_1} - \frac{\partial p_1^F}{\partial \Delta_1} = -\frac{b^2(2-b^2)}{2(1-b^2(1-\theta))(4-3b^2+2b^2\theta)} < 0.$$

These results imply Lemma 2 (ii).■

Proof of Lemma 3

From (7) and (11), we obtain

$$p_0^L - p_0^F = \frac{-(2-b^2-b^2\theta+2b^2\theta^2)(a(1-b)+bs_0-c_1)+(4-6b^2+3b^4+b^2(4-5b^2)\theta+2b^4\theta^2)\Delta_1}{2(1-b^2(1-\theta))(4-3b^2+2b^2\theta)} < 0$$

$$\Leftrightarrow \quad \Delta_1 < \bar{\Delta} := \frac{(2-b^2-b^2\theta+2b^2\theta)(a(1-b)+bs_0-c_1)}{4-6b^2+3b^4+b^2(4-5b^2)\theta+2b^4\theta^2}.$$

Comparing $\tilde{\Delta}$ and $\bar{\Delta}$, we obtain

$$\bar{\Delta} - \tilde{\Delta} = \frac{b^2(1-b^2)(1-\theta)^2(a(1-b)+bs_0-c_1)(4-3b^2+2b^2\theta)}{(2-2b^2+b^2\theta)(4-6b^2+3b^4+4b^2-5b^4\theta+2b^4\theta^2)} \geq 0.$$

The equality holds if and only if $\theta = 1$. These results imply Lemma 3(i).

From (12) and (8), we obtain

$$\begin{aligned} p_1^L - p_1^F &= \frac{b^2((1+2(1-b^2)(1-\theta))(a(1-b)+bs_0-c_1)-(2-b^2)\Delta_1)}{2(1-b^2(1-\theta))(4-3b^2+2b^2\theta)} > 0 \\ &\Leftrightarrow \quad \Delta_1 < \breve{\Delta} := \frac{(1+2(1-\theta)(1-b^2))(a(1-b)+bs_0-c_1)}{2-b^2}. \end{aligned}$$

Comparing $\tilde{\Delta}$ and $\breve{\Delta}$, we obtain

$$\breve{\Delta} - \tilde{\Delta} = \frac{(1-b^2)(1-\theta)(4-3b^2+2b^2\theta)(a(1-b)+bs_0-c_1)}{(2-b^2)(2-b^2(1-\theta))} \geq 0.$$

The equality holds if and only if $\theta = 1$. These results imply Lemma 3(ii).

Comparing $\breve{\Delta}$ and $\bar{\Delta}$, we obtain

$$\breve{\Delta} - \bar{\Delta} = \frac{2(1-b^2)(1-\theta)(4-3b^2+2b^2\theta)(1-b^2(1-\theta))(a(1-b)+bs_0-c_1)}{(2-b^2)(4-6b^2+3b^4+4b^2\theta-5b^4\theta+2b^4\theta^2)} \geq 0.$$

The equality holds if and only if $\theta = 1$. These results imply Lemma 3(iii).■

Proof of Proposition 1

From (5), (6), (9), (10), (13), and (14), we obtain

$$V_0^L - V_0^B = \frac{b^2[(1-b^2(1-\theta))(a(1-b)+bs_0-(c_1+\Delta_1))+(1-b^2)\Delta_1]^2}{2(1-b^2)(2-b^2(1-\theta))^2(4-3b^2+2b^2\theta)} \geq 0, \tag{23}$$

$$V_0^B - V_0^F = \frac{b^2(1-\theta)(a(1-b)+bs_0-c_1-b^2\Delta_1)X_6}{8(1-b^2)(1-b^2(1-\theta))^2(2-b^2(1-\theta))^2}, \tag{24}$$

$$V_1^L - V_1^B = \frac{b^4(1-\theta)^2(a(1-b)+bs_0-c_1-b^2\Delta_1)^2}{4(1-b^2)(1-b^2(1-\theta))(2-b^2(1-\theta))^2} \geq 0, \tag{25}$$

$$V_1^B - V_1^F = \frac{b^2[(1-b^2(1-\theta))(a(1-b)+bs_0-(c_1+\Delta_1))+(1-b^2)\Delta_1]X_7}{(1-b^2)(2-b^2(1-\theta))^2(4-3b^2+2b^2\theta)^2}. \tag{26}$$

The equality in (23) holds if and only if $\Delta_1 = \tilde{\Delta}$. The equality in (25) holds if and only if $\theta = 1$.

Proof of Proposition 1(i)

Bertrand ($t_0 = t_1 = 1$) is an equilibrium if and only if both (24) and (26) are nonnegative, and Bertrand is the unique equilibrium if both are positive.

Equation (24) is positive (negative, zero) if $X_6 > (<, =) 0$. Solving the equation $X_6 = 0$ with respect to Δ_1, we obtain

$$\Delta_1 = \hat{\Delta} := \frac{((1-b^2)(4-3b^2) + 5b^2(1-b^2)\theta + b^2(2+b^2)\theta^2 + b^4\theta^3)(a(1-b) + bs_0 - c_1)}{(1-b^2)(b^4 - 8b^2 + 8) + 3b^2(1-b^2)(4-b^2)\theta + 3b^4\theta^2 + b^6\theta^3}.$$

We now show that X_7 is positive. Because we assume an interior solution in the price competition stage, from (4), we obtain

$$a(1-b) + bs_0 - c_1 - b^2\Delta_1 > 0 \Leftrightarrow \Delta_1 < \frac{a(1-b) + bs_0 - c_1}{b^2}.$$

$X_7 > 0$ if

$$\Delta_1 < \frac{(b^4 - 7b^2 + 8 + 4b^2\theta - b^4\theta)(a(1-b) + bs_0 - c_1))}{6b^2 - 4b^4 + 3b^4\theta}.$$

We obtain

$$\frac{a(1-b) + bs_0 - c_1}{b^2} - \frac{(b^4 - 7b^2 + 8 + 4b^2\theta - b^4\theta)(a(1-b) + bs_0 - c_1))}{6b^2 - 4b^4 + 3b^4\theta}$$
$$= -\frac{(1-b^2)(2 - b^2(1-\theta))(a(1-b) + bs_0 - c_1)}{b^2(6 - 4b^2 + 3b^2\theta)} < 0.$$

These imply $X_7 > 0$.

Because X_7 is positive, equation (26) is positive (negative, zero) if $(1-b^2(1-\theta))(a(1-b)+bs_0-(c_1+\Delta_1))+(1-b^2)\Delta_1 > (<, =) 0$. Solving the equation $(1-b^2(1-\theta))(a(1-b)+bs_0-(c_1+\Delta_1))+(1-b^2)\Delta_1 = 0$ with respect to Δ_1, we obtain

$$\Delta_1 = \frac{(1-b^2(1-\theta))((1-b)a + bs_0 - c_1)}{2(1-b^2) + b^2\theta} = \tilde{\Delta}.$$

Therefore, both (24) and (26) are positive if $\Delta_1 < \Delta_a := \min\{\hat{\Delta}, \tilde{\Delta}\}$. Because $(1-b)a + bs_0 - c_1 > 0$, we obtain $\hat{\Delta} > 0$ and $\tilde{\Delta} > 0$. Thus, $\Delta_a > 0$. ■

Proof of Proposition 1(ii)

Two Stackelberg equilibria exist (both $(t_0, t_1) = (1, 2)$ and $(t_0, t_1) = (2, 1)$ are the equilibrium outcomes) if and only if both (24) and (26) are nonpositive. Only one Stackelberg equilibrium exists (either $(t_0, t_1) = (1, 2)$ or $(t_0, t_1) = (2, 1)$ is the equilibrium outcome) if and only if one of (24) and (26) is nonpositive and the other is positive. Let $\Delta_b := \max\{\hat{\Delta}, \tilde{\Delta}\}$. One of (24) and (26) is positive and the other is negative if $\Delta_1 \in (\Delta_a, \Delta_b)$. Both are negative if $\Delta_1 > \Delta_b$. These imply Proposition 1(ii). ■

Proof of Proposition 1(iii)

As shown in the proof of Proposition 1(i), private leadership is an equilibrium if $\Delta_1 \geq \hat{\Delta}$, and public leadership is an equilibrium if $\Delta_1 \geq \tilde{\Delta}$. Therefore, if $\hat{\Delta} > \tilde{\Delta}$ ($\hat{\Delta} < \tilde{\Delta}$), the unique Stackelberg is public leadership (private leadership) when $\Delta_1 \in (\Delta_a, \Delta_b)$.

From $\hat{\Delta}$ and $\tilde{\Delta}$, we obtain

$$\hat{\Delta} - \tilde{\Delta} = \frac{b^2(1-b^2)(1-\theta)(2-b^2(1-\theta))[(1-b^2)(1-2\theta)-b^2\theta^2](a(1-b)+bs_0-c_1)}{(2-b^2(2-\theta))[(1-b^2)(8-8b^2+b^4)+3b^2(1-b^2)(4-b^2)\theta+3b^4(2-b^2)\theta^2+b^6\theta^3]}. \quad (27)$$

Equation (27) is positive (negative, zero) if $(1-b^2)(1-2\theta)-b^2\theta^2 > (<,=)0$. Solving the equation $(1-b^2)(1-2\theta)-b^2\theta^2 = 0$, we obtain

$$(1-b^2)(1-2\theta)-b^2\theta^2 = 0 \to \theta = \frac{-(1-b^2) \pm \sqrt{1-b^2}}{b^2}.$$

The positive solution is

$$\theta_a := \frac{-(1-b^2)+\sqrt{1-b^2}}{b^2}. \quad (28)$$

Therefore, we obtain $\hat{\Delta} > \ (<,=)\tilde{\Delta}$ if $\theta < (>,=)\theta_a$.

From (28), we obtain

$$\begin{aligned}
\frac{d\theta_a}{db} &= -\frac{2(1-\sqrt{1-b^2})}{b^3\sqrt{1-b^2}} < 0, \\
\theta_a &= \frac{-(1-b^2)+\sqrt{1-b^2}}{b^2} \to 0 \quad (b \to 1), \\
\theta_a &= \frac{-(1-b^2)+\sqrt{1-b^2}}{b^2} \\
&= \frac{1-b^2}{\sqrt{1-b^2}+1-b^2} \to \frac{1}{2} \quad (b \to 0).
\end{aligned}$$

Thus, $\theta_a \in (0, 1/2)$. These imply Proposition 1(iii). ■

Proof of Proposition 2

Substituting $\theta = 1$ into (24) and (25) we obtain Proposition 2(i).

Bertrand equilibrium emerges if both (24) and (26) are nonnegative. (24) is always zero when $\theta = 1$. (26) is nonnegative if and only if $\Delta_1 \leq \tilde{\Delta}$. These imply Proposition 2(ii).

Public leadership is an equilibrium outcome if (23) is nonnegative and (26) is nonpositive. (23) is always nonnegative. (26) is nonpositive if and only if $\Delta_1 \geq \tilde{\Delta}$. These imply Proposition 2(iii).

Private leadership is an equilibrium if (24) is nonpositive and (25) is nonnegative. Both (24) and (25) are zero when $\theta = 1$. These imply Proposition 2(iv). ■

Proof of Lemma 4

First, we show that $\Delta_c > 0$. $\Delta_c > 0$ if the numerator of Δ_c is positive. Let $A := (1-b^2)(6-5b^2-2\theta(2-5b^2)) - b^2\theta^2(4-7b^2+2b^2\theta) - X_4$. We obtain

$$\begin{aligned} A &= (1-b^2)(6-5b^2-2\theta(2-5b^2)) - b^2\theta^2(4-7b^2+2b^2\theta) - X_4 \\ &= (1-b^2)(6-5b^2-2\theta(2-5b^2)) - b^2\theta^2(4-7b^2+2b^2\theta) \\ &\quad -2(1-b^2)(1-\theta)(1-b^2(1-\theta))\sqrt{4-3b^2+2b^2\theta} \\ &\geq (1-b^2)(6-5b^2-2\theta(2-5b^2)) - b^2\theta^2(4-7b^2+2b^2\theta) - 4(1-b^2)(1-\theta)(1-b^2(1-\theta)) \\ &= (1-b^2)(2-b^2) + 2b^2(1-b^2)\theta + b^4(3-2\theta)\theta^2 > 0. \end{aligned}$$

The inequality in the fourth line follows from $\sqrt{4-3b^2+2b^2\theta} \in (1,2)$. This implies $\Delta_c > 0$.

Next, we show that $\Delta_c < \Delta_e$. Comparing Δ_c and Δ_e, we obtain

$$\Delta_e - \Delta_c = \frac{4(1-b^2)(1-b^2(1-\theta))(1-\theta)(a(1-b)+bs_0-c_1)\sqrt{4-3b^2+2b^2\theta}}{(4-3b^4)(1-b^2)+8b^4(1-b^2)\theta(1-\theta)+b^4\theta^2(4-b^2-2b^2\theta)} > 0. \tag{29}$$

This implies $\Delta_c < \Delta_e$.

We then show that $\Delta_d > 0$. $\Delta_d > 0$ if the numerator of Δ_d is positive. Let $B := 5b^4 - 12b^2 + 8 + 8b^2\theta(1-b^2) + 4b^4\theta^2 - X_5$. We obtain

$$\begin{aligned} B &= 5b^4 - 12b^2 + 8 + 8b^2\theta(1-b^2) + 4b^4\theta^2 - X_5 \\ &= 5b^4 - 12b^2 + 8 + 8b^2\theta(1-b^2) + 4b^4\theta^2 - 2(1-b^2)(4-3b^2+2b^2\theta)\sqrt{1-b^2(1-\theta)} \\ &\geq 5b^4 - 12b^2 + 8 + 8b^2\theta(1-b^2) + 4b^4\theta^2 - 2(1-b^2)(4-3b^2+2b^2\theta) \\ &= b^2(2-b^2) + 4b^2\theta(1-b^2(1-\theta)) > 0. \end{aligned}$$

The inequality in the third line follows from $\sqrt{1-b^2(1-\theta)} \in (0,1)$. This implies $\Delta_d > 0$. ■

Proof of Proposition 3(i)

From (9) and (13), we obtain

$$V_0^L - V_0^F = \frac{b^2 X_8}{8(1-b^2)(1-b^2(1-\theta))^2(4-3b^2+2b^2\theta)}. \tag{30}$$

Equation (30) is positive (negative, zero) if $X_8 > (<,=)0$. Solving the equation $X_8 = 0$ with respect to Δ_1, we obtain

$$X_8 = 0 \rightarrow \Delta_1 = \frac{(a(1-b)+bs_0-c_1)[((1-b^2)(6-5b^2-2\theta(2-5b^2)-b^2\theta^2(4-7b^2+2b^2\theta)) \pm X_4]}{(1-b^2)(4-3b^2+b^4\theta(4(2-\theta)+7b^2\theta+2b^2\theta^2))}.$$

Note that Δ_c and Δ_e are solutions of this equation.

Then, we obtain

$$\begin{aligned} X_8 &< 0 \text{ (and thus } V_0^L < V_0^F), \text{ if } \Delta_1 \in (\Delta_c, \Delta_e), \\ X_8 &= 0 \text{ (and thus } V_0^L = V_0^F), \text{ if } \Delta_1 = \Delta_c \text{ or } \Delta_e, \\ X_8 &> 0 \text{ (and thus } V_0^L > V_0^F), \text{ otherwise.} \end{aligned}$$

This implies Proposition 3(i). ■

Proof of Proposition 3(ii)

We examine the profit ranking. If $\Delta_1 < \tilde{\Delta}$, then $p_0^B > p_0^L$. Because $p_1^B = R_1(p_0^B)$, $p_1^F = R_1(p_0^L)$, and $\pi_1(p_0, R_1(p_0))$ is increasing in p_0, we obtain $V_1^F < V_1^B$. Because $V_1^L \geq V_1^B$, we obtain $V_1^F < V_1^L$.

If $\Delta_1 > \bar{\Delta}$, then $p_0^F < p_0^L$. Because $p_1^F = R_1(p_0^L)$, $\pi_1(p_0, R_1(p_0))$ is increasing in p_0, and $\pi_1(p_0, R_1(p_0)) \geq \pi_1(p_0, p_1)$ for any p_1, we obtain $V_1^F > V_1^L$.

We now investigate profit ranking when $\Delta_1 \in [\tilde{\Delta}, \bar{\Delta}]$. From (10) and (14), we obtain

$$V_1^L - V_1^F = \frac{b^2 X_9}{4(1-b^2)(1-b^2(1-\theta))^2(4-3b^2+2b^2\theta)^2}. \tag{31}$$

Equation (31) is positive (negative, zero) if $X_9 > (<,=)0$. We obtain $X_9 = 0$ if $\Delta_1 = \Delta_d \in (\tilde{\Delta}, \bar{\Delta})$. If $\Delta_1 < \Delta_d$, then $X_9 > 0$ holds, and thus, we obtain $V_1^L > V_1^F$. If $\Delta_1 \in [\Delta_d, \bar{\Delta}]$, then $X_9 < 0$ holds, and thus, we obtain $V_1^L < V_1^F$. ■

Proof of Lemma 5

We show that $\Delta_c = \Delta_d = \Delta_e$ if $\theta = 1$. Substituting $\theta = 1$ into $\Delta_c(\theta)$, $\Delta_d(\theta)$ and $\Delta_e(\theta)$, we obtain

$$\Delta_c(1) = \Delta_d(1) = \Delta_e(1) = \frac{a(1-b)+bs_0-c_1}{2-b^2} = \tilde{\Delta}(1).$$

This implies Lemma 5. ■

Proof of Proposition 4

First, we show that $V_0^L \geq V_0^F$, and the equality holds if and only if $\Delta_1 = \Delta_d$. From (23), we obtain that $V_0^L \geq V_0^B$, and the equality holds if and only if $\Delta_1 = \tilde{\Delta}$. From (24), we obtain that $V_0^B = V_0^F$ when $\theta = 1$. $\Delta_d = \tilde{\Delta}$ when $\theta = 1$. These imply Proposition 4(i).

In the proof of Proposition 3(ii), we do not use the condition $\theta < 1$. Therefore, Proposition 3(ii) holds when $\theta = 1$. This implies Proposition 4(ii). ■

Appendix C

Under Cournot competition, each firm maximizes its payoff V_i with respect to q_i. The first-order conditions are

$$\frac{\partial V_0}{\partial q_0} = a - s_0 - q_0 - b(1-\theta)q_1 = 0, \tag{32}$$

$$\frac{\partial V_1}{\partial q_1} = a - c_1 - bq_0 - 2q_1 = 0. \tag{33}$$

The second-order conditions are satisfied. From the first order conditions, we obtain the following reaction functions for firm 0 and 1, respectively:

$$R_0(q_1) = a - s_0 - b(1-\theta)q_1, \tag{34}$$

$$R_1(q_0) = \frac{a - c_1 - bq_0}{2}. \tag{35}$$

These reaction functions lead to the following equilibrium quantities:

$$q_0^C = \frac{2(a-s_0) - b(1-\theta)(a-c_1)}{2 - b^2(1-\theta)}, \tag{36}$$

$$q_1^C = \frac{(a-c_1) - b(a-s_0)}{2 - b^2(1-\theta)}, \tag{37}$$

where superscript C denotes the equilibrium outcome in Cournot competition. The resulting welfare and firm 1's profit are, respectively,

$$V_0^C = \frac{X_{10}}{2(2 - b^2(1-\theta))^2}, \tag{38}$$

$$V_1^C = \left(\frac{a - c_1 - b(a-s_0)}{2 - b^2(1-\theta)}\right)^2. \tag{39}$$

In the sequential-move game with public leadership, firm 1 chooses $q_1 = R_1(q_0)$, and firm 0 maximizes its payoff, $V_0(q_0, R_1(q_0))$. We obtain

$$q_0^L = \frac{4(a-s_0) - 3b(a-c_1) + 2b(a-c_1)\theta + 2b\Delta_1}{4 - b^2(3-2\theta)}, \tag{40}$$

$$q_1^F = \frac{2(a-c_1) - 2b(a-s_0) - b^2\Delta_1}{4 - b^2(3-2\theta)}. \tag{41}$$

The resulting welfare and firm 1's profit are, respectively,

$$V_0^L = \frac{X_{11}}{2(4 - b^2(3-2\theta))}, \tag{42}$$

$$V_1^F = \left(\frac{2(a-c_1) - 2b(a-s_0) - b^2\Delta_1}{4 - b^2(3-2\theta)}\right)^2. \tag{43}$$

In the sequential-move game with private leadership, firm 0 chooses $q_0 = R_0(q_1)$, and firm 1 maximizes its payoff, $V_1(R_0(q_1), q_1)$. We obtain

$$q_0^F = \frac{2(a-s_0) - b(1-\theta)(a - c_1 - b(a-s_0))}{2(1 - b(1-\theta))}, \tag{44}$$

$$q_1^L = \frac{a(1-b) + bs_0 - c_1}{2(1 - b^2(1-\theta))}. \tag{45}$$

The resulting welfare and firm 1's profit are, respectively,

$$V_0^F = \frac{X_{12}}{8(1 - b^2(1-\theta))^2}, \tag{46}$$

$$V_1^L = \frac{(a - c_1 - b(a-s_0))^2}{4(1 - b^2(1-\theta))}. \tag{47}$$

From (38), (39), (42), (43), (46), and (47), we obtain

$$V_0^L - V_0^C = \frac{b^2(a - c_1 - b(a-s_0) - (2 - b^2(1-\theta))\Delta_1)^2}{2(2 - b^2(1-\theta))^2(4 - 3b^2 + 2b^2\theta)} > 0, \tag{48}$$

$$V_0^C - V_0^F = \frac{b^2(1-\theta)(a(1-b) + bs_0 - c_1)X_{13}}{8(1 - b^2(1-\theta))^2(2 - b^2(1-\theta))^2}, \tag{49}$$

$$V_1^L - V_1^C = \frac{b^4(1-\theta)^2(a - c_1 + b(a-s_0))^2}{4(1 - b^2(1-\theta))^2(2 - b^2(1-\theta))^2} \geq 0, \tag{50}$$

$$V_1^C - V_1^F = \frac{b^2[-(a - c_1 - b(a-s_0) - (2 - b^2(1-\theta))\Delta_1)]X_{14}}{(2 - b^2(1-\theta))^2(4 - 3b^2 + 2b^2\theta)^2}. \tag{51}$$

The equality in (50) holds if and only if $\theta = 1$.

Proof of Proposition 5(i)

Cournot ($t_0 = t_1 = 1$) is an equilibrium if and only if (49) and (51) are nonnegative, and Cournot is the unique equilibrium if both are positive.

(49) is positive (negative, zero) if $X_{13} > (<,=)0$. Solving $X_{13} = 0$ with respect to Δ_1, we obtain

$$\Delta_1 = \check{\Delta} := \frac{((1-b^2)(4-b^2) + b^2(7-3b^2)\theta - b^2(2-3b^2)\theta^2 - b^4\theta^3)(a-c_1-b(a-s_0))}{4((1-b^2)(2-b^2) + b^2(3-2b^2)\theta + b^4\theta^2)}.$$

We now show that X_{14} is positive. Since $a-c_1-b(a-s_0) > 0$, $X_{14} > 0$ if $2(a-c_1-b(a-s_0)) - b^2 s_0 > 0$. Because we assume an interior solution in the quantity competition stage, from (41), we obtain $2(a-c_1-b(a-s_0)) - b^2 s_0 > 0$. This implies $X_{14} > 0$.

Because X_{14} is positive, (51) is positive (negative, zero) if $-(a-c_1-b(a-s_0)-(2-b^2(1-\theta))\Delta_1) > (<,=)0$. Solving the equation $-(a-c_1-b(a-s_0)-(2-b^2(1-\theta))\Delta_1) = 0$ with respect to Δ_1, we obtain

$$\Delta_1 = \dot{\Delta} := \frac{a-c_1+b(a-s_0)}{2-b^2(1-\theta)}.$$

Therefore, both (49) and (51) are positive if $\Delta_1 > \Delta_f := \max\{\check{\Delta}, \dot{\Delta}\}$. ■

Proof of Proposition 5(ii)

Two Stackelberg equilibria exist (both $(t_0, t_1) = (1,2)$ and $(t_0, t_1) = (2,1)$ are the equilibrium outcomes) if and only if both (49) and (51) are nonpositive. Only one Stackelberg equilibrium exists (either $(t_0, t_1) = (1,2)$ or $(t_0, t_1) = (2,1)$ is the equilibrium outcome) if and only if one of (49) and (51) is nonpositive and the other is positive. Let $\Delta_g := \min\{\check{\Delta}, \dot{\Delta}\}$. One of (49) and (51) is positive and the other is negative if $\Delta_1 \in (\Delta_g, \Delta_f)$. Both are negative if $\Delta_1 < \Delta_g$. Because $a-c_1-b(a-s_0) > 0$, we obtain $\check{\Delta} > 0$ and $\dot{\Delta} > 0$. Thus, $\Delta_g > 0$. These imply Proposition 5(ii). ■

Proof of Proposition 5(iii)

As shown in the proof of Proposition 5(i), private leadership is an equilibrium if $\Delta_1 \leq \check{\Delta}$, and public leadership is an equilibrium if $\Delta_1 \leq \dot{\Delta}$. Therefore, if $\check{\Delta} < \dot{\Delta}$ ($\check{\Delta} > \dot{\Delta}$), the unique Stackelberg is public leadership (private leadership) when $\Delta_1 \in (\Delta_g, \Delta_f)$.

From $\check{\Delta}$ and $\dot{\Delta}$, we obtain

$$\check{\Delta} - \dot{\Delta} = \frac{b^2(1-\theta)(b^2(1-2\theta+\theta^2) - (1-2\theta))(a-c_1-b(a-s_0))}{4(1-b^2(1-\theta))(2-b^2(1-\theta))}. \tag{52}$$

Equation (52) is positive (negative, zero) if $b^2(1-2\theta+\theta^2)-(1-2\theta) > (<,=)0$. Solving the equation $b^2(1-2\theta+\theta^2)-(1-2\theta)=0$, we obtain

$$b^2(1-2\theta+\theta^2)-(1-2\theta)=0 \to \theta = \frac{-(1-b^2)\pm\sqrt{1-b^2}}{b^2}.$$

The positive solution is

$$\theta = \frac{-(1-b^2)+\sqrt{1-b^2}}{b^2} = \theta_a.$$

Therefore, we obtain $\check{\Delta} < \ (>,=)\dot{\Delta}$ if $\theta < (>,=)\theta_a$.

As proved in Proposition 1(iii), $\theta_a \in (0, 1/2)$. These imply Proposition 5(iii). ■

Proof of Proposition 6

Cournot equilibrium emerges if both (49) and (51) are nonnegative. (49) is always zero when $\theta = 1$. (51) is nonnegative if and only if $\Delta_1 \geq \dot{\Delta}$. Public leadership is an equilibrium outcome if (48) is nonnegative and (51) is nonpositive. (48) is always nonnegative. (51) is nonpositive if and only if $\Delta_1 \leq \dot{\Delta}$. These imply Proposition 6(i).

Private leadership is an equilibrium if (49) is nonpositive and (50) is nonnegative. Both (49) and (50) are zero when $\theta = 1$. These imply Proposition 6(ii). ■

Proof of Proposition 7(i)

Comparing Δ_h and Δ_j, we obtain

$$\Delta_j - \Delta_h = \frac{2(1-\theta)(a(1-b)+bs_0-c_1)\sqrt{4-3b^2+2b^2\theta}}{2(1-b^2(1-\theta))} > 0. \tag{53}$$

This implies $\Delta_h < \Delta_j$. ■

Proof of Proposition 7(ii)

From (42) and (46), we obtain

$$V_0^L - V_0^F = -\frac{b^2 X_{15}}{8(1-b^2(1-\theta))^2(4-3b^2+2b^2\theta)}. \tag{54}$$

Equation (54) is positive (negative, zero) if $X_{15} < (>,=)0$. Solving the equation $X_{15}=0$ with respect to Δ_1, we obtain

$$X_{15}=0 \to \Delta_1 = -\frac{(a(1-b)+bs_0-c_1)[(1-2\theta)\pm(1-\theta)\sqrt{4-3b^2+2b^2\theta}]}{2(1-b^2(1-\theta))}.$$

Note that Δ_h and Δ_j are solutions of this equation.

Then, we obtain

$$\begin{aligned} X_{15} &< 0 \text{ (and thus } V_0^L > V_0^F), \text{ if } \Delta_1 \in (\Delta_h, \Delta_j), \\ X_{15} &= 0 \text{ (and thus } V_0^L = V_0^F), \text{ if } \Delta_1 = \Delta_h \text{ or } \Delta_j, \\ X_{15} &> 0 \text{ (and thus } V_0^L < V_0^F), \text{ otherwise.} \end{aligned}$$

This implies Proposition 7(ii). ■

Proof of Proposition 7(iii)

Comparing (36) and (40), we obtain

$$\begin{aligned} q_0^C - q_0^L &= \frac{2b(a(1-b)+bc_0-c_1-(2-b^2(1-\theta))\Delta_1)}{(2-b^2(1-\theta))(4-3b^2+2b^2\theta)} \gtrless 0 \\ &\leftrightarrow \quad \Delta_1 \lessgtr \frac{a(1-b)+s_0-c_1}{2-b^2(1-\theta)} =: \dddot{\Delta}. \end{aligned}$$

If $\Delta_1 > \dddot{\Delta}$, then $q_0^C < q_0^L$. Because $q_1^C = R_1(q_0^C)$, $q_1^F = R_1(q_0^L)$, and $\pi_1(q_0, R_1(q_0))$ is decreasing in q_0, we obtain $V_1^F < V_1^C$. Because $V_1^L \geq V_1^C$, we obtain $V_1^F < V_1^L$.

Comparing (44) and (40), we obtain

$$\begin{aligned} q_0^F - q_0^L &= \frac{(2-3b^2+5b^2\theta-2b^2\theta^2)(a(1-b)+bs_0-c_1)-4(1-b^2(1-\theta))\Delta_1}{2(1-b^2(1-\theta))(4-3b^2+2b^2\theta)} \gtrless 0 \\ &\leftrightarrow \quad \Delta_1 \lessgtr \frac{(2-3b^2+5b^2\theta-2b^2\theta^2)(a(1-b)+s_0-c_1)}{4(1-b^2(1-\theta))} =: \ddot{\Delta}. \end{aligned}$$

If $\Delta_1 < \ddot{\Delta}$, then $q_0^L < q_0^F$. Because $q_1^F = R_1(q_0^L)$, $\pi_1(q_0, R_1(q_0))$ is decreasing in q_0, and $\pi_1(q_0, R_1(q_0)) \geq \pi_1(q_0, q_1)$ for any q_1, we obtain $V_1^F > V_1^L$.

We now investigate profit ranking when $\Delta_1 \in [\ddot{\Delta}, \dddot{\Delta}]$. From (43) and (47), we obtain

$$V_1^L - V_1^F = -\frac{b^2 X_{16}}{4(1-b^2(1-\theta))(4-3b^2+2b^2\theta)^2}. \tag{55}$$

Equation (55) is positive (negative, zero) if $X_{16} < (>, =) 0$. We obtain $X_{16} = 0$ if $\Delta_1 = \Delta_i \in (\ddot{\Delta}, \dddot{\Delta})$. If $\Delta_1 < \Delta_i$, then $X_{16} > 0$ holds, and thus, we obtain $V_1^L < V_1^F$. If $\Delta_1 \in [\Delta_i, \dddot{\Delta}]$, then $X_{16} < 0$ holds, and thus, we obtain $V_1^L > V_1^F$. ■

Proof of Proposition 8

We show that $\Delta_h = \Delta_i = \Delta_j$ if $\theta = 1$. Substituting $\theta = 1$ into $\Delta_h(\theta)$, $\Delta_i(\theta)$ and $\Delta_j(\theta)$, we obtain

$$\Delta_h(1) = \Delta_i(1) = \Delta_j(1) = \frac{a(1-b) + bs_0 - c_1}{2}.$$

This implies Proposition 8(i).

Next, we show that $V_0^L \geq V_0^F$, and the equality holds if and only if $\Delta_1 = \Delta_i$. From (48), we obtain that $V_0^L > V_0^C$. From (49), we obtain that $V_0^C = V_0^F$ when $\theta = 1$. These imply Proposition 8(ii).

In the proof of Proposition 7(iii), we do not use the condition $\theta < 1$. Therefore, Proposition 7(iii) holds when $\theta = 1$. This implies Proposition 8(iii). ■

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