Comparing Specific and Ad Valorem Taxes under Price-inelastic Demand with Quality Differentiation

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Abstract

We examine the superiority of a specific tax and an ad valorem tax in a fully covered model with vertical differentiation, in which the demand elasticities of the products are inelastic. By focusing on the influences of vertical differentiation and inelastic demand, we show that a specific tax is superior to an ad valorem tax in the short run, when the marginal cost of the high-quality product is relatively low or is relatively high but the ad valorem tax rate is high. We also show that a specific tax may not only be welfare superior to, but may also Pareto dominate an ad valorem tax in the long run, when the ad valorem tax rate is high. These two results imply that the government should impose a specific tax on products with price-inelastic demand if the tax revenue requirement is high.

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1. Introduction

The superiority of a specific tax and an ad valorem tax while collecting the same tax revenue is a longstanding issue in public economics. As the demand elasticity faced by firms in an ad valorem tax regime is greater than that faced by them in a specific tax regime, firms will produce a larger output in the former than in the latter under imperfect competition. Accordingly, pioneering studies, such as Suits and Musgrave (1953), Delipalla and Keen (1992), Skeath and Trandel (1994) and Schröder (2004), support the argument that, given an identical amount of tax revenues, an ad valorem tax is welfare superior to a specific tax under imperfect competition.\(^1\) In addition, Skeath and Trandel (1994) prove that an ad valorem tax also Pareto dominates a specific tax when the market is controlled by a monopoly.\(^2\)

While taking into account the differentials in marginal costs across firms, the low- (high-) cost firm’s market share will rise (decline) when the elasticity faced by firms becomes larger. In line with this argument, Denicolò and Matteuzzi (2000) and Anderson et al. (2001) find that, given a fixed total output, an ad valorem tax is superior

\(^1\) Several studies indicate that a specific tax can be welfare superior to an ad valorem tax. They include: Hamilton (1999), who compares the superiority under monopsony, Pirttila (2002) and Droge and Schröder (2009), who examine the choice between the two taxes when the production of a good creates harmful externalities, and Grazzini (2006), who shows that a specific tax is superior when the number of consumers is sufficiently high compared to the number of oligopolists.

\(^2\) The Pareto dominance of an ad valorem tax can be derived when all of the consumer’s surplus, total profit, and tax revenue under an ad valorem tax are no smaller than those under a specific tax, and meanwhile at least one of them under the former is larger than that under the latter.
to a specific tax under Cournot competition. This result arises because switching from a specific tax to an ad valorem tax scheme enlarges the demand elasticity faced by the firms, which increases (decreases) the low-cost (high-cost) firm’s production such that the production under an ad valorem tax becomes more efficient.

However, by employing the Hotelling model and assuming that the low-cost and high-cost firms locate at the opposite endpoints, respectively, Anderson *et al.* (2001) find that a specific tax may be superior to an ad valorem tax. In later studies, Wang and Zhao (2009) consider horizontal differentiations and cost differences across firms, while Wang *et al.* (2018) introduce quality differentiations and cost differences across firms. Both papers show that a specific tax can be welfare superior to an ad valorem tax under both Cournot and Bertrand competition.

A common characteristic arising in the above literature reveals that many of the studies focus on the superiority of the two tax schemes in the situation where the demand for a product is elastic. However, it can frequently be observed in the real world that there are many goods whose demand is inelastic or extremely insensitive to changes in prices, such as addictive products like tobacco and alcohol and certain necessities like fuel, etc. Accordingly, in this paper we shall examine the superiority of specific and ad valorem tax schemes in situations where the products are inherently

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3 The Hotelling case in Anderson *et al.* (2001) is an exception.
inelastic. It should be noted that although one may argue that the governments impose taxes on the above inelastic products for mainly health or environmental reasons, it is still worth studying which tax scheme is better.

By assuming that the market is fully covered to exhibit the characteristic that the consumers’ demand for the products is completely inelastic, we shall conduct our analysis based on the vertically differentiated model developed by Mussa and Rosen (1978). The reasons why we employ this model are as follows. First, each consumer only consumes one unit of product, and second, the change in a firm’s price only affects its own and its rival’s market shares but cannot induce new consumers or incumbent consumers to enter or exit the market. This model fulfills the characteristic that the consumers’ demand for the products is completely inelastic and the demand faced by firms is inelastic in the sense that the total demand for the products is insensitive to the price change.

By using a fully-covered market model with vertical differentiation, the main results derived in this paper are as follows. First, in the short-run model where firms’ quality levels are fixed, a specific tax is welfare superior to an ad valorem tax if the high-quality firm’s marginal cost is relatively lower. On the contrary, given that the

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high-quality firm’s marginal cost is relatively higher, a specific tax is welfare superior (inferior) to an ad valorem tax if the ad valorem tax rate is higher (lower) than a critical level. Second, in the long-run model where both firms can adjust their quality levels, a specific tax is welfare superior (inferior) to an ad valorem tax if the ad valorem tax rate is higher (lower) than a critical level. Finally, it is worth noting that when the ad valorem tax rate is sufficiently high, a specific tax can Pareto dominate an ad valorem tax.

The above results show that for those inherently inelastic products, the government should impose a specific tax if it needs to collect a larger amount of tax revenues, regardless of whether the firms’ quality levels are fixed or endogenously determined. The main results are supported by a report of the World Health Organization (2010), which states that a large number of countries (55 out of 182) impose only a specific tax on tobacco in the real world.\(^5\) We argue that collecting a larger amount of tax revenues is a possible reason why the governments of many countries impose a specific tax on these price-inelastic products.\(^6\)

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\(^5\) Please refer to the World Health Organization (2010, p. 35). In addition, about one quarter of countries (48 out of 182) levy both specific and ad valorem taxes. This report also points out that, in general, high-income countries are less likely to lean towards an ad valorem tax (only 2 of 38 high-income countries adopt an ad valorem tax, while 11 rely on a specific tax and 25 use a mixture of the two).

\(^6\) In addition to the fact that a large number of countries impose only a specific tax on tobacco in the real world, we also observe that the governments of the U.S. and Japan impose specific taxes on tobacco, alcohol and fuel, and the government of China levies specific taxes on alcohol and fuel. Please refer to the following websites: https://taxfoundation.org/state-cigarette-taxes/, https://taxfoundation.org/high-wine-taxes-state/, https://taxfoundation.org/state-gasoline-tax-rates-2017/, and www.nta.go.jp/kohyo/tokei/kokuzeicho/h27/h27.pdf.
The remainder of this paper is organized as follows. Section 2 develops the basic model. Section 3 examines the superiority of specific and ad valorem tax schemes in the short run where firms’ quality levels are fixed. Section 4 explores the welfare and Pareto rankings between specific and ad valorem tax schemes in a long-run model where firms’ quality levels are endogenously determined. Section 5 concludes the paper.

2. Basic model

Consider a vertically differentiated framework à la Mussa and Rosen (1978). Each consumer can buy at most one unit of the differentiated product, and has the same utility function $U = \theta s_i - p_i$ (and zero utility if it does not buy the differentiated product) where $s_i$ and $p_i$ denote firm $i$’s product quality and price, respectively. Consumers differ in their tastes for quality and are uniformly distributed along the taste line with unit density measured by $\theta \in [\theta, \theta + 1]$, where $\theta$ denotes the consumer’s taste for quality, and $\overline{\theta}$ denotes the consumer’s lowest taste for quality. Consumers with a higher $\theta$ will be willing to pay more for a higher quality good. We further assume that the consumer’s lowest taste is sufficiently large, i.e., $\overline{\theta} > 5/4$, to ensure that every consumer will definitely buy the product such that the market is fully covered.

The utility of the marginal consumer, who is indifferent between buying the high-
quality product and the low-quality product, can be derived by equating the utilities of buying these two products as $\theta_{12} = (p_2 - p_1)/(s_2 - s_1)$. Assume that there are two firms competing in Bertrand fashion in the industry where firm 1 produces the low-quality product and firm 2 the high-quality product, i.e., $s_2 > s_1$. Since the market is fully covered, the consumers in between the highest taste consumer and the marginal consumer will purchase the high-quality product while those in between the marginal consumer and the lowest-taste consumer will buy the low-quality product. Thus, the high- and low-quality firms’ demand functions can be defined as $q_2^D = \theta + 1 - \theta_{12} = \theta + 1 - (p_2 - p_1)/(s_2 - s_1)$ and $q_1^D = \theta_{12} - \theta = (p_2 - p_1)/(s_2 - s_1) - \theta$, respectively.

The cost function can be expressed as $TC_i = c_i q_i = s_i^2 q_i$, $i = 1, 2$, where $q_i$ is firm $i$’s output and $c_i$ denotes firm $i$’s marginal cost. This kind of setting implies that, given that output is fixed, the higher that the quality level is, the larger will be the quality cost, and meanwhile the marginal cost of the high-quality product will always be larger than that of the low-quality product. In order to collect tax revenues, we assume that the government imposes either a specific or an ad valorem tax on the

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7 A similar setting can be found in the literature, such as Crampes and Hollander (1995), Pepall (1997), Lambertini (1997, 2001), Lombardini-Riipinen (2005) and Wang (2003), whose cost function is expressed as $TC_i = q_i f(s_i)$, $f' > 0$, $f'' > 0$, while other studies, such as Maxwell (1998), Cheng (2014), Liao (2008) and Lambertini and Tampieri (2017), employ the following cost function: $TC_i = c q_i + f(s_i)$, $c \geq 0$, $f' > 0$, $f'' > 0$. 
products.

In what follows we shall study two situations, in which firms are unable to adjust their quality levels in the short run while they can adjust them in the long run. The game in the short run is a one-stage game where firms engage in Bertrand competition in the market, while the game in the long run consists of two stages where firms determine their quality levels in stage 1 and then compete in Bertrand fashion in stage 2.

3. The welfare superiority of the two tax schemes in the short run

In the short run, firms are incapable of adjusting their quality levels. They can only determine prices to maximize their profits. We first analyze the case where the government imposes a specific tax. Thus, the profit-maximizing problem can be described as follows:

$$\max_{p^t_i} \pi^t_i = (p^t_i - c_i - t)q^t_i, i = 1, 2,$$  

(1)

where the superscript “$t$” denotes variables associated with the specific tax scheme, and $t$ is the specific tax rate.

By solving the profit-maximizing conditions, we can obtain:

$$p^t_{1*} = \frac{2c_1 + c_2 + (1 - \theta)(s_2 - s_1)}{3} + t,$$  

(2)
\[ p_2^* = \frac{2c_2 + c_1 + (2+\theta)(s_2 - s_1)}{3} + t. \]  

(3)

Eqs. (2) and (3) show that given a fully-covered market, the specific tax will be completely transferred to the consumers. This result arises because the consumers’ demand is completely inelastic and both firms’ marginal costs are increasing by the same amount as the specific tax rate. By using (2) and (3), we can derive firms’ outputs and the location of the marginal consumer as follows:

\[ q_1^* = \frac{(c_2 - c_1 + (1-\theta)(s_2 - s_1))}{3(s_2 - s_1)}, \]  

(4)

\[ q_2^* = \frac{(c_1 - c_2 + (2+\theta)(s_2 - s_1))}{3(s_2 - s_1)}, \]  

(5)

\[ \theta_{12}^* = \frac{(c_2 - c_1)}{3(s_2 - s_1)} + \frac{(1+2\theta)}{3}. \]  

(6)

We find from (6) that the location of the marginal consumer is irrelevant with regard to the specific tax, i.e., \( \frac{\partial \theta_{12}^*}{\partial t} = 0 \). The explanation behind this result is as follows. We find that given that the quality levels are fixed, the location of the marginal consumer hinges upon the difference in the prices between the two products. Since the specific tax is completely transferred to the consumers, the imposition of a specific tax has no effect on the difference in the prices. As a result, the location of the marginal consumer will remain unchanged. Based on this result, from (5) and (6) we can derive that the imposition of a specific tax has no effect on the two firms’ outputs in the short run.
In order to ensure that the firms produce positive outputs and the market is fully covered, we make the following assumptions:

**Assumption 1.** \((\theta - 1)(s_2 - s_1) + c_1 \leq c_2 \leq (2 + \theta)(s_2 - s_1) + c_1\).

**Assumption 2.** \(c_2 \leq \theta(2s_1 + s_2) - (s_2 - s_1) - 3t - 2c_1\).

By using (4) and (5), we can derive the inequalities in Assumption 1 to ensure that the two firms’ outputs are positive in a fully-covered market. Assumption 2 shows that the lowest-taste consumer’s utility is not less than zero by buying the low-quality product, i.e., \(\theta s_1 - p_1^\tau \geq 0\). This assumption ensures that every consumer will purchase a product such that the market is fully covered.\(^8\)

Next, we examine the case where the government imposes an ad valorem tax. Given the tax rate \(\tau\), the firms’ profit-maximizing problems are as follows:

\[
\max_{p_i^\tau} \pi_i^\tau = ((1 - \tau)p_i^\tau - c_i)q_i^\tau, i = 1, 2, \tag{7}
\]

where the superscript “\(\tau\)” denotes variables associated with the ad valorem tax scheme.

By solving the profit-maximizing conditions, we can obtain the equilibrium prices of the products as follows:

\[p_1^\tau^* = \frac{2c_1 + c_2}{3(1 - \tau)} + \frac{(1 - \theta)(s_2 - s_1)}{3} , \tag{8}\]

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\(^8\) By assumptions 1 and 2, we can derive the restriction, \(t \leq \theta s_1 - c_1\), which points out that the specific tax rate has a cap to ensure that every consumer will purchase a product.
\[ p_2^{\tau^*} = \frac{2c_2+c_1}{3(1-\tau)} + \frac{(2+\theta)(s_2-s_1)}{3}, \quad (9) \]

Eqs. (8) and (9) show that the effects of the ad valorem tax on equilibrium prices are quite different from those of the specific tax. As \((1-\tau)p_i^{\tau^*} < p_i^{\tau^*}(\tau = 0)\), this indicates that part of the tax burden goes to the firms under an ad valorem tax scheme while the whole of the tax burden goes to the consumers under a specific tax scheme. This difference arises because an ad valorem tax can be regarded as a tax proportional to the firm’s revenue. The firm will be willing to absorb part of the tax burden, when it earns an excess profit.

From (8) and (9), we can derive the firms’ outputs and the location of the marginal consumer as follows:

\[ q_1^{\tau^*} = \frac{(c_2-c_1+(1-\tau)(1-\theta)(s_2-s_1))}{3(1-\tau)(s_2-s_3)}, \quad q_2^{\tau^*} = \frac{(c_1-c_2+(1-\tau)(2+\theta)(s_2-s_1))}{3(1-\tau)(s_2-s_3)}, \quad \text{and} \]
\[ \theta_{12}^{\tau^*} = \frac{(c_2-c_1)}{3(1-\tau)(s_2-s_1)} + \frac{(1+2\theta)}{3}. \quad (10) \]

By subtracting (6) from (10), we obtain that the location of the marginal consumer under an ad valorem tax is larger than that under a specific tax as follows:

\[ \theta_{12}^{\tau^*} - \theta_{12}^{t^*} = \frac{\tau(c_2-c_1)}{3(1-\tau)(s_2-s_1)} > 0. \quad (11) \]

Note that the movement of the location of the marginal consumer to the right means that the output of the low- (high-) quality product will rise (decline). It follows
from (11) that switching from a specific tax scheme to an ad valorem tax scheme will cause the low-quality firm to produce more, while it will cause the high-quality firm to produce less. The intuition behind this result can be stated as follows.

In a specific tax scheme, recall that the per unit tax of the high-quality product is identical to that of the low-quality product, and that the price difference between the high- and low-quality products remains unchanged by the imposition of a specific tax. It follows that the outputs of the low- and high-quality products (or the location of the marginal consumer) continue(s) to remain unchanged before and after the imposition of a specific tax.

Next, in an ad valorem tax scheme, recall that the per unit tax of the high-quality product is larger than that of the low-quality product. The output of the high- (low-) quality product will decrease (increase), i.e., the location of the marginal consumer will become larger. As a result, switching from a specific tax scheme to an ad valorem tax scheme will cause the high- (low-) quality firm to produce less (more). Moreover, we can derive from (11) that $\partial(\theta_{12}^{t_r} - \theta_{12}^{t_\ast})/\partial \tau = (c_2 - c_1)/(3(1 - \tau)^2(s_2 - s_1)) > 0$.

Recall from (6) that the location of the marginal consumer is fixed under a specific tax scheme, irrespective of the specific and ad valorem tax rates. Thus, when switching from a specific tax to an ad valorem tax scheme, the higher the ad valorem
tax rate is, the larger (smaller) will be the production of the low- (high-) quality product.

Based on the above analysis, we have Lemma 1 as follows:

**Lemma 1.** *Provided that the quality levels are fixed in the short run, switching from a specific tax to an ad valorem tax will cause the low-quality firm to produce more, while the high-quality firm will produce less. Next, the location of the marginal consumer is fixed under a specific tax scheme, irrespective of the specific and ad valorem tax rates. Moreover, when switching from a specific tax to an ad valorem tax scheme, the higher the ad valorem tax rate is, the larger (smaller) will be the production of the low- (high-) quality product.*

As has been stated in the Introduction, we measure the superiority of the two tax schemes by comparing the production distributions of the high- and low-quality products under these two tax schemes with the socially-optimal distribution. Thus, the socially-optimal distribution of the two products is essential in measuring the superiority. Accordingly, the social planner’s welfare-maximizing problem can be expressed as follows:
\[
\text{Max } SW = \int_0^{\hat{\theta}} (\theta s_1 - c_1) d\theta + \int_0^{1+\hat{\theta}} (\theta s_2 - c_2) d\theta,
\]

where \( \hat{\theta} \) denotes the location of the marginal consumer in the social planner’s problem.

By differentiating (12) with respect to \( \hat{\theta} \) and letting it equal zero, we obtain the welfare-maximizing condition for the socially-optimal location of the marginal consumer as follows:

\[
\frac{\partial SW}{\partial \hat{\theta}} = \hat{\theta}^* s_1 - c_1 - \hat{\theta}^* s_2 + c_2 = 0.
\]

As the market is fully covered, increasing the production of the low-quality product by one unit will meanwhile decrease the production of the high-quality product by one unit. Eq. (13) shows that the socially-marginal benefit of the low-quality product is \( \theta s_1 - c_1 \), which will equal the socially-marginal loss of the high-quality product equaling \( \theta s_2 - c_2 \). By solving (13), we can derive the socially-optimal location of the marginal consumer as:

\[
\hat{\theta}^* = \frac{c_2 - c_1}{s_2 - s_1}
\]

By subtracting (6) from (14), we have:

\[
\hat{\theta}^* - \theta_{12}^* = \frac{2(c_2 - c_1)}{3(s_2 - s_1)} - \frac{(1+2\theta)}{3}.
\]

Eq. (15) shows that the difference in the optimal locations of the marginal consumer between the social optimality and the specific tax scheme is ambiguous. It is
negative (positive) such that the production of the low-quality product under social optimality is smaller (greater) than that under a specific tax scheme, when \( c_2 < (>) (1 + 2\theta)(s_2 - s_1)/2 + c_1 \). Thus, we have:

**Lemma 2.** Compared with the socially-optimal production distribution of the high- and low-quality products, the low-quality product is over- (under-) produced under the specific tax scheme in the short run, when \( c_2 < (>) (1 + 2\theta)(s_2 - s_1)/2 + c_1 \).

(Insert Figure 1 here)

We find from Lemma 2 that the low-quality product is over-produced under a specific tax relative to the socially-optimal production distribution, when \( c_2 < (1 + 2\theta)(s_2 - s_1)/2 + c_1 \), and from Lemma 1 that the low-quality product will be produced even more while switching from the specific tax to the ad valorem tax scheme. Thus, we can conclude that a specific tax is definitely superior to an ad valorem tax, regardless of the tax rates in the two tax schemes, when \( c_2 < (1 + 2\theta)(s_2 - s_1)/2 + c_1 \). We can illustrate this result as shown in Figure 1.

In Figure 1, the line \( fg \) denotes the loci of the upper bound of \( c_2 \) in Assumption 1, i.e., \( c_2 = (2 + \theta)(s_2 - s_1) + c_1 \), line \( hg \) the lower bound of \( c_2 \) in Assumption 1, i.e.,
\((\theta - 1)(s_2 - s_1) + c_1 = c_2\), line ab the upper bound of \(c_2\) in Assumption 2, i.e., \(c_2 = \theta(2s_1 + s_2) - (s_2 - s_1) - 3t - 2c_1\), and line de the loci where the difference in optimal locations of the marginal consumer between the social optimality and the specific tax scheme equals zero, i.e., the sign of (15) equals zero. We find that only areas I and II in Figure 1 fulfill the restrictions imposed by Assumptions 1 and 2, where area I (II) denotes the area under which \(c_2 < (>) (1 + 2\theta)(s_2 - s_1)/2 + c_1\). Thus, area I in Figure 1 is the area where a specific tax is always welfare superior to an ad valorem tax.

We turn to discuss the case where \(c_2 > (1 + 2\theta)(s_2 - s_1)/2 + c_1\), i.e., area II in Figure 1. Recall that the low-quality product is under-produced under a specific tax relative to the socially-optimal production distribution in this case, and meanwhile the low-quality product is produced more while switching from a specific tax to an ad valorem tax scheme. Thus, the superiority of the two tax schemes is indeterminate in area II. By using (2)-(6), we can derive the social welfare under a specific tax. Next, by using (8)-(10), we can obtain the social welfare under an ad valorem tax. Thus, we obtain the difference in social welfare between the specific and the ad valorem tax schemes as follows:

\[
SW^\tau - SW^t = \frac{2(1-t)(s_2 - s_1)(1+2\theta - t)(c_2 - c_1)[4-5t](c_2 - c_1)}{18(1-t)^2(s_2 - s_1)}. \tag{16}
\]
We find from (16) that $SW^\tau < (>)SW^{\hat{\tau}}$ if and only if $\tau > (<)\hat{\tau}$ where $\hat{\tau} = (4(c_2 - c_1) + 2(s_2 - s_1) + 4\theta(s_2 - s_1))/(5(c_2 - c_1) + 2(s_2 - s_1) + 4\theta(s_2 - s_1))$.

Thus, in area II of Figure 1, a specific tax is superior (inferior) to an ad valorem tax if and only if the ad valorem tax rate is higher (lower) than $\hat{\tau}$, regardless of the magnitude of the specific tax rate. The explanation of this result is as follows.

In area II of Figure 1, the low-quality product is under-produced under a specific tax, and meanwhile more of the low-quality product is produced while switching from a specific tax to an ad valorem tax scheme. Provided that the ad valorem tax rate is low, although the production of the low-quality product under an ad valorem tax is greater than that under a specific tax, the production under the former tax scheme is still lower than the socially-optimal level, i.e., $\theta_{12}^{T} < \theta_{12}^{\tau} < \hat{\theta}^{*}$. In this situation, the social welfare under the ad valorem tax scheme will be higher than that under the specific tax scheme such that an ad valorem tax is superior to a specific tax.

Next, we find from Lemma 1 that a rise in the ad valorem tax rate will increase the optimal location of the marginal consumer and then raise the production of the low-quality product under an ad valorem tax. If it goes all the way through $\tau = \hat{\tau}$, not only will the production of the low-quality product under an ad valorem tax become larger than the socially-optimal level, i.e., $\theta_{12}^{T} < \hat{\theta}^{*} < \theta_{12}^{\tau}$, but it will also reach a critical level such that the distortion caused by the under-production of the low-quality good
under the specific tax scheme equals the distortion caused by the over-production of the low-quality good under the ad valorem tax scheme. Thus, we find from (16) that 
\[ SW^T = SW^f \] if and only if \( \tau = \hat{\tau} \).

Furthermore, when the ad valorem tax rate continues to rise to the level \( \tau > \hat{\tau} \), the distortion under an ad valorem tax will become larger than that under a specific tax such that a specific tax is welfare superior to an ad valorem tax.

Based on the above analysis, we can establish the following proposition:

Proposition 1. Provided that the quality levels are fixed in the short run, we can propose:

1. A specific tax is always welfare superior to an ad valorem tax when the marginal cost of the high-quality product is small, i.e., \( c_2 < (1 + 2\theta)(s_2 - s_1)/2 + c_1 \).

2. Given that the marginal cost of the high-quality product is large, i.e., \( c_2 > (1 + 2\theta)(s_2 - s_1)/2 + c_1 \), a specific tax is welfare superior (inferior) to an ad valorem tax when \( \tau > (<)\hat{\tau} \).

The result of (2) in Proposition 1 is similar to the part of Bertrand competition in Anderson et al. (2001), in which the Hotelling model is also employed. Anderson et al.
(2001) assume that the two cost-asymmetric firms locate at the opposite endpoints of the line segment, respectively. They derive the following characteristics. First, the low- (high-) cost firm under the specific tax scheme will produce less (more) than the socially-optimal level of output, which causes distortions. Second, when switching from the specific tax to the ad valorem tax scheme, the low- (high-) cost firm will produce more (less). Meanwhile, a rise in the ad valorem tax rate will increase (decrease) the low- (high-) cost firm’s output.

These two characteristics are the same as those in Lemma 1. Accordingly, they obtain the following results. First, given that the ad valorem tax rate is small, switching from a specific tax to an ad valorem tax scheme will reduce the distortion caused by the under-production in a specific tax such that an ad valorem tax is welfare superior to a specific tax. Second, provided that the ad valorem tax rate is sufficiently large, the imposition of an ad valorem tax will lead to the distortion caused by the over-production of the low-cost firm under an ad valorem tax. Thus, a specific tax will be welfare superior to an ad valorem tax, if the distortion caused by the over-production of the low-cost firm under an ad valorem tax exceeds that caused by the under-production of the low-cost firm under a specific tax.

It should be noted that the model in Anderson et al. (2001) is a covered market with horizontal differentiation, while ours is a covered market with vertical
differentiation. We can, however, obtain richer results in which (2) of Proposition 1 is the same as that in Anderson et al. (2001). In addition, we can derive the result that a specific tax is always welfare superior to an ad valorem tax as shown in (1) of Proposition 1. Compared to the covered market with horizontal differentiation, Proposition 1 implies that a specific tax is more likely to be welfare superior to an ad valorem tax scheme.

4. The superiority of the two tax schemes in the long run

In this section we explore the situation where firms are capable of adjusting their quality levels. An extra stage now arises prior to the price stage, with the equilibria of the price stage derived in the previous section carrying over to this section. In what follows, we shall directly analyze the optimal quality levels in stage 1. By substituting (2)-(5) and the marginal cost $c_i = s_i^2$ into (1) and then solving the profit-maximizing conditions, we can derive firms’ optimal quality levels, equilibrium prices, outputs, and the optimal location of the marginal consumer under a specific tax scheme as follows:

\[
\begin{align*}
    s_1^{t^*} &= \frac{4\theta - 1}{8}, s_2^{t^*} = \frac{5 + 4\theta}{8}, \\
p_1^{t^*} &= \frac{25 - 8\theta + 16\theta^2}{64} + t, p_2^{t^*} = \frac{49 + 40\theta + 16\theta^2}{64} + t, \\
q_i^{t^*} &= \frac{1}{2}, \theta_{12}^{t^*} = \frac{1}{2} + \theta.
\end{align*}
\]
Eq. (17) shows that the long-run equilibrium quality levels are not affected by the specific tax rate. We find from (18) that, given a fully-covered market, the specific tax will be completely transferred to the consumers in the long run. We also find from (19) that the high- and low-quality firms capture equal market shares.\(^9\) In addition, the intuition stated in the short-run model applies to the above results.

Next, we examine the equilibrium under an ad valorem tax scheme. By substituting (8)-(10) and \(c_i = s_i^2\) into (7) and then solving the profit-maximizing conditions, we can derive firms’ optimal quality levels, equilibrium prices, outputs, and the optimal location of the marginal consumer under an ad valorem tax scheme as follows:

\[
S_1^{T^*} = \frac{(4\theta - 1)(1 - \tau)}{8}, \quad S_2^{T^*} = \frac{(5 + 4\theta)(1 - \tau)}{8},
\]

\[
p_1^{T^*} = \frac{(25 - 8\theta + 16\theta^2)(1 - \tau)}{64}, \quad p_2^{T^*} = \frac{(4\theta + 40\theta + 16\theta^2)(1 - \tau)}{64},
\]

\[
q_i^{T^*} = \frac{1}{2}, \quad \theta_{12}^{T^*} = \frac{1}{2} + \theta.
\]

By comparing (17) with (20), we find that switching from a specific tax to an ad valorem tax scheme will cause the optimal quality levels of both the high- and low-quality products to fall. We can use the following three effects to explain why the

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\(^9\) From (17) and (18), we can derive the utility of the lowest taste consumer as being \(U = \theta s_1 - p_1 = \frac{16\theta^2 - 25}{64} - \tau\). Thus, by manipulation we can obtain that the location of the lowest taste consumer has to be greater than 5/4, i.e., \(\theta > 5/4\), to ensure that the market is fully covered.
quality levels of the two firms decline.

First, recall that the imposition of a specific tax can be fully transferred while the imposition of an ad valorem tax can only be partially transferred to the consumers. Thus, switching from a specific tax to an ad valorem tax will lead both the high- and low-quality levels to decline due to the reductions in the profit levels of the two products. Second, as the per unit tax of the high-quality product under an ad valorem tax is larger than that of the low-quality product, i.e., \( \tau p_2 > \tau p_1 \), this will enhance the competitiveness of the low-quality product and then raise the quality level of the low-quality product and lower the quality level of the high-quality product. Third, the quality levels chosen by the two firms are strategic complements.

For the high-quality firm, all of the above three effects will cause the high-quality firm’s quality level to decline. For the low-quality firm, the first and third effects will lower the low-quality firm’s quality level, while the second effect will raise its quality level. As the former two effects outweigh the latter effect, switching from a specific tax to an ad valorem tax scheme will reduce the low-quality firm’s quality level. In addition, by attributing the increase to the second effect, we can obtain that a rise in the ad valorem tax rate will cause the magnitude of the reduction in the low-quality firm’s quality level to be lower than that in relation to the high-quality firm’s quality level, which can be derived by (20) as \( |\partial s_1^*/\partial \tau| < |\partial s_2^*/\partial \tau| \).
By comparing (18) with (21), we find that switching from a specific tax to an ad valorem tax will decrease the prices of both products. This is because the imposition of a specific tax can be fully transferred, while the imposition of an ad valorem tax can only be partially transferred to the consumers, and meanwhile the quality levels of both products will fall by switching from a specific tax to an ad valorem tax. Eqs. (20) and (21) show that the imposition of an ad valorem tax will lower the quality levels and the prices of the two products proportionally. On the other hand, the location of the marginal consumer is determined by the difference in prices divided by the difference in quality levels between the high- and low-quality products. It follows that the equilibrium location of the marginal consumer will be located at the center of the taste line, i.e., $\frac{1}{2} + \theta$, so that the two firms equally share the market as shown in (22).

Next, we examine the socially-optimal quality levels and product distribution. By substituting the marginal cost $c_i = s_i^2$ into (12) and then differentiating (12) with respect to $\tilde{\theta}, s_1,$ and $s_2$, and letting them equal zero, we obtain the socially-optimal location of the marginal consumer and quality levels as follows:

$$\tilde{\theta}^{s*} = \frac{1}{2} + \theta, \quad s_1^{s*} = \frac{1+4\theta}{8}, \quad s_2^{s*} = \frac{3+4\theta}{8}. \quad (23)$$

We find from (19), (22), and (23) that the production distributions of the two products have equal shares in the long run under the specific tax scheme, the ad valorem
tax scheme, and the social optimality. Since the production distributions are identical in all three cases, the distortions caused by the imposition of a specific tax or an ad valorem tax will thereby arise solely from the deviations of the quality levels from their socially-optimal levels. Accordingly, we find from (17), (20), and (23) that the optimal quality level of the low-quality product under an ad valorem tax is lower than that under a specific tax, whose optimal quality level is also lower than that under social optimality, i.e., $s_1^s > s_1^t > s_1^\tau$. On the other hand, the optimal quality level of the high-quality product under a specific tax is higher than those under social optimality and under an ad valorem tax, i.e., $s_2^t > s_2^s$, and $s_2^t > s_2^\tau$. However, the difference in the optimal quality levels of the high-quality product between social optimality and an ad valorem tax scheme is indeterminate, i.e., $s_2^\tau > (<) s_2^s$.

From (17) and (19), we obtain the social welfare under a specific tax as follows:

$$SW^t = \frac{1+16\theta+16\theta^2}{64}. \quad (24)$$

Recall that the imposition of a specific tax generates no effect on the optimal quality levels of the two products and the production distribution. Thus, the social welfare is not affected either. This implies that the social welfare under a specific tax is identical to that in the absence of taxes. Thus, we have:
**Proposition 2.** Supposing that the market is fully covered and that the government imposes a specific tax, the specific tax has no effects on the optimal quality levels of the two products, the production distribution, and the social welfare in the long run. Thus, the specific tax is tax neutral in the long-run equilibrium.

We can derive the social welfare under an ad valorem tax from (20) and (22), and then, by subtracting this level of social welfare from (24), we obtain the difference in the levels of social welfare between the specific and ad valorem tax schemes as follows:

\[ SW^t - SW^T = -\frac{\tau \left( 12 - \tau \left( 13 + 16\theta + 16\theta^2 \right) \right)}{64} \geq (\leq)0, \text{if and only if} \]

\[ \tau \geq \frac{12}{13+16\theta+16\theta^2} \left( 0 \leq \tau \leq \frac{12}{13+16\theta+16\theta^2} \right). \quad (25) \]

Eq. (25) shows that a specific tax is welfare superior (inferior) to an ad valorem tax, when \( \tau > (\leq)12/(13 + 16\theta + 16\theta^2) \). Recall that the equilibrium outputs of the high- and low-quality firms are equal to 1/2 under both the specific and the ad valorem tax schemes. It follows that the superiority of the two tax schemes hinges upon the effects of the two tax schemes on the two firms’ quality levels, which can be pursued as follows. First, recall that the specific tax is tax neutral. We can thereby recognize that the two firms’ quality levels at \( \tau = 0 \) under an ad valorem tax scheme will be identical to those under a specific tax scheme. This argument can be proved by substituting \( \tau = \)
0 into (25), which gives the result that the difference in the social welfare between a specific and an ad valorem tax scheme equals zero.

Second, compared with the socially-optimal levels, the high- (low-) quality firm’s quality level is higher (lower) than the socially-optimal level under a specific tax scheme, i.e., $s_2^t > s_2^* \quad (s_1^* > s_1^t)$. Recall that switching from a specific tax to an ad valorem tax scheme will reduce the two firms’ quality levels. This means that the imposition of an ad valorem tax creates a benefit in terms of social welfare by pushing the high-quality firm’s quality level closer to the socially-optimal level, while generating damage through forcing the low-quality firm’s quality level farther away from the socially-optimal level. Note that we can derive from (20) that $0 > \partial s_1^* / \partial \tau > \partial s_2^* / \partial \tau$, indicating that a higher ad valorem tax rate will decrease the high-quality firm’s quality level more than the reduction in the low-quality firm’s quality level. This characteristic leads to the result that the benefit of the imposition of an ad valorem tax will be greater than the damage caused, such as that when the ad valorem tax rate is low, while the reverse will occur otherwise. Thus, we find from (25) that a specific tax is welfare superior to an ad valorem tax, when $\tau > (\leq) 12 / (13 + 16\theta + 16\theta^2)$.  

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10 When the ad valorem tax rate is sufficiently high, it may be the case that $s_2^t < s_2^*$. resulting in a distortion in that the high-quality firm’s quality level is lower than the socially-optimal level. However, we find that $s_2^t - s_2^* = (16\theta^2 - 16\theta - 17) / (52 + 64\theta + 64\theta^2) > 0$ when $\tau = 12 / (13 + 16\theta + 16\theta^2)$, indicating that switching from a specific tax to an ad valorem tax scheme continues to reduce the distortion caused by a higher quality level of the high-quality product at this tax rate.
Based on the above analysis, we can establish:

**Proposition 3.** *Supposing that the market is fully covered and that the quality levels are endogenously determined in the long run, a specific tax is welfare superior (inferior) to an ad valorem tax, when \( \tau \geq (\leq)12/(13 + 16\theta + 16\theta^2) \).*

Propositions 1 and 3 are sharply different from the traditional result. As the demand curve faced by the firms is flatter and the low-cost firm will produce more under an ad valorem tax, the existing literature, such as Suits and Musgrave (1953), Delipalla and Keen (1992), Skeath and Trandel (1994)), Schröder (2004), Denicolò and Matteuzzi (2000) and Anderson *et al.* (2001), derives the traditional result that an ad valorem tax is welfare superior to a specific tax.

By contrast, we obtain a different result when the consumers’ demands for the products are completely inelastic. Proposition 1 points out that, given that quality levels are fixed in the short run, the welfare superiority of the two tax schemes hinges upon the effects of the taxes on the production distributions of the high- and low-quality products. Moreover, Proposition 3 shows that, provided that the quality levels are endogenously determined in the long run, the welfare superiority of the two tax schemes
depends upon the effects of the taxes on the quality levels.

We proceed to an analysis of the Pareto ranking between the two taxations. First of all, we compare the producer’s surpluses between the two tax schemes. By substituting (17) and (18) into (1) and substituting (20) and (21) into (7), and then subtracting (7) from (1), we obtain:

$$\pi_1^t + \pi_2^t - (\pi_1^\tau + \pi_2^\tau) = \frac{3(2-\tau)\tau}{8} > 0.$$  (26)

Eq. (26) shows that the producer’s surplus under a specific tax is greater than that under an ad valorem tax. The intuition is as follows. As firms can fully shift the specific tax to consumers and the number of consumers is fixed, it follows that the imposition of a specific tax gives rise to no impact to the producer’s surplus. However, firms will need to bear part of the ad valorem tax. Thus, an ad valorem tax will lower the firms’ producer’s surplus. Based on the above analysis, it is clear that as long as the ad valorem tax rate is greater than zero, the producer’s surplus under a specific tax will always be greater than that under an ad valorem tax.

Next, we compare the consumer’s surpluses between the two tax schemes. In what follows we conduct the comparison based on the premise that the tax revenues under both tax regimes are identical. Given the same tax revenues, i.e., \(t = \tau(p_1^\tau + p_2^\tau)/2\), we can derive the relationship between the specific and the ad valorem taxes.
through the use of (21) as:

\[ t = \frac{\tau (1-\tau)(37+16\theta+16\theta^2)}{64}. \] (27)

By using (27), (17), (18), (20), and (21), we can obtain the differences in the consumer’s surpluses between the specific and the ad valorem taxes as follows:

\[ CS^t - CS^\tau = \frac{1}{64} \tau (-60 + 37\tau + 16\tau\theta + 16\tau\theta^2) \geq (\leq) 0, \text{ if and only if } \tau \geq \frac{60}{37+16\theta+16\theta^2} \] (0 \leq \tau \leq \frac{60}{37+16\theta+16\theta^2}) (28)

Eq. (28) shows that the consumer’s surplus under a specific tax is larger than that under an ad valorem tax, if $\tau \geq (\leq) \frac{60}{(37 + 16\theta + 16\theta^2)}$. The intuition behind this result can be stated as follows.

Recall that the marginal consumer and then the number of consumers for the high- and low-quality products are not affected by the specific and the ad valorem taxes. The changes in the specific and the ad valorem taxes can thereby affect the consumer’s surplus through the changes in the quality levels of the high- and low-quality products and prices.

Under the specific tax scheme, a rise in the specific tax will be fully transferred to the prices, but has no effect on the quality levels of the high- and low-quality products. Therefore, a hike in the specific tax will lower the consumer’s surplus only via raising the prices. Next, under the ad valorem tax regime, a higher ad valorem tax will decrease
the quality levels of both high- and low-quality products, and then reduce the prices of both products. As the impact from lowering the quality levels is greater than the impact from the declining prices, a rise in the ad valorem tax will reduce the consumer’s surplus.

Based on the above analysis, when the ad valorem tax rate is low, i.e., $\tau \leq 60/(37 + 16\theta + 16\theta^2)$, the impact of the reduced prices is relatively stronger such that the reduction in the consumer’s surplus under an ad valorem tax is smaller than the reduction under a specific tax. Thus, the consumer’s surplus under a specific tax is lower than that under an ad valorem tax. By contrast, the reverse occurs when the ad valorem tax rate is high i.e., $\tau \geq 60/(37 + 16\theta + 16\theta^2)$. Accordingly, we have:

**Proposition 4.** Supposing that the market is fully covered and that the quality levels are endogenously determined in the long run, a specific tax can Pareto dominate an ad valorem tax, when the ad valorem tax rate is high, i.e., $\tau \geq 60/(37 + 16\theta + 16\theta^2) \equiv \tau^p$.

Proposition 4 is sharply different from the result in the existing literature. Skeath and Trandel (1994) find that an ad valorem tax Pareto dominates a specific tax when the market is dominated by a monopoly. As Propositions 1, 3, and 4 have indicated,
when the ad valorem tax rate is high, a specific tax is definitely welfare superior to an ad valorem tax in the short-run and long-run equilibria. We can thereby induce the following corollary:

**Corollary 1.** *Supposing that the market is fully covered, the government should impose a specific tax scheme, if tax revenue requirements are high.*

We have examined the superiority of the specific and ad valorem tax schemes by employing a cost function where the quality cost is a variable cost. In what follows we extend our analysis to the case where the quality cost is a fixed cost. By referring to Maxwell (1998), Cheng (2014), Liao (2008), and Lambertini and Tampieri (2017), the cost function can be expressed as $TC_i = cq_i + s_i^2/2, c \geq 0$, which contains a production cost and a fixed quality cost. By using this cost function, the results derived in the existing literature are as follows. First, the low-quality firm will decrease its quality level to the lowest-taste level to reduce the competition. Second, the market shares captured by the high- and low-quality firms are $(2 + \theta)/3$ and $(1 - \theta)/3$, respectively, and they are irrelevant to the firms’ quality decisions. It is noteworthy that

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Note that the lowest-taste level, $\theta$, has to be smaller than unity in the model where the quality cost is a fixed cost. Otherwise, the high-quality firm will capture the whole of the market.
the above two results continue to be valid when a specific tax or an ad valorem tax is taken into account. We find that the quality levels of the high-quality product under both the specific and ad valorem taxes are lower than the socially-optimal level and meanwhile the level under the ad valorem tax is lower than that under the specific tax. We can thereby infer that a specific tax is always superior to an ad valorem tax.\textsuperscript{12} Thus, the main result derived in this paper, in which a specific tax may be superior to an ad valorem tax is robust regardless of whether the quality cost is a variable or a fixed cost.

5. Concluding remarks

It can be commonly observed in the real world that products such as tobacco, alcohol, and diesel fuel and gasoline are vertically differentiated, and meanwhile the demand for each of these products is inelastic. Unfortunately, these characteristics are ignored in the existing literature.

By taking into account these features, we have shown that a specific tax can be welfare superior to an ad valorem tax, regardless of whether the quality levels are fixed in the short run or endogenously determined in the long run. The key point that a specific tax can be welfare superior to an ad valorem tax is that less of the high-quality

\textsuperscript{12} The proof is available from the authors upon request.
product is produced under the ad valorem tax scheme in the short run, while it is the case that the imposition of an ad valorem tax will cause the quality level of the low-quality product to be lower than would be the case under a specific tax in the long run.

Moreover, we have proved that when the ad valorem tax rate is high, a specific tax may not only be welfare superior to, but may also Pareto dominate an ad valorem tax in the long run. It should be noted that the main reason why an ad valorem tax is welfare superior to a specific tax in the traditional literature is that the demand elasticity becomes greater under an ad valorem tax. However, the above three results indicate that this characteristic will vanish when the consumers’ demand for these products is completely inelastic and the demand faced by firms is inelastic.

The results of this paper also indicate that a specific tax is welfare superior to an ad valorem tax when governments would like to collect larger tax revenues, regardless of whether the imposition of the tax scheme can or cannot influence the quality levels. This may be the main reason why the governments of many countries in the real world impose specific taxes on tobacco, alcohol, gasoline and diesel fuel.

References


Figure 1. The superiority of a specific tax and an ad valorem tax in the short run