Abstract

We investigate the relationship between competition and privatization policies. Existing works measured the toughness of competition by the number of firms and showed that the optimal degree of privatization is higher when the number of firms is larger. We adopt the relative profit approach where each firm maximizes the relative profit rather than their own profit, and the resulting competition becomes tougher when firms care about the rivals’ profit more. We find that the optimal degree of privatization is smaller when each firm cares about the rival’s profit more (so the market competition is tougher).

**JEL classification numbers:** H42, H44, L13, L32

**Key words:** partial privatization, relative profit, mixed oligopoly, competition policy
1 Introduction

Since the 1980s, we have observed a worldwide wave of privatization. Nevertheless, public and semipublic (partially privatized) firms are still active. Many of them compete with private firms in developed, developing, and transitional economies.\(^1\) Recently, studies on mixed oligopoly involving both private and public enterprises, have increasingly become popular.\(^2\) In the literature on mixed oligopoly, the privatization of public enterprises is still an important research topic. In addition, in near future, the privatization of temporally nationalized firms in recent financial crisis must become important political and economic issues. In this paper we adopt partial privatization model formulated by Matsumura (1998) and discus the optimal degree of privatization in mixed oligopoly.\(^3\)

The privatization of public firms often takes place in conjunction with reforms of the competition policy. Thus, the relationship between the privatization policy and the market competition in a mixed oligopoly has attracted considerable attention among researchers. The existing works on mixed oligopoly show that the privatization is more likely to improve welfare when the number of private firms is large.\(^4\) However, since these works adopt the assumption of increasing marginal

\(^1\) See, e.g., Matsushima and Matsumura (2003).

\(^2\) This interest in mixed oligopolies is due to their importance in relation to European, Canadian, and Japanese economies. Although they are less significant in the United States, there are some examples of mixed oligopolies such as the packaging and overnight-delivery industries. The analysis of mixed oligopoly dates back to at least Merrill and Schneider (1966). Recently, the literature on mixed oligopoly has become richer and more diverse. See Ishibashi and Matsumura (2006), Gil-Moltó and Poyago-Theotoky (2008) and Ishida and Matsushima (2009) and the works cited by them for recent development in this field.


costs, the cost structure in the industry changes when the number of private firms changes.\(^5\) We investigate the relationship between market competition and privatization policy by adopting a different approach: the relative profit (performance) approach.

We now explain the relative profit approach we adopted. We consider a model where firms are concerned about relative profits rather than their own profits. Consider a symmetric \(n\) firm private oligopoly where all firms are private. Firm \(i\)’s payoff is \(\pi_i - \alpha(\sum_{j \neq i} \pi_j)\), where \(\pi_i\) is firm \(i\)’s profits. Firms independently choose their outputs. Then, the equilibrium outcome converges to the monopoly one when \(\alpha\) converges to \(-1\), it becomes the Cournot equilibrium when \(\alpha = 0\), and it converges to the perfectly competitive one (Walrasian) when \(\alpha\) is close to \(1/(n-1)\). Thus, we can interpret \(\alpha\) as a parameter indicating the severity of competition, a larger \(\alpha\) indicates a more competitive market.\(^6\) The relative performance approach enables us to treat the degree of market competition as a continuous variable.\(^7\) We introduce this approach into a mixed oligopoly setting and discuss the relationship between \(\alpha\) and the optimal degree of privatization. We find that optimal degree of privatization is decreasing in \(\alpha\), i.e., a tougher competition reduces the optimal degree of privatization. This result makes sharp contrast to the existing result that an increase in the number of firms increases the optimal degree of privatization.

We discuss the rationale for employing objective functions based on relative performance in

\(^5\) If the marginal cost is constant and all firms have identical cost functions, the number of firms does not affect the cost structure in the industry. Note that most works on mixed oligopoly including above three papers do not adopt this assumption. For the reason the mixed oligopoly literature does not adopt this assumption, see Matsumura (1998).

\(^6\) For the related discussions, see Symeonidis (2000, 2008). Under the standard conditions in Cournot oligopoly, the ratio between profit margin (price minus marginal cost) and the price, known as Lerner index, is decreasing in \(\alpha\). This index is intensively used in the empirical literature as a measure of market competition in product markets.

\(^7\) The conjectural variation approach is another approach that contains three models as special cases. However, the conjectural variation model assumes that firm 1’s output affects that of firm 2 and \textit{vice versa}. Needless to say, this assumption is inconsistent in any static model. The relative performance approach does not have this defect. This is an advantage of this approach.
a general context. First, evaluations of managerial performances are often based on the relative performance of managers as well as on their absolute performance. Outperforming managers often obtain good positions in the management job markets. This provides the rationale for considering positive $\alpha$ in our model. Second, many laboratory (experimental) works pointed out the spiteful behavior as well as reciprocal behavior or altruistic behavior, which is closely related to the objective functions based on relative performance (both positive and negative $\alpha$).\(^8\) Third, relative performance, especially the positive $\alpha$ case, is quite important from the viewpoint of evolutionary stability.\(^9\) Fourth, if private stockholders use $\alpha$ strategically, they adopt a positive $\alpha$.\(^10\) We believe that the relative performance approach is an important approach in the context of mixed oligopoly, too.

2 The Model

Firms produce perfectly substitutable commodities for which the market demand function is given by $p = 1 - Q$ (price as a function of quantity). Firm $i$’s cost function is given by $c_i(q_i) = (c/2)(q_i)^2$ where $q_i$ is the output quantity of firm $i$ ($i = 0, 1, ..., n$) and $c$ is a positive constant.\(^11\) We assume that $c > 1/2$ so as to ensure the interior solution in the following game. Firm 0 is a semipublic (partially privatized) firm, and it competes against $n$ private firms in the domestic product market.

Let $\pi_k$ ($k = 0, 1, ..., n$) be firm $k$’s profit. Let $V_i := \pi_i - \alpha(\sum_{j \neq i} \pi_j)$ be firm $i$’s relative profits.

\(^8\) See Coats and Neilson (2005), among others.

\(^9\) See Alchian (1950) and Vega-Redondo (1997). Vega-Redondo (1997) also shows that Cournot competition with relative performance objectives yields the Bertrand outcome even in duopoly and that this outcome is evolutionary stable.

\(^10\) See Kockesen et al. (2000). The payoff function based on relative wage or relative wealth status has been intensively discussed in the macroeconomics context, as well. Keynes (1936) discussed the rigidity of nominal wage based on relative wage. See also Akerlof and Yellen (1988) and Corneo and Jeanne (1997, 1999), and Futagami and Shibata (1998).

Firm $i$’s ($i = 1, 2, ..., n$) payoff is given by $V_i$ and firm 0’s payoff is given by $U_0 = \theta V_0 + (1 - \theta)W$, where $W$ is the total social surplus (consumer surplus + the profits of firms).\footnote{Another possible formulation of payoffs is $V_0 = \pi_0$ and $V_i = \pi_i - \alpha(\pi_1 + \pi_2 + \ldots + \pi_i)$ where $i = 1, 2, ..., n$ (each private firm is concerned with the private rivals’ profits only). Proposition holds in this setting unless $c$ is too small.} $\theta \in [0, 1]$ represents the degree of privatization.\footnote{This is a standard formulation of the payoff for a semipublic firm, if we replace $V_i$ with $\pi_i$. See Matsumura (1998). See also Fujiwara (2007) for recent works adopting this approach. We assume that $W$ is consumer surplus plus profits of firms rather than consumer surplus + $\sum_{i=0}^{n} V_i$. CEOs of firms might be concerned about the relative performance of their firms because the good relative perform increases their future income, but we simply regard this is just as income transfer and consider higher (lower) valuation for one CEO does not produce any net income in the society.} We assume that $\alpha \in [-1/n, 1/n)$. If $\alpha = 1/n$, the equilibrium outcome is Walrasian. We exclude the possibility of a negative price-cost margin (price minus marginal cost) by assuming that $\alpha < 1/n$.

In the first stage, the government sells shares in firm 0 and chooses $\theta$. In the second stage, after observing $\theta$, each firm $i$ chooses $q_i$ ($i = 0, 1, ..., n$) independently (Cournot competition). Let $\sum_{i=0}^{n} q_i$ be $Q$. The profit of firm $i$ is given by $\pi_i = p(Q)q_i - c_i(q_i) = (1 - Q)q_i - cq_i^2/2$.

### 3 Equilibrium Analysis and Results

We adopt the subgame perfection as the equilibrium concept and solve the game by backward induction. First, we investigate the second stage competition. Let $Q_{-i} := \sum_{j \neq i} q_j$ be the total outputs of firms other than firm $i$. The first-order conditions of firm 0 and firm $i$ ($i = 1, 2, ..., n$) are

$$1 - (c + 1 + \theta)q_0 - (1 - \alpha \theta)Q_{-0} = 0,$$

$$1 - (1 - \alpha)Q_{-i} - (c + 2)q_i = 0 \quad (i = 1, 2, ..., n).$$

Let superscript $S$ be the second stage equilibrium. Solving the above equations yields

$$q_0^S = \frac{c + 1 - \alpha(n - 1) + \alpha n \theta}{H}, \quad q_1^S = q_2^S = \ldots = q_n^S = \frac{c + \alpha + \theta}{H},$$

$$\sum_{i=0}^{n} q_i^S = Q.$$
where \( H := (c + 1)(c + 1 + \alpha) + cn(1 - \alpha) + \theta(c + n + 1 + \alpha - \alpha^2 n) \).

Second, we consider the first stage. The government chooses \( \theta \) so as to maximize \( W \). Let \( \theta^* \) be the optimal \( \theta \). The first-order condition is

\[
\frac{1}{J^3}(1 - \alpha n)^2(c + 1 + \alpha)(-\theta J + n(1 - \alpha)c) = 0,
\]

where \( J := (c + 1 + \alpha)(c + n + 1 - \alpha(n - 1)) - n(1 - \alpha)(1 + \alpha) > 0 \). From (4), we have:

\[
\theta^* = \frac{nc(1 - \alpha)}{J} \in (0, 1).
\]

We present our result.

**Proposition** (i) \( \theta^* \) is decreasing in \( \alpha \). (ii) \( \theta^* \) is increasing in \( n \).

**Proof** From (5), we have

\[
\frac{\partial \theta^*}{\partial \alpha} = -\frac{nc}{J^2} \left( J + (1 - \alpha) \frac{\partial J}{\partial \alpha} \right) = -\frac{2c}{J^2} (c^2 + 4c + (3 - \alpha)(1 + \alpha)) < 0
\]

\[
\frac{\partial \theta^*}{\partial n} = \frac{c(1 - \alpha)}{J^2} \left( J - n \frac{\partial J}{\partial n} \right) = \frac{c(1 - \alpha)(c + 1 - \alpha)^2}{J^2} > 0 \quad \blacksquare
\]

Proposition (i) and (ii) present a sharp contrast. Proposition (ii) is similar to the existing result (an increase of the number of private firms is more likely to improve welfare).\(^{14}\) Suppose that \( \theta = 0 \). The marginal cost of public firm is equal to the equilibrium price, while that of private firm is strictly lower than the price. Thus, in equilibrium the public firm’s marginal cost is higher than the private firm’s. An increase of \( \theta \) reduces the public firm’s output and increases the other firms’ outputs. In other words, privatization induces the production substitution from the public firm to the private firms. Since the marginal cost of private firm is lower than that of the public firm, this production substitution economizes the total production costs, thereby improving welfare.

\(^{14}\) For example, De Fraja and Delbono (1989) discuss the case of \( \alpha = 0 \) and show that there exists \( m \) such that private oligopoly \( \theta = 1 \) yields a larger welfare than the mixed oligopoly \( \theta = 0 \) if and only if \( n > m \).
At the same time, the privatization reduces the total output, and hence consumer surplus. This tradeoff determines the optimal degree of privatization $\theta^*$. An increase in the number of private firms reduces the output of each private firm (and hence reduces the firm’s marginal cost). Thus, an increase in the number of private firms strengthens the welfare-improving effect of production substitution mentioned above. This yields Proposition (ii).

On the contrary, given the number of private firms, an increase in $\alpha$ accelerates competition and increases the output of each private firm (thereby increasing the firm’s marginal cost). Thus, the welfare gain from production substitution from the public firm to the private firms is smaller when $\alpha$ is larger. In other words, if the competition is more severe in the market, the welfare gain from production substitution induced by the privatization is smaller. This yields Proposition (i).

4 Concluding Remarks

In this paper we adopted a relative performance approach and investigate the relationship between competition and privatization policy. We find that an increase of the number of private firms increases the optimal degree of privatization. However, given the number of firms, an increase of the intensity of competition reduces the optimal degree of privatization. The literature on free entry markets emphasized that measuring the intensity of competition by the number of firms yields misleading implications. More intensive competition makes the market less profitable. As a result, tougher competition results in a smaller number of entering firms. If we incorporate this effects, our result that the optimal degree of privatization can be smaller under a tougher competition is strengthen. In this paper we measure the intensity of competition by $\alpha$ (the weight of relative performance in the payoff of firms). An increase in $\alpha$ directly reduces the optimal degree of privatization $\theta^*$. At the same time, an increase in $\alpha$ reduces the number of firms at free entry market, and it indirectly reduces $\theta^*$. Thus, we guess that at free entry market, a tougher

15 See Etro (2007).
competition is more likely to reduce the optimal degree of privatization.\textsuperscript{16}

In this paper we assume that all private firms are domestic. However, public enterprises often compete against both domestic and foreign private firms. The competition against foreign firms often yields different implications in mixed oligopoly. Incorporating this aspect and fully investigate the relationship of three policies, privatization, competition, and international trade policies remains for future researches.\textsuperscript{17}

\textsuperscript{16} For the discussions of free entry market in mixed oligopoly, see Anderson et al (1997), Brandão and Castro (2007), Fujiwara (2007), and Matsumura and Kanda (2005).

\textsuperscript{17} For the discussion of foreign private firms, see Bárcena-Ruiz and Garzón (2005a,b), Corneo and Jeanne (1994), Fjell and Heywood (2002), Heywood and Ye (2009), Inoue et al (2009), Matsumura (2003), and Ogawa and Sanjo (2007).
References


