

Efficiency of joint enterprises with internal bargaining

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Abstract : In this paper we take a close look at those strategic incentives arising in a situation where firms share the costs and profits in a multi-firm project, and bargain for their respective (precommitted) split of cost- and profit-shares. We establish that, when each firm's effort contribution to the joint undertaking is mutually observable (which is often the case in closely collaborative operations) and hence can form basis of the contingent cost- and profit-sharing scheme, it is not the gross economic efficiency but the super-/sub-additivity of the nett returns from effort that directly affects the sustainability of a profile of firms' effort contributions. The (in)efficiency result we obtain in this paper is of different nature from so-called "free riding" or "team competition" problems: the set of sustainable outcomes with bargaining over precommitted cost- and profit-shares is generally neither a superset nor a subset of the sustainable set without bargaining.

Keywords : cost sharing, profit sharing, repayment, subgame perfection.

JEL classification : L13, G31, O32.

1 Introduction

Cost sharing between multiple economic bodies, such as a *research joint venture* undertaken by multiple firms operating in the same industry, often tends to be encouraged from policy- and welfare-points of view.¹ Apparently, the leading reason for such “official” encouragement is the assertion that cost sharing can enhance cost-efficiency, eliminating otherwise wasteful *effort duplication*. A relevant example is the possibility of carrying out joint R&D activities in the development of new products and processes (see Katz, 1986; d’Aspremont and Jacquemin, 1988; Katz and Ordover, 1990; Kamien *et al.*, 1992; Suzumura, 1992). On the other hand it is well noted that, generally, each participant in any joint project has an incentive to “free ride” the effort exerted by other participants, which tends to entail an inefficient outcome with a suboptimally low level of effort chosen strategically by each participant.

Perhaps one of the most natural and the most spontaneous “solution” to this incentive problem is to allow firms to make their cost- and profit-shares in the joint project *directly contingent upon their exerted effort levels*. In a joint project where participating firms closely collaborate with one another, it is often not at all unrealistic to assume that they can accurately monitor each other’s effort levels. In this paper we model such a situation by a simple two-stage game where each firm decides its own effort investment in the first stage, followed by the realisation of their respective nett profit shares in the ensuing stage. By precommitting their nett profit shares *before deciding their respective effort investment*, firms can reward or punish each other’s effort decisions and thereby enforce a certain profile of effort investment even if their decisions (in the first stage of the game) are simultaneously and noncooperatively made.

Our main finding in this paper is that, even though the aforementioned contingent profit-sharing scheme serves as a means to circumvent the classical free riding problem, it can harbour a different kind of inefficiency. The gist of the difference between preceding studies and our model is not the mere fact that firms can precommit with their nett profit shares and monitor each other’s effort, but that there can be room for *bargaining among firms* when they predetermine their nett profit shares. Essentially, each firm’s bargaining power is closely related to its “outside alternative” which, in a game-theoretic context,

¹See the National Cooperative Research Act in the US; EC Commission (1990); and Goto and Wakasugi (1988), *inter alia*.

is largely parallel to the firm’s *unilateral deviation incentives*. This inevitably implies that an outcome which is not susceptible to strong unilateral deviation incentives by any of the participating firms, can be sustained as a (subgame perfect) equilibrium whether economically efficient or not.

To summarise, if each firm’s cost share is determined independently of their respective contributions, it tends to entail the classical free riding problem. Alternatively, if profit sharing can be made contingent upon each firms’ strategic contribution decisions, then the bargaining between participating firms tends to give rise to multiple equilibria, among which the effort levels are *complementary between firms* (i.e., the locus of equilibria is downward-sloping) if nett *joint* returns to effort is *submodular* between firms ; or *complementary between firms* (i.e., the locus of equilibria is upward-sloping) if nett joint returns to effort is *submodular*.

The basic structure of our model, together with its general qualitative feature, is laid out in section 2. We present illustrative examples in section 3 to develop intuition on how precommitted bargaining can affect the sustainability of economically efficient outcomes. Section 4 concludes the paper. A glimpse of extension to a more quantitative decisions made by each of the participating firms in a joint project, is given in Appendix B.

2 Basic model

We start our analysis from a simple model of cost- and profit-sharing between two economic bodies, referred to as “firm 1” and “firm 2” henceforth. Although these two “firms” are to launch a jointly undertaken project, each of them is to decide, simultaneously and noncooperatively, whether or not to make an incremental marginal contribution to the joint project. The *nett joint returns* from their decisions can be summarised in the following table.

		firm 2	
		contribute	not contribute
firm 1	contribute	$Y[1, 1]$	$Y[1, 0]$
	not contribute	$Y[0, 1]$	$Y[0, 0]$

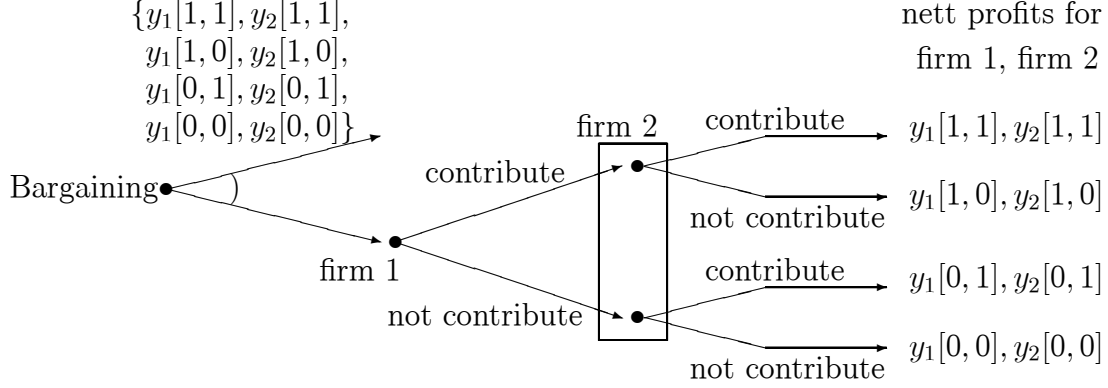
Throughout the paper we assume that contributions are *mutually observable*, so that firms can make their shares of nett profits $y_1[K]$ and $y_2[K]$ *contingent directly upon the*

profile of their contributions subject, obviously, to

$$y_1[K] + y_2[K] = Y[K] \quad [K] \in \{[1, 1], [1, 0], [0, 1], [0, 0]\}.$$

Hence, the procedural structure of this game can be summarised by the tree in figure 1.

Figure 1 : cost/profit sharing contingent upon contributions.



At the beginning, the two firms bargain² for the complete contingent set of nett profit shares. To retain as much generality as possible we avoid narrowly specifying the procedure of bargaining, other than imposing the following weak regularity requirement which seems plausible in any standard economic sense.

Definition : A bargaining function $b[\cdot, \cdot, \cdot] = (b_1[\cdot, \cdot, \cdot], b_2[\cdot, \cdot, \cdot])$ where

$$\begin{aligned} y_1[k_1, k_2] &= b_1 [y_1[\neg k_1, k_2], y_2[k_1, \neg k_2], Y[k_1, k_2]] & \{k_1, \neg k_1\} &= \{k_2, \neg k_2\} = \{0, 1\} \\ y_2[k_1, k_2] &= b_2 [y_1[\neg k_1, k_2], y_2[k_1, \neg k_2], Y[k_1, k_2]] \end{aligned}$$

is said to be **regular** if

- b_1 increases in $y_1[\neg k_1, k_2]$, decreases in $y_2[k_1, \neg k_2]$, and increases in $Y[k_1, k_2]$;
- b_2 decreases in $y_1[\neg k_1, k_2]$, increases in $y_2[k_1, \neg k_2]$, and increases in $Y[k_1, k_2]$;
- $b_1 = y_1[\neg k_1, k_2]$ and $b_2 = y_2[k_1, \neg k_2]$ whenever $y_1[\neg k_1, k_2] + y_2[k_1, \neg k_2] = Y[k_1, k_2]$.

Note that our definition of regularity implies

$$\text{sign} [y_1[k_1, k_2] - y_1[\neg k_1, k_2]] = \text{sign} [y_2[k_1, k_2] - y_2[k_1, \neg k_2]].$$

This automatically implies the following general feature.

²It deserves heightened attention that the presence of bargaining should not be mistaken as if we were invoking any sort of cooperative decision making. As is well known, for instance, Nash bargaining can be viewed as a limiting solution for Rubinstein's bargaining, which is in fact a genuinely noncooperative game and is entirely devoid of any form of cooperative decision making (Rubinstein, 1982).

Proposition 1 : For any regular bargaining function $b[\cdot, \cdot, \cdot]$ and nett joint profit schedule $Y[\cdot, \cdot]$, whenever a solution $\left\{ \left\{ y_i[k_1, k_2] \right\}_{k_1 \in \{0,1\}}^{k_2 \in \{0,1\}} \right\}_{i=1,2}$ exists, it sustains the equilibrium outcome such that :

- either both firms contribute or neither firm contributes if

$$Y[1, 1] + Y[0, 0] \geq Y[1, 0] + Y[0, 1]; \quad (2.1)$$

- only one of the two firms contributes if

$$Y[1, 1] + Y[0, 0] \leq Y[1, 0] + Y[0, 1]. \quad (2.2)$$

In words, {contribute, contribute} and {not contribute, not contribute} are equilibrium outcomes if the nett gains from the two firms' contributions are *superadditive* (as in inequality (2.1)).³ Otherwise, if the strategic contribution decisions are to yield *subadditive* nett profits (indicated by inequality (2.2), then {contribute, not contribute} and {not contribute, contribute} are equilibrium outcomes.⁴

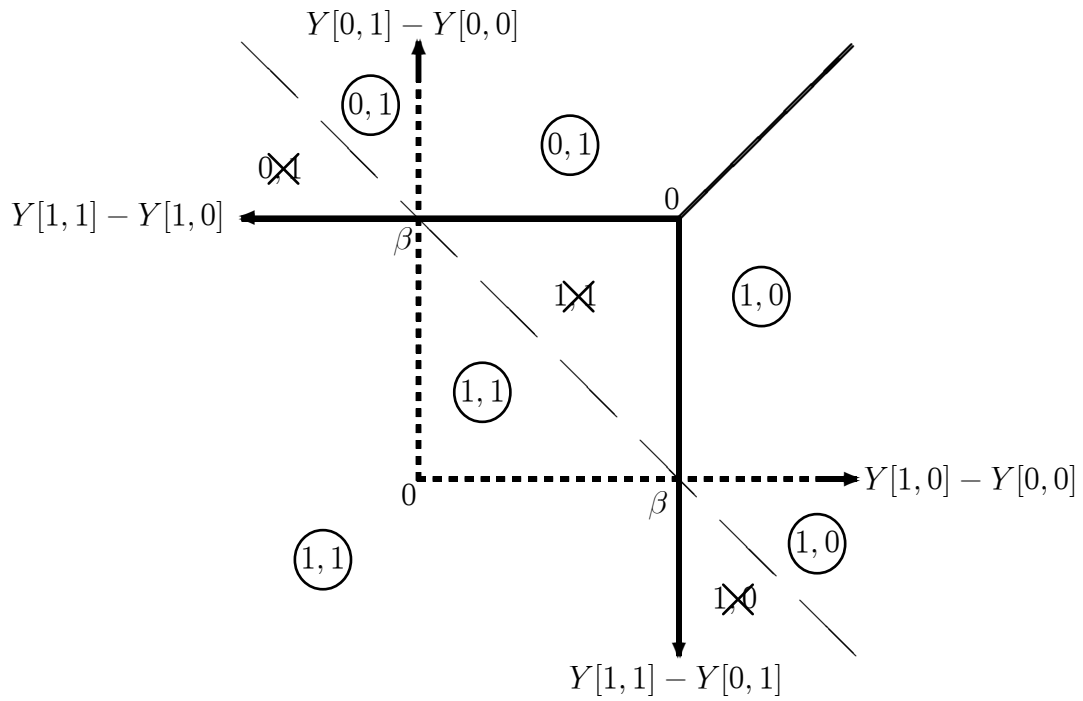
Economic implication : Proposition 1 implies that, in the prospect of bargaining between the participant firms inside the joint project, the equilibria resulting from each firm's "selfish" contribution decisions are determined solely on the grounds of super-/sub-additivity of the nett gains from contributions, not on the grounds of efficient outcomes *per se*. This can be illustrated in figure 2, where the most efficient contribution profiles change across thickened borders. They are sustainable through Nash bargaining where circled, unsustainable where crossed out. For, to the upper-right of the dashed diagonal the equilibrium profiles are [1, 0] and [0, 1], to its lower-left they are [1, 1] and [0, 0].

³Here, we refer to the discrete action version of superadditivity $Y[1, 1] - Y[0, 1] \geq Y[1, 0] - Y[0, 0]$ and submodularity $Y[1, 1] - Y[0, 1] \leq Y[1, 0] - Y[0, 0]$. In the former, the incentives for (or against) contribution are *supermodular* between the two firms (i.e., one firm's incentive to contribute is higher when the other firm does likewise than when the other firm does otherwise). In the latter, these incentives are *submodular*.

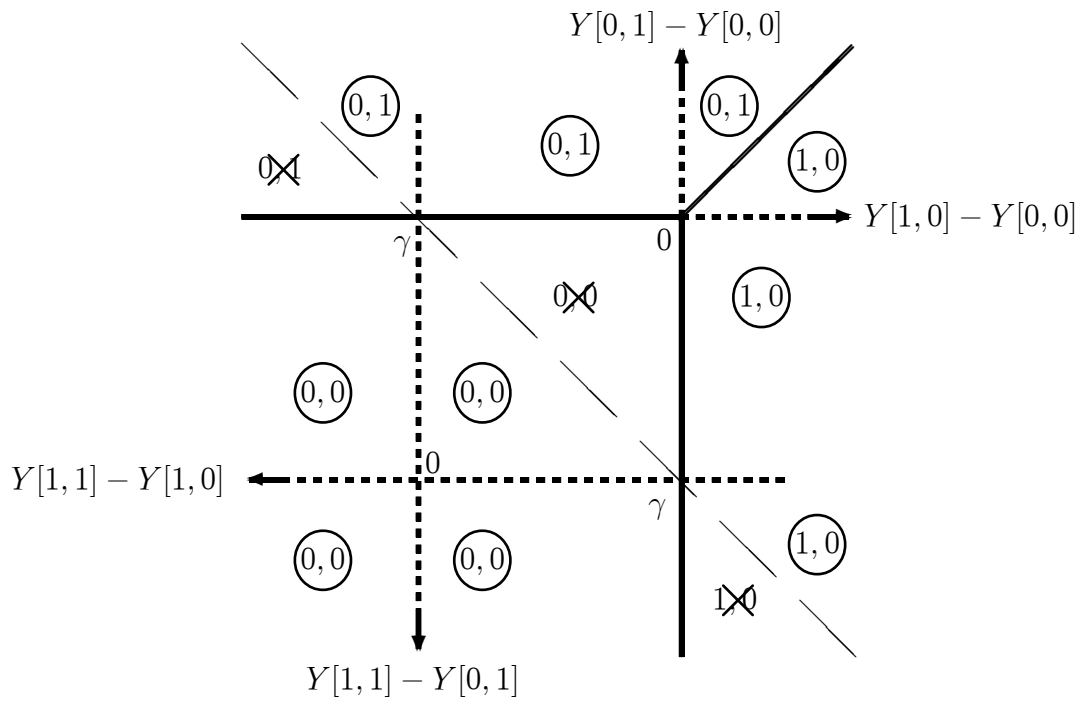
⁴Again, the analysis of R&D activity provides well known examples of both subadditivity and superadditivity, depending upon whether the choice of R&D efforts is followed by Cournot or Bertrand competition (see Brander and Spencer, 1983; Dixon, 1985; Bester and Petrakis, 1993).

Figure 2 : efficient outcomes and their sustainability via Nash bargaining :

(2a) when $Y[1, 1] - Y[0, 0] = \beta \geq 0$;



(2b) when $Y[1, 1] - Y[0, 0] = \gamma \leq 0$.



3 Illustrative examples

Probably the most popularly accepted bargaining solution concept is the Nash solution.⁵ Let the ratio of bargaining power between the two firms be $\psi_1 : \psi_2$. The two firms' nett profit shares when both firms contribute are determined as

$$\frac{y_1[1, 1] - y_1[0, 1]}{\psi_1} = \frac{y_2[1, 1] - y_2[1, 0]}{\psi_2}, \quad y_1[1, 1] + y_2[1, 1] = Y[1, 1]. \quad (3.1)$$

In words, firm 1's incentive (or disincentive) to "deviate" from contribution ($[1, \cdot]$) to no contribution ($[0, \cdot]$) should be matched against that for firm 2 (from $[\cdot, 1]$ to $[\cdot, 0]$), weighted by their respective bargaining power.⁶ Likewise,

$$\frac{y_1[1, 0] - y_1[0, 0]}{\psi_1} = \frac{y_2[1, 0] - y_2[1, 1]}{\psi_2}, \quad y_1[1, 0] + y_2[1, 0] = Y[1, 0], \quad (3.2)$$

$$\frac{y_1[0, 1] - y_1[1, 1]}{\psi_1} = \frac{y_2[0, 1] - y_2[0, 0]}{\psi_2}, \quad y_1[0, 1] + y_2[0, 1] = Y[0, 1], \quad (3.3)$$

$$\frac{y_1[0, 0] - y_1[1, 0]}{\psi_1} = \frac{y_2[0, 0] - y_2[0, 1]}{\psi_2}, \quad y_1[0, 0] + y_2[0, 0] = Y[0, 0]. \quad (3.4)$$

The above system of eight equations with eight unknowns leaves one degree of freedom and gives us the general solution

$$\begin{aligned} y_1[0, 0] + y_2[0, 0] &= Y[0, 0], \\ y_1[1, 1] &= y_1[0, 0] + \frac{Y[1, 1] - Y[1, 0] + Y[0, 1] - Y[0, 0]}{2}, \\ y_2[1, 1] &= y_2[0, 0] + \frac{Y[1, 1] - Y[0, 1] + Y[1, 0] - Y[0, 0]}{2}, \\ y_1[1, 0] &= y_1[0, 0] + \frac{\psi_1(Y[0, 1] + Y[1, 0] - Y[1, 1] - Y[0, 0])}{2(\psi_1 + \psi_2)}, \\ y_2[0, 1] &= y_2[0, 0] + \frac{\psi_2(Y[1, 0] + Y[0, 1] - Y[1, 1] - Y[0, 0])}{2(\psi_2 + \psi_1)}, \end{aligned}$$

⁵For algebraic simplicity, we treat Nash bargaining as sharing of the *nett excess surplus*, i.e., the nett profit differential in comparison to the "outside option" available to each firm, be the excess surplus positive or negative. The qualitative nature of the game would stand largely unaffected if we interpreted it as sharing of a *positive excess surplus* only, not adopting the same form of solution when the excess surplus is negative. The latter is technically more intricate but can be plausible depending upon what sort of economic circumstance is in question (see Appendix A).

⁶Again, in our metaphor to Rubinstein's bargaining and its limiting solution, this corresponds to the case where the two firms' degrees of patience, measured in terms of discount rates r_1 and r_2 , are in the ratio $r_1 : r_2 = \frac{1}{\psi_1} : \frac{1}{\psi_2}$. As we see hereinafter, we introduce these bargaining power parameters for the sake of completeness, or differently put, to demonstrate that our qualitative results do not hinge upon the distribution of bargaining power between the participating firms.

$$y_1[0, 1] = y_1[0, 0] + \frac{\psi_2(Y[1, 1] - Y[1, 0]) + (2\psi_1 + \psi_2)(Y[0, 1] - Y[0, 0])}{2(\psi_1 + \psi_2)},$$

$$y_2[1, 0] = y_2[0, 0] + \frac{\psi_1(Y[1, 1] - Y[0, 1]) + (2\psi_2 + \psi_1)(Y[1, 0] - Y[0, 0])}{2(\psi_2 + \psi_1)}.$$

Example 1 : *A priori* equal bargaining power $\psi_1 = \psi_2$, the private cost of contribution is £8 million per firm whilst the gross benefit from contribution is £10 million if only one firm contributes and £24 million if both contribute.

The nett joint return table becomes :

		firm 2	
		contribute	not contribute
firm 1	contribute	$Y[1, 1] = 8$	$Y[1, 0] = 2$
	not contribute	$Y[0, 1] = 2$	$Y[0, 0] = 0$

in £ million.

The nett joint return from a sole firm's contribution is £2 million whilst that from an additional firm's contribution is £6 million, whereby nett returns from contribution is *superadditive* between the two firms.

As aforementioned there are a continuum of bargaining solutions, but there is only one *symmetric* solution⁷ (in £ million):

$$y_1[0, 0] = y_2[0, 0] = 0, \quad y_1[1, 0] = y_2[0, 1] = -1,$$

$$y_1[0, 1] = y_2[1, 0] = 3, \quad y_1[1, 1] = y_2[1, 1] = 4.$$

This entails two pure strategy subgame perfect equilibria (simply “equilibria” hereinafter unless otherwise specified), in one of which neither firm contributes, in the other both firms contribute and share the profit equally. This reflects the fact that nett returns from contribution is superadditive (see Proposition 1 in section 2). Obviously, the latter equilibrium entails the most efficient (first best) outcome.

Were there no bargaining, instead if the two firms were to share the gross profit equally irrespective of their contributions, then the unique equilibrium outcome would be for neither firm to contribute. This is due to the classical *free riding problem* leading to underincentives for each firm to contribute. It is hereby concluded that the prospect of bargaining can help sustain the economically efficient outcome.

⁷We list the symmetric solution for nothing but concreteness. The set of equilibrium outcomes would, of course, be the same whether we selected the symmetric bargaining solution or an asymmetric solution.

Example 2 : *A priori* equal bargaining power $\psi_1 = \psi_2$, the private cost of contribution is £2 million per firm whilst the gross benefit from contribution is £10 million if only one firm contributes and £16 million if both contribute.

The nett joint return table becomes :

		firm 2	
		contribute	not contribute
firm 1	contribute	$Y[1, 1] = 12$	$Y[1, 0] = 8$
	not contribute	$Y[0, 1] = 8$	$Y[0, 0] = 0$

in £ million.

The nett joint return from a sole firm's contribution is £8 million whilst that from an additional firm's contribution is £4 million, whereby nett returns from contribution is *subadditive* between the two firms.

The unique *symmetric* solution (in £ million) is:

$$y_1[0, 0] = y_2[0, 0] = 0, \quad y_1[1, 0] = y_2[0, 1] = 1,$$

$$y_1[0, 1] = y_2[1, 0] = 7, \quad y_1[1, 1] = y_2[1, 1] = 6,$$

where the equilibria are for only one of the firms to contribute and take £3 million out of the gross profit of £10 million whilst the other firm takes the remainder, £7 million, without making contribution. This is the reflexion of the subadditivity of nett returns from contribution.

Obviously, the nett return from contribution is always positive, hence economically the most efficient outcome would be for both firms to contribute, which is nevertheless unsustainable through bargaining. Without bargaining, if the two firms were to split the gross profit always evenly, then the efficient outcome would indeed be the unique equilibrium outcome. In this case, unlike in our previous example, bargaining hinders the sustainability of economically the most efficient outcome.

Example 3 : *A priori* equal bargaining power $\psi_1 = \psi_2$, the private cost of contribution is nil for firm 1 and £10 million for firm 2, whilst the gross benefit from contribution is £4 million if only one firm contributes and £12 million if both contribute.

The nett joint return table is :

		firm 2	
		contribute	not contribute
firm 1	contribute	$Y[1, 1] = 2$	$Y[1, 0] = 4$
	not contribute	$Y[0, 1] = -6$	$Y[0, 0] = 0$

in £ million.

As this game is *a priori* asymmetric between the two firms, there is no “symmetric” solution. One of the solutions is

$$y_1[0, 0] = y_2[0, 0] = 0, \quad y_1[1, 0] = y_2[0, 1] = -1, \\ y_1[0, 1] = -5, \quad y_2[1, 0] = 5, \quad y_1[1, 1] = -4, \quad y_2[1, 1] = 6,$$

which accommodates two equilibria, in one of which neither firm contributes, in the other both firms contribute. Obviously in this case firm 1’s contribution is uniformly efficient whilst firm 2’s contribution is uniformly inefficient, hence the most efficient outcome is only for firm 1 not for firm 2 to contribute. This efficient outcome would be sustainable if gross profits are shared evenly all the time, but is not sustainable with the bargaining for profit sharing schedules.

4 Conclusion

Standard game-theoretic literature has it that, in any jointly undertaken productive activity, if the profit sharing schedule cannot be made contingent upon the level of effort exerted by each participating firm and hence each firm inevitably bears its own effort costs, then there arises systematic underincentives for efforts, entailing lower aggregate effort than economically efficient. This is the classical *free riding problem*.

What we have shown in this paper is that bargaining over a profit sharing schedule that is *contingent upon observed effort exerted by each participating firm* [I] can alleviate the free riding problem, yet [II] tends to entail a profile of effort levels based solely upon the super-/sub-additivity of nett returns to effort, irrespective of the nett joint productivity of effort. The latter inevitably implies that [IIa] when nett returns to effort are subadditive, a firm’s low effort tends to be traded for another firm’s high effort even when the joint nett returns to effort is uniformly positive (in which case the efficient outcome of all firms’ high effort becomes unsustainable) or when the joint nett returns is uniformly negative (where the efficient outcome would be all firms’ low effort), and that [IIb] when nett returns to

effort are superadditive, a firm's high effort tends to link with another firm's high effort, whereby the system fails to select for an efficient firm against an inefficient firm.

Appendix A

The alternative, technically more intricate, scenario is that firms must follow Nash solution only if each firm's excess surplus share is positive. The system of equations (3.1) through (3.4) is now replaced with :

$$\begin{aligned} \frac{y_1[1, 1] - y_1[0, 1]}{\psi_1} &= \frac{y_2[1, 1] - y_2[1, 0]}{\psi_2} & \text{if } & y_1[1, 1] + y_2[1, 1] = Y[1, 1] \geq y_1[0, 1] + y_2[1, 0], \\ \frac{y_1[1, 0] - y_1[0, 0]}{\psi_1} &= \frac{y_2[1, 0] - y_2[1, 1]}{\psi_2} & \text{if } & y_1[1, 0] + y_2[1, 0] = Y[1, 0] \geq y_1[0, 0] + y_2[1, 1], \\ \frac{y_1[0, 1] - y_1[1, 1]}{\psi_1} &= \frac{y_2[0, 1] - y_2[0, 0]}{\psi_2} & \text{if } & y_1[0, 1] + y_2[0, 1] = Y[0, 1] \geq y_1[1, 1] + y_2[0, 0], \\ \frac{y_1[0, 0] - y_1[1, 0]}{\psi_1} &= \frac{y_2[0, 0] - y_2[0, 1]}{\psi_2} & \text{if } & y_1[0, 0] + y_2[0, 0] = Y[0, 0] \geq y_1[1, 0] + y_2[0, 1]. \end{aligned}$$

It is intuitively clear that this leads to the same equilibrium result as in Proposition 1, although the exact set of sustainable solutions can now have one more degree of freedom than in section 2 (that is, two degrees of freedom in total).

Appendix B

Our basic analysis in section 2 can be straightforwardly extended to a discrete contributions space. Assume now that each of the two firms has a *trinary*, as opposed to the previously *binary*, choice of contributing either 0, 1, or 2 units to the jointly undertaken project. The nett joint returns table now becomes as follows.

		firm 2	2 units	1 unit	0 units
firm 1	2 units		$Y[2, 2]$	$Y[2, 1]$	$Y[2, 0]$
	1 unit		$Y[1, 2]$	$Y[1, 1]$	$Y[1, 0]$
	0 units		$Y[0, 2]$	$Y[0, 1]$	$Y[0, 0]$

Accordingly, our concept of regularity needs to be redefined as follows.

Redefinition : A bargaining function $b[\cdot, \cdot, \cdot] = (b_1[\cdot, \cdot, \cdot], b_2[\cdot, \cdot, \cdot])$:

$$\begin{aligned} y_1[k_1, k_2] &= b_1 [\max y_1[-k_1, k_2], \max y_2[k_1, -k_2], Y[k_1, k_2]], \\ y_2[k_1, k_2] &= b_2 [\max y_1[-k_1, k_2], \max y_2[k_1, -k_2], Y[k_1, k_2]], \end{aligned}$$

where

$$\begin{aligned} \max y_1[-k_1, k_2] &\equiv \max \{y_1[k_1^\bullet, k_2], y_1[k_1^\circ, k_2]\} & \{k_1, k_1^\bullet, k_1^\circ\} &= \{0, 1, 2\}, \\ \max y_2[k_1, -k_2] &\equiv \max \{y_2[k_1, k_2^\bullet], y_2[k_1, k_2^\circ]\} & \{k_2, k_2^\bullet, k_2^\circ\} &= \{0, 1, 2\}, \end{aligned}$$

is said to be **regular** if

- b_1 increases in $\max y_1[-k_1, k_2]$, decreases in $\max y_2[k_1, -k_2]$, and increases in $Y[k_1, k_2]$;
- b_2 decreases in $\max y_1[-k_1, k_2]$, increases in $\max y_2[k_1, -k_2]$, and increases in $Y[k_1, k_2]$;
- $b_1 = \max y_1[-k_1, k_2]$ and $b_2 = \max y_2[k_1, -k_2]$ whenever $\max y_1[-k_1, k_2] + \max y_2[k_1, -k_2] = Y[k_1, k_2]$.

Our interest is not in an exhaustive equilibrium comparative statics result on this game, but instead in the following analogue of our foregoing Proposition 1 (see section 2).⁸

Proposition 2 :

- The two firms' contributing the same number of units, either 0, 1, or 2 units each, is an equilibrium outcome if

$$Y[2, 2] + Y[1, 1] \geq Y[2, 1] + Y[1, 2]; \quad (\text{B.1})$$

$$Y[2, 2] + Y[0, 0] \geq Y[2, 0] + Y[0, 2]; \quad (\text{B.2})$$

$$Y[1, 1] + Y[0, 0] \geq Y[1, 0] + Y[0, 1]. \quad (\text{B.3})$$

- The two firms' contributing two units altogether, be it split 2-0 or 1-1, is an equilibrium outcome if

$$Y[2, 0] + Y[1, 1] \geq Y[2, 1] + Y[0, 1]; \quad (\text{B.4})$$

$$Y[2, 0] + Y[0, 2] \geq Y[2, 2] + Y[0, 0]; \quad (\text{B.5})$$

$$Y[1, 1] + Y[0, 2] \geq Y[1, 0] + Y[1, 2]. \quad (\text{B.6})$$

⁸Note that Proposition 2, unlike Proposition 1, does not exhaust the entire feasible range of parameter values $Y[\cdot, \cdot]$. For instance, it is possible that inequality (B.1) may be satisfied whilst inequalities (B.2) and (B.3) may be violated. Our intention here is to focus on those two cases listed specifically on Proposition 2, as these two are the relevant cases hereinafter.

Namely, when the nett gains from the two firms' contributions are *superadditive* in the sense of *positive affiliation* (as in (B.1) through (B.3)) the two firms' equilibrium contributions are *strategically perfectly complementary*; otherwise when the nett gains are *subadditive* (as in (B.4) through (B.6)) the equilibrium contributions are *strategically perfectly substitutable* between the two firms.

The reason why these two “extreme” cases appeal to our interest is because they enable us to extrapolate our analysis further to a continuous contributions space. An analogue of Propositions 1 and 2 is as below.

Proposition 3 :

- The two firms' effort levels k_1, k_2 are complementary along the locus of equilibria if $\frac{\partial^2 Y[k_1, k_2]}{\partial k_1 \partial k_2} > 0$;
- The two firms' effort levels k_1, k_2 are substitutional along the locus of equilibria if $\frac{\partial^2 Y[k_1, k_2]}{\partial k_1 \partial k_2} < 0$.

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