

Cost Differences, Strategic Location Decision, and Economic Welfare *

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Abstract

This paper examines non-traded goods producing firms' location in the economy where the firms locate in either of two markets with difference in production cost, and where they compete in a Cournot fashion in each market. The total number of firms is fixed. We establish: (1) firms insufficiently locate in the "low cost" market from the consumer's welfare viewpoint, while they excessively locate in the "low cost" market from the producer's and the whole economy's welfare viewpoints; (2) an increase in cost differences caused by the cost reduction (*resp.* enhancement) in low (*resp.* high) cost market improves (*resp.* hurts) welfare.

Keywords: Location Choice; Effective Cost Differences; Non-traded goods; Cournot Oligopoly; Inefficiencies

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1 Introduction

Many commodities have several segmented markets. Whereas a traded good may be supplied in all of segmented markets by its supplier located in a certain market, a non-traded good can be supplied only in a market where it locates. Due to managerial resource constraints, it is often difficult for a non-traded good supplier to locate in all segmented market. Therefore, a firm location decision is more closely related to its profitability when it is a non-traded good supplier than a traded good supplier.

There are the differences in wage rate and that in other factors' prices among the markets. These differences result in a non-traded goods supplier's production cost differences among them. In this situation, the supplier faces to a trade-off of location in the "low cost" market: the market is profitable because of low cost, whilst it is less profitable because competition becomes intensive. Thus, the non-traded producing firm's location decision depends on its rival's location as well as its effective production cost level.

We address the location decision of firms supplying non-traded goods: (1) Is the firm's location decision efficient from economic welfare viewpoint? (2) Does the reduction of cost differences improve economic welfare? The purpose of this paper is to examine the problems.

We construct a simple model: There are two segmented markets, whose effective production cost is different, called a *low cost market* and a *high cost market*. The total number of firm is fixed. Each firm determines where it locates. Given firms' location decision, each firm competes in Cournot fashion. We establish: (1) Firm's location in the low cost markets is insufficient from consumer's welfare viewpoint, while it is excessive from producer's and the whole economy's welfare viewpoint. (2) The increases in cost differences caused by the cost reduction (*resp.* enhancement) in low (*resp.* high) cost market improves (*resp.* hurts) welfare.

The problem of firm location has been extensively studied mainly in the economic geography for the last two decades. The literature (*e.g.*, Krugman, 1991; Fujita, Krug-

man and Venables, 1999; Baldwin, *et al.*, 2003) investigated firm's location choice. Our analysis differs from the literature in the following point: Those article explain firm's location choice by the movement of labor and these mainly examine how agglomeration or dispersion emerges in the equilibrium. These papers also assume that monopolistic competition prevails in the markets, implying that there is no strategic interaction among firms. On the other hand, our paper assume that Cournot competition prevails in the markets, and that firms *strategically* competes in each market.

Our result relates closely to the literature about the welfare effect of entry. Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) obtained *excess entry theorem*: a marginal decrease in long-run equilibrium number of firms improves economic welfare in a single free-entry oligopolistic market. Our paper differs from those articles in two points. The total number of firms is fixed in our analysis ¹, while the number is endogenously determined in the "excess entry" papers. Our paper analyzes where markets each firm locates, whilst the "excess entry" papers focus on whether each firm enters a *single* market. That is, our paper analyzes the *location* of each firm, while those literatures examines the *entry* of each firm.

The remaining of our paper is organized as follows: Section 2 presents our model and derives the equilibrium firm location. Section 3 examines whether the equilibrium firm location is excessive or insufficient. Section 4 examines the welfare effect of changes of cost differences. Section 5 concludes.

2 The Model and the Equilibrium

There are two segmented non-traded good markets with production cost differences, called *low cost* market L and *high cost* market H . The cost differences of firms comes from the fact that each market holds its intrinsic characters, such as wage rate, factor prices. We express production cost differences as differences in marginal cost. Cost

¹Elberfeld (2003) analyzes the firm's the technology choice with the fixed number of firms and demonstrates that cost-reducing investment is excessive from the welfare viewpoint. We apply his approach on the analysis of firm location.

function when a firm locates in market i is given by $C_{ij} = c_i x_{ij}$, where x_{ij} is quantity supplied by firm j in the market i ($i = L, H$; $j = 1, \dots, N$), $c_i \geq 0$ is a marginal cost with $c_H > c_L$.

A total number of firms is assumed to be fixed N . We assume that each firm can locate only in a single market because of managerial resource constraint.² Since goods are non-traded, each firm can supply the product only in the market where it locates.

An inverse demand functions are identical in both markets. Its function in market i is provided as follows:

$$p_i = p(X_i) = 1 - X_i^{\alpha+1}, \quad \alpha > -1, \quad (1)$$

where p_i and X_i are a price and a total output in market i , respectively ($i = L, H$), α represents the *price elasticity of the slope of demand*, i.e., $\alpha \equiv \frac{p'' X_i}{p'}$. This class of demand function keeps α constant.

We consider the following two stage game: In the first stage, each firm simultaneously determines which market it enters. In the second stage, given firms' location choices, it competes in the Cournot fashion.

We derive a subgame perfect equilibrium of the game by backward induction. At the second stage, given the number of firms in market i , n_i determined in the first stage, the profit of each firm in market i is

$$\pi_i = (p_i(X_i) - c_i) x_i. \quad (2)$$

The profit-maximizing condition for each firm in market i , is given by

$$p'_i(X_i) x_{ij} + p_i(X_i) - c_i = 0. \quad (3)$$

We focus on the symmetric equilibrium hereafter, i.e., $x_{ij} = x_i$. From equations (2) and (3), each firm's equilibrium output, the equilibrium total output, and the resulting profit π_i can be written as a function of the number of firms in the each market; that is, $x_i = x_i(n_i)$, $X_i = X_i(n_i) = n_i x_i(n_i)$, and $\pi_i = \pi_i(n_i)$. Under the demand function (1),

²This assumption is adopted in Barros and Cabral (2000) and Fumagalli (2003).

the *quasi-competitiveness*; i.e., $\frac{\partial X_i}{\partial n_i} > 0$, and the *business-stealing effect*; i.e., $\frac{\partial x_i}{\partial n_i} < 0$, are satisfied.³ In the following, we impose

Assumption 1

$$\pi_L(N) < \pi_H(1).^4$$

No agglomeration arises and some firms locate in each market under this assumption.

For the equilibrium profits, we can prove the following results.

Lemma 1

The equilibrium profit in either market decreases with the number of firms, i.e., $\pi'_i(n_i) < 0$.

Proof. See Appendix A. □

Lemma 2

For any given number of firms n , $\pi_L(n) > \pi_H(n)$ holds.

Proof. See Appendix B. □

Lemma 2 shows that a firm located in the low cost market earns more profit than that in the high cost one when the number of firms is identical between two markets.

Now let us introduce the *firm-number elasticity of firm's output in market i* by $\theta_i(n_i) \equiv -\frac{n_i}{x_i} \frac{\partial x_i}{\partial n_i}$ ($i = L, H$). Under the elasticity, $\theta_i(n_i)$ has the following properties.

Lemma 3

The firm-number elasticity of firm's output in market i is given by

$$\theta_i(n_i) = -\frac{n_i}{x_i} \frac{\partial x_i}{\partial n_i} = \frac{\alpha + n_i}{\alpha + n_i + 1} \tag{4}$$

for $i = L, H$. The following properties hold:

- (i) $\theta_i(n_i)$ is in the interval $(0, 1)$,*
- (ii) $\theta_i(n_i)$ is independent of the cost differences, and*
- (iii) $\theta_i(n_i)$ is increasing in n_i .*

³These properties are assumed in Mankiw and Whinston (1986).

⁴This assumption provides the ceiling of the difference in production cost.

Proof. See Appendix C. □

Let us go back to the first stage. If the resulting profit differs across the markets for a pattern of the firms location, any firm located in the market with less profit has an incentive to move another market. At the equilibrium location, the resulting profit must be equalized in both markets.⁵ We thus define the equilibrium location as follows.

Definition: The equilibrium location

The equilibrium location is a pair of (n_L^e, n_H^e) such that

- (i) $n_L^e + n_H^e = N$
- (ii) $\pi_L(n_L^e) = \pi_H(n_H^e)$, and
- (iii) For given n_i^e , equation (3) holds.

We then obtain the following property of the equilibrium location.

Proposition 1

- (i) *The equilibrium location is uniquely determined.*
- (ii) *The equilibrium number of firms in the low cost market exceeds that in the high cost market.*

Proof. See Appendix D. □

3 The Efficiency of the Equilibrium Location

In this section, we investigate whether the equilibrium location is efficient. In particular, we focus on whether the equilibrium number of firms in the low cost market is excessive or insufficient from the welfare viewpoint of producer, consumer, and the whole economy.

⁵We ignore the “integer problem” of the game.

3.1 Producer

We consider the efficiency of firm location from producer's welfare viewpoint. The producer's welfare of the economy W_P is producer surplus, which consists of the sum of firm's profit in each market, $PS_i = PS_i(n_i) \equiv n_i\pi_i$. That is, $W_P = n_L\pi_L + n_H\pi_H$. The effect of n_i on the producer surplus in each market is given by

$$PS'_i(n_i) = \pi_i + n_i p'_i x_i \frac{\partial X_i}{\partial n_i} + n_i(p_i - c) \frac{\partial x_i}{\partial n_i}. \quad (5)$$

The first term of equation (5) represents a *direct effect* of relocation. The second one shows a *price effect* for producers on the incumbents' revenue associated with the relocation. The third one is a *strategic effect*; *i.e.*, a change in incumbents' profits through strategic interaction against the relocation. Using equations (4) we rewrite (5) as

$$PS'_i(n_i) = \pi_i - n_i\pi_i(1 - \theta_i) - \pi_i\theta_i = (1 - n_i)(1 - \theta_i) \leq 0 \quad (6)$$

Equation (6) means that both price and strategic effects dominate direct effect.

Let introduce the new parameter z such that $\frac{dn_L}{dz} = 1$ and $\frac{dn_H}{dz} = -1$. From equation (6), we derive

$$\begin{aligned} \frac{\partial W_P}{\partial z} &= \frac{\partial}{\partial z}(PS_L + PS_H) \\ &= (\pi_L - \pi_H) - [n_L\pi_L(1 - \theta_L) - n_H\pi_H(1 - \theta_H)] - (\pi_L\theta_L - \pi_H\theta_H). \end{aligned} \quad (7)$$

The first term of equation (7) represents a differences in the direct effect, and The second (*resp.* the third) one is that in the price (*resp.* the strategic) effect.

Evaluating equation (7) at the equilibrium location, we have the following result.

Proposition 2

From producer's welfare viewpoint, firms excessively locate in the low cost market.

Proof. See Appendix E. □

Thus, the firms locate too concentrated in the low cost market from producer's standpoint. Proposition 2 is explained as follows. A relocating firm solely considers the

change in its own profit associated with relocation, *i.e.*, differences in direct effect. The firm is not concerned about differences in both price and strategic effects. The relocation from the market H to the market L brings about incumbents' revenues reduction (*resp.* enhancement) associated with price decrease (*resp.* increase) in market L (*resp.* H). This revenue reduction in the market L dominates that in market H because $n_L > n_H$. This implies that differences in price effect become negative.

The relocation also generates incumbents' profit reduction (*resp.* enhancement) in the market L (*resp.* H) through business-stealing effect. The former negative external effect dominates the latter positive external effect because $n_L > n_H$. Therefore, differences in price effect also become negative. Since the sign of both differences is negative at the equilibrium location, firms excessively locate in the low cost market.

Let (n_L^P, n_H^P) be the efficient location for producer, which maximizes producers' surpluses. Therefore, we obtain

Proposition 3

Suppose that demand function is not too concave. There exists at least two number of firms for producers, n_L^P , which locally maximizes producers' surpluses. One is in the interval $(1, \frac{N}{2})$, and another is in the interval $(\frac{N}{2}, n_L^e)$.

Proof. See Appendix F. □

3.2 Consumer

Next, we examine the efficiency from consumer viewpoint. The consumers welfare of the economy W_C is the sum of consumer surplus in each market, *i.e.* $W_C = CS_L + CS_H$, where CS_i is the consumer's surplus in the market i . The consumer surplus in the market i is defined as

$$CS_i = \int_0^{X_i} p_i(s)ds - p_i(X_i)X_i. \quad (8)$$

Differentiating equation (8) with respect to n_i and considering $X_i = n_i x_i$, we have

$$\frac{\partial CS_i}{\partial n_i} = -p'_i(X_i)X_i \frac{dX_i(n_i)}{dn_i} > 0. \quad (9)$$

Equation (9) implies that firm relocation affects consumers' expenditures in the market i . Since consumers' expenditures is total revenue of firms in market i , the RHS of (9) represents a *price effect* for consumers.

Using π_i and θ_i , we obtain the effect of relocation z on W^C as follows:

$$\begin{aligned}\frac{\partial W_C}{\partial z} &= \frac{\partial}{\partial z}(CS_L + CS_H) \\ &= \pi_L(n_L)n_L(1 - \theta_L) - \pi_H(n_H)n_H(1 - \theta_H).\end{aligned}\quad (10)$$

Evaluating equation (10) at the equilibrium location, we have the following result.

Proposition 4

From consumer's welfare viewpoint, firms insufficiently locate in the low cost market.

Proof. See Appendix G. □

Each firm does not pay attention to the price effect shown in (9) when it decides its location. The relocation generates consumers' expenditures saving (*resp.* enhancing) effect in market L (*resp.* H). The former positive external effect dominates the latter negative one, because the price effect for producers is negative. The differences in price effect for consumers are positive at the equilibrium location. That is why equilibrium location number of firms in the low cost market is inefficient.

3.3 The whole economy

Finally, we consider the efficiency of the equilibrium location from the welfare viewpoint of whole economy. The whole economy welfare, W is defined as a sum of consumer surplus and producer surplus, *i.e.* $W = W_C + W_P$. The whole economy welfare effect of relocation from market H to market L obtains

$$\frac{\partial W}{\partial z} = \frac{\partial W_C}{\partial z} + \frac{\partial W_P}{\partial z} = (\pi_L - \pi_H) - (\pi_L\theta_L - \pi_H\theta_H).\quad (11)$$

Since the positive price effect for consumers is canceled out by negative price effect for producers in each market, the effects of relocation on total surpluses consist of differences in both direct and strategic effects.

Evaluating equation (11) at the equilibrium location, we establish:

Proposition 5

From whole economy welfare viewpoint, firms excessively locate in the low cost market.

Proof. See Appendix H. □

The intuition behind Proposition 5 is as follows: In the equilibrium location, the moving firm's profits in the both market are equal. Only the strategic effect contributes to the welfare change. Note that the strategic effect works negatively (*resp.* positively) for existing firms in the market L (*resp.* market H). This strategic effect means externality. Since $n_L^e > n_H^e$ from Proposition 1, the negative externalities in market L dominate the positive externalities in market H . Therefore, the firms excessively locate in the low cost market from the whole economy viewpoint.

Let (n_L^T, n_S^T) be the efficient location for total surpluses viewpoint. We obtain

Proposition 6

n_L^T uniquely exists in the interval $(\frac{N}{2}, n_L^e)$.

Proof. See Appendix I. □

3.4 Market intervention in service FDI

We consider a following service FDI economy where all of firms belong to a certain country, called *source country*, and two markets belong to another country, called *host country*. Suppose that the source country (*resp.* the host country) government maximizes producers surpluses, W_P (*resp.* consumers surpluses, W_C).

We consider the policy intervention in which source country can assign each firm where to locate. From Lemmas 1 and 2, we derive

$$\pi_L(n_L^P) > \pi_L(n_L^e) = \pi_H(n_H^e) > \pi_H(n_H^P).$$

It shows that the policy implementation enhances a firm's profit in the low cost market, but reduces a firm's profit in the high cost market.

From Proposition 3 and quasi-competitiveness, the policy implementation reduces (*resp.* expands) total output in the low (*resp.* high) cost market. It implies that the policy narrows price differences between two markets.

Therefore, we establish

Remark 1

(i) *The source country policy intervention is beneficial for firms in the low cost market, but harmful for those in the high cost market.*

(ii) *The source country policy intervention is beneficial for the consumers in the high cost market, but harmful for those in the low cost market.*

Next, we consider a policy intervention in which host country can determines location choices. Let (n_L^C, n_H^C) be the efficient location for consumer, which maximizes consumers surpluses. That is, (n_L^C, n_H^C) is a pair of number of firms such that $\frac{\partial W_C}{\partial z} = 0$. At the efficient location, $\pi_L(n_L^C) < \pi_H(n_H^C)$ holds, because $\pi_L(n_L^C) < \pi_L(n_L^e) = \pi_H(n_H^e) < \pi_H(n_H^C)$ from Lemmas 1 and 2.

From quasi-competitiveness, the policy implementation increases (*resp.* reduces) total output in the low (*resp.* high) cost market. Thus, this policy expands price differences between two markets.

Therefore, we obtain

Remark 2

(i) *The host country's market intervention is beneficial for firms in the high cost market, but harmful for those in the low cost market.*

(ii) *The host country's market intervention is beneficial for consumers in the low cost market, but harmful for those in the high cost market.*

4 Welfare Effect of Cost Differences

We examine the welfare effect of a change in c_i at the equilibrium location. From (3) and definition of equilibrium location, we derive

Lemma 4

- (i) A marginal reduction of c_i increases (resp. decreases) n_i (resp. n_k).
- (ii) A marginal reduction of c_i increases (resp. decreases) X_i (resp. X_k).

Proof. See Appendix J. □

First we consider the effect of a marginal reduction of c_i on producers' surpluses W_P . From Lemmas 1 and 4(i) and equation (6), a marginal reduction of c_i increases firm's profit in the market k as well as producer surplus in the market k . A marginal reduction of c_i enhances profitability in the market i , which is a direct effect. An increase in profitability in the market i induces relocation from market k to market i . The relocation deteriorates competition in market k , and accordingly increases in both price and individual output, implying that it enhances firm's profit in market k . These are both price and strategic effects. Since equation (6) means that direct effect is dominated by both price and strategic effects, a marginal reduction of c_i enhances producer surplus in market k .

A marginal reduction of c_i must increase firm's profit in market i at an equilibrium location because of the increases in π_k . Since the relocation caused by a marginal reduction in c_i raises the number of firms in market i , the level of producer surplus in market k increases. Thus, we summarize

Proposition 7

A marginal reduction in market i 's marginal cost improves producers' welfare.

Next we consider the effect of a marginal reduction in c_i on consumers' surpluses W_C . From Lemma 4(ii) and equation (9), we establish

Proposition 8

A marginal reduction in market i 's marginal cost improves consumers' welfare.

Proof. See Appendix K. □

Lemma 4(ii) indicates that a marginal reduction in c_i increases consumer surplus in market i , but decreases it in market k . The former effect dominates the latter one,

because cost reduction in c_i directly increases X_i whilst it indirectly alters total outputs in both markets through relocation.

Furthermore we consider the effect of a marginal reduction in c_i on total surpluses. We establish the following result from Propositions 7 and 8:

Proposition 9

A marginal reduction in market i 's unit cost improves the whole welfare.

Proposition 9 states that an increases in cost differences caused by the cost reduction in low cost market improves the whole welfare, whereas the increases caused by the cost raise in high cost one hurts it.

Proposition 9 contrasts the result obtained in the model whose cost structure of each firm is fixed. In the situation, the social welfare may reduce even though the marginal cost of less efficient firm improves. The production substitution effect generates this seemingly counter-intuitive results.⁶ Our result means that this counter-intuitive phenomenon never appears when there are fixed total number of firms and when a firm can choose its cost structure.

5 Concluding Remarks

We have considered the situation where firms locate in either of two markets with difference in effective production cost under a given total number of firms, and addressed to the following problem: (1) Is firm's location choice efficient for viewpoint of economic welfare? (2) Does an enlargement in cost differences improve welfare? We establish: (1) Firms insufficiently locate in the low cost market from the consumer viewpoint, whilst they excessively locate in the low cost market from the producer viewpoint and the whole economy viewpoint. (2) An enlargement in cost differences caused by the cost reduction (*resp.* enhancement) in low (*resp.* high) cost market improves (*resp.* hurts) welfare.

⁶See Lahiri and Ono (1988), for instance.

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Mathematical Appendices

A The Proof of Lemma 1

Differentiating equation (2), and considering quasi-competitiveness and business-stealing effect yield

$$\pi'_i(n_i) = (p_i - c_i) \frac{\partial x_i}{\partial n_i} + p'_i(X_i) x_i \frac{\partial X_i}{\partial n_i}.$$

From equation (3), we transform the above equation into

$$\pi'_i(n_i) = p'_i(X_i) x_i \left(\frac{\partial X_i}{\partial n_i} - \frac{\partial x_i}{\partial n_i} \right). \quad (\text{A.1})$$

The sign of $\pi'_i(n_i)$ is thus negative because, under the demand function (1), $\frac{\partial x_i}{\partial n_i} < 0$ and because $\frac{\partial X_i}{\partial n_i} > 0$.

B The Proof of Lemma 2

Suppose that the number of firms in market i is n . Differentiating the above equation with respect to c_i and considering the first-order condition of profit-maximization (3) yield

$$\frac{\partial \pi_i}{\partial c_i} = (np'_i x + p_i - c_i) \frac{\partial x_i}{\partial c_i} = (n-1)p'_i \frac{\partial x_i}{\partial c_i}. \quad (\text{B.1})$$

Also, the first order condition (3) provides

$$\frac{\partial x_i}{\partial c_i} = -\frac{n}{p'_i(\alpha + n_i + 1)} < 0. \quad (\text{B.2})$$

From (B.1), (B.2), and $\alpha > -1$, $\frac{\partial \pi}{\partial c_i} < 0$, implying that $\pi_L(n) > \pi_H(n)$ for a given n .

C The Proof of Lemma 3

From equation (3), we obtain

$$\frac{\partial X_i}{\partial n_i} = -\frac{p_i - c_i}{p''_i X_i + (n_i + 1)p'_i}. \quad (\text{C.1})$$

Also, from equation (3), $p_i - c_i = -p'_i x_i$ holds at the equilibrium. Using the relation $\frac{\partial X_i}{\partial n_i} = x_i + n_i \frac{\partial x_i}{\partial n_i}$, equation (C.1) is transformed into

$$\theta_i(n_i) = -\frac{n_i \frac{\partial x_i}{\partial n_i}}{x_i \frac{\partial n_i}{\partial n_i}} = \frac{\alpha + n_i}{\alpha + n_i + 1}. \quad (\text{C.2})$$

Since $\alpha > -1$, therefore, θ_i is positive and less than one. It is straightforward to see that $\theta(n_i)$ is independent of the cost differences from equation (C.2).

Differentiating equation (C.2) with respect to n_i , we have

$$\theta'(n_i) = \frac{1}{(\alpha + n_i + 1)^2} = (1 - \theta)^2 > 0. \quad (\text{C.3})$$

D The Proof of Proposition 1

Since $n_H + n_L = N$ and Lemma 1, $\pi'(n_L) < 0$ and $\frac{\partial \pi_H}{\partial n_L} > 0$. From Lemma 2, we derive

$$\pi_L\left(\frac{N}{2}\right) > \pi_S\left(\frac{N}{2}\right). \quad (\text{D.1})$$

Thus, Assumption 1 ($\pi_L(N-1) < \pi_H(1)$) and (D.1) ensures that n_L^e uniquely exist in the interval $(\frac{N}{2}, N-1)$.

E The Proof of Proposition 2

From (C.2), we rewrite equation (7) as

$$\begin{aligned} \frac{\partial W_P}{\partial z} &= \pi_L[(1 - n_L)(1 - \theta_L) - (1 - n_H)(1 - \theta_H)] \\ &\quad + (\pi_L - \pi_H)(1 - n_H)(1 - \theta_H) \\ &= \pi_L \left[\left(\frac{n_H}{n_H + \alpha + 1} - \frac{n_L}{n_L + \alpha + 1} \right) + (\theta_H - \theta_L) \right] \\ &\quad + (\pi_L - \pi_H)(1 - n_H)(1 - \theta_H). \end{aligned} \quad (\text{E.1})$$

Equation (E.1) states that $\frac{\partial W_P}{\partial z} < 0$ at $n_L = n_L^e$. Therefore, firms excessively locate in low cost market.

F The Proof of Proposition 3

Equation (E.1) shows that (n_L^P, n_S^P) satisfies $\frac{\partial W_P}{\partial z} = 0$. Since $\frac{\partial W_P}{\partial z} >$ (resp. $<$)0 when $n_L = \frac{N}{2}$ (resp. $\forall n_L \in (n_L^e, N)$), there is at least one efficient location for producers is in the interval $(\frac{N}{2}, n_L^e)$.

When $N_L = 1$, $\frac{\partial W_P}{\partial z} > 0$ from equation (7). It implies that if producers' surpluses in the location pattern $(1, N - 1)$ denoted by $W_P(1)$ is greater than those in the pattern $(\frac{N}{2}, \frac{N}{2})$ denoted by $W_P(\frac{N}{2})$, then there is at least one efficient location for producers is in the interval $(1, \frac{N}{2})$. From now, we will prove that $W_P(1) \geq W_P(\frac{N}{2})$.

Differentiating equation (5) with respect to n_i and considering equations (3) and (C.2) yield

$$\begin{aligned} PS_i''(n_i) &= (p_i''X_i + 2p_i') \left(\frac{\partial X_i}{\partial n_i} \right)^2 + (p_i'X_i + p - c_i) \frac{\partial^2 X_i}{\partial n_i^2} \\ &= p_i'x_i^2 \left[(\alpha + 2)(1 - \theta_i)^2 - \frac{n_i - 1}{n_i}(1 - \theta_i)\theta_i - (n_i - 1)\theta_i' \right]. \end{aligned} \quad (\text{F.1})$$

From equations (C.2) and (C.3), we rewrite equation (F.1) as

$$\begin{aligned} PS_i''(n_i) &= p_i'x_i^2(1 - \theta_i) \left[(\alpha + 2)(1 - \theta_i) - \frac{n_i - 1}{n_i}(1 - \theta_i)\theta_i - (n_i - 1)(1 - \theta_i)^2 \right] \\ &= p_i'x_i^2(1 - \theta_i)^2 \left[\frac{(\alpha + 1) - 2(n_i - 1)^2}{n_i(n_i + \alpha + 1)} \right]. \end{aligned} \quad (\text{F.2})$$

From equation (F.2), we obtain the followings:

$$PS_i''(n_i) \leq 0 \quad \text{for } n_i \in [1, \frac{\sqrt{1+\alpha}}{2} + 1], \quad (\text{F.3})$$

$$PS_i''(n_i) > 0 \quad \text{for } n_i \in (\frac{\sqrt{1+\alpha}}{2} + 1, N]. \quad (\text{F.4})$$

We assume that $\frac{\sqrt{1+\alpha}}{2} + 1 \leq \frac{N}{2}$. This assumption ensures that, from equation (F.3), PS_i is a strictly convex function of $n_i \in (\frac{N}{2}, N]$. Note that this assumption implies that demand function is not too concave.

From equations (F.3) and (F.4), we obtain

$$\begin{aligned} \int_{\frac{N}{2}}^1 PS'_L(n_L)dn_L &> \int_{N-1}^{\frac{N}{2}} PS'_L(n_L)dn_L \\ PS_L(1) - PS_L(\frac{N}{2}) &> PS_L(\frac{N}{2}) - PS_L(N-1). \end{aligned} \quad (\text{F.5})$$

Suppose that n_i is fixed. Using equation (B.2), we differentiate equation (5) with respect to c_i as

$$\begin{aligned} \frac{\partial PS'_i(n_i)}{dc_i} &= (p''_i X_i + 2p'_i)n_i \frac{\partial x_i}{\partial c_i} x_i(1 - \theta_i) - x_i(1 - \theta_i) + (p'_i X_i + p_i - c_i) \frac{\partial x_i}{\partial c_i} x_i(1 - \theta_i) \\ &= x_i(1 - \theta_i)(n_i - 1) \left[\frac{(\alpha + 3)n_i + \alpha + 1}{n_i + \alpha + 1} \right] > 0. \end{aligned} \quad (\text{F.6})$$

Since $PS'_L(n) < PS'_H(n)$ for any n from equation (F.6), we obtain

$$\begin{aligned} \int_{N-1}^{\frac{N}{2}} PS'_L(n_L)dn_L &> \int_{N-1}^{\frac{N}{2}} PS'_H(n_H)dn_H \\ PS_L(\frac{N}{2}) - PS_L(1) &> PS_H(\frac{N}{2}) - PS_H(N-1). \end{aligned} \quad (\text{F.7})$$

From equations (F.5) and (F.7), therefore, we have

$$\begin{aligned} \int_{\frac{N}{2}}^1 PS'_L(n_L)dn_L &> \int_{N-1}^{\frac{N}{2}} PS'_H(n_H)dn_H \\ PS_L(1) - PS_L(\frac{N}{2}) &> PS_H(\frac{N}{2}) - PS_H(N-1) \\ W_P(1) = PS_L(1) + PS_H(N-1) &> PS_L(\frac{N}{2}) + PS_H(\frac{N}{2}) = W_P(\frac{N}{2}). \end{aligned}$$

G The Proof of Proposition 4

At the equilibrium location,

$$\frac{\partial W_C}{\partial z} = \pi \left\{ \frac{n_L}{n_L + \alpha + 1} - \frac{n_H}{n_H + \alpha + 1} \right\} > 0.$$

This represents that firms insufficiently locate in low cost market for consumer viewpoint.

H The Proof of Proposition 5

We find that at the equilibrium location,

$$\frac{\partial W}{\partial z} = \pi(\theta_H - \theta_L) < 0,$$

whose sign is derived from Lemma 3(iii).

I The Proof of Proposition 6

We transform the RHS of (11) into $\pi_L(1 - \theta_L) - \pi_H(1 - \theta_H)$. From Lemmas 1 and 3, we derive

$$\frac{\partial \pi_L(1 - \theta_L)}{\partial z} = \pi'_L(1 - \theta_L) - \pi_L \theta'_L < 0, \quad (\text{I.1})$$

$$\frac{\partial \pi_L(1 - \theta_L)}{\partial z} = -[\pi'_H(1 - \theta_H) - \pi_H \theta'_H] > 0. \quad (\text{I.2})$$

We also show that from equation (11) $\frac{\partial W}{\partial z} >$ (resp. $<$) 0 for $n_L = \frac{N}{2}$ (resp. n_L^e). From (I.1) and (I.2), n_L^T uniquely exists in $(\frac{N}{2}, n_L^e)$.

J The Proof of Lemma 4

Differentiating the equations consisting of equilibrium location totally and rearranging terms yield

$$\begin{bmatrix} -1 & 1 & -\frac{n_i-1}{x_i} & \frac{n_k-1}{x_k} \\ x_i(1 + \frac{\alpha}{n_i}) & 0 & n_i + \alpha + 1 & 0 \\ 0 & x_k(1 + \frac{\alpha}{n_k}) & 0 & n_k + \alpha + 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} dn_i \\ dn_k \\ dx_i \\ dx_k \end{bmatrix} = \begin{bmatrix} -\frac{1}{p'_i x_i} \\ \frac{1}{p'_i} \\ 0 \\ 0 \end{bmatrix} dc_i,$$

for $(i, k) = (L, H), i \neq k$. The determinant of LHS denoted by Δ is

$$\Delta = -(n_L + \alpha + 1)(1 + \frac{\alpha}{n_H}) - (n_H + \alpha + 1)(1 + \frac{\alpha}{n_L}) < 0$$

for $(i, k) = (L, H), i \neq k$. Using Cramer's rule, we obtain

$$\frac{\partial n_i}{\partial c_i} = -\frac{\partial n_k}{\partial c_i} = -\frac{1}{\Delta p'_i x_i} (2 + \alpha)(n_k + \alpha + 1) < 0, \quad (\text{J.1})$$

$$\frac{\partial x_i}{\partial c_i} = \frac{1}{\Delta p'_i} \left[\frac{\alpha(n_k + \alpha + 1)}{n_i} - \frac{\alpha}{n_k} - 2 \right], \quad (\text{J.2})$$

$$\frac{\partial x_k}{\partial c_i} = -\frac{x_k}{\Delta p'_i x_i} (2 + \alpha) \left(1 + \frac{\alpha}{n_k} \right), \quad (\text{J.3})$$

Equation (J.1) shows the statement in Lemma 4(i).

We will prove Lemma 4(ii). From equations (J.1)-(J.3), we derive the effect of c_i on total output in each market.

$$\frac{\partial X_i}{\partial c_i} = \frac{\partial n_i}{\partial c_i} x_i + n_i \frac{\partial x_i}{\partial c_i} = -\frac{1}{\Delta p'_i} \left[2(N + \alpha + 1) + \frac{n_i}{n_k} \alpha \right] < 0, \quad (\text{J.4})$$

$$\frac{\partial X_k}{\partial c_k} = \frac{\partial n_k}{\partial c_i} x_k + n_k \frac{\partial x_k}{\partial c_i} = \frac{(\alpha + 2)x_k}{\Delta p'_i x_i} > 0. \quad (\text{J.5})$$

Equations (J.4) and (J.5) show the statement in Lemma 4(ii).

K The Proof of Proposition 8

From equations (8), (J.4), and (J.5), we have

$$\begin{aligned} \frac{\partial W_C}{\partial c_i} &= -p'_i X_i \frac{\partial X_i}{\partial c_i} - p'_k X_k \frac{\partial X_k}{\partial c_i} \\ &= -\frac{\pi n_k}{\Delta p'_i x_i} \left[2(n_i + N) + \left(1 + \frac{n_i}{n_k} \right) \right] < 0, \end{aligned}$$

which implies that a decrease in c_i raises consumers' surpluses.