Effects of discount factor distribution in repeated games — fads, fashion, and novelty fair —

Taku Masuda[1]

1 Introduction

NEW MODELS tend to be released almost every year, by almost every manufacturer in industries such as automobiles, electric and electronic appliances, let alone clothing and fashion apparels. Some of these new models do indeed incorporate material improvements, embodying some of the latest technological advancements of the year. In many if not most cases, however, new models offer only superficial, very marginal, or otherwise trivial alterations from their predecessors, even though they are invariably announced big-time as the very latest models of the year, not of the previous year or even several years before which they substantively are.

Without doubt, such marketing gimmicks exploit consumer psychology. Presumably some, if not all, consumers are temporarily attracted to what appears to be novelle. Given this tendency, it is hardly surprising that frequent release of new models turn out profitable insofar as their development costs are manageable, which indeed further explains why many of these new models involve no more than minor incremental improvements.

Viewed closely, however, there also exists a counterforce. Namely, it is not as if releasing trivial new models every day, or every hour, might raise extreme profits. For one, excessively frequent release would hurt the credibility of new models and thus would defeat the novelty purpose. For another, most of these consumer goods are quite durable: that is, not only cars and electronics, but even clothing items – how many of us discard our clothes after exactly one single season? Consumers who have just purchased what they want, tend to disappear from the market for awhile, so that it is best for the next release to wait until these customers return.

Still, our vastly common intuition goes that new models tend to be released more frequently than they "should" in the aforesaid double meanings: that their excessive frequency often compromises their attractiveness, and also that they fail to wait for the customers to return.

[1]Student of Graduate School of Economics, University of Tokyo. Email: taku121281@gmail.com
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In this paper we contemplate an extremely oversimplified model which nonetheless illustrates incisively how the excess release phenomenon can be sustained as an equilibrium, in the framework of repeated oligopoly.

2 Basic model

Duopolists operate in a repeated market over time $t = 0, \pm 1, \pm 2, \cdots$.[2] Their stage profits are π_C per firm if they collude; otherwise a firm can temporarily earn π_D by deviating from collusion but thereafter each firm earns π_N per period, where $\pi_D > \pi_C > \pi_N$. For simplicity, we normalise $\pi_N = 0$ unless specified otherwise, although our qualitative findings would stand intact insofar as π_N remains sufficiently low.[3]

In addition, each firm can temporarily enhance its stage profit by $1 + k$ times, where $k > 0$, by announcing a new model. This is effective if and only if the firm did not announce a new model in the previous period. The announcement is assumed to be costless for further simplicity, although the main spirit of our model would stand unaffected even if the announcement were slightly, not prohibitively, costly.

Given this structure, the most profitable path would be for firms to collude with new models announced every other period.

Lemma i : Collusion with new models released in periods $2n$, where $n = 0, \pm 1, \pm 2, \cdots$, is sustained as a subgame perfect equilibrium if and only if each firm has a discount factor no less than

$$\delta^*[k] = \frac{\pi_C}{2\pi_D}\left(-\frac{1}{1+k} + \sqrt{\left(\frac{1}{1+k}\right)^2 + \frac{4\pi_D}{\pi_C}\left(\frac{\pi_D}{\pi_C} - 1\right)}\right).$$

Sketched proof: It is intuitively straightforward that deviation incentives in periods $2n$ are to be checked; those in other periods are slack.

[2] Our main objective is to inspect the subgame perfection of *stationary* (including *periodic*) profiles. For believers that a game must commence from an unambiguously defined initial node, an easy alternative is to assume that the game unfolds over time $t = 0, 1, 2, \cdots$ with $t = 0$ being the default initial stage where both firms exogenously release new models and collude.

[3] These exhaust all relevant stage payoffs as we abstract away which specific collusive scheme these firms might adopt: whether simple trigger strategies with static Nash reversion *à la* Friedman (1971), or optimal punishment as in Abreu (1986, 1988) or Abreu, Pearce and Stacchetti (1986). According to Hæckner (1996), optimal punishment *à la* Abreu *et al* is to be sustained via a penal code which gives each firm what is referred to as security level profits, i.e., that level of profits whereby the participation constraint exactly binds. This corresponds to our π_N.

In periods $2n$, continuation profits discounted by factor δ need to beat the deviation profit, i.e.,

$$\sum_{\nu=0}^{\infty} \delta^{2\nu}(1+k+\delta)\pi_C = \frac{1+k+\delta}{1-\delta^2}\pi_C \;\geq\; (1+k)\pi_D\,.$$

Corollary i : The collusion in Lemma i requires a higher critical discount factor than that in standard repeated duopoly without new model release.

Sketched proof: The standard repeated duopoly without new model release corresponds to the extreme case in our model setting where $k = 0$. It is straightforward to confirm that the critical discount factor $\delta^*[k]$ in Lemma i is an increasing function in k, the parameter representing the effectiveness of new model release.

For the range of discount factors δ falls below $\delta^*[k]$, how can the firms possibly collude ?[4] Whilst a spontaneous glance might envision the firms colluding without ever releasing any new models at all, this proves even less sustainable than the aforesaid collusion in Lemma i.

Lemma ii : Collusion without ever releasing new models can be sustained if and only if each firm has a discount factor (weakly) higher than

$$\delta^{**}[k] = 1 - \frac{\pi_C}{(1+k)\pi_D}\,.$$

Sketched proof: Obviously the most profitable deviation from this collusive scheme is for a firm to release a new model and deviate at a time. Collusive continuation profits discounted by δ beat the deviation profit if and only if

$$\sum_{\nu=0}^{\infty} \delta^{\nu}\pi_C = \frac{\pi_C}{1-\delta} \;\geq\; (1+k)\pi_D\,. \qquad (\clubsuit)$$

Corollary ii : $\delta^{**}[k] > \delta^*[k]$.

Instead, when the discount factor falls short of $\delta^*[k]$, the firms can still collude by releasing new models <u>every period</u> – even though their release is utterly ineffective – or rather, precisely because it is ineffective.

Lemma iii : Collusion with new release every period is sustainable if and only if each firm has a discount factor higher than

$$\delta^*[0] = 1 - \frac{\pi_C}{\pi_D}\,.$$

[4] Albeit outside the scope of our present discussion, depending upon the specific form of oligopolistic interactions it might alternatively be feasible for the colluding firms to opt for what has been known as partial collusion, i.e., to collude on a path that is intermediate in between the fully collusive monopoly-like profile and the static Nash equilibrium.

Proof: Even the most profitable deviation from this collusive path would bring no more than π_D to the deviating firm. Hence the equilibrium condition would be our foregoing inequality (♣) with its right hand side replaced with simply π_D.

Corollary iii : This collusive scheme is more sustainable than that in Lemma i in that $\delta^*[0] < \delta^*[k]\ \forall k > 0$.

Proof follows from the fact that $\delta^{[}k]$ increases in k, in conjunction with $\delta^*[0] = \lim_{\ell \downarrow 0} \delta^*[\ell]$.

The upshot here is that, precisely because these firms know that excessively frequent release of new models is not an effective way of marketing, they deliberately opt for it as a means to curtail deviation opportunities.

The most profitable form of collusion *vis-à-vis* the discount factor δ can be summarised as follows, based upon Lemmata i and iii.

Proposition I : Collusion is
[i] sustainable, with each firm releasing a new model *every other period*, if $\delta \geq \delta^*[k]$,
[ii] sustainable, with each firm releasing a new model *every period*, if $\delta^*[0] \leq \delta < \delta^*[k]$,
[iii] unsustainable if $\delta < \delta^*[0]$.

3 Extension (1) : deviation incentives

As a general rule, sustainability of tacit collusion hinges upon the profitability on the collusive path comparative to that from deviation.

Previously, we have analysed the most profitable collusion as a function of the discount factor δ, taking the profit variables π_C, π_D (and π_N) as given. We now contemplate the other dimension, that is, to vary the profit structure, specifically the ratio $\frac{\pi_D}{\pi_C}$, keeping time preferences δ fixed.

The profit structure may be affected by a number of distinct factors, including:
- strategic variables – whether Bertrand, Cournot, or other supply functions;
- product differentiation – $\frac{\pi_D}{\pi_C}$ tends to increase in product substitutability;
- competition – $\frac{\pi_D}{\pi_C}$ tends to increase in the number of firms.

Reflecting the fact that $\delta^*[k]$ increases in $\frac{\pi_D}{\pi_C}$ for all $k \geq 0$, Proposition I can be reformulated as follows.

Proposition II : For any given discount factor $\delta \in (0, 1)$, the firms can

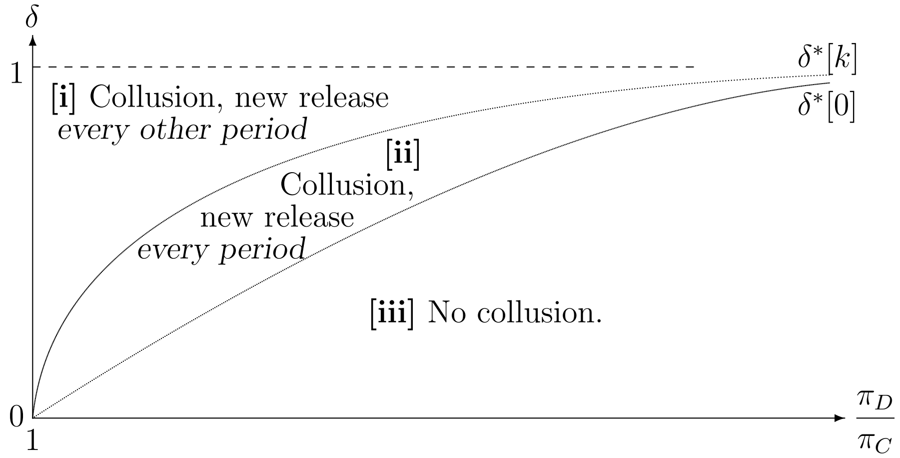
[i] collude, with each firm releasing a new model *every other period*, if $1 < \dfrac{\pi_D}{\pi_C} \leq k^*[\delta]$,

[ii] collude, with each firm releasing a new model *every period*, if $k^*[\delta] < \dfrac{\pi_D}{\pi_C} \leq 0^*[\delta]$,

[iii] not collude if $\dfrac{\pi_D}{\pi_C} > 0^*[\delta]$,

where $0^*[\delta]$ and $k^*[\delta]$ denote those values of the profit ratio $\dfrac{\pi_D}{\pi_C}$ which make $\delta^*[0] = \delta$ and $\delta^*[k] = \delta$ respectively.

Propositions I and II can be jointly illustrated as below.



4 Extension (2) : heterogeneous firms

We have hereinbefore confined our attention to symmetrical collusion, which can be sustained if and only if *all* colluding firms have discount factors above a certain threshold.

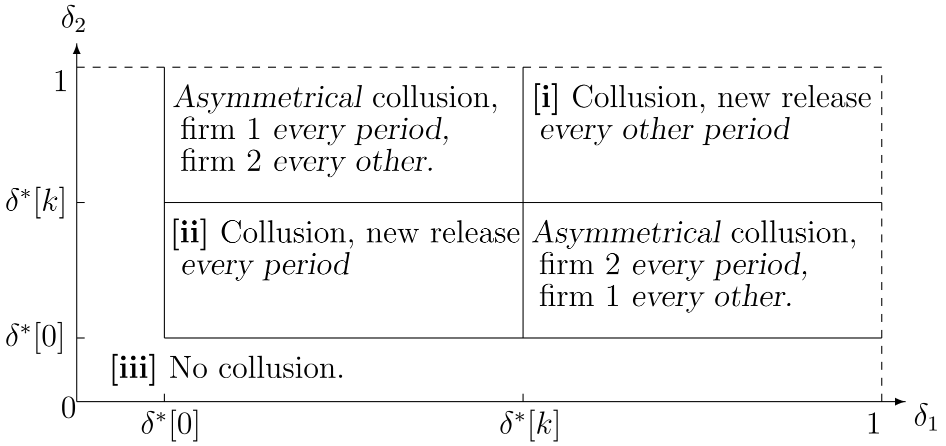
What if firms have heterogeneous time preferences? If they collude symmetrically in that all participant firms take identical actions on the collusive path, the only binding constraint is the discount factor held by the most myopic firm, whilst incentives of other firms, whose discount factors are higher, are slack. This implies the scope that asymmetrical collusion might prove better sustainability.

In our overly simplified model, asymmetrical collusion turns out extremely straightforward to contemplate. We now revisit our basic model from section 2 except that the duopolists are now allowed to have heterogeneous time preferences.

Noting that our foregoing analysis in Lemmata i through iii and Proposition I carries on with respect to each firm's incentives separately, the following result emerges.

Proposition III : Collusion is sustainable if and only if both firms have discount factors no less than $\delta^*[0]$, in which case any firm with $\delta \geq \delta^*[k]$ can collude with a new release every other period whilst any firm in the range $\delta^*[0] \leq \delta < \delta^*[k]$ have no choice but to collude with a new release every period.

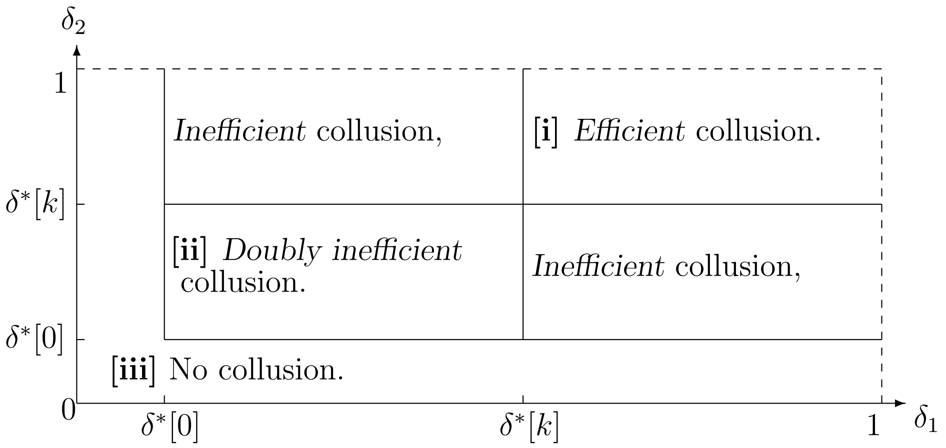
Configurations of the most profitable collusion can be illustrated as follows, where δ_1 and δ_2 represent the two firm's idiosyncratic discount factors, and [i] through [iii] correspond to those sub-cases in Propositions I and II.



5 Welfare comparative statics

Obviously, our oversimplified model is reduced-form, not tailored for welfare analysis. Welfare implications of our findings hinge critically upon [a] the deadweight loss caused by collusion, and [b] whether the promotion effect parametrised by k embodies substantive consumer satisfaction or a mere illusion which is to be offset by later disillusionment.

Nonetheless, insofar as we accept that releasing new models every period is wasteful, the following welfare appraisal emerges. Again, δ_1 and δ_2 denote the two firm's respective discount factors, and [i] through [iii] refer to corresponding equilibrium configurations in Propositions I and II.



It can be read from the above diagramme that welfare efficiency of the equilibrium outcomes proves non-monotone in δ_1 and δ_2 . This reflects the feature that, for the intermediate range of discount factors $\delta^*[0] \leq \delta < \delta^*[k]$, firms need to distort their actions in order to sustain collusion.

6 Discussion

We have been discussing the aforementioned firm behaviour mostly from the viewpoint of the colluding firms themselves, i.e., in terms of their time preferences and profit structures. On the flip side of the same coin, however, is the fact that our qualitative results are structurally driven by consumer preferences, parametrised by k and also the assumption that new release cannot promote the sales in two consecutive periods.

Put differently, the unit time period in our model serves to interlink two structural variables together. On one hand, it represents the duration of consumer memory. On the other, it functions as a unit of measurement against which the firms' time preferences are parametrised.

In this light, our findings can be reinterpreted that, when consumer memory is long comparative to firms' time preferences, the firms are forced to release new models more frequently than they "should" in order to sustain collusion. This is likely when :

- the good is very durable ;
- consumers intertemporarily seek product variety ;
- consumers have keen tastes for fashion and novelty (k high).

7 Conclusion

We have observed that excessively frequent release of new models, which *per se* is an ineffective marketing tactic, may nevertheless be opted for by tacitly colluding oligopolists. The intuition behind this seemingly paradoxical result is that, precisely because of its ineffectiveness, it can serve as a means to sustain collusion by curtailing deviation prospects.

Our findings exploit the assumption that consumers are not exactly memoryless. This assumption is realistic in markets for goods which are somewhat durable. Even in markets for nondurable consumption goods, this assumption may not be entirely unrealistic insofar as consumer tastes are temporarily satiable.

The major difficulty in analysing such markets with consumer memory, is the technical complication stemming from the structural time-inseparability. This is why we have confined our attention to the overly simplified reduced-form model. Analysis with more structure, technical intricacy notwithstanding, shall be highly longed for as a subject of prospective future research.

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