Competitive Grouping and Tacit Coordination of Complex Equilibria in GBM Mechanisms with Two Endowment Levels

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Abstract.

Purpose and approach: We examine theoretically and experimentally how unequal abilities to contribute affect incentives and efficiency when players compete for membership in stratified groups based on the contributions they make. Players have either a low or a high endowment. Once assigned to a group based upon the contribution they have made, players share equally in their group's collective output. Depending upon the parameters, the mechanism has several distinct equilibria that differ in efficiency.

Findings: The theoretical analysis indicates that as long as certain assumptions are satisfied, efficiency should increase rather than decrease the more abilities to contribute differ. The paper's general theoretical analysis suggests numerous follow-up experiments about equilibrium selection, tacit coordination, and the effect of unequal abilities in systems with endogenous grouping. The experiment shows that subjects tacitly coordinate the mechanism's asymmetric payoff-dominant equilibrium with precision; this precision is robust to a change in the structure and complexity of the game.

Implications: The results indicate that people respond to merit-based grouping in a natural way, and that competitive contribution-based grouping encourages social contributions even when abilities to contribute differ, which is the case in all communities and societies.

Keywords. Endogenous group formation; cooperation; meritocracy; mechanism design; experiment; social dilemma; game theory; policy.

JEL Classification. D20, C72, C92, H41.

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1. Introduction

Can competitive grouping based upon individuals' group contributions attenuate or even overcome social dilemmas? Recent behavioral research has answered this question with a clear "yes":[1] Experimental findings about the effects of endogenous group formation on cooperation levels indicate that the degree of excludability of public goods or team goods (Buchanan 1965) is not the only factor that matters. The method by which players are assigned to their cooperative units might be equally important. In this paper we theoretically analyze and experimentally test a formal mechanism of competitive endogenous grouping, called "Group-based Meritocracy Mechanism" (GBM).

Applying the principle of payoff dominance (Harsanyi and Selten 1988), one can make a precise prediction about the aggregate behavior of GBM participants. If payoff dominance holds empirically, the GBM should lead to high social contributions and efficiency in most instances (it does not do so in all instances because of its complexity, which gives rise to many different cases). Notwithstanding complexity, Gunnthorsdottir, Vragov, Seifert, and McCabe (2009) (henceforth GVSM), who analyzed and tested a basic version of the GBM, found that the payoff-dominant, asymmetric "near-efficient equilibrium" (henceforth, NEE) was reliably and precisely coordinated in the laboratory, even though it is unlikely that experimental subjects can consciously grasp such a complex equilibrium.

The current study builds upon GVSM's introductory work; the three main contributions here are as follows: (1) we show that GVSM's findings of precise tacit coordination of the payoff dominant asymmetric equilibrium are robust to an increase in the complexity of the game, (2) we increase the realism of GVSM's original model by introducing unequal abilities to contribute and (3) we provide a general theoretical analysis which suggests an array of future experimental tests, as well as extensions of the current model.

(1) GVSM's subjects all had the same endowment and thus equal ability to make a contribution. We increase complexity by introducing two different endowment levels while keeping everything else (including the median/mean endowment) the same as in GVSM's experiments. Under two endowment levels, the asymmetric NEE is more complex; it consists of three different strategies, while in GVSM's setup it consisted of only two. We have discovered only one reliable method of finding the game's equilibria involving positive contributions: the gradual elimination of possible strategy combinations by searching for incentives to deviate (Section 3 and Appendix A). However, our experimental results show that GVSM's initial findings about the "magical" (Kahneman 1988, p.12) coordination of the asymmetric payoff-dominant equilibrium are robust to the change we implemented.

(2) Unequal ability to contribute is a reality in communities and societies, and should be incorporated in any design intended to increase cooperation. Our experimental results indicate that even when abilities to contribute are unequal, competitive, contribution-based team formation remains an effective and precise mechanism to raise social contributions, at least in the controlled environment of the laboratory.

(3) The general theoretical analysis of a GBM Mechanism with two endowment levels (henceforth 2-Type GBM) suggests that the effect of unequal abilities to contribute on contribution-based grouping is not straightforward: Group size, the exact distribution of players with high or low endowments in the system, and the degree of inequality all impact efficiency. Interestingly, we find that efficiency

[1] See, e.g. Ahn, Isaac, and Salmon (2008); Charness and Yang (2009); Croson, Fatas, and Neugebauer (2007); Güth, Levati, Sutter, and der Heijden (2007); Cabrera, Fatas, Lacomba, and Neugebauer (2007); Page, Putterman, and Unel (2006); Gachter and Thoni (2005); Cinyabuguma, Page, and Putterman (2005); see Maier-Rigaud, Martinsson, and Staffiero (2005) for a comprehensive overview of endogenous group formation games where the rules of the game are common knowledge. Endogenous grouping also has an impact if players do not even know that they are being grouped (e.g., Ones and Putterman 2004; Gunnthorsdottir, Houser, and McCabe 2007 .

increases when the difference in abilities to contribute increases. Our theoretical analysis suggests an array of further experimental tests of competitive endogenous grouping when abilities to contribute differ. By changing the game's parameters experimenters can create many different cases, which allow the examination of **(a)** theories of equilibrium selection, in particular payoff dominance (Harsanyi and Selten 1988), **(b)** tacit coordination of various types of asymmetric equilibria which are non-obvious to subjects and which, depending upon the parameters, have different properties, and **(c)** the impact of different degrees of inequality with regard to players' ability to contribute on equilibrium structure and subject behavior.

Overview

Section 2 describes the GBM Mechanism, and compares it to the Voluntary Contribution Mechanism (VCM) (Isaac, McCue, and Plott 1985). We suggest that the VCM and the GBM can serve as rough models of privilege-based and merit-based social stratification, respectively. Section 2 also contains a brief overview of the equilibrium structure of the basic GBM and its extension under study here, the 2-Type GBM. Section 3 formally analyzes the 2-Type GBM. The examples in Section 3, with parameters commonly used in experiments, suggest an array of further experimental tests.

Section 4 describes a GBM experiment where subjects have two different endowment levels. Section 5 contains the results and shows that the payoff dominant Nash equilibrium organizes aggregate behavior very well. In Section 6 we detail possible follow-up studies based on our theoretical analysis, discuss the sociological and policy implications of our findings, and address shortcomings and potential criticisms.

2. The Group Based Meritocracy Mechanism (GBM) with Two Different Endowments

A group-based meritocracy (GBM) is a society in which participants are assigned to groups based on their contributions to a group account. The game shares features with the Voluntary Contribution Mechanism (VCM), the standard experimental model to examine free-riding, but with competitive contribution-based grouping added. We first briefly describe the VCM, before addressing how the GBM differs.

The VCM

In a VCM n participants are randomly assigned to G groups of fixed size ϕ. After grouping, players each decide simultaneously and anonymously how much of their individual endowment w_i to keep for themselves, and how much to contribute to a group account. Contributions to the group account are multiplied by a factor g representing the gains from cooperation before being equally divided among all ϕ group members. In the remainder of this paper, we denote the rate g/ϕ by m. m is the Marginal Per Capita Return (**MPCR**) to each group member from an investment in the group account. As long as $1/\phi < m < 1$, this game is a social dilemma: efficiency is maximized if all participants contribute fully to their group, but each individual's dominant strategy is to contribute nothing. In experimental tests of the VCM, mean group contributions start at about half of the total endowments and fall toward the dominant-strategy equilibrium of non-contribution by all within about ten repetitions (for overviews see, e.g., Ledyard 1995; Davis and Holt 1993).

The Basic GBM Mechanism with Homogeneous Endowments w_i

The GBM's equilibrium structure differs from the VCM's because in the GBM group membership is competitively based on individual contributions. As in the VCM, payoff functions, group size, and other parameters are fixed. However, a GBM player has considerable control over her placement through her public contribution decisions.

Participants first make their contribution decisions, and then get ranked according to their contributions to the group account. Based on this ranking, participants are partitioned into equal-sized groups. For the game's equilibrium analysis it is important to note that ties for group membership are broken at random. Finally, individual earnings are computed taking into account the group a subject has been assigned to. All this is common knowledge.[2]

The GBM also differs from the VCM in how the entire society is modeled. In the VCM each arbitrarily composed group exists in isolation. Since team assignment is random, there is no social mobility either. The GBM, in contrast, is not just about a single isolated group, but about a society consisting of multiple groups, where socially mobile players are linked via a cooperative-competitive mechanism. Through their contribution decisions they compete for membership in units with potentially different collective output and payoffs. The GBM's equilibrium analysis must therefore extend over the multiple groups that make up an organizationally stratified society.

The VCM and the GBM as models of social grouping and stratification

In the VCM, the choices a participant makes do not affect her placement in the experimental mini-society: each VCM player must accept what has been handed to her in the random grouping process. As Rawls (1971) points out each individual must accept the “Lottery of Birth” with regard to factors that are fixed at the beginning of life and over which the individual has no control, such as race or gender. In privilege-based societies however the Lottery of Birth remains disproportionally important throughout a person's life since these unalterable characteristics determine her organizational membership and place in society, and through it, her payoffs. This is why the VCM, where players' grouping is random, can be viewed as a model of an ascriptive (Linton 1936), privilege-based society where the Lottery of Birth looms large. The GBM in contrast with its competitive contribution-based grouping can serve as a model of meritocratic social organization where people are grouped and stratified based on their choices; high-contributors join more productive cooperative units where payoffs are higher. The GBM's incentive structure generates competition and increases efficiency. This is reflected in its equilibrium structure.

The equilibria of the GBM with homogeneous endowments

In contrast to the VCM with its dominant strategy equilibrium of non-contribution by all, GVSM show that in the relatively simple case when endowments, and thus abilities to contribute, are equal, the

[2] Gunnthorsdottir, Houser, and McCabe (2007); see also Gunnthorsdottir (2001)) use a somewhat related game where like-contributors are grouped together. With the goal of identifying player types who vary in reciprocity, Gunnthorsdottir et al. created a purposefully vague and brief version of a VCM with contribution based grouping, so that subjects, ignorant about the grouping method, can project their personality (cooperator or free rider) into this ambiguous situation. Thus their design and its purpose differ from ours. The current study tests a specific equilibrium prediction based on a precise game-theoretic model. In established communities and societies the grouping method is usually known, as is the case in the current study. Gunnthorsdottir (2009) found that behavior is very different when subjects know the grouping method compared to situations where they don't.

GBM has two pure-strategy equilibria[3] which differ in efficiency. An equilibrium of non-contribution by all remains omnipresent, reflecting the fact that the GBM retains some social dilemma properties. However, with competitive grouping the social dilemma features are much attenuated, and the equilibrium of non-contribution changes from a dominant-strategy equilibrium to a best-response equilibrium. The GBM with equal endowments always has a second, payoff-dominant and highly efficient, asymmetric equilibrium. In this equilibrium, as long as the within-group interaction has social dilemma properties (or $1/\phi < m < 1$), all n players contribute fully with the exception of $c_R < \phi$ players[4] who contribute nothing. GVSM call this payoff dominant equilibrium a "near-efficient equilibrium" (NEE) because it asymptotically approaches full efficiency as the number of players becomes large. The GBM's payoff-dominant equilibrium becomes more complex when unequal endowments are added:

A GBM with two different endowment levels (2-TYPE GBM)

We now change the basic GBM so that there are two different endowment levels.[5] Some players have high endowments, others low endowments. This is common knowledge. We henceforth denote the high endowment w_i as H and the low w_i as L.

Incentives under two different endowment levels. Recall that as long as the within-group interaction has social dilemma properties, the mechanism always has a best-response equilibrium of non-contribution by all. With the unequal distribution of endowments common knowledge, players with endowment $w_i = L$ (henceforth, "Lows") might not feel motivated to contribute. This in turn would affect the expected payoffs of players with endowment $w_i = H$, ("Highs"), and could drive the system toward the inefficient equilibrium rather than the NEE. However, this is not the case in our experiment: Even though Lows can never aspire to the earnings level that Highs can achieve, the 2-Type GBM elicits high social contributions from Highs and Lows alike, and the NEE is reliably realized.

Increased NEE complexity under two different endowment levels. One might expect that the 2-Type GBM's NEE might be hard to coordinate because of its complexity. High demands are put on subjects' ability to tacitly coordinate. In the game tested experimentally in Sections 4 and 5, the NEE consists of three corner strategies. Subjects thus must **(1)** somehow grasp that they should not play strategies drawn from the interior of their strategy spaces, $\{0, 180\}$ for Lows, and $\{0, 1, 120\}$ for Highs, respectively, **(2)** tacitly coordinate the three equilibrium strategies, 0, 80, and 120 in the correct proportions. This is complicated by the fact that **(3)** this NEE is not obvious, as reflected by the length of the analytical derivation of the conditions for its existence (Section 3). (As mentioned above, we

[3] Additionally and depending on the parameters, there exist mixed-strategy equilibria. Their existence is briefly discussed in GVSM (2009). Mixed strategies are beyond the scope of the current paper since 1) the pure strategy equilibrium predicts very well here, 2) mixed strategies are intuitively implausible when there is no stringent need to play unpredictably and pure equilibrium strategies are available to players (see, e.g., Kreps 1990, pp. 407-410; Aumann 1985, p. 19). 3) Even in games with a unique equilibrium in mixed strategies, proper mixing (both the right proportions of choices and their serial independence) is usually beyond regular subjects' abilities (see e.g., Palacios-Huerta and Volij 2008; Walker and Wooders 2001; Brown and Rosenthal 1990; Erev and Roth 1998. 4) GVSM report that their subjects do not play mixed strategies.

[4] GVSM denote c_R by z.

[5] By introducing unequal endowments, we make players' world less fair even though it is not exactly an ascriptive (Linton 1936) system. Note though that Rawls (1971) explicitly included differing abilities in the Lottery of Birth. Unequal abilities to contribute still allow players some control over their grouping, but within constraints which are again Lottery of Birth based (exactly what a meritocracy often claims to overcome). In a meritocracy with differential abilities to contribute, ability constitutes a ceiling to what an individual can aspire to, even though within these constraints, she determines her contribution levels and with it, her social position. Fair or not, ability to contribute is a significant determinant of social position in contemporary societies. For example, IQ is the strongest single predictor of socio-economic status (see, e.g., Grusec, Lockhart, and Walters 1990; Herrnstein and Murray 1996, Ch. 3).

ourselves have discovered only one reliable method of finding this NEE—the gradual elimination of strategy combinations by searching for incentives to deviate focusing first on the necessary conditions for an equilibrium with positive contributions, then on the sufficient conditions.) 4) The 2-Type GBM's NEE can be ephemeral in that its exact structure, even its existence, is often parameter dependent (see Examples 2 and 5 in Section 3; see also Section 3.5). We show that different equilibrium predictions can be generated by slightly modifying the experimental parameters. Since both GVSM and the authors of this paper find that subjects coordinate the GBM equilibria quite precisely, such parameter changes should lead to discernibly different aggregate behavior.

3. Theory

Before formally describing the equilibria of the game and their properties, we provide (1) an intuitive account of the equilibria of the 2-Type GBM, and (2) a brief overview of the formal steps by which the equilibria are derived, highlighting some of the theoretical findings and the examples that suggest future experimental tests.

We first introduce three terms, formally defined in **Section 3.1**. A **group** is the cooperative unit whose members equally share the earnings from their public account. (Ranking all players by their contributions from highest to lowest with ties broken at random and then grouping them into G groups, one can define three general kinds of groups: the first group, Group 1, contains the top ϕ contributors, the last group, Group G, contains the bottom ϕ contributors, and any group in between is designated as an "intermediate group".) A player's **type** is defined by her endowment, so that a player is either a "High" or a "Low". A **class** is a subset of players whose public contributions are identical. The first class C_1 is the subset whose members contribute the most, C_2 the next class whose members contribute less, and so on; the last class C_R is the subset of those who contribute least.

An intuitive account of the 2-Type GBM's equilibria

We next provide an intuitive account of how GBM equilibria are found. We focus first on the simpler (GVSM's) version of the mechanism where all endowments w_i are equal, then extend the same reasoning to the 2-Type case.[6] Firstly, non-contribution by all is clearly an equilibrium—no single individual has an incentive to increase her contribution if everyone else contributes nothing. Are there equilibria with positive contributions? It can be verified that in an equilibrium with positive contributions, a group cannot contain players from three classes, since each player in the middle class could decrease her contribution by a small ε and remain in the same group. Therefore, if an equilibrium with positive contributions exists, each group must contain either one or two classes of players.

We now examine the three different kinds of groups separately: Group 1 can only contain one class, C_1: if it had two classes, any member of C_1 would have an incentive to decrease her contribution by a small ε and remain in Group 1 nonetheless, enjoying the top earnings associated with such a position. For the same reason the number of players in C_1 must be greater than the group size ϕ and not divisible by ϕ. It is also easy to show that members of C_1 must contribute their full endowments: If they do not contribute fully, each C_1 member has an incentive to increase her contribution and thus her earnings, because her expected earnings are higher if she is with certainty in Group 1 than if she is grouped with some positive probability with lower classes in a lower group.

We now examine whether the first intermediate group, Group 2, could possibly contain individuals from the next class, C_2. We already know from the previous paragraph that Group 2 must already

[6] For illustration purposes we describe a case with three or more groups. The case with two groups only is easily inferred in a similar fashion.

contain at least one full contributor. Since groups can contain either one or two classes, there are two cases to consider with regard to the composition of the other players in Group 2. **(1)** All other members of Group 2 also contribute fully, or **(2)** all its other members belong to the next class, C_2, whose members contribute less. We next examine case (2) and show that it is impossible if endowments are equal: Following similar logic as laid out with regard to Group 1 membership, if there were C_2 players in Group 2, C_2 must extend into the next intermediate group (Group 3) else there cannot be an equilibrium; if C_2 did not extend into Group 3, any C_2 player could decrease her contribution and stay in Group 2. Assume now C_2 does extend to Group 3: in such a case any C_2 player will increase her contribution so that she can be in Group 2 with certainty, and can free ride off the full contributor(s) already in Group 2. This shows that in an equilibrium with positive contributions members of the intermediate group must contribute fully.

What about Group G? It is clear that Group G cannot contain one class only, because from above it follows that it already has at least one full contributor. If all members of Group G are full contributors, then everyone has an incentive to free ride and contribute nothing. Hence, Group G must contain two classes. Also, the individuals in its lower class CR contribute nothing, else any one of them has an incentive to lower her contribution since she remains in Group G nonetheless.

In order to find a point where earnings from the different strategies are equal and the system is in equilibrium, one needs to determine how many zero-contributors are needed in Group G. GVSM derived the conditions for the existence of such an equilibrium for the case with homogeneous endowments, and called it a NEE.

Does a similar equilibrium exist when there are two endowment levels? Following the same logic as above, one can verify that non-contribution by all is still an equilibrium; in an equilibrium with positive contributions each group still must have either one or two classes; Group 1 can still only have one class of full contributors; the number of C_1 players must still be greater than the group size ϕ and not divisible by ϕ. However, differences arise in the first intermediate group, Group 2, which might contain players which are in C_2 by necessity, because of their lower endowment. Group 2 can thus have either **(1)** one class or **(2)** two classes, if some Group 2 members are Lows who would want to but cannot contribute as much as the Highs do. It follows that one intermediate group with two classes must exist in an equilibrium with positive contributions if there are more than ϕ Lows and more than ϕ Highs in the system. It is easy to see that in this case C_2, consisting of fully contributing Lows, must extend to the intermediate groups below this mixed group, and that all intermediate groups below the mixed group can have only one class.

What about Group G—the last group? Since we showed that a group can never contain more than two classes, we know that Group G has either **(1)** one or **(2)** two classes. By the logic laid out above for the case with homogeneous endowments, in case (2) the lower-class players must contribute zero in equilibrium. We will show formally here below that both (1) and (2) can be equilibria depending on the parameters. We call (1), the configuration where Group G consists of full contributors only, a "fully efficient equilibrium" (FEE). (2) corresponds to the NEE originally defined by GVSM. We now provide a brief overview of our formal analysis and highlight its most important findings about the impact of unequal endowments.

The game defined

In **Assumption 1** we formally restrict the endowment w_i to two levels, H or L. Without loss of generality we let L = 1 and H = (1+Δw) where Δw >0. We will examine the effect of change in Δw in depth.[7] In

[7] In the experimental test in Sections 4 and 5 $L = 80$ tokens and $H = 120$ tokens so that $\Delta w = 0.5$.

Assumption 2 we restrict the distribution of player types, Highs and Lows, in the following manner: type count is not fully divisible by group size, and for each type its count, nH or nL, must exceed the group size ϕ.

The reason for these restrictions is as follows: **(1)** The current section and Appendix A make it clear that even with these assumptions in place the process of finding the equilibria of the 2-Type GBM is lengthy and cumbersome. Relaxing Assumptions 1 and 2 would mean that there would be numerous additional cases to consider, each of which requires the same detailed examination of all possible strategy combinations as contained in Section 3.[8] **(2)** Cases that satisfy Assumption 2 are the most interesting since a distribution of types as stipulated by Assumption 2 encourages competition for group membership. Recall that, in any GBM, ties for group membership are broken at random, and that equilibrium payoffs are expected payoffs, computed before the random resolution of ties puts players in specific groups. For an equilibrium with positive contributions in the cases of the GBM studied so far (GVSM's and ours) there must be competition between players for group membership.

The equilibrium of non-contribution by all

In **Section 3.2** we first show the omnipresence of an equilibrium of non-contribution by all. This is the only equilibrium of the game where all players use the same strategy. This equilibrium is always present as long as the MPCR m is within the bounds that make the within-team interaction a social dilemma (**Lemma 1**).

Equilibria with positive contributions

We focus first on the necessary conditions for equilibria with positive contributions. **Theorem 1** states that there are only two equilibrium configurations with positive contributions possible; both are asymmetric and consist of corner strategies.: **(1)** a FEE where both types contribute fully. **(2)** A NEE where all players contribute fully with the exception of $c_R < \phi$ players[9] who contribute zero. The two equilibria are depicted in Fig. 3.1. Appendix A contains the proof of Theorem 1; it involves the usual gradual process of elimination, including the step-by step elimination of initial "equilibrium candidate" E′ by searching for incentives by individual players to deviate from this particular strategy combination.)

We apply Theorem 1 to three examples relevant to experimental testing or previous literature: In **Example 1** we derive the equilibrium with positive contributions of the version of the 2-Type GBM experimentally tested in Sections 4 and 5, and show that it must be a NEE. **Example 2** illustrates that not all 2-Type GBMs have an equilibrium with positive contributions: We slightly modify the type composition of the experimental game in Example 1 so that only the equilibrium of non-contribution by all remains. In **Example 3** we connect our general analysis to GVSM's original analysis of a GBM when endowments are all equal. We show that if endowments are equal a FEE cannot exist, only a NEE is possible.

When is a fully efficient equilibrium (FEE) possible?

In **Sections 3.3 and 3.4** we explore the conditions for the existence of FEE and NEE by examining the incentives to deviate for all players, always starting with the lowest class. While lengthy and cumbersome, this process is rather straightforward. We draw attention to **Theorem 2** (Eqn. 3) in

[8]Some simple examples of cases where Assumption 2 is relaxed: n_H and n_L are divisible by ϕ; n_H or n_L equals ϕ; $n_H < \phi$; $n_L < \phi$, etc.. Relaxing Assumption 1, too, creates a large array of different cases. Many of these cases are interesting, and are being developed in separate papers.

[9]As originally shown by GVSM, c_R ,which GVSM denote as z, is MPCR dependent.

Section 3.3, which states (subject to the constraints in Remark 2 and 3 below) that the existence of a FEE depends on a combination of parameters including the group size ϕ, the count of Highs and Lows in the system (nH and nL, respectively), as well as the MPCR m. A FEE's existence also depends on Δw, the difference between the high and the low endowment. The Theorem implies that if the difference between Highs and Lows, Δw, increases, efficiency increases rather than decreases until a fully efficient equilibrium (FEE) rather than a NEE, is possible.

Theorem 2 has practical implications: it allows building a mechanism that is fully efficient by intervening upon the parameters. In the field, Δw may be fixed at least in the short run; same for nH and nL, the distribution of the two types in a community or society. The gains from cooperation m and with it, M, could for example be changed through managerial tools that increase team productivity. It might however be easiest to intervene through the team size ϕ, which in turn determines $h = n_H \mod \phi$ and $\ell = n_L \mod \phi$.

Three remarks elaborate further on Theorem 2: if the MPCR m approaches 1 from below, full contribution by all becomes an equilibrium (**Remark 1**). (Of course, if $m > 1$, it is a dominant strategy to contribute fully as it is in the VCM). **Remarks 2 and 3** focus on the effect of Δw, the difference in ability to contribute: If Δw is small, a FEE is impossible (Remark 2, compare to Example 3). However, while a large Δw is a necessary condition for a FEE, it is not sufficient. Cases can be found where Δw is large yet no FEE exists (Remark 3). **Example 4** illustrates how a FEE can be found combining Theorem 2 with a graphical approach. In **Example 5** we apply Theorem 2 to our experimentally tested version of the mechanism, where $L = 80$, and $H = 120$, and find that if H were raised to $200(2.5 \times L)$, a FEE would replace the current NEE.

Existence of a NEE

The exact type composition of a NEE is parameter dependent with regard to the last class of $c_R < \phi$ non-contributors: In our experimental game, the last class CR consists of Lows. However, as the bottom right of Fig. 3.1 shows, if the group size or the number of groups increases, CR might also contain Highs. However, $c_R < \phi$ does not change with this, so that the NEE's efficiency is not affected much. To our knowledge a NEE can be discovered only through a gradual elimination process of strategy configurations. The length and complexity of the analysis can be seen in **Theorem 3** in **Section 3.4**. We also use specific examples to show that a NEE exists and to illustrate as best we can the conditions under which this happens (see Examples 1, 2, 5).

Can NEE and FEE coexist?

Section 3.5 demonstrates that it is possible to construct a case where FEE and NEE co-exist. Example 5 already illustrated that if $H \geq 2.5$, our experimental game would have a FEE rather than a NEE. Section 3.5 shows that at the exact point where $H = 2.5$, a weak NEE and a weak FEE co-exist: one player is indifferent between contributing and not contributing.

3.1. **Model**

The set of players is $N \equiv \{1, \ldots, n\}$. Each player $i \in N$ has an endowment $w_i > 0$. The distribution of endowments is common knowledge. Each player $i \in N$ makes a contribution $s_i \in [0, w_i]$ to a public account, and keeps the remainder $(w_i - s_i)$ in her private account. The return from the private account is without loss of generality set to 1, the return from the public account is the Marginal per Capita Return (MRCP) $m \in \big(1/\phi, 1\big)$. So far, this game is a standard VCM.

☐ *Players Compete for Group Membership*

Our model however differs from the VCM in the following way: After their investment decisions, all players are ranked according to their public contributions and divided into G groups of equal size ϕ, so $G = n/\phi$. Ties for group membership are broken at random. The ϕ players with the highest contributions are put into Group 1; then ϕ players with the next highest contributions are put into Group 2, and so on. Payoffs are computed after players have been grouped. Each player's payoff consists of the amount kept in her private account, plus the total public contribution of all players in the group she has been assigned to multiplied by the MPCR m.

Given the other players' contributions $(s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n) \equiv \boldsymbol{s}_{-i}$, let $U_i(s_i, \boldsymbol{s}_{-i})$ be player i's expected payoff from contributing s_i. Let $\Pr(k \mid s_i, \boldsymbol{s}_{-i})$ be i's probability of entering group k when the contribution profile is $(s_i, \boldsymbol{s}_{-i}) \equiv \boldsymbol{s}$, where $k = 1, \ldots, G$; for simplicity we henceforth denote this probability by $\Pr(k \mid s_i)$. Let S^k_{-i} be the total contribution in group k except for player i. Therefore, player i's expected payoff $U_i(s_i, \boldsymbol{s}_{-i})$ from a contribution combination $\boldsymbol{s} = (s_i, \boldsymbol{s}_{-i})$ can be expressed as follows:

$$U_i(s_i, \boldsymbol{s}_{-i}) = (w_i - s_i) + \sum_{k=1}^{G} \Pr(k \mid s_i, \boldsymbol{s}_{-i}) \cdot \left[m \cdot \left(S^k_{-i} + s_i \right) \right]. \tag{1}$$

☐ *Formally Defining the Game*

We can now transform this into a normal form game. The set of players is N; each player i's strategy is her contribution s_i. Her strategy space is the interval $[0, w_i] \subseteq \mathbb{R}$; finally, player i's payoff function is defined by (1) for all $i \in N$. The Nash equilibrium is defined as follows:

Definition 1 (**Nash equilibrium**). A contribution profile $\boldsymbol{s} = (s_1, \ldots, s_n)$ is a *Nash equilibrium* if and only if

$$U_i(\boldsymbol{s}) \geq U_i\left(s'_i, \boldsymbol{s}_{-i}\right),$$

for all $s'_i \neq s_i$ and all $i \in N$.

So far this game is a standard GBM as originally defined by GVSM, where w_i is the same for all players. We now increase the game's complexity with the following two assumptions:

Assumption 1 (**Two different endowment levels**). Each player's endowment is either $w_i = H$ or $w_i = L < H$.

For what follows, we apply the following simplification without loss of generality: we normalize $L = 1$, and let $\Delta w \equiv H - 1 > 0$ be the gap between the high endowment H and low endowment $L = 1$. We call a player with endowment H a "High", and a player with endowment 1 a "Low". N_H is the set of Highs. N_L is the set of Lows. Their respective counts are $n_H \equiv |N_H|$ and $n_L \equiv |N_L|$. It follows that $N_H \cup N_L = N$, or equivalently, $n_H + n_L = n$. Further, one can find some nonnegative integers A, B, $h < \phi$, and $\ell < \phi$, such that the counts of Highs and Lows can be expressed as:

$$n_H = A\phi + h, \quad \text{and} \quad n_L = B\phi + \ell.$$

Assumption 2 (**Distribution of player types whose endowments differ**). The count of each type, High and Low, is more than, and not a multiple of the group size ϕ, that is,

- $A \geq 1$, $B \geq 1$, and $A + B = G - 1$;
- $h \geq 1$, $\ell \geq 1$, and $h + \ell = \phi$.

We need to define one more basic concept, which will be crucial when we identify all the game's equilibria.

□ *The Concept of Class*

Definition 2 (**Class**)**.** Let $C_r \subseteq N$. We call C_r a *class* if each player $i \in C_r$ contributes the same, that is, $i, j \in C_r$ if and only if $s_i = s_j$. We call a player $i \in C_r$ a C_r-*player*.

Given a contribution profile $\boldsymbol{s}$, the players can be divided into $R(\boldsymbol{s}) \leq n$ classes; we henceforth omit the argument $\boldsymbol{s}$. Let $\mathscr{C}$ be the family of all classes, i.e., $\mathscr{C} \equiv \{C_1, \ldots, C_R\}$. Both $\mathscr{C}$ and $\{N_H, N_L\}$ partition N, that is, $\bigcup_{r=1}^{R} C_r = N_H \cup N_L = N$. In a class $C_r \in \mathscr{C}$, there are c_r players; the contribution of each player in C_r is s^r, that is, $|C_r| \equiv c_r$, and $s_i = s^r$ for all $i \in C_r$. We index the classes such that $s^{r+1} < s^r$, where $r + 1 \leq R$; hence, C_1 is the class consisting of the highest contributors, and C_R is the class consisting of the lowest contributors. For each class C_r, we can find nonnegative integers D_r and $\widetilde{c}_r < \phi$ such that the count of C_r-players can be expressed as

$$c_r \equiv |C_r| = D_r \cdot \phi + \widetilde{c}_r. \tag{2}$$

3.2. **Equilibria**

□ *The Equilibrium of Non-Cooperation by All Is Always Present*

Lemma 1 (**Equilibrium of non-contribution by all**)**.** *$s_i = 0$ for all player $i \in N$ is a Nash equilibrium. This is the only equilibrium satisfying $|\mathscr{C}| = 1$.*

Proof. Let $s_j = 0$ for all players $j \neq i$. Player i obtains $(w_i - s_i) + ms_i = w_i - (1 - m)\, s_i$ if she contributes s_i. Her best response is therefore $s_i = 0$.

To verify that $s_i = 0$ for all player $i \in N$ when $|\mathscr{C}| = 1$, let $s^1 > 0$. Consider any player $i \in N$. She gets $\left(w_i - s^1\right) + m\phi s^1$ if she contributes s^1, but if she deviates and contributes 0, she enters the last group G, and gets

$$w_i + m\,(\phi - 1)\, s^1 = \left(w_i - ms^1\right) + m\phi s^1 > \left(w_i - s^1\right) + m\phi s^1$$

since $m < 1$. Hence, $s_i = 0$ for each player $i \in N$ in an equilibrium with only one class. ■

The equilibrium with $s_i = 0$ for all $i \in N$ always exists as long as the MPCR $m < 1$. It is however not a dominant response equilibrium. Theorem 1 here below defines the *necessary* conditions for equilibria with positive contributions. Since $s_i = 0$ for all $i \in N$ if $|\mathscr{C}| = 1$ by Lemma 1, in any equilibrium with positive contributions it must be that $|\mathscr{C}| \geq 2$.

□ *The Two Equilibria Involving Positive Contributions*

This section will show that there are two equilibria involving positive contributions: (1) a fully efficient equilibrium (FEE), and (2) a near-efficient equilibrium (NEE):

***FEE*:** There are two classes: C_1 is identical to N_H, and C_2 is identical to N_L. All players contribute fully, that is:
- *Classes*: $|\mathscr{C}| = 2$, where $C_1 = N_H$ and $C_2 = N_L$.
- *Strategies*: $s_i = \begin{cases} H, & \text{if } i \in C_1 \\ 1, & \text{if } i \in C_2. \end{cases}$.

***NEE*:** There are three classes: C_1 consists of Highs, C_2 consists of Lows, and C_3 consists of the players who are not in C_1 or C_2. Both C_1 and C_2-players contribute fully, but C_3-players contribute nothing. The sum of C_2 and C_3-players together is greater than and not a multiple of group size; the count of C_3-players is less than the group size, that is:

- *Classes*: $|\mathscr{C}| = 3$, where $\begin{cases} C_1 \subseteq N_H, c_1 > \phi \text{ and } \tilde{c}_1 > 0 \\ C_2 \subseteq N_L, c_2 + c_3 > \phi, \text{ and } \tilde{c}_2 + \tilde{c}_3 \neq \phi \\ C_3 = N \setminus (C_1 \cup C_2) \text{ and } c_3 < \phi. \end{cases}$
- *Strategies*: $s_i = \begin{cases} H, & \text{if } i \in C_1 \\ 1, & \text{if } i \in C_2 \\ 0, & \text{if } i \in C_3. \end{cases}$

In both equilibria with positive contributions, strategies only take one of three forms: full contribution of the high endowment (H), full contribution of the low endowment (1) or zero contribution. Fig. 3.1 illustrates FEE and NEE. The dark gray sections represent Highs, the light gray sections Lows. The players' strategies s_i are shown on top of the horizontal bars. The segments in the bars represent groups. For illustration purposes and without loss of generality, only four groups are shown.

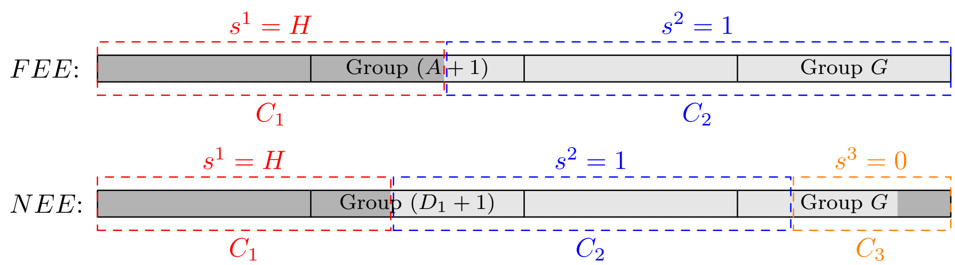


Fig. 3.1. The two equilibrium configurations with positive contributions

Theorem 1. *If there is an equilibrium with positive contributions, then it is a FEE or NEE.*

Proof. Appendix A. ■

☐ *Applications of Theorem 1*

In Example 1 we derive the equilibrium of the game tested experimentally in Section 4 and 5. Example 2 shows that a specific version of the 2-Type GBM does not have an equilibrium with positive contributions. In Example 3 we apply Theorem 1 to a situation where all endowments are equal and show that the only equilibrium with positive contributions possible in such a situation is a NEE.

Example 1 (Deriving the experimental NEE). Let $n = 12$, $n_H = n_L = 6$, $\phi = 4$, $L = 1$ and $H = 1.5$ (in our experimental test, $L = 80$ tokens and $H = 1.5L = 120$ tokens). According to Theorem 1, we only need to consider FEE and NEE:

There is no FEE since any player $i \in C_2$ has an incentive to reduce her contribution: If i contributes 1, she enters the second group with probability $2/6$, and the third group with probability $4/6$, so the expected payoff is $0.5 \times \left(\frac{2}{6} \times 5 + \frac{4}{6} \times 4\right) = 13/6$, but if she contributes 0, she enters the third group with certainty and obtains $1 + 0.5 \times 3 = 5/2 > 13/6$.

Hence, if there exists an equilibrium with positive contributions, it must be a NEE. As the following table shows, the unique equilibrium with positive contributions is

$$(\langle 1.5, 1.5, 1.5, 1.5 \rangle, \langle 1.5, 1.5, 1, 1 \rangle, \langle 1, 1, 0, 0 \rangle).$$ [10]

c_1	c_2	c_3	NEE?	**Deviator**	**Deviation** $(s_i \longrightarrow s_i')$
5	6	1	No	$i \in C_2 \subseteq N_L$	$1 \to 0$
5	5	2	No	$i \in C_3 \cap N_H$	$0 \to 1+\varepsilon$
5	4	3	No	$i \in C_3 \cap N_H$	$0 \to 1+\varepsilon$
6	5	1	No	$i \in C_2 \subseteq N_L$	$1 \to 0$
6	4	2	Yes	$\varnothing$	
6	3	3	No	$i \in C_3 \cap N_L$	$0 \to 1$

Example 2 (**No equilibrium with positive contributions exists**). In a game with parameters as in Example 1, now let $n_H = 7$ instead of previously 6. It can be verified that there is no FEE. By Theorem 1, it suffices to show that there is no NEE either. There are eight cases to consider:

c_1	c_2	c_3	NEE?	**Deviator**	**Deviation** $(s_i \longrightarrow s_i')$
5	5	2	No	$i \in C_3 \cap N_H$	$0 \to 1+\varepsilon$
5	4	3	No	$i \in C_3 \cap N_H$	$0 \to 1+\varepsilon$
6	5	1	No	$i \in C_2 \subseteq N_L$	$1 \to 0$
6	4	2	No	$i \in C_3 \cap N_H$	$0 \to 1+\varepsilon$
6	3	3	No	$i \in C_3 \cap N_H$	$0 \to 1+\varepsilon$
7	4	1	No	$i \in C_2 \subseteq N_L$	$1 \to 0$
7	3	2	No	$i \in C_1 = N_H$	$H \to 1+\varepsilon$
7	2	3	No	$i \in C_1 = N_H$	$H \to 1+\varepsilon$

Example 3 (**If endowments are all equal, the only equilibrium with positive contributions possible is a** $\boldsymbol{NEE}$). This example relies on some results in Appendix A. The general method developed so far can be used to reprove Observation 2 in GVSM (2009). GVSM's parameter z corresponds to $c_R = |C_R|$, the number of players in the last class. If $H = L = 1$ and if there exists an equilibrium with positive contributions, it can be characterized as follows:

$$|\mathscr{C}| = 2, \quad s^1 = 1, \quad s^2 = 0, \quad \text{and} \quad c_2 < \phi.$$

Proof. By Lemma A.1(a) (in Appendix A), in any equilibrium with positive contributions $c_1 > 0$, $\widetilde{c}_1 > \phi$, and $s^1 = 1$. Now consider the last class C_R:

(1) If $c_R > \phi$ and $\widetilde{c}_R > 0$ in equilibrium, then $s^R = 1$ by Claim 1 (Appendix A). However, this means that $|\mathscr{C}| = 1$ and $\widetilde{c}_1 = 0$, a contradiction to Lemma A.1(a).

(2) Assume $\widetilde{c}_R = 0$ in equilibrium. Then $s^2 = 0$ by Lemma A.1(e). By the same logic as in Lemma A.1(c), there cannot exist a class C_r satisfying $0 < s^r < 1$; hence, $|\mathscr{C}| = 2$. According to Lemma A.1(a) $\widetilde{c}_1 > 0$. If $\widetilde{c}_2$ were zero, it would contradict our initial assumption at the beginning of Section 3.1 that the total number of players $n = G \cdot \phi$.

(3) Thus, it must be that $c_R < \phi$. It follows that $s^R = 0$ by Lemma A.1(e). An argument analogous to Lemma A.1(c) shows that $|\mathscr{C}| = 2$.

■

[10] This corresponds to $(\langle 120, 120, 120, 120\rangle, \langle 120, 120, 80, 80\rangle, \langle 80, 80, 0, 0\rangle)$ in experimental tokens.

3.3. Existence of a *FEE*

A fully efficient equilibrium (FEE) exists if and only if

- Player $i \in C_2$ has no incentive to reduce her contribution from 1 to 0, and
- Player $i \in C_1$ has no incentive to reduce her contribution from H to $1+\varepsilon$, 1, or 0, where ε is a small positive real number;

We first consider C_2, then C_1. We use $U_{s_i}^{w_i}(C_r)$ to denote player i's expected payoff when her endowment is $w_i \in \{H, 1\}$, she contributes $s_i \in [0, w_i]$, and is in class C_r. We develop our analysis with the help of Fig. 3.2.

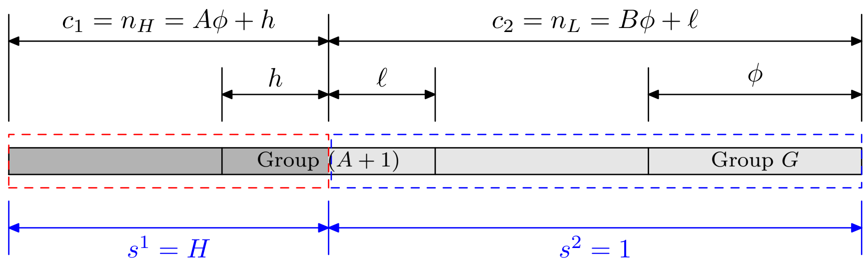


Fig. 3.2. The distribution of players in a FEE

Theorem 2. *Let $M \equiv \frac{1-m}{m}$. A FEE exists if and only if*

$$\frac{M \cdot n_L}{\Delta w \cdot \ell} \leq h \leq \min\left\{ \frac{\left[(\phi - 1)\,\Delta w - MH\right] \cdot n_H}{\Delta w \cdot \ell}, \frac{(\ell - M) \cdot n_H}{\ell} \right\}. \tag{3}$$

In the remainder of this section we account for Theorem 2 by examining players' incentives to deviate.

□ *Incentives to Deviate for C_2-Players in a FEE*

Fix the contribution profile $\boldsymbol{s}_{-i} \equiv (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$ satisfying $s_j = w_j$ for all $j \in N \setminus \{i\}$. For any player $i \in C_2 = N_L$, if she contributes 1, she enters the following groups with positive probabilities: $A+1, A+2, \ldots, G$ (see Fig. 3.2). The probabilities are:

$$\Pr(k \mid 1) = \begin{cases} \ell / n_L, & \text{if } k = A+1 \\ \phi / n_L, & \text{if } k = A+2, \ldots, G. \end{cases}$$

Since $\sum_{k=A+1}^{G} \Pr(k \mid 1) = 1$, we have $\sum_{k=A+2}^{G} \Pr(k \mid 1) = 1 - \Pr(A+1 \mid 1) = 1 - \frac{\ell}{n_L}$. For ease of expression, let

$$S^{A+1} \equiv hH + \ell,$$

that is, S^{A+1} is the sum of contributions in Group $(A+1)$ from the full contribution profile $\boldsymbol{s} = (s_i = 1, \boldsymbol{s}_{-i})$. By (1), player i's expected payoff from contributing $s_i = 1$ is

$$\begin{aligned} U_1^L(C_2) &= (w_i - s_i) + m\left\{\Pr(A+1 \mid 1) \cdot S^{A+1} + \sum_{k=A+2}^{G} \left[\Pr(k \mid 1) \cdot \phi\right]\right\} \\ &= (1-1) + m\left\{\Pr(A+1 \mid 1) \cdot S^{A+1} + \left[\sum_{k=A+2}^{G} \Pr(k \mid 1)\right] \cdot \phi\right\} \\ &= m\left[\frac{\ell}{n_L} S^{A+1} + \left(1 - \frac{\ell}{n_L}\right)\phi\right] \\ &\overset{\langle 1 \rangle}{=} m\left(\phi + \frac{h\ell\Delta w}{n_L}\right), \end{aligned}$$

where equality $\langle 1 \rangle$ holds since $S^{A+1} - \phi = (hH + \ell) - (h + \ell) = h(H-1) = h\Delta w$.

If player $i \in C_2$ deviates and contributes $s_i < 1$, she enters group G, and her payoff is

$$(1 - s_i) + m\left[(\phi - 1) + s_i\right] = 1 + m(\phi - 1) - (1-m)s_i;$$

hence, the optimal deviation is $s_i = 0$ since $1 - m > 0$ with payoff is $U_0^L(C_2) = 1 + m(\phi - 1)$.

Hence, player $i \in C_2$ has no incentive to reduce her contribution from 1 to 0 if and only if $U_1^L(C_2) \geq U_0^L(C_2)$, that is,

$$h \geq \frac{(1-m)\,n_L}{m\ell \cdot \Delta w} \equiv \frac{M \cdot n_L}{\ell \cdot \Delta w}, \tag{4}$$

where $M \equiv (1-m)/m$. Because $m \in (1/\phi, 1)$, we know that $M \in (0, \phi - 1)$.

□ *Incentives to Deviate for C_1-Players in a FEE*

Since we now consider a player $i \in C_1 = N_H$, we rewrite the full contribution profile as $\boldsymbol{s} = (s_i = H, \boldsymbol{s}_{-i})$, where $s_j = w_j$ for any $j \in N \setminus \{i\}$. If player $i \in C_1$ contributes $s_i = H$, she enters Group $1, 2, \ldots, A, A+1$ with positive probabilities, which are

$$\Pr(k \mid H) = \begin{cases} \phi/n_H, & \text{if } k = 1, \ldots, A \\ h/n_H, & \text{if } k = A+1. \end{cases}$$

Hence, i's expected payoff from contributing $s_i = H$ is

$$\begin{aligned} U_H^H(C_1) &= (H - H) + m\left\{\left[\sum_{k=1}^{A} \Pr(k \mid H)\right] \cdot \phi H + \Pr(A+1 \mid H) \cdot S^{A+1}\right\} \\ &\overset{\langle 1 \rangle}{=} m\left[\left(1 - \frac{h}{n_H}\right)\phi H + \frac{h}{n_H} S^{A+1}\right] \\ &\overset{\langle 2 \rangle}{=} m\left(\phi H - \frac{h\ell\Delta w}{n_H}\right), \end{aligned}$$

where $\langle 1 \rangle$ holds because $\sum_{k=1}^{A} \Pr(k \mid H) = 1 - \Pr(A+1 \mid H) = 1 - h/n_H$, and $\langle 2 \rangle$ holds because $\phi H - S^{A+1} = \phi H - (hH + \ell) = \ell H - \ell = \ell\Delta w$.

If player $i \in C_1$ contributes $s_i \in (1, H)$, she enters group $(A+1)$ with certainty and obtains

$$\begin{aligned} U_{s_i}^H(C_1) &= (H - s_i) + m\left[(h-1)H + \ell + s_i\right] \\ &= H + m\left[(h-1)H + \ell\right] - (1-m)s_i. \end{aligned} \tag{5}$$

From (5) we know that the optimal deviation is $s_i = (1+\varepsilon) \to 1$ if player $i \in C_1$ wants to contribute $s_i \in (1, H)$. Thus,

$$\begin{aligned} \lim_{\varepsilon\to 0} U_{1+\varepsilon}^H(C_1) &= \lim_{\varepsilon\to 0}\left\{H + m\left[(h-1)H+\ell\right] - (1-m)(1+\varepsilon)\right\} \\ &= H + m\left(S^{A+1} - H\right) - (1-m) \\ &= mS^{A+1} + (1-m)\Delta w. \end{aligned}$$

Hence, player $i \in C_1$ has no incentive to reduce her contribution from H to $1+\varepsilon$ if and only if $U_H^H(C_1) \geq \lim_{\varepsilon\downarrow 0} U_{1+\varepsilon}^H(C_1)$, that is

$$h \leq n_H\left(1 - \frac{M}{\ell}\right). \tag{6}$$

Note that (6) is *independent* of H or Δw: it is fully determined by the distribution of player types and the MPCR m.

Lemma 2 here below indicates that we do not need to consider whether $i \in C_1$ has an incentive to contribute 1 if she has no incentive to contribute $1+\varepsilon$.

Lemma 2. *If a player $i \in C_1$ has no incentive to reduce her contribution from H to $1+\varepsilon$, she also has no incentive to reduce her contribution from H to 1.*

Proof. If player $i \in C_1$ contributes 1, she enters Group $A+1, A+2, \ldots, G$ with positive probabilities. Therefore, her expected payoff from contributing 1 is

$$\begin{aligned} U_1^H(C_1) &= (H-1) + m\left\{\Pr(A+1 \mid 1)\cdot\left[(h-1)H + \ell + 1\right] + \sum_{k=A+2}^{G}\left[\Pr(k \mid 1)\cdot\phi\right]\right\} \\ &= \Delta w + m\left\{\Pr(A+1 \mid 1)\cdot\left(S^{A+1} - \Delta w\right) + \left[\sum_{k=A+2}^{G}\Pr(k \mid 1)\right]\cdot\phi\right\} \\ &\overset{\langle 1\rangle}{\leq} \Delta w + m\left\{\Pr(A+1 \mid 1)\cdot\left(S^{A+1} - \Delta w\right) + \left[1 - \Pr(A+1 \mid 1)\right]\cdot\left(S^{A+1} - \Delta w\right)\right\} \\ &= mS^{A+1} + (1-m)\Delta w \\ &= \lim_{\varepsilon\to 0} U_{1+\varepsilon}^H(C_1), \end{aligned}$$

where $\langle 1\rangle$ holds because $S^{A+1} - \Delta w = (hH+\ell) - (H-1) = \left[hH + (\phi - h)\right] - H + 1 \geq (H + \phi - 1) - H + 1 = \phi$. Therefore, $U_H^H(C_1) \geq U_1^H(C_1)$ when $U_H^H(C_1) \geq \lim_{\varepsilon\to 0} U_{1+\varepsilon}^H(C_1)$. ■

Finally, if player $i \in C_1$ wants to contribute $s_i < 1$, she should contribute $s_i = 0$, so that her payoff is $U_0^H(C_1) = H + m(\phi - 1)$. Hence, she has no incentive to contribute 0 if and only if $U_H^H(C_1) \geq U_0^H(C_1)$, that is,

$$h \leq \frac{\left[(\phi-1)\Delta w - MH\right]\cdot n_H}{\Delta w \cdot \ell}. \tag{7}$$

Combining (4), (6) and (7), one obtains Theorem 2.

☐ *Comparative Statics of the FEE and Two Examples*

Remark 1. It can be seen from (3) that when m is large enough, the FEE is an equilibrium ifor all possible parameters of the game. To illustrate, consider the extreme case: Let $m \to 1$, then $\lim_{m\to 1} M = \lim_{m\to 1}\left(\frac{1-m}{m}\right) = 0$. Then the left-hand side (LHS) of (3) approaches 0, the right-hand side (RHS) of (3) becomes

$$\min\left\{\frac{(\phi-1)\,n_H}{\ell}, n_H\right\} = n_H,$$

and $0 \le h \le n_H$ always holds. This result is intuitive: $m \to 1$ means that if a player puts one dollar into the public account, her strategic risk becomes negligible.

Remark 2. In a FEE, the gap between Highs and Lows, Δw, cannot be very small. This result might strike the reader as counterintuitive since it implies that equality (in w_i) prevents a fully efficient solution. Consider once again the extreme case. Fixed all other parameters and let $\Delta w \to 0$, then

$$\lim_{\Delta w\to 0}\frac{M\cdot n_L}{\Delta w\cdot \ell} = +\infty > h$$

so that (3) is violated. This result corresponds to GVSM (2009): when all players have the same endowment, it is not an equilibrium that all contribute fully.

Remark 3. Although a large enough Δw, or H, is a *necessary* condition for the existence of a FEE, it is not *sufficient*. To see this, let $H \to +\infty$, so that $\Delta w \to +\infty$, too; then (3) becomes

$$0 \le h \le \min\left\{\frac{(\phi-1-M)\,n_H}{\ell}, \frac{(\ell-M)\,n_H}{\ell}\right\} = \frac{(\ell-M)\,n_H}{\ell}. \tag{3'}$$

We can see that there exist ℓ and M such that (3′) fails. In particular, if $M \to (\phi-1)$, or equivalently $m \to 1/\phi$, then there is clearly no FEE no matter how high H is and no matter what the distribution of types is, since $\ell \le (\phi-1)$.

Example 4 (Numerical application of Theorem 2). Let $m = 0.5$ [so $M \equiv \frac{1-m}{m} = 1$], $\phi = 4$, $n = 24$, $H = 3$. We refer to Fig. 3.3 below. In the figure, each point n_H on the horizontal axis determines a particular ℓ according to the equation $n_L = n - n_H = B\phi + \ell$, and such an ℓ determines: **(a)** the h by the equation $h = \phi - \ell$ [the *black dashed* line], **(b)** the (3)-LHS [the *blue* curve], and **(c)** the (3)-RHS [the *orange* curve]. Thus, if there is a h determined by a n_H that lies between the blue and orange curve, then there exists a FEE by Theorem 2.

Fig. 3.3 indicates that there is a FEE if and only if $n_H = 18$. Note that $n_H = 4A + h$ yields $h = \ell = 2$ [the *red* point in the figure]; furthermore, $n_L = n - n_H = 6$, (3)-LHS $= 1.5$, and

$$\text{(3)-RHS} = \min\left\{\frac{(3\times 2-3)\times 18}{2\times 2}, \frac{(2-1)\times 18}{2}\right\} = 9;$$

thus, $1.5 < h = 2 < 9$, that is, (3) holds. We now show it is indeed an equilibrium:

In equilibrium, $i \in C_2$ gets $0.5 \times \left(\frac{2}{6}\times 8 + \frac{4}{6}\times 4\right) = 2.7$. If she contributes 0, she gets $1 + 0.5\times 3 = 2.5 < 2.7$. Hence, $i \in C_2$ has no incentive to deviate.

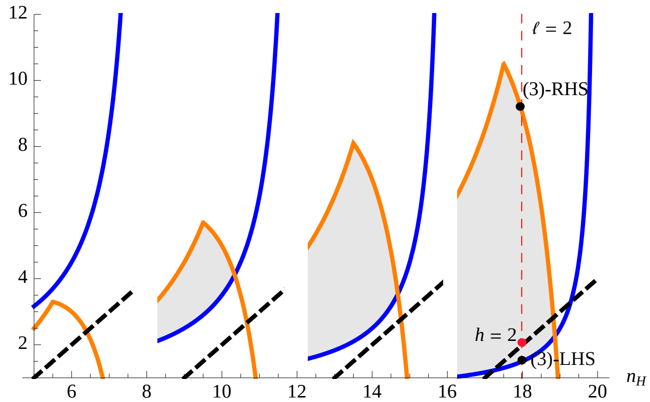


Fig. 3.3. *FEE*

In equilibrium, $i \in C_1$ gets $0.5 \times \left(\frac{16}{18} \times 12 + \frac{2}{18} \times 8\right) = 5.8$; If she contributes $1 + \varepsilon$, she gets no more than $0.5 \times 8 + (1 - 0.5) \times 2 = 5$, which is less than 5.8; finally, if she contributes 0, she gets $3 + 0.5 \times 3 = 4.5 < 5.8$. Hence, $i \in C_1$ also has no incentive to deviate.

Example 5 (**Finding the experimental *FEE***)**.** In a game with parameters as in Example 1, now let H be unspecified. We want to find an H such that there exists a FEE. According to (3), H has to satisfy $h = 2 \geq \frac{6}{2(H-1)}$, which solves for $H \geq 2.5$. Because (3)-RHS holds when $H \geq 2.5$, this concludes the calculation. In light of this, in our experimental setup where Lows have an endowment of 80 tokens each, and Highs 120 tokens, the endowment of the Highs would need to be raised from 120 tokens to at least 200 tokens for a FEE rather than a NEE to emerge.

3.4. **Existence of a *NEE***

The near-efficient equilibrium (NEE) exists if and only if

- player $i \in C_3 \cap N_L$ has no incentive to increase her contribution from 0 to 1.
- player $i \in C_3 \cap N_H$ has no incentive to increase her contribution from 0 to $1 + \varepsilon$ or H.
- player $i \in C_2 \cap N_L$ has no incentive to reduce her contribution from 1 to 0.
- player $i \in C_1 \cap N_H$ has no incentive to reduce her contribution from H to $1 + \varepsilon$ or 0.

Since Example 1 (Deriving the Experimental NEE, Section 3.3) already showed that this equilibrium is possible in some cases, there is no real existence problem. However we provide here a general overview of the conditions under which it exists.

Let c_3^H be the count of Highs in C_3, and c_3^L be the count of Lows in C_3. Then $c_3 = c_3^H + c_3^L < \phi$ and $c_3^H \neq h$, otherwise $\tilde{c}_1 = 0$, which contradicts Lemma A.1(a). We have

$$\begin{aligned} c_1 &= n_H - c_3^H \\ &= \begin{cases} A\phi + h - c_3^H & \text{if } c_3^H < h \\ (A-1)\,\phi + h + \left(\phi - c_3^H\right) & \text{if } c_3^H > h, \end{cases} \end{aligned} \tag{8}$$

and

$$c_2 = n_L - c_3^L$$
$$= \begin{cases} B\phi + \ell - c_3^L & \text{if } c_3^L \le \ell \\ (B-1)\,\phi + \ell + \left(n - c_3^L\right) & \text{if } c_3^L > \ell. \end{cases} \tag{9}$$

It is obviously impossible that $c_3^H > h$ and $c_3^L > \ell$ hold simultaneously since $h + \ell = \phi$. It also can be seen from (8) and (9) that there are three situations to consider: (1) $c_3^H < h$ and $c_3^L \le \ell$, (2) $c_3^H < h$ and $c_3^L > \ell$, and (3) $c_3^H > h$ and $c_3^L \le \ell$. In this paper we only analyze the simplest case, in category (1):

$$c_3^H < h, \quad c_3^L < \ell, \quad \text{and} \quad c_3^H + c_3^L < \phi.$$

The other cases can be analyzed in the same manner. We develop our analysis with the help of Fig. 3.4, which illustrates the distribution of players in a *NEE*.

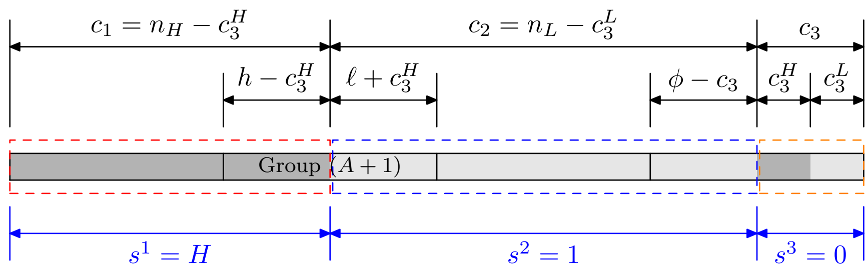


Fig. 3.4. The distribution of players in a *NEE*

☐ *Incentives to Deviate for C_3-Players in a NEE*

Firstly, for player $i \in C_3 \cap N_L$, her payoff from contributing 0 is

$$U_0^L\left(C_3\right) = 1 + m\left(\phi - c_3\right). \tag{10}$$

If she contributes 1, then there are $c_2 + 1$ players contributing 1 and player i enters Group $A+1, \ldots, G$ with positive probabilities, which are

$$\Pr\left(k \mid 1\right) = \begin{cases} \left(\ell + c_3^H\right)/\left(c_2+1\right), & \text{if } k = A+1 \\ \phi/\left(c_2+1\right), & \text{if } k = A+2, \ldots, G-1 \\ \left(\phi - c_3 + 1\right)/\left(c_2+1\right), & \text{if } k = G. \end{cases}$$

Let $S \equiv \left(h - c_3^H\right) H + \left(\ell + c_3^H\right)$. Thus, player i's expected payoff from contributing 1 is

$$\begin{aligned} U_1^L\left(C_3\right) &= m\left\{\Pr\left(A+1 \mid 1\right)\cdot S + \left[\sum_{k=A+2}^{G-1}\Pr\left(k \mid 1\right)\right]\cdot\phi + \Pr\left(G \mid 1\right)\cdot\left(\phi - c_3 + 1\right)\right\} \\ &\overset{\langle 1\rangle}{=} \frac{m}{c_2+1}\left[\left(\ell + c_3^H\right)S + \left(n_L - \phi - \ell\right)\phi + \left(\phi - c_3 + 1\right)^2\right], \end{aligned} \tag{11}$$

where $\langle 1 \rangle$ holds because

$$\sum_{k=A+1}^{G-1} \Pr(k \mid 1) = 1 - \frac{\ell + c_3^H}{c_2 + 1} - \frac{\phi - c_3 + 1}{c_2 + 1} = \frac{\left(c_2 + c_3^L\right) - \phi - \ell}{c_2 + 1} = \frac{n_L - \phi - \ell}{c_2 + 1}.$$

Hence, player $i \in C_3 \cap N_L$ has no incentive to deviate from contributing 0 to contributing 1 if and only if $U_0^L(C_3) \geq U_1^L(C_3)$.

Secondly, for $i \in C_3 \cap N_H$, her payoff from contributing $s_i = H$ is

$$U_0^H(C_3) = H + m(\phi - c_3). \tag{12}$$

If player i contributes $1 + \varepsilon$, she enters group $(A+1)$ and obtains

$$\begin{aligned} \lim_{\varepsilon \to 0} U_{1+\varepsilon}^H(C_3) &= \lim_{\varepsilon \to 0} \left\{ (H - 1 - \varepsilon) + m\left[\left(h - c_3^H\right) H + \left(\ell + c_3^H - 1\right) + (1 + \varepsilon)\right] \right\} \\ &= \Delta w + mS. \end{aligned} \tag{13}$$

If player i contributes H, then there are $c_1 + 1$ players contributing H; player i enters Group $1, \ldots, A+1$ with positive probabilities, which are

$$\Pr(k \mid H) = \begin{cases} \phi / (c_1 + 1), & \text{if } k = 1, \ldots, A \\ \left(h - c_3^H + 1\right) / (c_1 + 1), & \text{if } k = A + 1. \end{cases}$$

Thus, player i's expected payoff is

$$\begin{aligned} U_H^H(C_3) &= m\left\{ \sum_{k=1}^{A} \left[\Pr(k \mid H) \phi H\right] + \Pr(A+1 \mid H) \left[\left(h - c_3^H + 1\right) H + \left(\ell + c_3^H - 1\right)\right] \right\} \\ &= m\left[\left(1 - \frac{h - c_3^H + 1}{c_1 + 1}\right) \phi H + \frac{h - c_3^H + 1}{c_1 + 1} (S + \Delta w)\right] \\ &= \left(\frac{m}{c_1 + 1}\right) \left[(n_H - h) \phi H + \left(h - c_3^H + 1\right)(S + \Delta w)\right]. \end{aligned} \tag{14}$$

Hence, player $i \in C_3$ has no incentive to deviate if and only if the following conditions are satisfied:

$$\begin{cases} (10) \geq (11): & i \in C_3 \cap N_L \text{ has no incentive to deviate from 0 to 1} \\ (12) \geq (13): & i \in C_3 \cap N_H \text{ has no incentive to deviate from 0 to } 1 + \varepsilon \\ (12) \geq (14): & i \in C_3 \cap N_H \text{ has no incentive to deviate from 0 to } H. \end{cases} \tag{IC$_3$}$$

☐ *Incentives to Deviate for C_2-Players in a NEE*

Recall that C_2 consists of Lows. If $i \in C_2 \subseteq N_L$ contributes 1, she gets

$$U_1^L(C_2) = \frac{m}{c_2}\left[\left(\ell + c_3^H\right) S + \left(c_2 - \ell - c_3^H - \phi + c_3\right) \phi + (\phi - c_3)^2\right]; \tag{15}$$

if she contributes 0, she gets

$$U_0^L(C_2) = 1 + m(\phi - c_3 - 1). \tag{16}$$

Thus, $i \in C_2 \cap N_L$ has no incentive to deviate if and only if

$$(15) \geq (16): \quad i \in C_2 \subseteq N_L \text{ has no incentive to deviate from 1 to 0.} \tag{IC$_2$}$$

□ *Incentives to Deviate for C_1-Players in a NEE*

C_1 consists of Highs. For $i \in C_1 \subseteq N_H$, if she contributes H, her expected payoff is

$$U_H^H(C_1) = m\left[\left(1 - \frac{h - c_3^H}{c_1}\right)\phi H + \frac{h - c_3^H}{c_1} S\right]. \tag{17}$$

If she contributes $1 + \varepsilon$, she obtains

$$\begin{aligned} \lim_{\varepsilon \to 0} U_{1+\varepsilon}^H(C_1) &= \lim_{\varepsilon \to 0}\left\{(H - 1 - \varepsilon) + m\left[\left(h - c_3^H - 1\right)H + \left(\ell + c_3^H\right) + (1 + \varepsilon)\right]\right\} \\ &= mS + (1 - m)\Delta w. \end{aligned} \tag{18}$$

A similar argument as in Lemma 2 shows that we need not consider whether $i \in C_1 \cap N_H$ has any incentive to contribute 1 if she has no incentive to contribute $1 + \varepsilon$. We can therefore immediately consider the last possible deviation. If player i contributes 0, she obtains

$$U_0^H(C_1) = H + m(\phi - c_3 - 1). \tag{19}$$

Thus, $i \in C_1 \subseteq N_H$ has no incentive to deviate if and only if

$$\begin{cases} (17) \geq (18): & i \in C_1 \subseteq N_H \text{ has no incentive to deviate from } H \text{ to } 1 + \varepsilon \\ (17) \geq (19): & i \in C_1 \subseteq N_H \text{ has no incentive to deviate from } H \text{ to } 0. \end{cases} \tag{IC$_1$}$$

Theorem 3 summarize this section's findings:

Theorem 3. *The NEE exists if and only if* (IC$_3$), (IC$_2$), *and* (IC$_1$) *are all satisfied.*

3.5. Coexistence of *NEE* and *FEE*?

So far we know that if there are equilibria with positive contributions, it is a FEE or NEE. Can these two equilibria with positive contributions ever coexist? We will now show with an example that this is possible. Our analysis focuses on the version of the 2-Type GBM tested experimentally in this paper. Example 1 demonstrated that this game has a NEE. Example 5 showed that the game has a FEE if and only if $H \geq 2.5$. We now show that if $H = 2.5$ there exists, in addition to the FEE, the following NEE:

$$(\langle H, H, H, H\rangle, \langle H, H, 1, 1\rangle, \langle 1, 1, 1, 0\rangle).$$

- For player $i \in C_3 \subseteq N_L$, her equilibrium payoff is $U_0^L(C_3) = 1 + 3/2 = 5/2$; if she contributes 1, the expected payoff is $U_1^L(C_3) = \frac{1}{2} \times \left(\frac{2}{6}S + \frac{4}{6} \times 4\right) = \frac{5}{2} = U_0^L(C_3)$.
- For player $i \in C_2 \subseteq N_L$, her equilibrium payoff is $U_1^L(C_2) = \frac{1}{2} \times \left(\frac{2}{5} \times 7 + \frac{3}{5} \times 3\right) = 2.3$; if she contributes 0, the payoff is $U_0^L(C_2) = 1 + \frac{1}{2} \times 2 = 2 < U_1^L(C_2)$.
- Finally, for player $i \in C_1 = N_H$, she gets $U_H^H(C_1) = \frac{1}{2} \times \left(\frac{4}{6} \times 4H + \frac{2}{6}S\right) = 4.5$ in equilibrium; if she contributes $1 + \varepsilon$, the payoff is $\lim_{\varepsilon \to 0} U_{1+\varepsilon}^H(C_1) = S/2 + (H - 1)/2 = 4.25 < U_H^H(C_1)$; if she contributes 0, the payoff is $U_0^H(C_1) = H + 2/2 = 3.5 < U_H^H(C_1)$.

Note however that the unique equilibrium with positive contributions is the FEE if $H > 2.5$: Since it is required that $c_1 > 4$ and $\widetilde{c}_1 > 0$ in any equilibrium with positive contributions, c_3^H can only take two possible values: either $c_3^H = 1$ or $c_3^H = 0$. However, $c_3^H = 1$ is impossible. This is because if a High has no incentives to contribute 0 in the FEE, she also has no incentive to contribute 0 when there is at least one Low in Group G contributing 0. Hence, we only need to consider the case of $c_3^H = 0$. By (10),

$$U_0^L(C_3) = 1 + \frac{4 - c_3}{2} = \frac{6 - c_3}{2}. \tag{10'}$$

By (11),

$$U_1^L(C_3) = \frac{4H + 4 + (5 - c_3)^2}{14 - 2c_3}, \tag{11'}$$

where $c_3 = 1, 2, 3$. Then

$$\begin{aligned}(11') - (10') &= \frac{3c_3 + 4H - 13}{14 - 2c_3} \\ &> \frac{3(c_3 - 1)}{14 - 2c_3} \\ &> 0,\end{aligned}$$

for any $c_3 = 1, 2, 3$, which means that $U_0^L(C_3) < U_1^L(C_3)$, that is, any C_3-player will deviate no matter how many players contribute 0 in Group G. We thus proved that no player will contribute 0 if $H > 2.5$, in other words, the FEE is the unique equilibrium with positive contributions if $H > 2.5$.

4. Method

Experimental game parameters and experimental NEE

The 2-Type GBM was examined under MPCR $m = 0.5$. The number of participants per session was twelve, group size was four. Six participants were randomly selected as Lows and received $L = 80$ tokens, the remaining six Highs received $H = 120$ tokens per round. Once assigned, a subject's type did not change over the experiment's 80 rounds. Most parameters here are the same as in GVSM including the mean endowment over twelve subjects. The only difference is that in GVSM's study endowments are uniform.

Our experimental parameter configuration does not allow a FEE since H is less than 2.5 times L (by Theorem 2; see also Example 5). However, there exists the following NEE: $\{120, 120, 120, 120, 120, 120, 80, 80, 80, 80, 0, 0\}$. This NEE is calculated in Example 1 (Section 3), and shown in Fig. 3.2. As usual in a GBM, there also exists a risk-dominant equilibrium of non-contribution by all (by Lemma 1).

Design and participants

Participants were undergraduates at City University of New York, recruited from the general student population for a two-hour experiment with payoffs contingent upon the decisions they and other participants made during the experiment. Subjects were seated in front of computer terminals separated by blinders. There were four experimental sessions with twelve participants each, 48 subjects in total. Each session lasted two hours. The show-up fee was \$10. The exchange rate was 700 tokens for a dollar or conversely, 0.143 cents per token. In addition to the show-up fee, mean earnings of Highs were \$25; mean earnings of Lows were \$16.

Procedure

Investment decision. At the beginning of each round, each subject received the type-appropriate amount of integer tokens, to be divided between a public account and a private account. For every token invested the private account, the account returned one token to the investor alone. For every token invested in the public account, the return was 0.5 tokens to everyone in the investor's group including herself. Appendix B contains the experimental instructions.

Group assignment. In each round, after all subjects had made their investment decisions, they were partitioned in three groups of four. The four highest investors to the public account were placed into one group, the fifth through the eighth highest investor into a second group, and the four lowest investors into a third group. Ties were broken at random. After grouping, subjects' earnings were calculated based on the group to which they had been assigned. Note that group assignment depended only on the subjects' current contributions in that round, not on contributions in previous rounds. Subjects were regrouped according to these criteria in each decision round (See Appendix B).

End-of-round feedback. After each round, a subject's computer screen displayed her private and public investment in that round, the total investment made by the group she had been assigned to, and her total earnings. The screen also displayed an ordered series of the current round's group account contributions by all n participants, with a subject's own contribution highlighted so that she could see her relative standing. This ordered series was visually split into three groups of four, which further underscored that the participants in the experiment had been grouped according to their contributions and that ties had been broken at random.

5. Results and Discussion

The main purpose of this analysis is to establish whether the 2-Type GBM is an effective mechanism when abilities to contribute differ, and whether GVSM's results about the precise coordination of the payoff dominant equilibrium are robust to such inequality.

Result 1 (Observed mean contributions correspond to the NEE mean contributions). The broken lines in Fig. 5.1 represent the NEE mean contributions per round (86.67 tokens). The solid lines are the observed mean contributions. Mean contributions over all four sessions (solid lines) closely trace their predicted values, and trace them particularly closely after Round 20. This pattern also emerges in the single sessions shown in the lower half of Fig. 5.1.

Adjustment in initial rounds. There is some adjustment in the initial rounds, particularly up to round 20. In GVSM's experiments with homogeneous endowments, subjects coordinated the payoff-dominant Nash equilibrium as well, but did it more quickly: GVSM's subjects reached NEE means by Round 2. Here however, a comparable level of consistent precision is only achieved after Round 20, even though sporadic mean precision is seen as early as Round 6. Since GVSM's experiments and the present experiment were run at different universities, it is not possible to attribute the slower convergence here to the fact that the NEE of the 2-Type GBM has a more complex structure (three strategies) than the NEE in GVSM's homogeneous-endowment game (two strategies).

Result 2 (Strategies that are part of the NEE are predominantly selected, and selected with precision; there is slightly more precision after about Round 20). The experiment's NEE consists of the two corner strategies from among a set 81 choices $\{0, 1, \ldots, 80\}$ for Lows, and only one of 121 available choices $\{0, 1, \ldots, 120\}$ for Highs. Fig. 5.2 shows the strategy space on the horizontal axis and the observed percentages of choices over four sessions on the vertical axis. Red bars show the NEE proportions. Part A shows choice frequencies for Rounds 1-80. Part B shows the same for Rounds

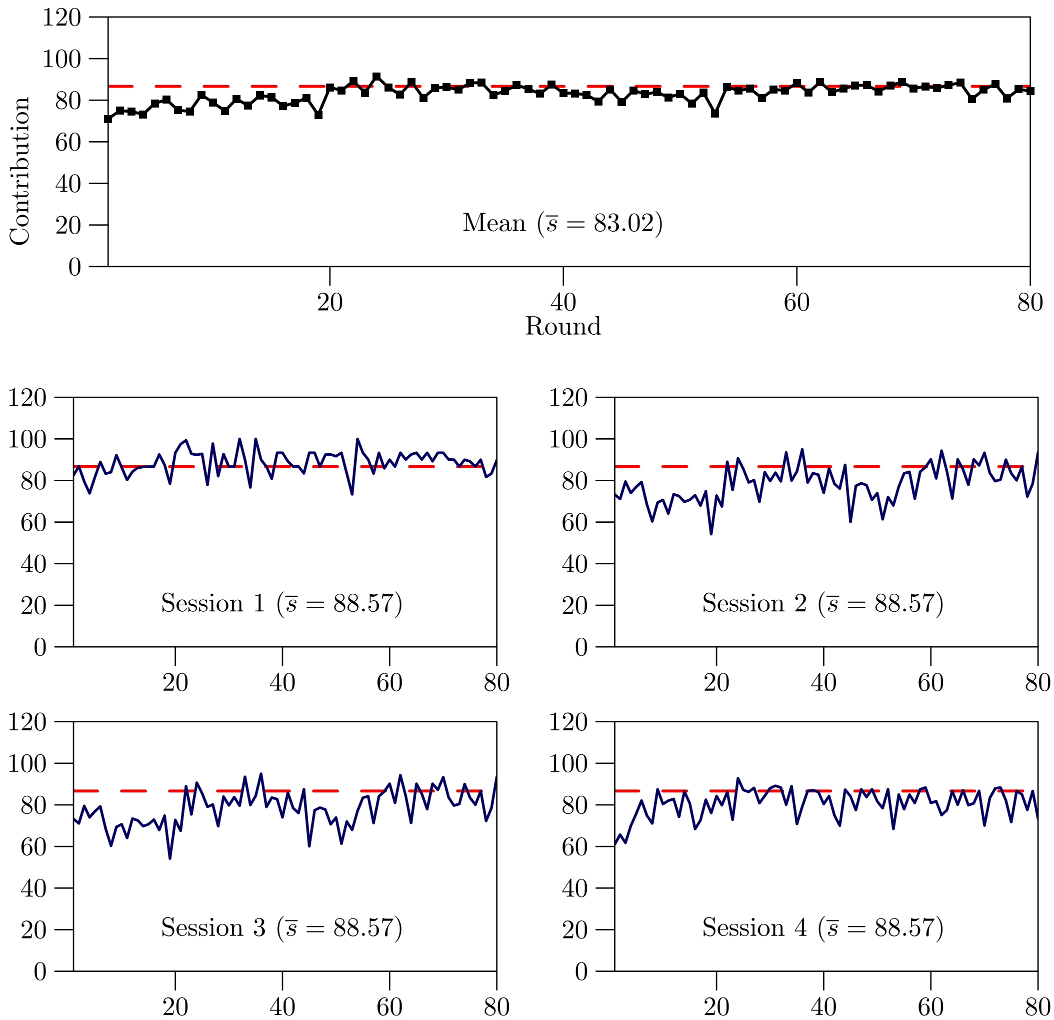


Fig. 5.1. Mean contributions per round over four sessions and for each session

21-80 only, and once again highlights that the equilibrium strategies are executed with more precision after Round 20.

We include a comparable graph from GVSM as Part C. A comparison of Parts A-B with Part C shows that in both series of experiments the *NEE* strategy proportions were coordinated quite precisely.

Coding the data. In Fig. 5.2 and in all subsequent analysis, we classify choices ≥ 77 as 80, choices ≥ 117 as 120, and choices ≤ 3 as zero contribution. We recode the raw data this way since GVSM did the same, so that the two studies can be properly compared. Note however that GVSM report that this minor recoding, while grounded in behavioral theory about prominence (Selten 1997) and neighboring strategies (Erev and Roth 1998), barely changed their results. The same applies to our data. Table 5.1 displays the raw frequencies of the exact *NEE* strategies and of their neighboring strategies that were recoded, separately for Rounds 1-80, Rounds 21-80., and Rounds 1-21. The

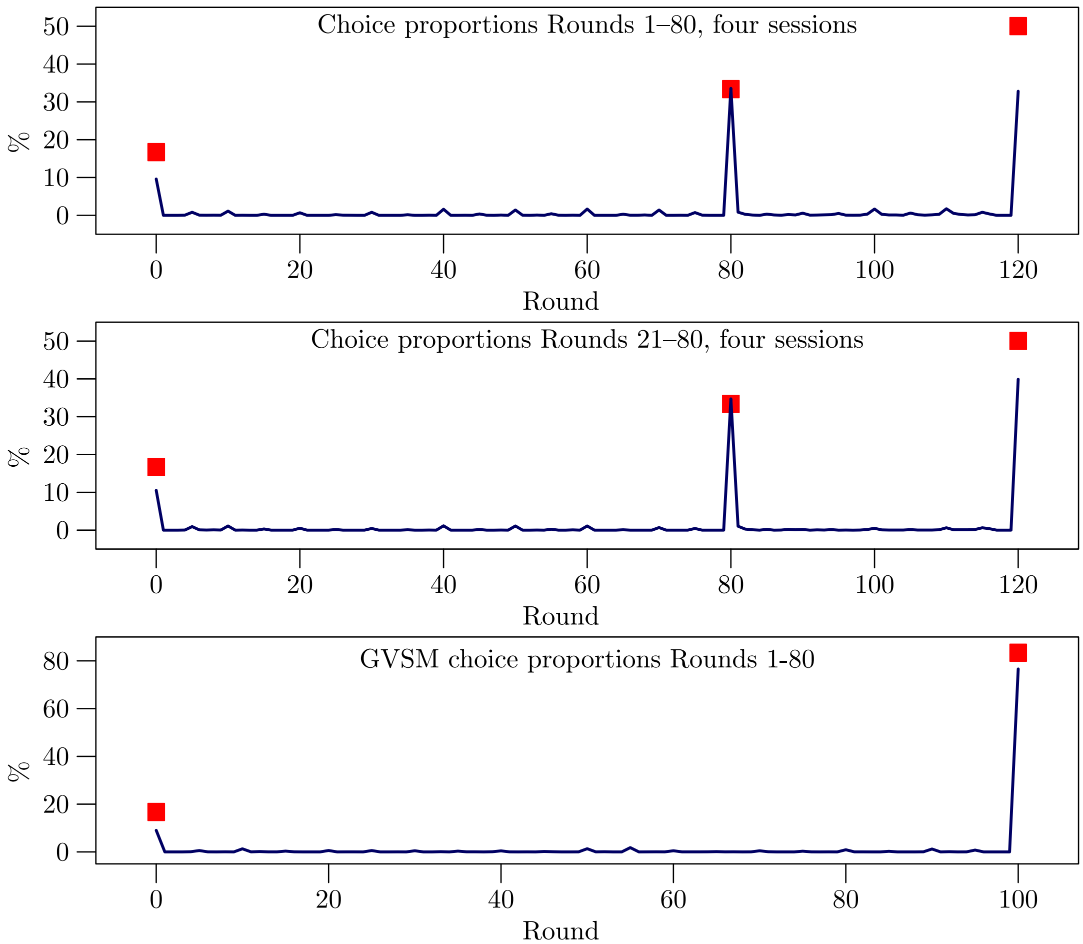


Fig. 5.2. Observed proportion of choices in the current study (top two graphs) and in GVSM's experiment (*NEE* choice proportions as red blocks)

precision with which the *NEE* was realized becomes once again clear, as well as the increased precision after Round 20. What is this increased precision in later rounds due to? For this purpose, we next examine choice strategies by Type.

Result 3 (**The aggregate frequencies with which equilibrium strategies were selected by the two different types are close to the *NEE***). In the experimental game's *NEE*, all Highs contribute fully; four out of six Lows also contribute fully while the other two Lows contribute nothing. Thus, Lows have a choice between two strategies but Highs must play one specific strategy. Fig. 5.3 displays, separately for Highs and Lows, the frequency with which equilibrium strategies were chosen in each round over four sessions. Broken red lines show the frequencies of a given strategy as predicted by the *NEE* over four sessions (For example, for Highs, the *NEE*-based prediction is 4 x 6 = 24 observations of full contribution per round).

Table 5.1. Raw frequencies of choices neighboring NEE strategies

A: Raw frequencies of choices before recoding (Rounds 1-80)

Strategy	**Raw %**	**Strategy**	**Raw %**	**Strategy**	**Raw %**
0	8.0	80	32.8	120	31.3
1	1.2	79	0.5	119	0.9
2	0.2	78	0.2	118	0.5
3	0	77	0.0	117	0.1
Totals	9.6		33.6		32.8

B: Raw frequencies before recoding (Rounds 21-80)

Strategy	**Raw %**	**Strategy**	**Raw %**	**Strategy**	**Raw %**
0	8.8	80	34.1	120	38.4
1	1.5	79	0.4	119	0.8
2	0.2	78	0.1	118	0.4
3	0	77	0	117	0.2
Totals	10.5		34.7		39.9

C: Raw frequencies of choices before recoding (Rounds 1-21)

Strategy	**Raw %**	**Strategy**	**Raw %**	**Strategy**	**Raw %**
0	5.9	80	28.8	120	10.0
1	0.7	79	0.8	119	0.9
2	0.1	78	0.3	118	0.3
3	0.1	77	0.2	117	0.1
Totals	6.9		30.1		11.6

It can be seen that the number of fully contributing Lows is quite close to the NEE prediction by Round 20. Many Highs on the other hand only gradually appear to discover that, since the game is converging to the NEE rather than the alternative equilibrium of non-contribution by all, their optimal strategy is full contribution.

Appendix C displays the individual choice path of each subject over 80 rounds. Column headings on top of each page indicate the session. Numbers on the left hand side alongside each page identify the subject. Within each session, Subjects 1-6 are Highs, Subjects 7-12 are Lows. We henceforth refer to subjects by these two numbers, so that for example Subject 4-3 is Subject 4 (a High) in Session 3. The straight horizontal line in each chart shows the endowment; the lower, red line represents the subject's group contribution; the jagged green line in the top part of each graph shows the associated earnings. An initial glance over all graphs shows support for the NEE: The contribution paths of Highs, who in the NEE must contribute fully, are flat in particular in later rounds, and often on or close to the straight endowment line. Lows often oscillate between their two NEE strategies of full contribution and non-contribution.[11] Appendix C again underscores that a noticeable proportion of Highs experimented in

[11]The focus here is on the 2-Type GBM's payoff-dominant equilibrium rather than on individual strategies. We note however that Lows' oscillations between their two equilibrium strategies 1) are similar to what GVSM's subjects with homogeneous

early rounds before settling on their sole optimal strategy. Notice the slow learning of Highs 1-1, 2-3, 4-1, 4-3 and particularly 2-6, and the consistent "confusion" (Andreoni 1995) of Highs 2-4, 2-6, 3-6 and particularly 3-2. Among Lows, 4-9 is a slow learner. Notice consistently confused Lows 3-12 and 4-12. Finally, the charts show that no Low is a permanent non-contributor. GVSM similarly found no steady free-riders in their study where the proportion of non-contributors in the *NEE* is the same as here.

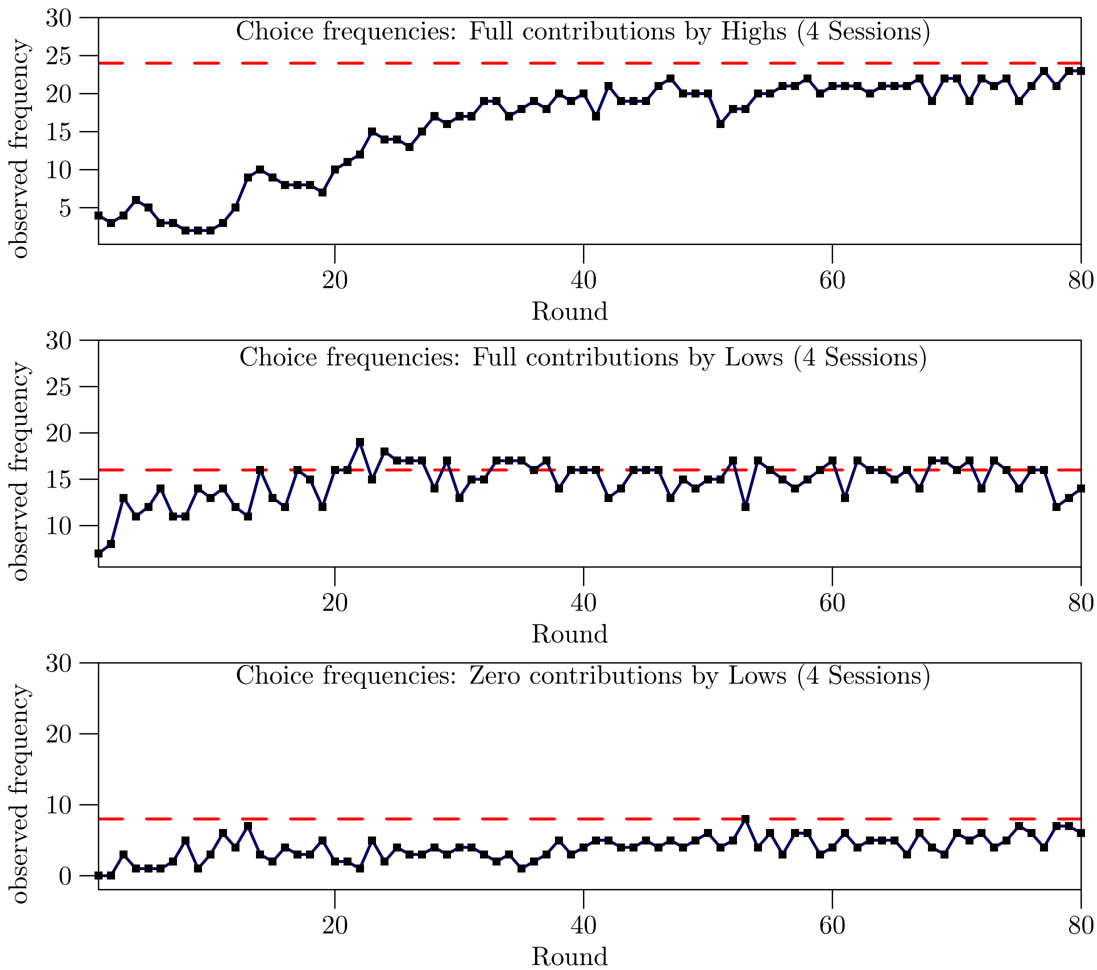


Fig. 5.3. Frequencies of full contributions by Highs, full contributions by Lows, and zero contributions by Lows, over four sessions

endowments, who thus all had a choice between two *NEE* strategies, did. GVSM compute the game's complete mixed-strategy equilibrium and report that neither the individual choice proportions over 80 rounds nor the sequence of choices is consistent with mixing. 2) are similar to what is found in Market Entry Games where individual strategies over rounds oscillate unpredictably but aggregate choice proportions are close to the asymmetric equilibrium (for overviews, see, e.g., Ochs 1999; Camerer and Fehr 2006).

Result 4 (Deviations from the NEE strategies are penalized by lowered earnings). NEE earnings are 227 tokens for Highs, 140 tokens for contributing Lows, and 160 tokens for non-contributing Lows. A subject's mean earnings over 80 rounds are written in the lower right corner of her chart. Overall, individual mean earnings over 80 rounds are close to NEE earnings. The mean earnings of subjects who do not select their NEE corner strategies are lower than the earnings of subjects who do. A similar pattern can be detected by examining he green lines in the upper part of the Appendix C charts, which show a subject's earnings per round.

See for example confused High 3-2 who consistently does not quite contribute fully and whose mean earnings over 80 rounds are only 197 tokens; see also the lowered mean earnings of Highs 2-4 and 2-6. The reason for their lowered mean earning is that, as long as most other players choose NEE strategies, Highs who contribute > 80 and < 120 can never enter the High-only top group, and are instead put into the mixed middle group consisting of Highs and Lows, where Lows can free-ride off them.

Lows who consistently select strategies from the interior of their strategy space such as Lows 3-12 and 4-12, also make less than they otherwise would, had they selected their NEE strategies. Since the NEE is quite consistently played by most participants, Lows who contribute between 0 and 80 are usually placed in the lowest group with certainty, get no chance to free-ride off Highs in the middle group, and get free-ridden by the zero-contributors in the bottom group.

6. Conclusion

Unequal abilities to contribute are an important feature of real-world societies. We use a formal mechanism to examine the impact of endogenous group formation in the context of mechanism design and rational choice, and study the impact of unequal ability to contribute on contribution behavior and efficiency. In our game, some players ("Lows") are naturally disadvantaged due to low endowments. They can never aspire to membership in the most productive and rewarding teams, nor can their earnings ever match those of players with high endowments ("Highs"). Our theoretical and experimental results show that despite of this, competitive contribution-based grouping is an effective and precise tool to raise social contributions by the advantaged and disadvantaged alike. Not only do our behavioral results show that unequal abilities to contribute are not deleterious to efficiency, but our theoretical analysis shows that when the difference between the high and low endowments increases, efficiency can increase until full Pareto optimality is achieved.

The predictive power of the Nash equilibrium. In our experiment, subjects' strategy sets are quite large; the payoff-dominant "near-efficient" equilibrium (NEE) is asymmetric, and consists of three different strategies. Discovering the NEE analytically is a long, involved process (as reflected in the length of Section 3 and Appendix A) that requires the step-by-step elimination of configurations involving positive contributions. It is therefore unlikely that a subject can compute or understand this equilibrium. Yet subjects reliably tacitly coordinate it in a "magical" (Kahneman 1988, p. 12) way. It further underscores the predictive power of the Nash equilibrium that (1) aggregate behavior conforms to the NEE even though many Lows, who, in a NEE have a choice between two different corner strategies, oscillate erratically between their strategies over rounds, and (2) the experimentally tested version of the 2-Type GBM does not lead to full efficiency since the latter is not an equilibrium.

In a study of a simpler form of the mechanism with homogeneous endowments, GVSM, using a different subject pool, also found that subjects coordinated the NEE with precision. This indicates that the precise coordination of the GBM's asymmetric equilibrium is likely robust. Since this payoff-dominant equilibrium predicts so well, we do not apply explanatory concepts such as reciprocity, competitiveness and the like, which only allow for a directional prediction rather than a point prediction.

Policy relevance. Our results suggest efficiency gains if a system is organized according to meritocratic rather than ascriptive principles. Since the nature of the GBM's group-based output is broadly defined, our theoretical and experimental findings could apply to a wide variety of settings such as teams, firms, or academic departments.[12] The empirical confirmation that the GBM's payoff-dominant equilibrium, however complex, is easily coordinated in the laboratory even if abilities to contribute vary and an alternative equilibrium of non-cooperation by all is still present, might add to our understanding of how many societies and organizations have become increasingly meritocratic, as evidenced for example by the gradual abolition of monarchies, the trend away from family firms and toward professional management, and the reduced relevance of gender, race or class in many industrialized or developing countries. We note however that we have found cases of the mechanism where only an equilibrium of non-contribution by all exists (Example 2). This raises the question whether and how the efficiency-enhancing effects of meritocratic organization are dependent upon social structure.

Criticisms

Do lags need to be built into the model? Our model is one of instantaneous, perfect mobility based on current performance, with no lags between performance and grouping, or between grouping and reward: Players decide, get grouped and rewarded, all in the same round. Lags would represent system imperfections in the form of delays, e.g., if information needs to be collected over periods that are longer than the reward cycles. In an ideal Group-based Meritocracy there should be no lags since positions and associated rewards should be instantaneously adjusted based upon performance. Individuals' occasional mistakes would thus be immediately reflected in group membership and associated rewards; on the other hand, a slacker could instantaneously redeem herself if she increases her contribution. A trend to shorten employment contracts or to increase the frequency of performance reviews, could be interpreted as a move toward such a model. However, it is clear that our current model remains extreme in this regard since in the real world, grouping and reward is based on past behavior and reputation. Note however that introducing lags into the model would make this game dynamic. The game's equilibrium structure is already quite complex in the current static version, and introducing reputation, more complex institutional rules, and other complications would make the model very difficult, perhaps even impossible, to solve analytically.

We acknowledge that in the current version of the model, and in its experimental test, boundedly rational players are not overloaded with information and additional complications that exist in the field such as reputation and lags. We also do not incorporate possible effects of homogeneity of class, race or gender on in-group cohesion and thus, cooperation. Our model thus provides a favorable environment for a payoff-dominant Nash equilibrium to be realized. The impact of lags and other complications therefore merits systematic exploration, but this does not detract from the finding that performance-based group mobility makes provision levels of collective goods efficient even if players' abilities to contribute are not equal. The current paper is part of a research program that studies the rational-aspects of endogenous group formation. While lags and other complicating aspects should at some point be built into the mechanism, we consider the following extensions more pressing.

[12]Usually, a system is considered a meritocracy when each member is rewarded individually according to his output. In a modern organization-based economy however a significant proportion of rewards are shared, for example: overall firm salary levels, profit sharing payments, health care coverage, leave policy, and intangibles such as firm reputation, location, premises, or work atmosphere.

Extensions

The main purpose of this paper's experiment was to test whether GVSM's finding that the GBM Mechanism's NEE is precisely coordinated in the lab is robust to inequality and the added complexity that goes with it. The general theoretical analysis of the 2-Type GBM in Section 3 however can form the base for numerous other experimental tests. The sensitivity of the mechanism's equilibrium structure to a change in parameters, as illustrated in the examples in Section 3, together with the precision with which subjects have so far coordinated the mechanism's payoff-dominant equilibrium, should yield distinctive experimental results that closely reflect the underlying equilibrium structure.

Full efficiency with sufficient inequality? The theoretical finding that if the difference between the advantaged and disadvantaged types is large enough, the disadvantaged, far from getting discouraged, might increase their social contributions even more so that a fully efficient, rather than merely a near-efficient solution results (Theorem 2) invites testing. Payoff dominance (Harsanyi and Selten 1988 suggests that full efficiency should occur in this case. However, payoff dominance and other theories of equilibrium selection are not entirely uncontested (see, e.g., Binmore 1989; Aumann 1988; Crawford and Haller 1990; Harsanyi 1995; van Damme 2002, Section 5). A useful method to distinguish among a game's multiple equilibria is therefore to test with experiments which equilibrium subjects actually pick.

From a policy viewpoint, could one increase inequality in order to raise efficiency? It would all depend upon how it is done: Lowering the ability of the Lows to the point where they all contribute fully (leading to a FEE) might be counterproductive: In our experiment for example it would require lowering the low endowment L to only 40% of the high endowment H, from 80 tokens to 48 tokens. This however does not increase overall social contributions or earnings: Subjects' total earnings per round in the NEE experimentally tested in this paper are 2240 tokens, but would only be 2016 in the FEE that would result if L were reduced to 48 tokens only. Increasing H however could achieve the dual goal of higher overall earnings and of full efficiency. However, we do not know whether at some point Lows revolt and gravitate toward the alternative equilibrium of non-contribution by all. An experiment could provide indications.

Type counts as critical elements. Type count can be manipulated so that both NEE and FEE disappear (See Example 2). In such a case, will subjects indeed converge to the only remaining equilibrium of non-contribution by all?

Full heterogeneity. Our current model allows for inequality only in the form of a 2-type society. An obvious further extension of the current model is to increase the number of types, eventually up the number of players.

Concluding remarks

If endogenous group formation is intended as a policy tool, the question of unequal abilities must be addressed. The findings of the current paper indicate that unequal abilities to contribute are not detrimental to a system where grouping is competitively based upon contributions.

Acknowledgments

We thank Jim Andreoni and Carlos Pimienta for helpful comments, and the Australian Research Council (ARC) for financial support.

Appendix

A. Proof of Theorem 1

The proof of Theorem 1 relies upon the five auxiliary results summarized in Lemma A.1:

Lemma A.1. *If an equilibrium with positive contributions exists, it has the following properties:*

(a) *The count of C_1-players is larger than and not a multiple of group size ϕ, and each C_1-player contributes fully. Formally, $c_1 > \phi$, $\widetilde{c}_1 > 0$, and $s_i = w_i$ if $i \in C_1$.*
(b) *C_1 consists of Highs only, that is, $C_1 \subseteq N_H$.*
(c) *There is no class C_r satisfying $1 < s^r < H$.*
(d) *If the equilibrium consists of only two classes, it is a FEE.*
(e) *If the count of C_R-players is less than or a multiple of the group size, then each C_R-player contributes nothing. Formally, if $c_R < \phi$ or $\widetilde{c}_R = 0$, then $s^R = 0$.*

Proof. **(a)** If $\widetilde{c}_1 = 0$, then $c_1 \equiv |C_1| = D_1 \cdot \phi$ by (2). Consider any player $i \in C_1$. If $s_i = s^1$, she is always grouped with $(\phi - 1)$ players contributing s^1 and gets $\left(w_i - s^1 + m\phi s^1\right)$; if she contributes $s_i' = s^1 - \varepsilon > s^2$ where $\varepsilon \in \mathbb{R}$, she is in Group D_1 but is still grouped with $(\phi - 1)$ players contributing s^1, and gets

$$\left(w_i - s^1 + \varepsilon\right) + m\left[(\phi - 1)\, s^1 + s^1 - \varepsilon\right] = \left(w_i - s^1\right) + m\phi s^1 + (1 - m)\,\varepsilon$$
$$> w_i - s^1 + m\phi s^1$$

since $m < 1$. Thus i has an incentive to deviate. It follows that $\widetilde{c}_1 > 0$ as claimed.

To see that $c_1 > \phi$, note that if $c_1 < \phi$, player $i \in C_1$ is in the first group where the total contribution except for player i is S^1_{-i}. If she reduces her contribution from s^1 to $s^1 - \varepsilon > s^2$, she remains in the first group, but her payoff increases from $\left[w_i - s^1 + m\left(S^1_{-i} + s_i\right)\right]$ to

$$w_i - s^1 + m\left(S^1_{-i} + s_i\right) + (1 - m)\,\varepsilon.$$

Thus i has an incentive to deviate. This proves that $c_1 > \phi$.

To verify that each C_1-player contributes fully, note that we now have $c_1 = D_1 \cdot \phi + \widetilde{c}_1$, where $D_1 \geq 1$ and $\widetilde{c}_1 > 0$; hence, every C_1-player has a strictly positive probability of entering Group $(D_1 + 1)$, that is, $\Pr\left(D_1 + 1 \,\middle|\, s^1\right) = \widetilde{c}_1 / c_1 > 0$. Given a contribution profile $\boldsymbol{s}$ satisfying $s_i = s^1 < w_i$ for some $i \in C_1$, let $S = \phi s^1$ be the total contribution in Group $1, \ldots, D_1$, and let $S' \leq \widetilde{c}_1 s^1 + \left(\phi - \widetilde{c}_1\right) s^2$ be the total contribution in Group $(D_1 + 1)$.[13] Then $S > S'$ since $s^1 > s^2$. Hence, if a C_1-player contributes

[13] We use a weak inequality here because it is not clear at this stage if there are players from classes after C_2 in group $(D_1 + 1)$.

$s_i = s^1 < w_i$, her payoff is

$$\begin{aligned}
&(w_i - s_i) + m\left\{\left[\sum_{k=1}^{D_1}\Pr\left(k \,\middle|\, s^1\right)\right]\cdot S + \Pr\left(D_1+1 \mid s^1\right)\cdot S'\right\}\\
&\qquad = \left(w_i - s^1\right) + m\left\{\left[1-\Pr\left(D_1+1\,\middle|\, s^1\right)\right]\cdot S + \Pr\left(D_1+1\,\middle|\, s^1\right)\cdot S'\right\}\\
&\qquad < \left(w_i - s^1\right) + mS.
\end{aligned}$$

However, if she increases her contribution from s^1 to $s^1 + \varepsilon < w_i$, she enters the first group with certainty and obtains:

$$\left(w_i - s^1 - \varepsilon\right) + m\left(S+\varepsilon\right) = \left[\left(w_i - s^1\right) + mS\right] - (1-m)\,\varepsilon.$$

This deviation is profitable as long as ε is small enough. We thus proved that $s^1 = w_i$ if player i is in the first class.

(b) We first show that there is at least one High in C_1. Suppose this is not true, that is, suppose that $C_1 \subseteq N_L$. Then $s^1 = 1$ since each C_1-player contributes fully. We can show that in such a situation any C_2-player has an incentive to deviate. There are three cases to consider:

i). $\boldsymbol{\widetilde{c}_1 + c_2 \le \phi}$; see Fig. A.1(i). Since we assume that $C_1 \subseteq N_L$, there are more than $n_H > \phi$ players outside of C_1, so that $|\mathscr{C}| \ge 3$ and $s^2 > 0$. In such a case, each C_2-player can reduce her contribution from s^2 to $s^2 - \varepsilon > s^3$ and remain in Group (D_1+1). By the same reasoning as in Lemma A.1(a), this is a profitable deviation.

ii). $\boldsymbol{\widetilde{c}_1 + c_2 > \phi}$ **and** $\boldsymbol{\widetilde{c}_1 + \widetilde{c}_2 = \phi}$; see Fig. A.1(ii). Consider any player $i \in C_2$. If $s_i = s^2 < 1$, her payoff is

$$\begin{aligned}
&\left(w_i - s^2\right) + m\left\{\Pr\left(D_1+1\,\middle|\, s^2\right)\cdot\left[\widetilde{c}_1 + \left(\phi-\widetilde{c}_1\right)s^2\right] + \left[1-\Pr\left(D_1+1\,\middle|\, s^2\right)\right]\cdot\phi s^2\right\}\\
&\qquad < \left(w_i - s^2\right) + m\left[\widetilde{c}_1 + \left(\phi-\widetilde{c}_1\right)s^2\right]
\end{aligned}$$

because $\widetilde{c}_1 + \left(\phi-\widetilde{c}_1\right)s^2 > \phi s^2$. However, if she contributes $s^2+\varepsilon < s^1$, she enters Group (D_1+1) with certainty and obtains

$$\left(w_i - s^2-\varepsilon\right) + m\left[\widetilde{c}_1 + \left(\phi-\widetilde{c}_1\right)s^2+\varepsilon\right] = \left(w_i - s^2\right) + m\left[\widetilde{c}_1 + \left(\phi-\widetilde{c}_1\right)s^2\right] - (1-m)\,\varepsilon,$$

which is greater than her original payoff when ε is small enough. Thus, player $i \in C_2$ has an incentive to increase her contribution.

iii). $\boldsymbol{\widetilde{c}_1 + c_2 > \phi}$ **and** $\boldsymbol{\widetilde{c}_1 + \widetilde{c}_2 \ne \phi}$; see Fig. A.1(iii). This cannot be an equilibrium since any player $i \in C_2$ will increase her contribution for the same reason as in ii).

Hence, there is at least one High i in C_1. Together with Lemma A.1(a) this implies that $s_i = H$. We thus conclude that $s^1 = H$ and $C_1 \subseteq N_H$.

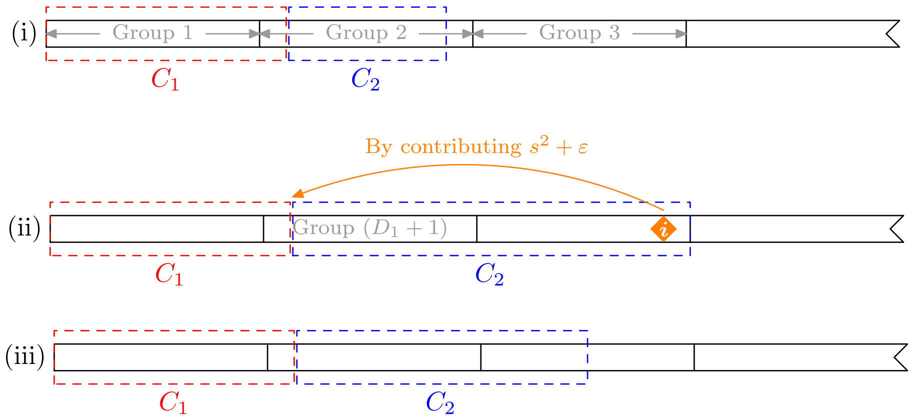


Fig. A.1. There is at least one High in C_1

(c) Suppose there exists a Class C_r satisfying $1 < s^r < H$. Since $s^r < H = s^1$, Class C_1 is ranked above Class C_r; since $s^r > 1$, there is at least one class after C_r and $C_r \subseteq N_H$. A similar argument as in Lemma A.1(b) shows that **(1)** C_1 is the immediate predecessor class of C_r, and **(2)** any C_r-player has an incentive to deviate. This proves the nonexistence of a Class C_r where $1 < s^r < H$.

(d) Let $\mathscr{C} = \{C_1, C_2\}$. Then $s^2 \leq 1$ because of the existence of Lows, and $N_L \subseteq C_2$ since $C_1 \subseteq N_H$ by Lemma A.1(a). Hence, $c_2 \geq n_L > \phi$, $\tilde{c}_1 + c_2 > \phi$ and $\tilde{c}_1 + \tilde{c}_2 = \phi$, which is exactly Case ii) in Lemma A.1(b); therefore, $s^2 = 1$ and $N_H \subseteq C_1$. This conclusion together with the fact that $C_1 \subseteq N_H$ implies that $C_1 = N_H$, and consequently $C_2 = N_L$.

(e) Let $c_R < \phi$ and $s^R > 0$. Then Class C_R is in Group G, and each C_R-player gets $\left(w_i - s^R\right) + m \cdot S^G$, where S^G is the total contribution in Group G. If $i \in C_R$ reduces her contribution from s^R to 0, her payoff becomes $w_i + m \cdot \left(S^G - s^R\right) > \left(w_i - s^R\right) + m \cdot S^G$. Therefore, $s^R = 0$ in equilibrium when $c_R < \phi$.

Let $\tilde{c}_R = 0$ and $s^R > 0$. Consider any C_R-player. If she reduces her contribution from s^R to 0, she enters the Group G, but is still grouped with $(\phi - 1)$ players contributing s^R, so that her payoff increases by deviating this way. ■

Proof of Theorem 1:

By Lemma 1, $|\mathscr{C}| \geq 2$ in any equilibrium with positive contributions. Since $|\mathscr{C}| \leq n$ in any equilibrium, we can characterise the last class C_R, which can only take one of the following three forms:

(a). $c_R = D_R \cdot \phi + \tilde{c}_R$, where $D_R \geq 1$, and $\tilde{c}_R > 0$;
(b). $c_R < \phi$; or
(c). $c_R = D_R \cdot \phi$, where $D_R \geq 1$.

Also note that $s^R \leq 1$ in any equilibrium because of the existence of Lows. The proof will be given by the following four claims:

Claim 1. *If (a) holds, then the equilibrium candidate is a FEE.*

Let $c_R = D_R \cdot \phi + \tilde{c}_R > \phi$ with $\tilde{c}_R > 0$, and suppose that $s^R < 1$. Since $\tilde{c}_R > 0$ and $n = G\phi$, we have $c_R \neq n$. So there exists at least one class C_{R-1} before C_R satisfying $s^{R-1} > s^R$. In this case, each C_R-player has an incentive to increase her contribution so that she can be grouped with the C_{R-1}-players with certainty. In equilibrium it must be that each C_R-player cannot increase her contribution further, i.e., $s^R = 1$ and $C_R \cap N_H = \varnothing$. Therefore, C_1 is the immediate predecessor class of C_R by Lemma A.1(c), i.e., $|\mathscr{C}| = 2$. Lemma A.1(d) implies that this is a FEE.

Claim 2. *If (b) holds, then the equilibrium candidate is a NEE.*

Suppose that $c_R < \phi$ in equilibrium. In this case $|\mathscr{C}| \geq 3$ since $|\mathscr{C}| = 2$ implies that $c_R = n_L > \phi$ by Lemma A.1(d). Also note that $s^R = 0$ by Lemma A.1(e). Consider Class C_{R-1}. There are three cases to consider:

i). $c_{R-1} + c_R \leq \phi$; see Fig. A.2(i). This is impossible since C_{R-1} is in the last group and any C_{R-1}-player has an incentive to reduce her contribution for the same reason as in Lemma A.1(e).

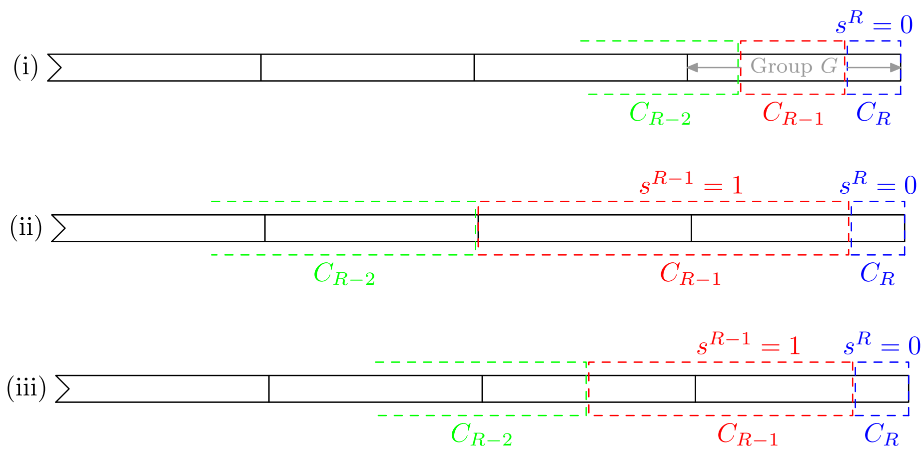


Fig. A.2. The last class C_R

ii). $c_{R-1} + c_R > \phi$ and $\tilde{c}_{R-1} + \tilde{c}_R = \phi$; see Fig. A.2(ii). With the following two steps we show that in this case $s_{R-1} = 1$:

Step 1. Suppose that $s^{R-1} > 1$. Then Lows cannot be in C_{R-1} or the classes, if any, before C_{R-1} since $s^1 > \cdots > s^{R-1} > 1$, which means that $n_L \leq c_R < \phi$. This contradicts Assumption 2 that $n_L > \phi$.

Step 2. Suppose that $s^{R-1} < 1$ and consider any player $i \in C_{R-1}$. If player i contributes $s_i = s^{R-1} < 1$, her expected payoff is

$$w_i - s^{R-1} + m\left[\left[1 - \Pr\left(G \,\middle|\, s^{R-1}\right)\right]\phi s^{R-1} + \Pr\left(G \,\middle|\, s^{R-1}\right)\tilde{c}_{R-1}s^{R-1}\right] < w_i - s^{R-1} + m\phi s^{R-1}$$

because $\tilde{c}_{R-1} < \phi$. But if she increases her contribution from s^{R-1} to $s^{R-1} + \varepsilon < \min\left\{1, s^{R-2}\right\}$, she enters the *first* group in class C_{R-1}, and gets

$$\left(w_i - s^{R-1} - \varepsilon\right) + m\left(\phi s^{R-1} + \varepsilon\right) = \left(w_i - s^{R-1}\right) + m\phi s^{R-1} - (1 - m)\,\varepsilon,$$

which is greater than her original payoff as long as ε is small enough.

The above two steps proved that $s^{R-1} = 1$ when $c_{R-1} + c_R > \phi$ and $\tilde{c}_{R-1} + \tilde{c}_R = \phi$. It follows from Lemma A.1(c) that C_1 is the immediate predecessor class of C_{R-1}, that is, $|\mathscr{C}| = 3$. The fact that $\bigcup_{r=1}^{3} C_r = N$ implies:

$$\begin{aligned} n &= c_1 + c_2 + c_3 \\ &= (D_1 + D_2 + D_3)\,\phi + (\tilde{c}_1 + \tilde{c}_2 + \tilde{c}_3) \\ &\overset{\langle 1\rangle}{=} (D_1 + D_2 + D_3)\,\phi + (\tilde{c}_1 + \phi) \\ &= (D_1 + D_2 + D_3 + 1)\,\phi + \tilde{c}_1, \end{aligned}$$

where $\langle 1\rangle$ holds because $\tilde{c}_2 + \tilde{c}_3 = \phi$. The above equation implies that n is not a multiple of the group size ϕ because $0 < \tilde{c}_1 < \phi$ from Lemma A.1(a). This contradicts the assumption at the beginning of Section 3.1 that $n = G\phi$, where $G \in \mathbb{N}$.

iii). $c_{R-1} + c_R > \phi$ and $\tilde{c}_{R-1} + \tilde{c}_R \neq \phi$; see Fig. A.2(iii). In this case, $s^{R-1} = 1$ and $C_{R-1} \subseteq N_L$, otherwise any C_{R-1}-player will increase her contribution so that she can be grouped with C_{R-2}-players and avoid entering the last group. Lemma A.1(c) implies that C_1 is the immediate predecessor class of C_{R-1}, i.e., $|\mathscr{C}| = 3$. We know the composition of the first two classes in terms of their members' endowments but we do not know for sure the composition of the third class, that we cannot exclude the possibility that $N_H \cap C_3 \neq \varnothing$ or that $N_L \cap C_3 \neq \varnothing$, so that $C_1 \subseteq N_H$ and $C_3 \subseteq N_H \cup N_L$.

Claim 3. *If (c) holds, then there is an equilibrium candidate, called E', which is not an equilibrium.*

Suppose that $c_R = D_R \cdot \phi$. We first verify that $|\mathscr{C}| \neq 2$: if $|\mathscr{C}| = 2$, then $c_1 = n - c_2 = (G - D_2)\,\phi$, which implies that $\tilde{c}_1 = 0$, and contradicts Lemma A.1(a).

We next show that $|\mathscr{C}| = 3$ if $c_R = D_R \cdot \phi$. Note that $|\mathscr{C}| \geq 3$ and $s^R = 0$ [Lemma A.1(e)] imply $\tilde{c}_{R-1} > 0$ and $c_{R-1} > \phi$, else any C_{R-1}-player has an incentive to reduce her contribution, which further implies that $s^{R-1} = 1$ and $C_{R-1} \subseteq N_L$ since each C_{R-1}-player wants to be grouped with C_{R-3}-players. Once again, Lemma A.1(c) implies that C_1 is the immediate predecessor class of C_{R-1}; thus, the equilibrium structure is as in Fig. A.3.

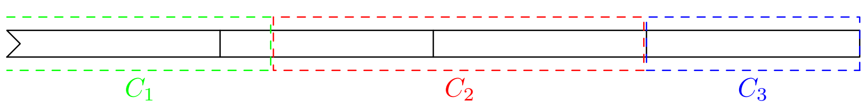


Fig. A.3. $\tilde{c}_R = 0$

We will prove in Claim 4 that E' is not an equilibrium, but for now, we content ourselves with proving that $C_3 \subseteq N_L$: Suppose there exists a player i such that $i \in C_3 \cap N_H$. It follows that her payoff is H. But if she deviates and contributes $1 + \varepsilon$, she enters group $(D_1 + 1)$, and since there exists at least one player contributing H in Group $(D_1 + 1)$ by Lemma A.1(a), player i can guarantee

$$(H - 1 - \varepsilon) + m\left[H + (\phi - 2) + (1 + \varepsilon)\right] > H + (m\phi - 1) - (1 - m)\,\varepsilon > H,$$

when $\varepsilon < \left(m\phi - 1\right) / (1 - m)$, where the first strict inequality holds because $H > 1$, and the second one *can* hold because $m\phi > 1$. This proves that $C_3 \subseteq N_L$. Because $\bigcup_{r=1}^{3} C_r = N_H \cup N_L = N$, $C_2 \subset N_L$, and $C_3 \subset N_L$, we thus have $C_1 = N_H$ and $C_2 \cup C_3 = N_L$.[14]

Claim 4. *E' is not an equilibrium.*

E' is an equilibrium if and only if:

- Player $i \in C_3 \subseteq N_L$ has no incentive to increase her contribution from 0 to 1;
- Player $i \in C_2 \subseteq N_L$ has no incentive to reduce her contribution from 1 to 0; and
- Player $i \in C_1 = N_H$ has no incentive to reduce her contribution from H to $1 + \varepsilon$, 1, or 0, where $\varepsilon \to 0$.

Here below we examine the incentives of all players starting with the last class, and will show that there exists *no* equilibrium satisfying all these constraints.

Recall from Claim 3 that if E' is an equilibrium, we must have **(a).** $c_3 = D_3 \cdot \phi$, and **(b).** $c_2 + c_3 = n_L$ since $C_2 \cup C_3 = N_L$. Let $b \equiv (B - D_3)\,\phi$. This allows us to write c_2 as follows:

$$c_2 = n_L - c_3 = \left(B\phi + \ell\right) - D_3 \cdot \phi = b + \ell.$$

☐ *Incentives to Deviate for C_3-Players in E'*

Consider any player $i \in C_3 \subseteq N_L$. Her payoff from contributing 0 is $U_0^L\,(C_3) = 1$. If i wants to deviate, she should contribute $s_i = 1$; then there would be $(c_2 + 1)$ players contributing 1, and i would enter Group $A + 1, \ldots, A + D_2 + 2$ with positive probabilities, which are:

$$\Pr\left(k \,|\, 1\right) = \begin{cases} \ell / \left(c_2 + 1\right), & \text{if } k = A + 1 \\ \phi / \left(c_2 + 1\right), & \text{if } k = A + 2, \ldots, A + D_2 + 1 \\ 1 / \left(c_2 + 1\right), & \text{if } k = A + D_2 + 2. \end{cases}$$

Because $\sum_{k=A+1}^{A+D_3+2} \Pr\left(k \,|\, 1\right) = 1$, we have

$$\begin{aligned} \sum_{k=A+2}^{A+D_3+1} \Pr\left(k \,|\, 1\right) &= 1 - \Pr\left(A + 1 \,|\, 1\right) - \Pr\left(A + D_3 + 2 \,|\, 1\right) \\ &= \frac{c_2 - \ell}{c_2 + 1} \\ &= \frac{b}{b + \ell + 1}. \end{aligned}$$

Recall that $S^{A+1} \equiv hH + \ell$, so that player i's expected payoff from contributing 1 is

$$\begin{aligned} U_1^L\,(C_3) &= (1 - 1) + m \left\{ \Pr\left(A + 1 \,|\, 1\right) \cdot S^{A+1} + \left[\sum_{k=A+2}^{A+D_3+1} \Pr\left(k \,|\, 1\right) \right] \cdot \phi + \Pr\left(A + D_3 + 2 \,|\, 1\right) \right\} \\ &= m \left(\frac{\ell}{c_2 + 1} S^{A+1} + \frac{c_2 - \ell}{c_2 + 1} \phi + \frac{1}{c_2 + 1} \right) \\ &= \left(\frac{m}{b + \ell + 1} \right) \left(\ell S^{A+1} + b\phi + 1 \right). \end{aligned}$$

[14] More precisely, $i \in N_H \Longrightarrow i \notin N_L \Longrightarrow i \notin C_2 \cup C_3 \Longrightarrow i \in C_1$, so that $N_H \subseteq C_1$. Combining this conclusion with the fact that $C_1 \subseteq N_H$ in Lemma A.1(b) results in $C_1 = N_H$.

Therefore, player $i \in C_3$ has no incentive to deviate if and only if $U_0^L(C_3) \geq U_1^L(C_3)$, that is

$$b \leq \frac{\ell + 1 - m\ell S^{A+1} - m}{m\phi - 1}. \tag{A.1}$$

The above equation shows that there cannot be too many players in Class C_2 (recall that $c_2 = b + \ell$), else some players in class C_3 will have an incentive to try to go to C_2.

□ *Incentives to Deviate for C_2-Players in E'*

Consider any player $i \in C_2 \subseteq N_L$. If i contributes 1, she enters Group $A+1, \ldots, A+D_2+1$ with positive probabilities, which are:

$$\Pr(k \mid 1) = \begin{cases} \ell / c_2, & \text{if } k = A+1 \\ \phi / c_2, & \text{if } k = A+2, \ldots, A+D_2+1. \end{cases}$$

Her expected payoff is

$$\begin{aligned} U_1^L(C_2) &= m\left(\frac{\ell}{c_2} S^{A+1} + \frac{c_2 - \ell}{c_2}\phi\right) \\ &= \left(\frac{m}{b+\ell}\right)\left(\ell S^{A+1} + b\phi\right). \end{aligned}$$

If $i \in C_2$ wants to deviate, she will contribute $\varepsilon \to 0$ in order to stay in Group $(A + D_2 + 1)$, and her expected payoff is

$$\begin{aligned} \lim_{\varepsilon \to 0} U_\varepsilon^L(C_2) &= \lim_{\varepsilon \to 0}\left[1 + m(\phi - 1) - (1-m)\varepsilon\right] \\ &= 1 + m(\phi - 1). \end{aligned}$$

Therefore, $i \in C_2$ has no incentive to deviate if and only if $U_1^L(C_2) \geq \lim_{\varepsilon \downarrow 0} U_\varepsilon^L(C_2)$, that is,

$$b \leq \frac{m\ell S^{A+1} - (1 + m\phi - m)\ell}{1 - m}. \tag{A.2}$$

The reason why b cannot be very large is as follows: Consider $i \in C_2$. If b is large, her probability of entering Group $(A+1)$ is small, and her expected payoff from contributing 1 is small, so that her incentive to deviate is large.

Note that by Claim 3, we also require $b \geq \phi$, otherwise $i \in C_2$ will reduce her contribution. Combining this requirement, (A.1), and (A.2), we observe that m has to satisfy the following conditions:

$$\frac{\phi + \ell}{\ell S^{A+1} - \ell\phi + \phi + \ell} \leq m \leq \frac{\phi + \ell + 1}{\ell S^{A+1} + \phi^2 + 1}. \tag{A.3}$$

The intuition behind (A.3) is as follows: m is the return from the group investment, so it cannot be very small because if it is very small C_2-players will have no incentive to contribute. At the same time, m cannot be very large because this would give C_3-players an incentive to contribute. These two constraints determine the bounds of m in (A.3).

For ease of expression, define

$$\frac{\phi + \ell}{\ell S^{A+1} - \ell\phi + \phi + \ell} \equiv \underline{m}, \quad \text{and} \quad \frac{\phi + \ell + 1}{\ell S^{A+1} + \phi^2 + 1} \equiv \overline{m}.$$

(A.3) implies that $\underline{m} \leq \overline{m}$; thus given all other parameters, S^{A+1} must satisfy

$$S^{A+1} \geq \frac{-\ell^2 - \ell\phi + \ell^2\phi - \phi^2 + 2\ell\phi^2 + \phi^3}{\ell}.$$

Substituting the above inequality to $\overline{m}$, we obtain

$$\overline{m} \leq \frac{1 + \ell + \phi}{1 - \ell^2 - \ell\phi + \ell^2\phi + 2\ell\phi^2 + \phi^3}. \tag{A.4}$$

☐ *Incentives to Deviate for C_1-Players in E'*

$C_1 = N_H$ in E'. We have shown in Section 3.3 that a C_1-player has no incentive to reduce her contribution from H to $1 + \varepsilon$ if and only if:

$$h \leq n_H \left(1 - \frac{M}{\ell}\right). \tag{6}$$

It can be seen that if (6) holds, then $1 - M/\ell > 0$, which means that

$$m > \frac{1}{\ell + 1}. \tag{A.5}$$

E' is not an equilibrium because (A.4) and (A.5) are incompatible: If E' is an equilibrium, m must satisfy $1/\left(\ell + 1\right) < m \leq \overline{m}$, so we must have $1/\left(\ell + 1\right) < \overline{m}$; however,

$$\overline{m} - \frac{1}{\ell + 1} \leq \frac{-\ell\left(\phi - 2\right) - \left(\phi^2 - \phi\right)}{\left(1 + \ell\right)\left[1 + \ell(\phi - 1) + \phi^2 - \phi\right]} < 0.$$

A contradiction.

Conclusion: Equilibrium candidate E' is not a equilibrium.

B. Experimental Instructions

This is an experiment in the economics of group decision-making. You have already earned $10.00 for showing up at the appointed time. If you follow the instructions closely and make decisions carefully, you will make a substantial amount of money in addition to your show-up fee.

Number of periods and endowments

There will be many decision-making periods. In each period, you are given an endowment of experimental tokens. You receive the same endowment in each round of the experiment. By a random process, half of the participants receive 80 tokens per round, and half receive 120 tokens per round.

The decision task

In each period, you need to decide how to divide your tokens between two accounts: a private account and a group (public) account. The latter account is joint among all members of the group that you are assigned to in that period. See below for the group assignment process and for how earnings from your accounts are calculated.

How earnings from your two different accounts are calculated in each period

- Each token you place in the private account stays there for you to keep.
- All tokens that group members invest in the **group (public) account** are added together to form the so-called "group investment". The group investment gets doubled before it is equally divided among all group members. Your group has 4 members (this includes yourself).

A numerical example of the earnings calculation in any given period

Assume that your endowment per period is 80 tokens. In a given period, you decide to put 30 tokens into your private account and 50 tokens into the group (public) account. The other three members of your group together contribute an additional 300 tokens to the group (public) account. This makes the total group investment 350 tokens, which gets doubled to 700 tokens ($350 \times 2 = 700$). The 700 tokens are then split equally among all four group members. Therefore, each group member earns 175 tokens from the group investment ($700/4 = 175$). In addition to the earnings from the group (public) account, each group member earns 1 token for every token invested in his/her private account. Since you put 30 tokens into your private account, your total profit in this period is $175 + 30 = 205$ tokens.

How each decision-making period unfolds and how you are assigned to a new group in each of the periods

First, you make your investment decision. Decide on the number of tokens to place in the private and in the group (public) account, respectively. To make a private account investment, use the mouse to move your cursor to the box labeled "Private Account". Click on the box and enter the number of tokens you wish to allocate to this account. Do likewise for the box labeled "Public Account" Entries in the two boxes must sum up to your endowment. To submit your investment click on the "Submit" button. Then wait until everyone else has submitted his/her investment decision.

Second, you are assigned to the group that you will be a member of in this period. Once every participant has submitted his or her investment decision, you will be assigned to a group with 4 members (including yourself). The group assignment proceeds in the following manner: All participants' contributions to the group (public) account are ordered from the highest contribution to the lowest contribution. Participants are then grouped based on this ranking:

- The four highest contributors are grouped together(for example, if four of the participants all contributed 120 tokens they are all put together into one group).
- Participants whose contributions rank from 5-8 form the second group.
- The four lowest contributors form the third group.

As said, you will be grouped based on your public account investment. If there are ties for group membership because contributions are equal, a random draw decides which of these equal-contributors are put together into one group and who goes into the next group below. For example, if 5 participants each contributed 120 tokens, a random draw determines which four participants form a group of like-contributors and who is the one participant who goes into the next group below.

Recall that group membership is determined anew in each period based on your public contribution in that period. group membership does not carry over between periods!

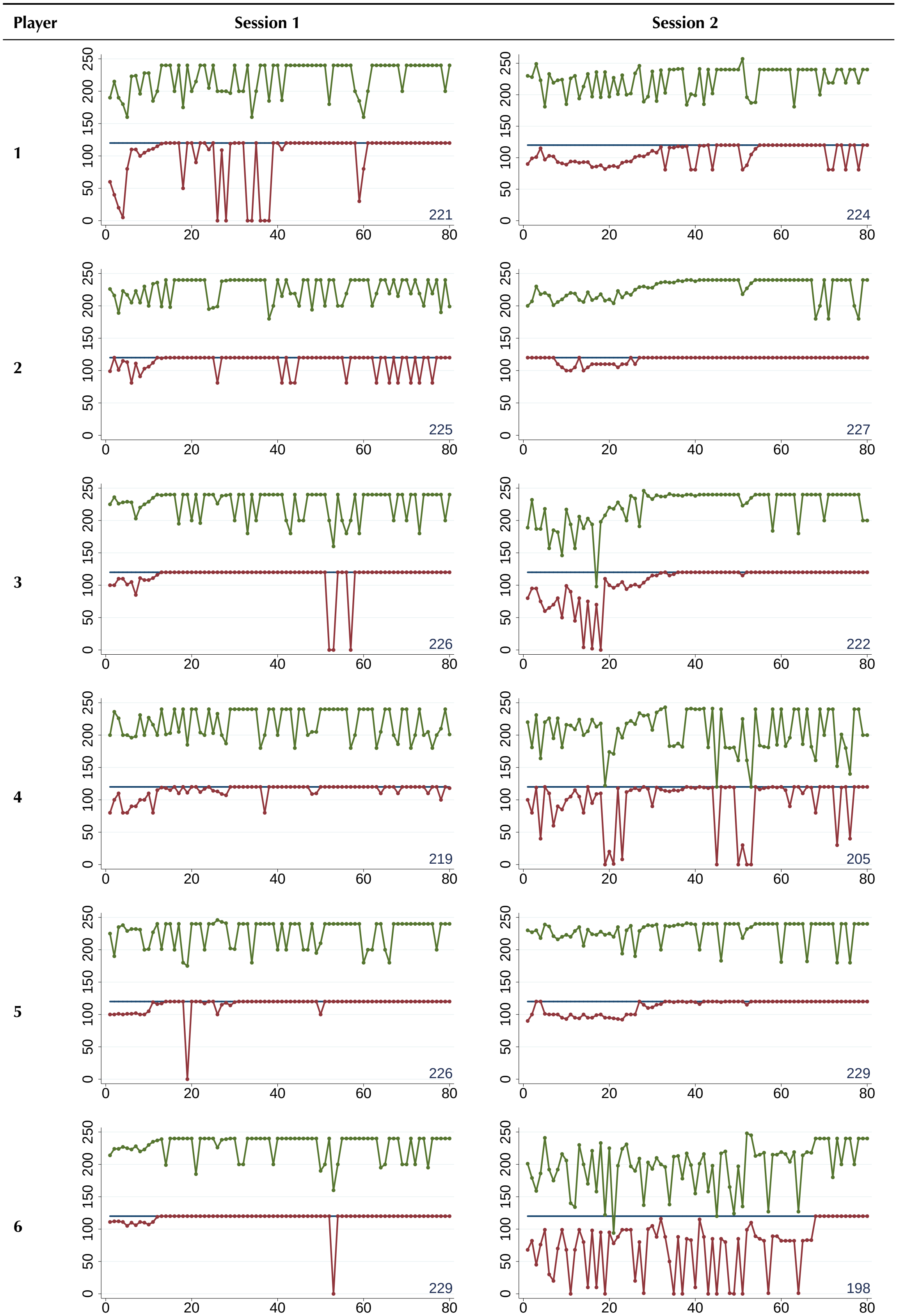
After the group assignment, your earnings for the round are computed. Experimental earnings from a given round are computed after you have been assigned to your group. See the numerical example above for details of how earnings are computed after you have been assigned to a group.

End-of period message. At the end of each period you will receive a message with your total experimental earnings for the period (total earnings = the earnings from the group (public) and from your private account added together). This information also appears in your Record Sheet at the bottom of the screen. The Record Sheet will also show the group (public) account contributions of all participants in the experiment in a given round in ascending order. Your contribution will be highlighted.

A new period begins after everyone has acknowledged his or her earnings message.

At the end of the experiment your total token earnings will be converted into US$ at a rate of 700 **tokens for** 1 **US$.**

C. Individual Graphs with Earnings


Player
Session 1
Session 2
1
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6
221
224
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227
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222
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205
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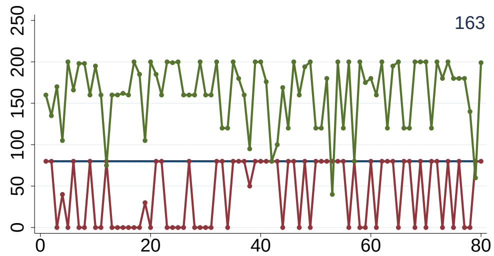
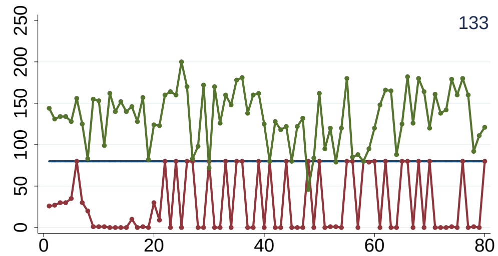
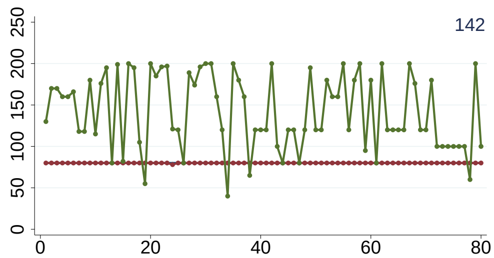
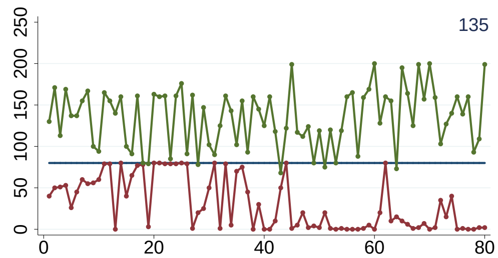
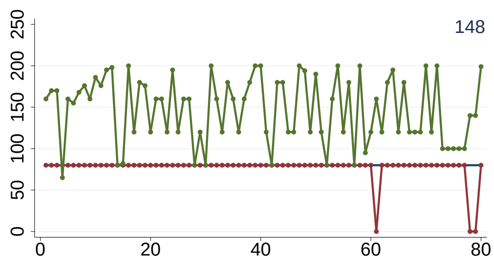
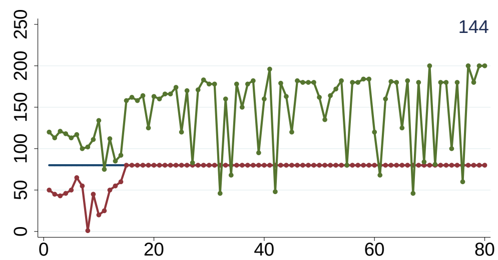
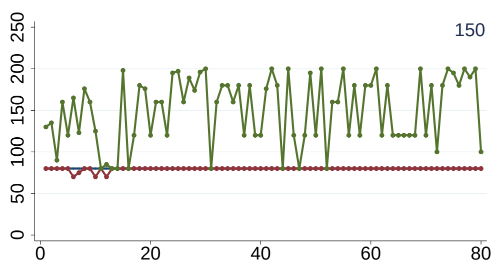
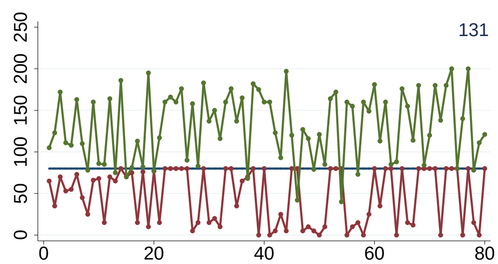
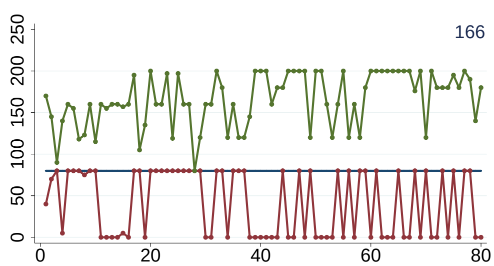
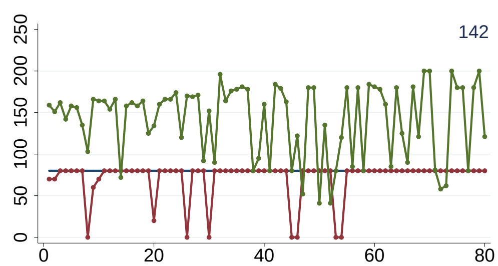
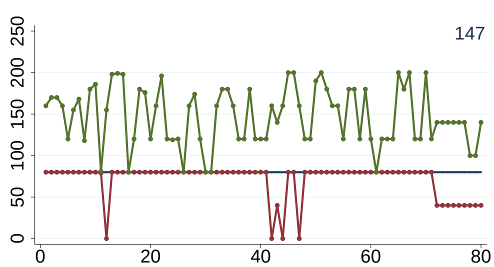
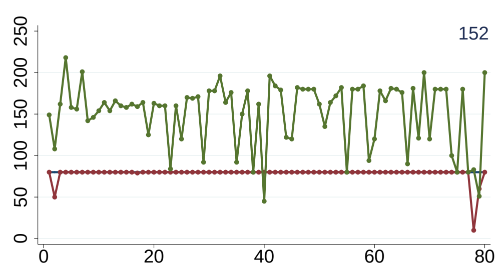
198
0 50 100 150 200 250
0 20 40 60 80

896

Player	Session 1	Session 2
7		
8		
897 **9**		
10		
11		
12		

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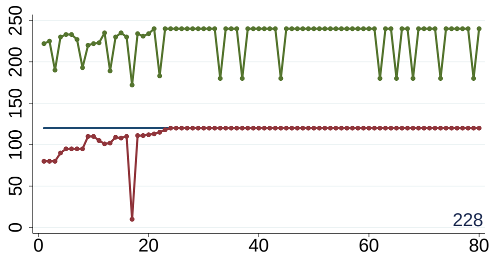
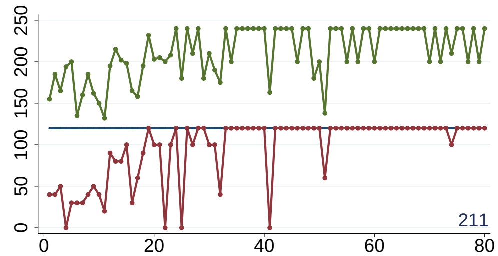
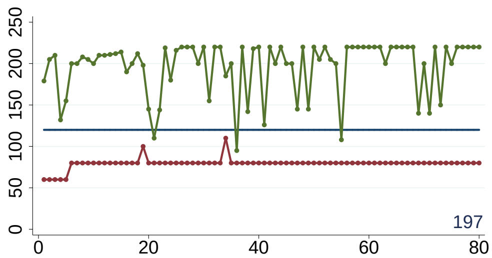
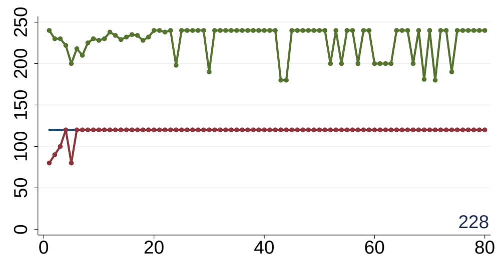
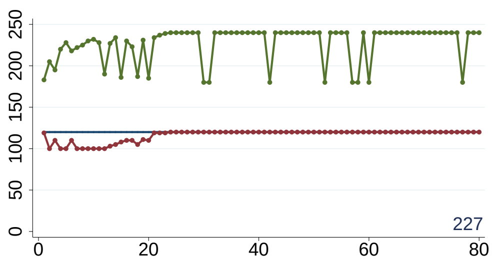
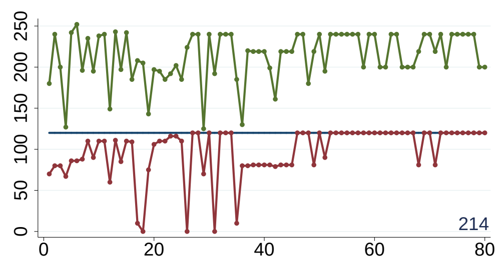
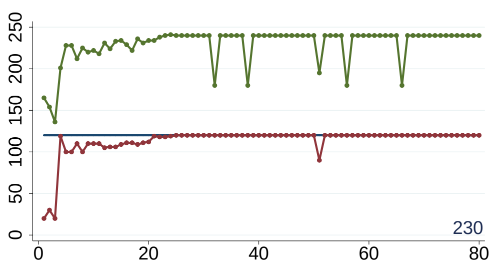
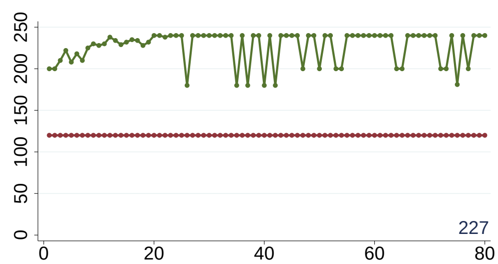
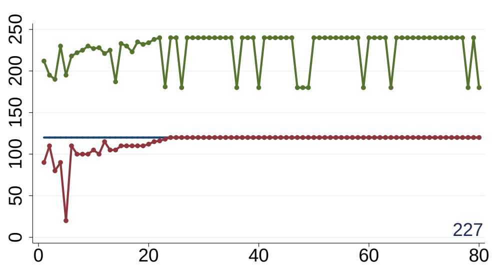
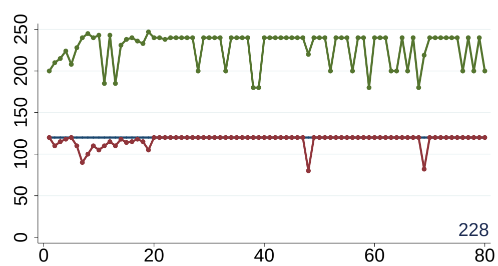
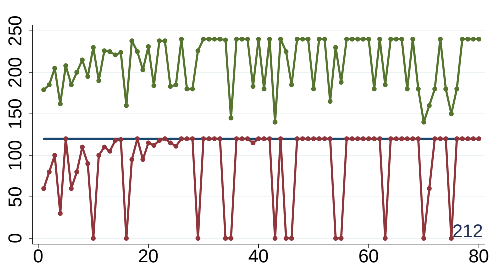
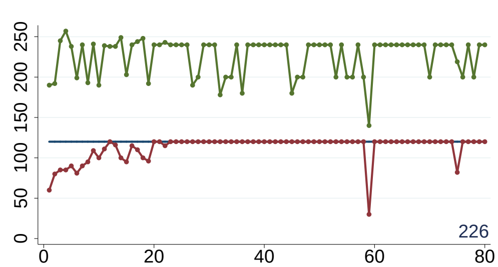
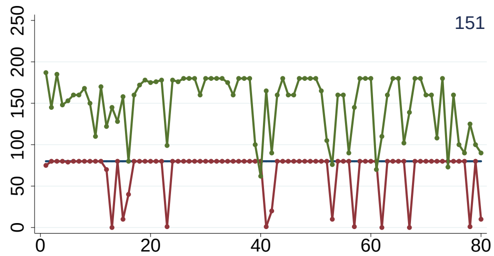
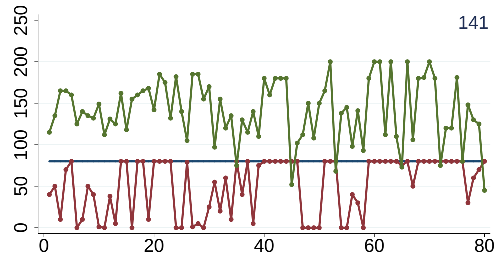
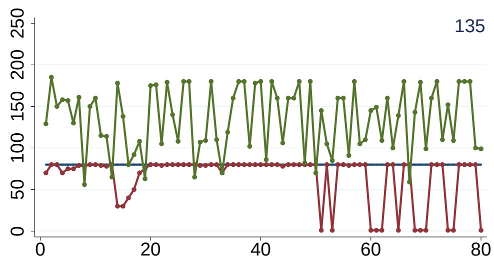
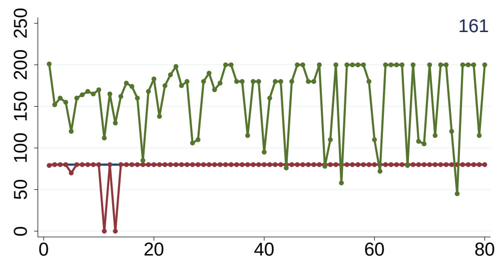
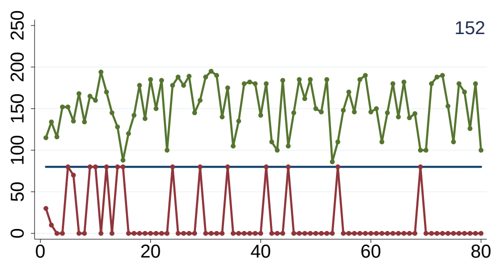
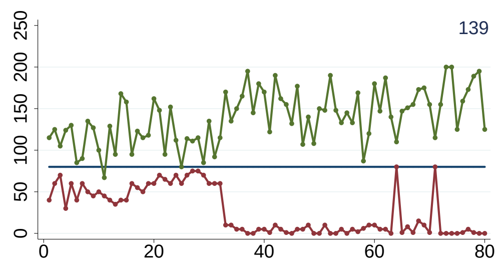
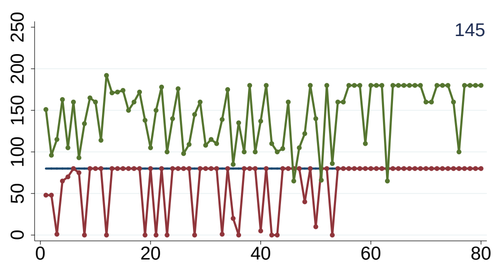
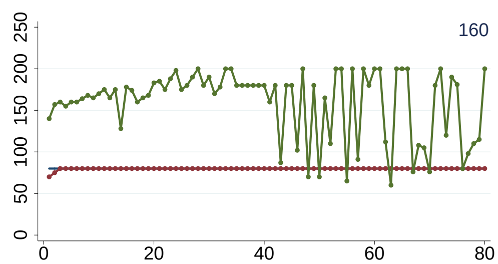
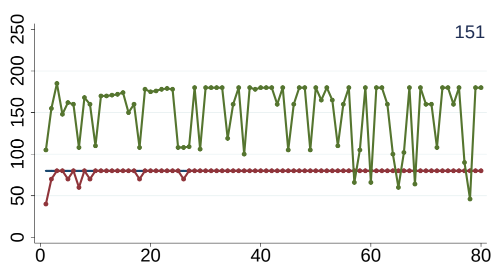
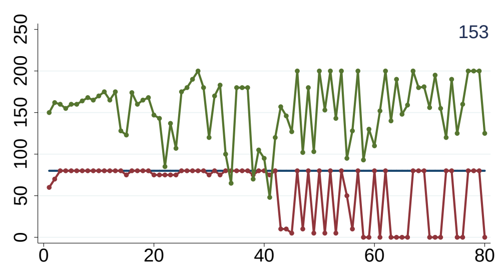
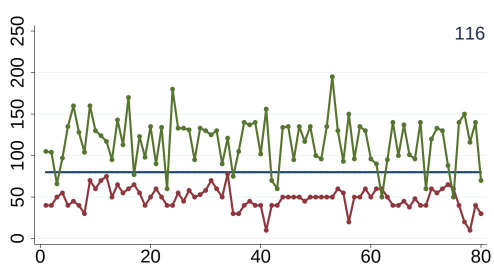
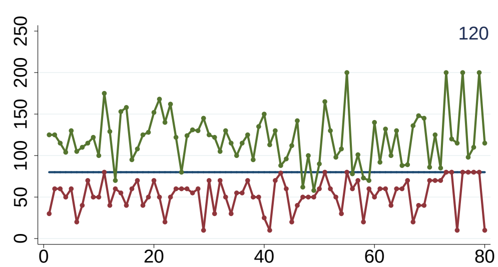
1		
2		
3		
4		
5		
6		

Player	Session 3	Session 4
7		
8		
9		
10		
11		
12		

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