

# Sequential mergers under product differentiation in a partially privatized market \*

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## Abstract

This paper studies the condition for the emergence of sequential mergers in a partially privatized oligopoly with differentiated goods. The purpose of this study is to examine (i) the optimal merger strategies by potential merging firms and (ii) the optimal merger policy and (iii) privatization policy for the policymaker. First, under the subgame perfect Nash equilibrium, sequential mergers either completely emerge or do not emerge at all. The parameter range that leads to complete sequential mergers becomes larger as the market is more privatized. Second, the policymaker is better off to at least partially privatize the public firm, unless the goods are perfect substitutes or independents. Third, the policymaker can halt privatization, diminishing the private incentive for further sequential mergers, leading to a higher level of welfare. Furthermore, given that some mergers have already taken place, there may be a case such that further mergers would actually improve welfare. These welfare-improving mergers may not be privately profitable, implying there may be room for merger-friendly policies. Our results are applicable to Japanese life insurance industry and the partial privatization of Japan Post Insurance.

**JEL classification:** G34; H42; L13; L41

**Keywords:** Sequential mergers, Privatization, Merger policy, Welfare improving merger.

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# 1 Introduction

Sequential mergers, in which the merging of one set of firms leads to more mergers in the same industry, have become more prominent in recent years. Such examples are rampant in many industries, including both goods producers and service providers. In particular, the pharmaceutical industry is filled with news of mergers and acquisitions, reaching \$126 billion in the first quarter of 2015, with players such as Shire, Valeant, AbbVie, Mylan, Perrigo, and Teva making purchases or purchase offers exceeding \$5 billion. Today, when firms propose mergers, the prospective proposers must examine not only with whom to merge but also how the market structure would materialize after other firms in the industry may choose to follow with more mergers. Consequently, merger strategies have become much more complicated than in the past.

This recent movement has also affected decision making by antitrust authorities. Regarding most issues in competition policy such as cartels, tie-in sales, and resale price maintenances, the antitrust authorities make judgments based on current performance and past evidences. Merger cases, on the other hand, require serious consideration of the future market structure forecasting. The antitrust authorities must make judgments not only reflecting the validity of current merger proposals but also attempting to consider their effect on future forecasts. Their decisions must also be prudent, as it is difficult to nullify mergers once they are approved. These two issues make merger decisions very difficult compared to other antitrust cases. Thus, understanding the characteristics behind sequential mergers is crucial from the perspective of competition policy. However, even though there is a rich variety of papers on mergers in industrial organization and marketing, the number of papers on sequential mergers is very small.

Since sequential mergers are rampant, there are industries in which sequential mergers have occurred while the public firm is partially privatized. One recent example is the life insurance industry in Japan. In the industry that have not seen mergers since Meiji Life Insurance and Yasuda Life Insurance merging in 2004, the biggest news in 2015 was the listing of Japan Post Insurance (JPI) on the Tokyo Stock Exchange in November. Japanese life insurance industry has long been

led by about ten private firms, in which Nippon Life Insurance led, Dai-ichi Life Insurance closely followed, and Sumitomo Life Insurance and Meiji-Yasuda Life Insurance rounded out the top four. JPI was a government-owned firm created in 2006 from the aftermath of Postal Reform by the Koizumi Administration.<sup>1</sup> At its creation, JPI was the largest insurer in the world in terms of total assets, passing Nippon Life. In April, 2014, the Japanese government started discussing how JPI along with its parent Japan Post Holdings be listed on the Tokyo Stock Exchange (It was officially announced in December that this would take place the following November). This shocked the industry and became one of the reasons for the following sequential M&As to occur: Dai-ichi acquiring U.S.-based Protective Life Corporation in June, 2014; Meiji-Yasuda Life acquired the U.S.-based StanCorp Financial Group in July, 2015; Sumitomo Life acquired the U.S.-based Symetra Financial Corporation in August, 2015; and Nippon Life acquired Mitsui Life Insurance (one of the aforementioned top ten insurers) in September, 2015 and 80% stake in the life insurance business of MLC Limited, a subsidiary of National Australia Bank in November, 2015. Thus it is of interest to see how the existence of a public firm and its privatization would affect the private firms' incentive to sequentially merge.<sup>2</sup>

Thus in this paper, we consider sequential mergers in an industry with a public firm.<sup>3</sup> Although the above example shows that there is a huge market where a public firm and its privatization policy would affect the private firms' sequential merger incentives, there has been no previous research on sequential merger in an industry with a public firm. We examine (i) the optimal merger strategies by potential merging firm pairs and (ii) the optimal merger policy and (iii) the privatization policy

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<sup>1</sup>Before the creation of JPI, Postal Services had sold "simple insurance" called Kampo (which is the name of current JPI in Japanese), which required fewer restriction for the insured such as no requirements for doctor's examination or occupational limitations, but only provided coverage up to 7 to 13 million yen depending on the age of the insured. This amount is about a half or a third of the average coverage for Japanese male between 20 and 55.

<sup>2</sup>There are many other examples of sequential mergers in the pharmaceuticals, steel, banking, and automobile industries. As some of these mergers are international in nature, we must proceed with care while suggesting policy implications using the results of our model. We discuss this issue in more detail in the concluding section.

<sup>3</sup>Public and private firms compete in the markets of most countries other than the United States. According to Ishida and Matsushima (2009), among the developed countries, a mixed market in, for example, the packaging and overnight delivery industries, is prevalent in the EU, Canada, and Japan. Matsumura and Kanda (2005) also noted the presence of many industries in a mixed oligopoly, such as the airline, telecommunications, natural gas, electricity, steel, banking, home loans, health care, life insurance, hospitals, broadcasting, and education industries.

for the policymaker. The important aspect the firms must consider in such a market is to make merger decisions while forecasting whether or not the other firms would merge, given the level of partial privatization set by the government. Meanwhile, the antitrust authority must consider how its decision on privatization would affect the postmerger market structure and the resulting welfare level. Thus the purpose of our paper is to consider the private merger incentives ((i)) and the two policies for the policymaker under dynamic merger formation ((ii) and (iii)).

To accomplish our three purposes, we constructed a model that can tackle these points. Given parameter values such as the number of insider (potentially merging) firm pairs, number of already-merged firm pairs, number of outsider (non-merging) firms, degree of product differentiation<sup>4</sup>, and the level of partial privatization, we derive the condition in which sequential mergers do or do not emerge. Here, we also conduct comparative statics on the latter two parameters and examine welfare. Furthermore, we endogenize the level of partial privatization and show a possible relation between it and the merger incentive of firms affecting the policymaker's decision on setting this level.

Our results are as follows. First, with respect to (i), the subgame perfect Nash equilibrium reveals either of the two eventualities: further sequential mergers or no further mergers. This depends on the parameter values (given in the preceding paragraph), and one equation serves as a threshold determining whether or not further mergers occur. This result is in line with that of Ebina and Shimizu (2009, 2016) in a four firm competition under product differentiation and Salvo (2010) considering cross-border sequential mergers. Our contribution here is that we introduce the degree of partial privatization and that this result holds for any value describing the privatization level. In addition, we also show that the firms are more eager to merge and sequential mergers are more likely to occur, as the number of firm pairs that have an opportunity to merge and the number of firms that does not participate in mergers for some external reason increase.

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<sup>4</sup>In order to shy away from fierce price competition and to maximize profit, firms often decide to differentiate their goods from the competition. For example, in the pharmaceuticals and gaming industries, the areas of expertise often characterize the firm, and how they choose such expertise can be considered as a way of differentiating their goods. In the retail and service industries, location of stores can be interpreted as product space, which can also be viewed as product differentiation. As the differentiation between goods and services in most markets increases, it is very important to consider the effects of product differentiation when making policy implications.

Second, with respect to (ii), from the policymaker's perspective, selecting no mergers compared to complete sequential mergers would result in a higher level of welfare. However, if some mergers have already taken place, there may be a case such that further mergers would actually improve welfare. In fact, even when the firms have no incentive to merge further, further mergers could actually improve welfare. We believe that our paper is the first to theoretically derive these somewhat counter-intuitive results.

Third, with respect to (iii), we find that the set of parameters in which sequential mergers occur becomes larger as the partial privatization further proceeds. In other words, sequential mergers are more likely to occur the more privatized the public firm is. Combined with the usual welfare-deteriorating effect of the mergers, we find that privatization may worsen welfare owing to the consideration of future sequential mergers.

Furthermore, if the policymaker can choose the level of privatization, it is better off at least partially to privatize, unless the goods are perfect substitutes or independents. Privatization should proceed further as the number of the currently merged firm pairs, potential pairs, and the outsider firms increase. One example we derived regarding the relationship between privatization and sequential mergers is that by stopping privatization the policymaker can also stop the firms from further sequential mergers, leading to a higher level of welfare.

The results here are applicable to the aforementioned life insurance market in Japan now in Section 7.2. The degree of privatization of JPI is in the range of our model's suggestion. Also, this privatization of JPI may have been a trigger that started further sequential mergers. Here, it may be that due to the characteristic of the market such as the potentially shrinking customer base, the policymaker should encourage further mergers. Our result implies that further privatization may take place in the future. We explain this in more detail using our results in Subsection 7.2.

The remainder of this paper proceeds as follows. Section 2 introduces the related literature. Section 3 sets up our model. Section 4 derive the equilibrium of the model and the condition affecting the sequential merger decision. Comparative statics and welfare analysis are also conducted. Section

5 endogenizes the level of partial privatization by creating a game with this decision added in stage 0, making the analysis up to here a subgame. Section 6 provides numerical analyses supporting our theoretical results. Section 7 provides discussions on cost reduction and our model's implication on the current life insurance industry in Japan. Concluding remarks follow.

## 2 Related Literature

Merrill and Schneider (1966), De Fraja and Delbono (1989, 1990), and Fershtman (1990) are pioneering works on the mixed market. De Fraja and Delbono (1989) showed that the social welfare level may be higher when the public firm maximizes profit rather than welfare. This result implies that there are cases where it may be better for society as a whole if the public firm becomes privatized and maximizes its own profit.<sup>5</sup>

Several scholars have studied mixed oligopoly incorporating product differentiation. Most of the early works, such as Cremer *et al.* (1991), Matsumura and Matsushima (2003, 2004), and Matsushima and Matsumura (2003, 2006), have been based on Hotelling-type spatial competition.<sup>6</sup> Bárcena-Ruiz and Garzón (2003) and Fujiwara (2007) adopted the differentiated demand function of Dixit (1979) and Singh and Vives (1984) (which we also adopt in this paper) to analyze mixed markets.<sup>7</sup>

Researchers have focused on mergers that are not sequential in terms of how firms would choose to merge under mixed oligopoly. Bárcena-Ruiz and Garzón (2003) considered a situation with one public firm and one private firm, and by merging, they created a multi-product monopoly where the government possesses an exogenous share level, similar to Matsumura (1998). They showed that the occurrence of mergers depends on the degree of product differentiation and the aforementioned exogenous share level.

Mendez-Naya (2008) and Artz *et al.* (2009) extended this setting to a competition between a

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<sup>5</sup>Matsumura (1998) and Bennett and Maw (2003) discussed partial privatization, in which a public firm maximizes a mixture of profit and social welfare as its objective.

<sup>6</sup>Anderson *et al.* (1997) employed free entry under the CES representative consumer setting and examined the effect of a public firm being privatized.

<sup>7</sup>Haraguchi and Matsumura (2015) present more recent research on mixed oligopoly with a differentiated demand function.

public firm and  $n$  private firms competing in Cournot fashion, albeit with no product differentiation. In the pure oligopoly, it is well known that a merger paradox exists, in that a two-firm merger with no cost synergy never occurs unless there are no outsiders (non-merging firms), that is, a duopoly becomes a monopoly.<sup>8</sup> Mendez-Naya (2008) and Artz *et al.* (2009) showed that a merger other than a duopoly-to-monopoly case can occur under mixed oligopoly. In particular, the sustainability of a merger is dependent on both the degree of privatization of the merged firm and the number of outsider firms. Fujiwara (2010) allowed mergers between two or more firms and obtained similar results.

Mendez-Naya (2012) examined a mixed oligopoly market with one public firm and two private firms. Here, mergers between public-private firms and two private firms are compared. In addition, by introducing a timing game, the public-private firm merger leads to a Stackelberg sequential result, whereas the mergers of the private firms (with the public firm outside) lead to a Cournot simultaneous result.

While all these earlier works considered mergers, none of them are sequential. Also, only Bárcena-Ruiz and Garzón (2003) considered product differentiation. As sequential mergers are becoming much more prevalent in the real world, decision makers on merger policies must consider how the market structure would change in the future and the importance of mixed markets in many countries. Thus, we must consider sequential mergers in mixed oligopoly markets. In fact, the banking sector and non-life insurance industry mentioned above are both examples of sequential mergers in mixed oligopoly under product differentiation, although the degree of differentiation in the banking sector is low. The literature on sequential mergers in pure oligopoly has been growing recently.<sup>9</sup> However, as far as we can find, no study has focused on sequential mergers under mixed oligopoly. It is also very important to consider product differentiation when determining a merger policy wherein the goods are indeed differentiated, and the policymaker must determine the market scope of the goods considered. In

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<sup>8</sup>See Salant *et al.* (1983).

<sup>9</sup>Nilssen and Sørgaard's (1998) paper is a seminal work on sequential mergers. Nocke and Whinston (2010) introduced analysis of sequential mergers from the welfare perspective. Ebina and Shimizu (2009, 2016) and Salvo (2010) considered product differentiation in the setting of sequential mergers.

sum, from the policymakers' viewpoint, it is vital to undertake an analysis of sequential mergers under mixed oligopoly with product differentiation.

### 3 Model

Assume an economy with a competitive sector producing a numeraire good and an oligopolistic sector with  $2n + k + 1$  firms, each producing differentiated goods. Let  $F$  be the set of firms, with  $F = \{0, 1, \dots, 2n, 2n + 1, \dots, 2n + k\}$ . Firm 0 is considered to be a public firm, which maximizes the convex combination of its own profit and social welfare. We will discuss firm 0's objectives later in the model. All other firms are private firms, maximizing their own profit. One caveat is that when a merger occurs, both firms engaging in the merger maximize their joint profit level. Let  $n$  represent the number of firm pairs contemplating their own pairwise mergers and outsider firms, and  $k$  be the number of outsider firms that do not participate in mergers owing to some exogenously given reasons. Let us assume that  $k$  is a nonnegative integer and  $n$  is an integer greater than or equal to 2, so that the analysis on sequential mergers is meaningful. Let  $N$  be the set of these  $n$  firms, that is,  $N = \{1, \dots, 2n\} \subset F$ .

From this setting, let the number of firm pairs currently merged be denoted as  $m \leq n$ . Finally, we assume that when firms merge, firm  $h$  merges with firm  $h + 1$ , where  $h$  is an odd number. Thus, for example, when  $m = 2$ , firms 1 and 2 have merged, as have firms 3 and 4.

Consumers maximize  $U(q_0, q_1, \dots, q_{2n+k}) - \sum_{i \in F} p_i q_i$ , where  $q_i$  is firm  $i$ 's output, and  $p_i$ , its price. We let gross utility  $U$  take the form

$$U(\bar{q}) = \hat{q} + a \sum_{i \in F} q_i - \frac{1}{2} \sum_{i \in F} q_i^2 - \beta \sum_{i \in F} \sum_{j > i} q_i q_j,$$

where  $a > 0$ , and  $\bar{q} = (q_0, q_1, \dots, q_{2n+k})$  and  $\hat{q}$  are the vector of quantity and the quantity of the numeraire good, respectively. Since this is a "quasi-linear" utility, we can proceed with the partial equilibrium analysis.  $\beta \in (0, 1]$  is a parameter describing the degree of product differentiation among the goods produced by the firms.  $\beta = 1$  implies the firms are perfect substitutes, whereas  $\beta = 0$  implies the goods are independent.

Next, we describe the timing structure of the game. At the first stage, the first pair of firms in  $N$  jointly decides whether to merge. At stage 2, the second pair of firms in  $N$  jointly decides whether to merge. This continues until stage  $n$ , when the final pair makes the merger decision. Note that all the merger proposals are one-time offer and cannot be renegotiated.<sup>10</sup> At stage  $n + 1$ , differentiated Cournot competition ensues. We assume that mergers with three or four firms do not occur.<sup>11</sup>

Following the above set up, we derive the inverse demand function. From the consumer utility maximization problem, the system of inverse demand functions for each good is given by the following equations.

$$p_i = a - q_i - \beta \sum_{j \neq i} q_j \quad i \in F.$$

Then, the profit of firm  $i \in F$  is

$$\begin{aligned} \pi_i(\bar{q}) &= (p_i - c)q_i, \\ &= \left( a - q_i - \beta \sum_{j \neq i} q_j - c \right) q_i, \end{aligned}$$

where  $c$  denotes a marginal cost common to all firms. The social welfare is

$$\begin{aligned} W(\bar{q}) &= U(\bar{q}) - \sum_{i \in F} p_i q_i + \sum_{i \in F} \pi_i(\bar{q}), \\ &= U(\bar{q}) - c \sum_{i \in F} q_i, \end{aligned} \tag{1}$$

whereas consumer surplus is  $CS = U(\bar{q}) - \sum_{i \in F} p_i q_i$ .

Finally, firm 0, the public firm, maximizes the convex combination of its own profit and social welfare, following Matsumura (1998)'s formulation. The expression to be maximized is  $V = \theta W + (1 - \theta)\pi_0$ ,  $\theta \in [0, 1]$ . Thus this can be considered a partial privatization model, with the pure oligopoly competition  $\theta = 0$  and fully mixed oligopoly competition  $\theta = 1$  as special cases. Using these equations, we solve for the overall subgame perfect equilibrium in the next section.

<sup>10</sup>The main results in this paper are not affected by this assumption. This is set to keep the analysis simple.

<sup>11</sup>One reason for this assumption may be barriers from antitrust authorities. Scherer and Ross (1990) observed that most mergers in the United States since World War II have been two-firm combinations.

## 4 Equilibrium analysis and welfare analysis

### 4.1 Equilibrium analysis

In this section, we derive the equilibrium of the game described in the previous section. Consider the situation in which  $m \geq 0$  pairs of firms have already merged. Let  $N^m \subset N$  be the set of already merged firms, that is,  $N^m = \{1, 2, \dots, 2m - 1, 2m\}$ . Since the goods are differentiated, both merged firms still produce a positive amount. However, the firms in set  $N^m$  maximize the joint profit of the two firms that have merged. Note that superscript  $m$  denotes the case where  $m$  pairs of firms have already merged. We later compare the profit levels at different levels of  $m$ . Thus, we have the following equations that the firms maximize:

$$\begin{aligned} \max_{q_0} V(\bar{q}) &= \theta W(\bar{q}) + (1 - \theta)\pi_0(\bar{q}), \\ \max_{q_i} \Pi_i(\bar{q}) &= \pi_i(\bar{q}) + \pi_{i+1}(\bar{q}), \quad i \in N^m, \quad \text{if } i \text{ is odd} \\ \max_{q_i} \Pi_i(\bar{q}) &= \pi_{i-1}(\bar{q}) + \pi_i(\bar{q}), \quad i \in N^m, \quad \text{if } i \text{ is even} \\ \max_{q_j} \Pi_j(\bar{q}) &= \pi_j(\bar{q}), \quad j \in F \setminus (N^m \cup \{0\}). \end{aligned}$$

The set  $F \setminus (N^m \cup \{0\})$  is the set of private firms that have not (yet) merged. Since merged and non-merged firms are respectively symmetric, the solution we obtain is respectively symmetric. Focusing on firms 0,  $i$  (odd-numbered merged firm) and  $j$  (not yet merged firm), we have

$$\begin{aligned} \frac{\partial V}{\partial q_0} &= a - (2 - \theta)q_0 - \beta \sum_{l \in F, l \neq 0} q_l - c = 0, \\ \frac{\partial \Pi_i}{\partial q_i} &= a - 2q_i - \beta \sum_{l \in F, l \neq i} q_l - c - \beta q_{i+1} = 0, \quad i \in N^m, \\ \frac{\partial \pi_j}{\partial q_j} &= a - 2q_j - \beta \sum_{l \in F, l \neq j} q_l - c = 0, \quad j \in F \setminus (N^m \cup \{0\}). \end{aligned} \tag{2}$$

The equilibrium quantity and price are

$$q_0^m = \frac{(2 - \beta)(a - c)}{(2 - \beta - \theta)(2 + \beta(2n + k) - \beta^2 m) + \beta\theta}, \quad (3)$$

$$q_i^m = \frac{(2 - \beta)(2 - \beta - \theta)(a - c)}{2[(2 - \beta - \theta)(2 + \beta(2n + k) - \beta^2 m) + \beta\theta]}, \quad i \in N^m, \quad (4)$$

$$q_j^m = \frac{(2 - \beta - \theta)(a - c)}{(2 - \beta - \theta)(2 + \beta(2n + k) - \beta^2 m) + \beta\theta}, \quad j \in F \setminus (N^m \cup \{0\}). \quad (5)$$

It is easy to see that  $q_i^m \leq q_j^m \leq q_0^m$ . The equilibrium profits are

$$\begin{aligned} \pi_0^m &= \frac{(2 - \beta)^2(1 - \theta)(a - c)^2}{[(2 - \beta - \theta)(2 + \beta(2n + k) - \beta^2 m) + \beta\theta]^2}, \\ \pi_i^m &= \frac{(2 - \beta)^2(2 - \beta - \theta)^2(1 + \beta)(a - c)^2}{4[(2 - \beta - \theta)(2 + \beta(2n + k) - \beta^2 m) + \beta\theta]^2}, \quad i \in N^m, \\ \pi_j^m &= \frac{(2 - \beta - \theta)^2(a - c)^2}{[(2 - \beta - \theta)(2 + \beta(2n + k) - \beta^2 m) + \beta\theta]^2}, \quad j \in F \setminus (N^m \cup \{0\}). \end{aligned}$$

Note the superscript  $m$  on  $q$  and  $\pi$  also signify the number of merged firm pairs  $m$  in the current setting. Also, the subscript  $i$  is used when we discuss the merged firms, and  $j$  is used when we discuss the outsider firms.

Now, we present the two important properties that are satisfied under Cournot competition with substitute goods. The first concerns the effect of the number of merged pairs on the profit levels.

**Lemma 1** (i)  $\pi_i^0 \leq \pi_i^1 \leq \dots \leq \pi_i^m$ ,  $i \in N^m$ . (ii)  $\pi_j^0 \leq \pi_j^1 \leq \dots \leq \pi_j^m$ ,  $j \in F \setminus (N^m \cup \{0\})$

Lemma 1 (ii) is similar to property (1) in Salvo (2010), in that the merger of a pair of firms has an externality that is positive and increasing on the profit of outside independent firms. In addition, we show that the firms that have already merged also receive this positive externality in Lemma 1 (i). A merger causes the two firms to internalize their profit-maximizing decisions on the output levels. If the merger creates little or no cost synergies, the output decreases, benefiting the outside firms. This effect also applies to the already merged firms.

We can easily check that these properties hold. An increase in  $m$  affects  $\pi_i^m$  and  $\pi_j^m$  only through the term  $-\beta^2 m$  in the denominators; thus, they are both increasing in  $m$ .

The second property concerns the successive profitability of the merger. We show that given a profitable merger at  $m = s$ , the potentially successive merger at  $m = s + 1$  is also profitable.

**Lemma 2** *If  $\pi_i^{s+1} \geq \pi_j^s$  for  $i \in N^{s+1}$  and  $j \in F \setminus (N^s \cup \{0\})$ , then  $\pi_i^{s+2} \geq \pi_j^{s+1}$  for  $i \in N^{s+2}$  and  $j \in F \setminus (N^{s+1} \cup \{0\})$ .*

**Proof.** See the Appendix.

This property is similar to property (2) in Salvo (2010), which shows that the pro-competitive free-riding response of outside firms decreases as the number of mergers increases, and thus, the number of outsider firms decreases. The reduction in competition created by each merger dampens the competitive response and free riding by the other firms, giving rise to a strategic wave of mergers.

Using these lemmas, we have our proposition.

**Proposition 1** *Given that  $m$  pairs of firms have already merged, either sequential mergers are fully completed or no further mergers occur in the subgame perfect Nash equilibrium. In addition, further sequential mergers occur if and only if  $\pi_i^n \geq \pi_j^{n-1}$ , where  $i$  is a merged firm and  $j$  is an outsider firm.*

*This condition is equivalent to*

$$\frac{(1 + \beta)(2 - \beta)^2}{4} \left[ 1 + \frac{\beta^2(2 - \beta - \theta)}{(2 - \beta - \theta)(2 + \beta(2n + k) - \beta^2n) + \beta\theta} \right]^2 \geq 1. \quad (6)$$

**Proof.** See the Appendix.

The sequential-merger-or-no-merger result is consistent with those obtained by Ebina and Shimizu (2009, 2016) and Salvo (2010), where only pure market competition are analyzed. This proposition shows that to determine the equilibrium outcome, we only need to check whether the last merger pair of firms has an incentive to merge, even when there are many merger firm pairs, outsider firms, and partial privatization. Therefore  $m$ , the number of initially merged firm pairs, is irrelevant in determining whether sequential mergers stop or continue to the end.

Note that since  $\pi_j^m$  is increasing in  $m$ , sequential mergers in equilibrium imply  $\pi_i^n \geq \pi_j^{n-1} > \pi_j^0$ , which means that sequential mergers are more profitable than no mergers.<sup>12</sup> Thus, if firms do indeed

<sup>12</sup>Unlike Ebina and Shimizu (2009, 2016), sequential mergers are not always more profitable than no mergers, since there are  $k$  outsider firms in our model looking to free ride on the price increase caused by the mergers.

merge sequentially, they do so because mergers are profitable. On the other hand, when there are no mergers in equilibrium, there can be a region of parameters such that  $\pi_j^{n-1} \geq \pi_i^n \geq \pi_j^0$ , which means the firms do not merge even though sequential mergers are more profitable than no mergers. Thus, there is a coordination failure in such a case.

Using this condition, we show that there always exist some regions of  $\beta$  where the further sequential mergers occur in equilibrium when  $\beta$  is low enough.

**Proposition 2** *There necessarily exist some regions where further sequential mergers occur (do not occur) when the level of  $\beta$  is low (high) enough, respectively.*

**Proof.** Let the left-hand side of equation (6) be denoted  $f(\beta)$ . If  $\beta$  converges to zero or one, we have

$$f(0) = 1, \quad f(1) = \frac{1}{2} \left[ 1 + \frac{1}{2 + n + k + \theta/(1 - \theta)} \right]^2 (< 1).$$

Thus we have shown the negative part. As for the positive part, differentiating function  $f$  with respect to  $\beta$  and converging  $\beta$  to zero, we have

$$f'(0) = 0, \quad f''(0) = \frac{1}{2} > 0. \tag{7}$$

Thus, the value of the ratio  $f(\beta)$  initially becomes greater than one as  $\beta$  increases from 0, implying that sequential mergers occur at least when  $\beta$  is in the neighborhood of 0. ■

We have shown that further sequential mergers always occur when the level of  $\beta$  is low enough, whereas they never do when it is high enough. Notice that this is only an existence proposition. It simply claims that under the condition that  $\beta$  is in the neighborhood of zero (one), at least some regions where further sequential mergers occur (do not occur) are guaranteed to exist.

Finally, we examine the comparative statics on the equilibrium regarding  $\theta$ , the parameter describing how much weight firm 0 places on social welfare compared to its own profit.

**Proposition 3** *(i) The parameter range of  $\{\beta, k, n\}$  that leads to further sequential mergers becomes larger as  $\theta$  decreases. In other words, further sequential mergers are more likely to occur when*

the regime is closer to the pure oligopoly regime rather than the mixed oligopoly regime with an unprivatized public firm.

(ii) The parameter range of  $\{\beta, \theta\}$  that leads to further sequential mergers becomes large as  $n$  or  $k$  decreases.

**Proof.** See the Appendix.

The first part of this proposition shows that as privatization proceeds further, it becomes more tempting for the firms to engage in further sequential mergers. The intuition behind this is as follows. As  $\theta$  increases, firm 0 is more considerate of social welfare than its own profit. Thus it produces more than its private counterparts. This can be seen from the best response function of firm 0:

$$q_0^r = \frac{a - c - \beta \sum_{l \in F, l \neq 0} q_l}{2 - \theta}$$

Note that when  $\theta = 0$ , firm 0 is fully privatized and acts similarly as the private firms. The best response function of firm 0 changes as  $\theta$  changes, whereas the functions of the rest of the private firms, merged and outside firms, are the same. Thus, we have focused only on the reaction function of firm 0.

Since Cournot competition is assumed, strategic choices of quantity among the firms move in opposite directions; thus, they are strategic substitutes. When a merger occurs, there is a reaction by the outsider firms, known as the free-rider effect. The merging firms limit their production, trying to induce a higher price, which the outsiders exploit by increasing their own production, enhancing their profit. A welfare-maximizing public firm would further pursue this goal, as it already has an tendency to overproduce. As this would cause the initial gain from merger to be reduced, further sequential mergers would not be profitable in such a case.

## 4.2 Welfare analysis

In this subsection, we look at social welfare and consumer surplus and examine how the changes in parameter values affect them. The following proposition examines how the welfare level changes when  $m$  the number of merger pairs increase.

**Proposition 4** (i) *Consumer surplus is nonincreasing with respect to the number of merger pairs  $m$ .* (ii) *Social welfare with completed sequential mergers is worse from the welfare perspective than that with no mergers.* (iii) *The welfare function can have at most one point, which we call  $\tilde{m}$ , in which its derivative with respect to  $m$  is equal to 0. If  $\tilde{m}$  exists, welfare is minimized at  $\tilde{m}$  and an increase in  $m$  increasingly increases welfare thereafter. Otherwise, social welfare is strictly decreasing in  $m$ .*

The first two parts of this proposition are typical ones seen in the merger literature. The last part is the more interesting and novel result, derived by introducing  $\beta$  the product differentiation parameter.  $W$  can take one of two forms regarding  $m$ : monotone decreasing and decreasing to some  $\tilde{m}$  and then increasing in a convex sense thereafter. The existence of product differentiation leads to this seldom-seen improvement in welfare caused by mergers. The intuition for this result is as follows.

In the usual non-differentiated model, when the firms merge it is as though one firm drops out of the market. This direct effect (so-called elimination effect) leads to a lower quantity level for the whole market, although the aforementioned free-rider effect exists as an indirect effect, mitigating the loss of quantity produced.

In our model, products are differentiated, so both firms engaged in a merger still actively produce goods. Of course, they still internalize the externalities caused by quantity competition, lowering the quantity produced after merger, but this elimination effect is weaker when the goods are more differentiated. The free-rider effect is also weakened as the goods are more differentiated. First we show the intuition on part (i) using this setting.

The following equation is the total quantity produced when  $m$  firm pairs have merged.

$$Q^m = q_0^m + 2mq_i^m + (2n - 2m + k)q_j^m. \quad (8)$$

If  $m$  increases, the only one term,  $(2n - 2m)$ , causes a decrease, whereas all other terms lead to an increase.

$$\underbrace{\frac{\partial Q^m}{\partial m}}_{\text{Total effect on quantity}} = \underbrace{\left( \frac{\partial q_0^m}{\partial m} + k \frac{\partial q_l^m}{\partial m} \right)}_{\text{Indirect effect (Outsiders)}} + 2 \underbrace{\left[ m \frac{\partial q_i^m}{\partial m} + (n - m) \frac{\partial q_j^m}{\partial m} \right]}_{\text{Indirect effect (Potential insiders)}} - \underbrace{2(q_j^m - q_i^m)}_{\text{Direct effect}} \quad (9)$$

The change of  $m$  on  $Q^m$  are composed of three parts. The result in Proposition 4(i) implies that the direct effect causing a loss on total quantity, which affects consumer surplus, dominates the sum of the two indirect effects.

As for welfare, it is more complicated as we must incorporate producer surplus. Although all profits are increasing in  $m$ , the total may not be, as there is a switch during a merger from being an outsider to an insider.

This proposition implies that when the parameters are set such that (iii) holds, it is better not to put a stop to sequential mergers after some mergers have already taken place. From (i) and (ii), it is always better to prohibit merger altogether. However, when some mergers have already taken place, the intuitive anti-merger stance by the antitrust authorities may be harmful to social welfare.<sup>13</sup>

We also compare the levels of consumer surplus and social welfare with respect to  $\theta$ .

**Proposition 5** *For given parameter values of  $\{\beta, m, n, k\}$ , consumer surplus is an increasing function of  $\theta$ .*

**Proof.** See the Appendix.

We can show from numerical analysis that when further sequential mergers occur, social welfare is greater in the case in which firm 0 is unprivatized. From Propositions 3, 4, and 5, we infer a case in which full privatization (from  $\theta = 1$  to 0) causes welfare loss in two ways: The first is from a pure switch in the regimes from Proposition 5, and the other is by switching from the mixed regime to the pure regime; thus, sequential mergers that would not have occurred can occur, leading to welfare

<sup>13</sup>The result stating welfare becomes an increasing function after hitting the bottom is similar in structure to Matsumura and Shimizu (2010). In their paper, multiple public firms exist and their privatization is analyzed. This privatization may worsen welfare temporarily, but after it becomes socially beneficial, further privatization is increasingly beneficial for the society.

loss. We examine a similar issue when we endogenize the level of privatization in the next section. Policymakers must consider this issue when deciding whether to privatize a public firm in markets where sequential mergers could arise.

## 5 Incorporating optimal level of privatization in stage zero

In this section, we consider an optimal level of privatization and investigate how the level of privatization affects a decision of sequential mergers or no mergers. This decision of the policymaker can be applied to a situation in which the government can purchase or sell its stock and the share of a privatized firm changes. To examine this behavior, let us consider a new game extended in the previous sections that the policymaker can choose the optimal level of privatization  $\theta^{**}$ , given the current state of economy. Concretely, the policymaker can choose  $\theta^{**}$  at stage zero. After stage 1, the firms can decide whether to merge or not as stated in the previous sections.

At stage zero, the policymaker chooses an optimal level of privatization. Substituting equations (3) to (5) into equation (1), we have

$$\begin{aligned} \max_{\theta \in [0,1]} W(\theta; \beta, k, m, n, a, c) &\equiv W(q_0^m, q_1^m, \dots, q_{2n+k}^m) \\ &= \frac{(a-c)^2 \times H_1}{4 \{(2-\beta-\theta)[2+\beta(2n+k)-\beta^2 m] + \beta\theta\}^2} \end{aligned}$$

where

$$\begin{aligned} H_1 &= (2-\beta)^2 [6-4\beta m-5\beta^2 m+\beta^3 m+2\beta^3 m^2+2(2n+k)(3+\beta+2\beta n+\beta k-2\beta^2 m)] \\ &\quad - 2(2-\beta) [4-2\beta-4\beta m-3\beta^2 m+\beta^3 m+2\beta^3 m^2+2(2n+k)(3+2\beta n+\beta k-2\beta^2 m)] \theta \\ &\quad + [(2n+k)\{6+\beta(4n+2k-4\beta m-1)\}-\beta m(4+\beta-\beta^2-2\beta^2 m)] \theta^2 \end{aligned}$$

In order to derive  $\theta^{**}$ , we proceed as follows. From the discussion in the last section, the merger outcome is either completed sequential mergers or no further mergers. Let the former outcome be denoted as  $m^* = n$  and the latter  $m^* = m$ . Here, first we hold  $m$  constant and derive the optimal level of privatization  $\theta^*$  given  $m$  in Proposition 6. Then, since, as we show later, sequential merger

incentive arises as  $\theta$  decreases, it may be the case such that further sequential mergers occur when  $\theta^*$  is chosen but no further mergers occur when a higher level of  $\theta$  is chosen. By comparing welfare in the cases with and without further mergers arising, we finally obtain  $\theta^{**}$  in Proposition 7.

**Proposition 6** (i) Suppose  $n > 2/[(1 - \beta)\beta^2]$  holds. The optimal level of privatization given the level of  $m$  is

$$\theta^* = \begin{cases} 0 & \text{if } m \in \left[ \frac{2}{(1-\beta)\beta^2}, n \right] \text{ for } \beta \in (0, 1), \\ \theta^+ & \text{if } m \in \left[ 0, \frac{2}{(1-\beta)\beta^2} \right) \text{ for } \beta \in (0, 1), \\ 1 & \text{if } \beta = 0, 1. \end{cases}$$

where

$$\theta^+ \equiv \frac{(2 - \beta)^2[2 - (1 - \beta)\beta^2 m]}{(4 - \beta)[2 - (1 - \beta)\beta^2 m] + 2\beta[(1 - \beta)(2n + k) - 3 + \beta]}$$

(ii) Suppose  $n \leq 2/[(1 - \beta)\beta^2]$  holds. The optimal level of privatization given  $m$  is

$$\theta^* = \begin{cases} \theta^+ & \text{if } m \in [0, n] \text{ for } \beta \in (0, 1), \\ 1 & \text{if } \beta = 0, 1. \end{cases}$$

**Proof.** See the Appendix.

Proposition 6 states that firm 0 should be kept perfectly public to maximize social welfare in the extreme cases in which all products are perfectly independent or homogeneous. If the current merger status  $m$  is sufficiently high, full privatization is optimal. Otherwise, the level of privatization should be set at a point between 0 and 1 inclusive.

To clarify this threshold of  $m$ , we consider the shape of the expression  $m = 2/[(1 - \beta)\beta^2]$  on  $\beta \in (0, 1)$ . This is a convex function on  $\beta \in (0, 1)$  and takes the minimum value  $27/2$  at  $\beta = 2/3$ . Thus, the optimal level of the privatization may become zero depending on the other parameters *only if* the number of the pair of merger candidate  $n$  is larger than 14. When  $n \leq 13$ , the corner solution  $\theta^* = 0$  never exists and only the interior solution  $\theta^* = \theta^+$  exists when  $\beta \in (0, 1)$  in the equilibrium.

Thus we have the optimal level of  $\theta$  given  $m$ . Depending on the merger incentives, the merger outcome may be  $m^* = n$  or  $m^* = m$ . Thus now we examine how the level of  $\theta$  affects merger

incentive by the firms. Equation (6) of Proposition 1 determines whether further sequential mergers take place. This can be rewritten as follows:

$$SM(\theta) \equiv \frac{(1 + \beta)(2 - \beta)^2}{4} \left[ 1 + \frac{\beta^2}{2 + \beta(2n + k) - \beta^2 n + \frac{\theta}{2 - \beta - \theta} \beta} \right]^2 - 1 \geq 0 \quad (10)$$

Since the fraction  $\theta/(2 - \beta - \theta)$  is increasing in  $\theta$ , the left-hand side  $SM(\theta)$  is decreasing in  $\theta$ . That is, as  $\theta$  decreases, it is more likely that the firms have incentive to sequentially merge.

This result implies that when the firms will sequentially merge given  $\theta^*$ , the social planner must also check for  $\theta > \theta^*$  that would maximize social welfare  $W$  given that equation (10) is violated and no further mergers would occur. On the other hand, when the firms will not merge further given  $\theta^*$ , the social planner must also check for  $\theta < \theta^*$  that would maximize  $W$  given (10) is satisfied and sequential mergers would fully occur.

We summarize this result in the following proposition.

**Proposition 7** *In order to obtain the optimal level of privatization  $\theta^{**}$ , the social planner must consider the following two situations.*

(i) *When further sequential mergers would arise by setting  $\theta^*$ , that is when  $SM(\theta^*) \geq 0$ , the social planner must search for  $\hat{\theta}$  such that  $\hat{\theta} \in \arg \max_{\bar{\theta}} W(\bar{\theta})$ , where  $\bar{\theta} \in \{\theta \mid SM(\theta) < 0\}$  holds and no further mergers occur.*

(ii) *When no further sequential mergers would arise by setting  $\theta^*$ , that is when  $SM(\theta^*) < 0$ , the social planner must search for  $\hat{\theta}$  such that  $\hat{\theta} \in \arg \max_{\bar{\theta}} W(\bar{\theta})$ , where  $\bar{\theta} \in \{\theta \mid SM(\theta) \geq 0\}$  holds and further mergers emerge.*

(iii) *Then the social planner must choose between the two levels of  $\theta$ . That is,*

$$\theta^{**} = \begin{cases} \theta^* & \text{if } W(\theta^*) \geq W(\hat{\theta}), \\ \hat{\theta} & \text{otherwise.} \end{cases} \quad (11)$$

We show a numerical example of case (i) of Proposition 7 occurring in Figure 5 in the next section. More concretely, the optimal  $\theta^*$  given  $m$  leads to further sequential mergers and by setting a higher  $\theta$  the social planner can prevent mergers from occurring and achieve a higher level of welfare. Thus the overall optimal  $\theta^{**}$  is higher than  $\theta^*$ .

## 6 Numerical Analysis

In this section, we conduct a numerical analysis to understand the basic mechanism behind our propositions.

### 6.1 Optimal level of privatization

First, let us illustrate the relationship between the degree of product differentiation  $\beta$  and the optimal level of partial privatization  $\theta^{**}$ , which is defined in Proposition 7. Figure 1 (a) shows the optimal levels of privatization  $\theta^*$  for  $m = 0, 5, 10, 15,$  and  $20$  when  $n = 21$  and  $k = 1$  hold. This figure illustrates how much the change in the number of already merged pair  $m$  affects the optimal level of the privatization, when the number of total firms  $2n + k$  is kept constant. One can see that as  $m$  increases, the optimal level of privatization decreases, implying that firm 0 should be more privatized.

Figure 1 (b) depicts the optimal levels of privatization  $\theta^*$  when  $n = 2, 6, 11, 16,$  and  $21$  when  $m = 1$  and  $k = 1$ . This figure shows how much the number of potential merger pairs  $n$  affects  $\theta^*$  as  $m$  and  $k$  is kept constant. The optimal level of privatization  $\theta^*$  decreases in  $n$ . One can see that as  $n$  increases, the optimal level of privatization decreases, yielding firm 0 should be more privatized to increase total welfare.

Figure 1 (c) depicts the optimal levels of privatization  $\theta^*$  when  $n = 2, 6, 11, 16, 21,$  and  $m = n - 1$  and  $k = 1$ . This figure shows how much the number of merger pairs  $n$  affects  $\theta^*$  as  $n - m$  and  $k$  are kept constant. Here, the merger process has continued and all potential firm pairs but one have already merged. The figure obtained is similar to the former two. Figure 1 (d) is explained in the next section.

Finally, one can confirm using the three figures the various properties behind  $\theta^*$  obtained in Propositions 6 and 7. First, when  $\beta = 0$  or  $1$ , the optimal level of privatization  $\theta^*$  equals  $1$ . As  $m$  increases past  $13.5$ , there exists a corner solution in which  $\theta^*$  equals  $0$  and firm 0 should be perfectly privatized. Finally, these examples show that the optimal level of privatization is decreasing at first, and then is increasing in  $\beta$ .

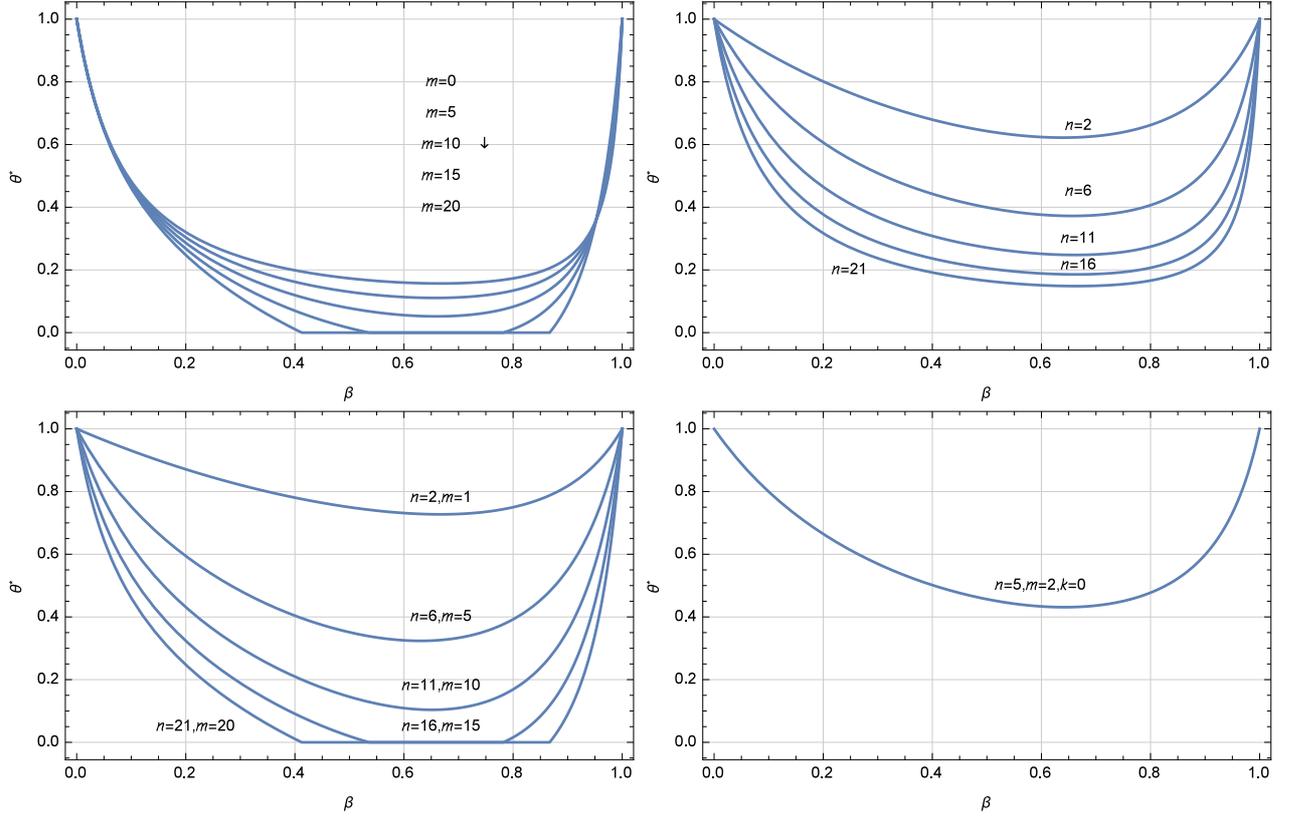


Figure 1: The relationship between the degree of product differentiation  $\beta$  and optimal degree of privatization  $\theta^*$  (a) when  $n = 21$ ,  $m = 0, 5, 10, 15, 20$ , and  $k = 1$  (Upper left), (b) when  $n = 2, 6, 11, 16, 21$ ,  $m = 1$ , and  $k = 1$  (Upper right), (c) when  $n = 2, 6, 11, 16, 21$ ,  $m = n - 1$ , and  $k = 1$  (Lower left), and (d)  $n = 5$ ,  $m = 2$ , and  $k = 0$  (Lower right).

## 6.2 The threshold on further sequential mergers or no mergers: Extended version of Proposition 1

In this subsection, let us consider the effects of the privatization level on the threshold whether further sequential mergers or no mergers occur. Figure 2 depicts the threshold  $SM$  when  $(n, m, k) = (2, 1, 0)$ ,  $(2, 1, 1)$ , and  $(6, 1, 1)$ , when optimal privatization level  $\theta^*$  is employed. Further sequential mergers (no further mergers) occur if and only if  $SM \geq (<) 0$ , respectively. This figure shows that further sequential mergers are likely to occur when  $n$ ,  $m$ , or  $k$  becomes smaller. This result remains in a similar fashion when the optimal level of privatization is given exogenously to the policymaker

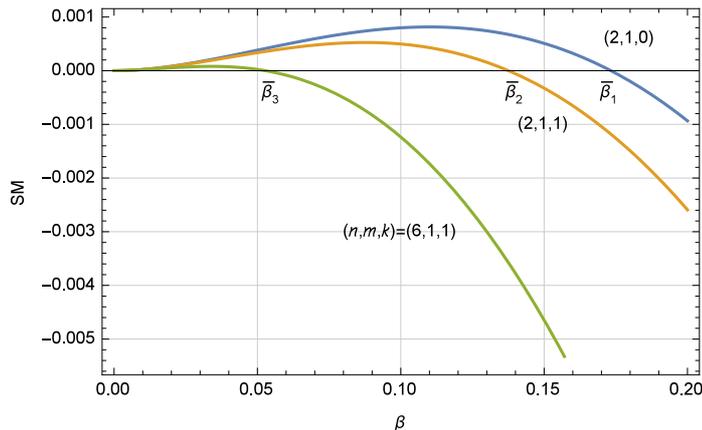


Figure 2: The relationship between the degree of product differentiation  $\beta$  and the threshold  $SM$  on whether or not further sequential mergers would occur.

as in Section 4. Alternatively, if we look at  $\beta$  to be the threshold, there is an upper bound of  $\beta$  whether further sequential mergers and no mergers are indifferent for the firms, which is denoted by  $\bar{\beta}$ . That is,  $SM = 0$  at  $\bar{\beta}$  and further (no) sequential mergers occur if  $\beta \leq (>)\bar{\beta}$ , respectively. We show three cases of  $\bar{\beta}$  in this figure.

In addition, conducting the similar numerical analysis, we illustrate that the number of the merged pair  $m$  affects the threshold only a little:  $SM = 0.0000232082$  when  $n = 6$ ,  $m = 1$ ,  $k = 1$ ;  $SM = 0.0000232144$  when  $n = 6$ ,  $m = 2$ ,  $k = 1$ ;  $SM = 0.0000232207$  when  $n = 6$ ,  $m = 3$ ,  $k = 1$ .

### 6.3 Policy implications on sequential mergers and privatization levels

Next, we investigate economic implications of sequential mergers. The policymaker may have the power to (1) stop or encourage firms to merge and (2) control the level of privatization. Thus there are three regimes describing which policies are available to the policymaker: the policymaker can decide (i) both (1) and (2) (called Regime (i)), (ii) only (1) (Regime (ii)), and (iii) only (2) (Regime (iii)).

Regime (ii) is our analysis up to Section 4. From here, we compare Regimes (i), the first best solution, and (iii), one of the two second best solutions. Suppose that  $n = 2$ ,  $m = 0$ ,  $k = 1$ , for

simplicity. If further sequential mergers occur in the equilibrium,  $m^* = 2$  hold, whereas if no further mergers occur,  $m^* = 0$  would hold.

Figure 3 shows the relationship between the degree of product differentiation and welfare under Regime (iii). With respect to Figure 3 (a), the dotted line represents the case of unprivatized public firm  $\theta = 1$ , the straight line represents that where the policymaker can choose  $\theta^*$ , and the broken line represents the case of fully privatized firm  $\theta = 0$ . The thresholds of  $\beta$  are  $\bar{\beta}|_{\theta=1} \approx 0.129$ ,  $\bar{\beta}|_{\theta=\theta^*} \approx 0.138$ , and  $\bar{\beta}|_{\theta=0} \approx 0.168$ , respectively, and  $\theta^* \approx 0.853$ . The values of the social welfare jump higher at these points of  $\bar{\beta}$ , since the equilibrium outcome changes from the sequential-merger phase to the no-merger case and the latter's welfare is higher with same sets of parameters in these cases.

We need to notice that the level of the welfare with  $\theta^*$  is not necessarily the highest of all cases. At almost all values of  $\beta$ , consisting of  $\beta \in (0, \bar{\beta}|_{\theta=1})$  and  $\beta \in [\bar{\beta}|_{\theta=\theta^*}, 1)$ , the value of the social welfare with  $\theta = \theta^*$  has the maximum value among the three cases of  $\theta = 0, \theta^*, 1$ . However, in particular, when  $\beta \in [\bar{\beta}|_{\theta=1}, \bar{\beta}|_{\theta=\theta^*})$ , the level of social welfare with  $\theta = 1$  is the highest. This is because in this region of  $\beta$ , no further mergers occur under  $\theta = 1$ , whereas further mergers would occur under  $\theta = \theta^*$  and 1. This means that if the policymaker can forecast the future merger formation precisely, it has an incentive to set  $\theta$  higher than  $\theta^*$ .

How high would this better  $\theta$ ,  $\theta^{**}$ , be? For example, Figure 3 (c) adds the case where  $\theta$  is set at 0.9. There is a jump in welfare around when  $\bar{\beta}_{\theta=0.9} \approx 0.135$ . In fact, setting  $\theta$  close to 0.9 would be optimal for  $\beta$  near here. Figure 3 (d) shows 7 dots describing the highest welfare (with no mergers) with varying values of  $\theta$ . By employing this method to all values of  $\beta$  in this region, we have the thick line of Figure 3 (e), which corresponds to  $\theta^{**}$  derived in Proposition 7.

To summarize, on the one hand, if  $\beta \in (0, \bar{\beta}_{\theta=1})$  or  $\beta \in [\bar{\beta}_{\theta=\theta^+}, 1)$ , the policymaker should choose the level of privatization  $\theta^*$  and the level of the welfare is maximized. On the other hand, if  $\beta \in (\bar{\beta}_{\theta=1}, \bar{\beta}_{\theta=\theta^+}] \approx (0.129, 0.138)$ , by choosing  $\hat{\theta}$ , which is higher than  $\theta^*$ , welfare is maximized.

Let us now focus on Regime (i) and consider a numerical example that supports our result of

Proposition 4. This proposition states that, if a merger process is in midstream, there is a parameter region in which the policymaker should encourage sequential mergers to be completed even though firms may have no incentive to merge more. To investigate the situation where the firms do not have an incentive to merge but the policymaker (AA) has an incentive to make firms merge due to higher level of welfare, we offer Table 1. It shows that the optimal decisions of the firms and the policymaker when  $n = 100$ ,  $\beta = 0.25, 0.5, 0.75$ ,  $m = 10, 30, 50, 90$ , and  $k = 10$ . “No” by the firms indicates that the firms do not have an incentive to merge, meaning that no further sequential mergers occur. With respect to the policymaker, “No” means that it should stop sequential mergers at the current stage. On the other hand, “Yes” means that the policymaker should encourage the mergers to be completed. This finding indicates that once a process of sequential mergers has started and is now at a midstream, then it may be better for the policymaker not to stop the merger. This point is a contribution that has not arose in the earlier works in the sequential mergers literature.

$\beta$	$m$	$\theta^*$	$m^*$	Firms	AA
0.25	10	0.0564	10	No	No
0.5	10	0.0161	10	No	No
0.75	10	0.0120	10	No	No
0.25	30	0.0228	30	No	No
0.5	30	0	30	No	No
0.75	30	0	30	No	No
0.25	50	0	50	No	No
0.5	50	0	100	No	No
0.75	50	0	100	No	Yes
0.25	90	0	90	No	No
0.5	90	0	100	No	Yes
0.75	90	0	100	No	Yes

Table 1: The optimal decisions of the firms and policymaker (AA) whether to stop or encourage the firms to merge, when  $n = 100$ ,  $\beta = 0.25, 0.5, 0.75$ ,  $m = 10, 30, 50, 90$ , and  $k = 10$ . “No” represents having no further mergers ( $m^* = m$ ) is preferred to any other mergers ( $\tilde{m} \in (m, n]$ ) for the firms or AA. “Yes” represents sequential mergers to be completed ( $m^* = n$ ) is most preferred for AA.

## 7 Discussion and concluding remarks

In this section, first we offer discussions on how the results would be affected when cost reducing synergy effect from mergers is incorporated. Then we offer concluding remarks.

### 7.1 Discussion on cost reduction

One issue we have abstracted from in this paper that is important when mergers are considered is cost reduction when mergers take place owing to synergy. We can consider a model where the marginal cost level post merger is  $\underline{c} < c$ . Firms have a greater incentive to merge and the outsiders' free-riding increase in profit would be reduced. The policymaker would tend to welcome mergers and the lower marginal costs.

The analysis can be proceeded similarly. This time, the term  $(a - c)$  is now relevant and we must include its level on any numerical analysis.

One benefit of this extension is that the threshold of  $\beta$  that does or does not lead to further sequential mergers becomes more realistic. For example, when there are no cost synergies, this threshold is around 0.1 and 0.08 for the privatized ( $\theta = 0$ ) and public ( $\theta = 1$ ) cases, respectively, when  $n = 4$  and  $k = 0$ . These values decrease as  $n$  and  $k$  increase. However, by introducing synergy, we get more realistic values of  $\beta$ . Let  $a = 10, c = 1$ , and  $\underline{c} = 0.9$  (a 10% decrease). Then, the thresholds are 0.465 and 0.461, respectively. There can be more different values of  $\beta$  depending on  $n, k$ , and  $\underline{c}$ . For example, in the privatized case, when  $n = 4$  and  $k = 0$ , the threshold is around 0.7 when  $\underline{c}$  is 0.8, 0.9 when  $\underline{c}$  is 0.7, and all mergers are profitable when  $\underline{c}$  is 0.6.

The welfare-reducing effects mentioned after Proposition 5 can still hold under cost synergies. Let  $(a, c, \underline{c}, n, k, \beta) = (10, 1, 0.95, 4, 0, 0.325)$ . Here, the firms choose to merge when  $\theta = 0$  but not when  $\theta = 1$ . As a result, the welfare level in the former case is 95.1453 and that in the latter case is 96.0843. Thus, the welfare is reduced if the cost synergy is not too great.

Of course, given large cost synergies, sequential mergers become possibly better for welfare than no mergers. In fact, the opposite results may emerge, in that the public regime is more likely to

lead to further sequential mergers, resulting in worse welfare. For example, let  $(a, c, \underline{c}, n, k, \beta) = (10, 1, 0.97, 10, 3, 0, 246)$ . Then, the case with  $\theta = 1$  leads to further sequential mergers but the case with  $\theta = 0$  does not. The welfare level under the latter case is 143.5, while that under the former case is 142.762. Thus, privatization would be welfare-enhancing in such a case.

## 7.2 Discussion on our model's implications and the current examples

Here we show how the implications derived from our model can be interpreted in the current real-world examples. As mentioned in the Introduction, since the merger of Meiji Life and Yasuda Life in 2004, there had not been any major domestic mergers in the life insurance industry in Japan. However in September of 2015, Nippon Life acquired Mitsui Life, one of the top ten life insurance firms in Japan. Nippon Life had been the top revenue insurer (excluding Japan Post Insurance (JPI), explained in the next paragraph) until 2014 when Dai-ichi Life took the top spot by acquiring the U.S.-based Protective Life Corporation.

The hot issue in Japanese life insurance of 2015 was the listing of JPI in the Tokyo Stock Exchange in November. A subsidiary of Japan Post Holdings, JPI was partially privatized along with Japan Post (postal service) and Japan Post Bank. 11% of stock was sold in the stock market. Initially when postal privatization was pushed by Prime Minister Koizumi in 2007, the plan had been that privatization be complete in 10 years after the initial sale of stock. This plan has since been retracted and the future cabinet will decide the level of privatization. The Japanese government holds 80.49% of Japan Post Holdings, which in turn holds 89% of JPI, making  $\theta = 0.72$ .

This result is consistent with our model with  $n = 5, m = 2$ , and  $k = 0$  from Propositions 6 and 7, which leads to  $\theta^{**} = 0.43$  to 1 when  $\beta \in (0, 1)$  and 0.43 to 0.8 when  $\beta \in (0.1, 0.9)$  (See Figure 1 (d)). Namely,  $\theta = 1$  is not optimal unless the goods are perfect substitutes or independent, as in this case. The obtained result implies that further privatization may take place in the future.

In addition, further M&As are discussed in the industry, but other than the Nippon Life and Mitsui Life, the other are all cross-bordered cases. One issue hindering domestic M&As is that most

life insurance companies in Japan are mutual companies, in which the insured policyholders are their owners. Unlike a stock company that can be purchased using public tender offers, the purchase procedures are complicated. This is one of the reasons that only one major merger (Meiji and Yasuda) had occurred for a long time. In addition, many acquisitions of foreign life insurance firms have proceeded since 2015, as this complication by mutual insurance is not relevant when merging internationally. In addition to Dai-ichi purchasing Protective Life, Meiji-Yasuda Life acquired StanCorp, Sumitomo Life acquired Symetra, and Nippon Life acquired 80% stake in the life insurance subsidiary of National Australia Bank, all in 2015.

These events can be examined under the results of our model in two ways. First, as Proposition 3 explains, the promotion of partial privatization leads to possibly more sequential mergers. It may not have been worthwhile to merge but as JPI's stock were (partially) offered to the public, the merger incentives emerged. Another point is Proposition 4 (iii), Table 1, and the mutual insurance issue. As the domestic market for life insurance in Japan is or will be shrinking due to a decrease in the population in the near future, it may be better from the viewpoint of social welfare for the firms to merge, considering excess entry. In fact, as mentioned in the last paragraph, the major players are purchasing foreign insurers to reach markets outside Japan. As we discussed using Table 1, the policymaker may want to encourage further mergers, perhaps by relaxing the merger approval procedures in the Insurance Business Act by, for example, lowering the required level of post-merger firm's ability to pay insurance claims to the beneficiaries.

### **7.3 Concluding remarks**

In this paper, we examine how and what kind of sequential mergers could emerge in an industry with partially privatized firms with product differentiation. As sequential mergers are becoming more prevalent in markets where goods are possibly differentiated and public and private firms compete globally, it is very important for policymakers to understand the implications of our model. The main findings of our paper are that given some mergers have already occurred, further sequential

mergers are completed or no further mergers would arise in the subgame perfect Nash equilibrium in both regimes. Sequential mergers are more likely to emerge in the pure oligopoly regime than in the mixed oligopoly regime. Combined with the welfare-deteriorating effect of the mergers, we find that privatization may worsen welfare owing to the existence of sequential mergers. This is evident when we endogenize the level of partial privatization. As we show in Proposition 6 (ii), it may be better for the policymaker to limit privatization to keep firms from merging and worsening welfare.

Another important implication of our model is that given sufficient number of mergers have already occurred, it may be better for the policymaker to encourage mergers to be completed even when the remaining potential merger pairs do not have incentive to merge (Proposition 4 (iii)). However, we note here that this situation may not be subgame perfect, as we show in Proposition 1 if some pairs have incentive to merge, further pairs must also have incentive to merge. Thus this situation is possibly applicable when there are some outside shock to the model that gave early pairs incentive to merge but the later pairs do not have incentive to merge. Also, if we extend our model to allow asymmetric cost reduction by synergy, this situation may be more appropriate.

Finally, we mention one possible direction of future research. Since many of the observed sequential mergers involve the acquisition of firms across borders, a consideration of cross-border mergers may be relevant. We can first consider the simple domestic merger case, with the possible presence of foreign outsiders. For example, the steel industry in Japan was characterized by six major firms in the 1990s. In 2000, Kawasaki Steel and NKK Corporation merged, creating JFE Steel. Thereafter, in 2012, the number one firm, Nippon Steel, and the number three firm, Sumitomo Metal Industries, merged to create Nippon Steel & Sumitomo Metal Corporation. The latter merger would have been deemed unlawful under the antitrust laws in Japan. However, it was approved because the antitrust authority considered the industry's fierce competition with foreign firms from China, India, and South Korea. Our proposed analysis would fit perfectly in such a case. We can also consider a situation where the mergers occur between domestic and foreign firms. For example, automobile and airline industries have seen many instances of international mergers and alliances. The welfare

analysis would likely be complicated, but the implication should be very meaningful.

## Appendix

**Proof of Lemma 2:** To prove this, we show that  $\pi_i^{s+r+1}/\pi_j^{s+r}$  is an increasing function of  $r$ . This can be rewritten as

$$\begin{aligned} \frac{\pi_i^{s+r+1}}{\pi_j^{s+r}} &= \frac{(1+\beta)}{4} \left[ \frac{(2-\beta)(2-\beta-\theta)(a-c)}{[(2-\beta-\theta)(2+\beta(2n+k)-\beta^2(s+r+1))+\beta\theta]} \right]^2 & (12) \\ &/ \left[ \frac{(2-\beta-\theta)(a-c)}{(2-\beta-\theta)(2+\beta(2n+k)-\beta^2(s+r))+\beta\theta} \right]^2 \\ &= \frac{(1+\beta)(2-\beta)^2}{4} \left[ \frac{(2-\beta-\theta)(2+\beta(2n+k)-\beta^2(s+r))+\beta\theta}{(2-\beta-\theta)(2+\beta(2n+k)-\beta^2(s+r+1))+\beta\theta} \right]^2 \\ &= \frac{(1+\beta)(2-\beta)^2}{4} \left[ 1 + \frac{\beta^2(2-\beta-\theta)}{(2-\beta-\theta)(2+\beta(2n+k)-\beta^2(s+r+1))+\beta\theta} \right]^2, & (13) \end{aligned}$$

which is increasing in  $r$ . Thus, we have the desired result. ■

**Proof of Proposition 1:** We first show that  $\pi_i^n \geq \pi_j^{n-1}$  is a sufficient condition for sequential mergers. We later show that  $\pi_i^n < \pi_j^{n-1}$  is a sufficient condition for no mergers. Finally, the uniqueness of  $\beta$  is derived.

Let  $\pi_i^n \geq \pi_j^{n-1}$  hold. Since  $\pi_j^m$  is increasing in  $m$  by Lemma 1 (ii), we have  $\pi_i^n \geq \pi_j^r$  for all  $r \in [0, n-1]$ . In the subgame perfect Nash equilibrium, on the path where all other firms choose to merge, the pair is better off merging, as the complete sequential merger payoff is greater than any outcome where the firm pairs are outsiders.

More concretely, consider the final merger after all other firm pairs have merged. Since  $\pi_i^n \geq \pi_j^{n-1}$ , the pair merges. Working backwards, the penultimate firms obtain  $\pi_i^n$  if they merge and either  $\pi_j^{n-1}$  or  $\pi_j^{n-2}$  if they do not. Thus, they do merge. Similarly, working backwards, the choice is between  $\pi_i^n$  and  $\pi_j^r$  for some  $r \in [0, n-1]$  depending on the (subgame perfect Nash equilibrium) decisions of the others, and merging prevails in each case regardless of the value of  $r$ . Thus, the unique subgame perfect Nash equilibrium is where all firm pairs sequentially merge. Thus, we have proved the first part.

Now, let  $\pi_i^n < \pi_j^{n-1}$  hold. Using the contrapositive of Lemma 2, we have  $\pi_i^r < \pi_j^{r-1}$  for all  $r \in [1, n]$ . Consider the decision by the last merger pair. Regardless of the decisions of the other firm pairs, the last pair does not merge in this case. The penultimate pair has a similar decision, knowing that the last pair will not merge. Thus, they do not merge. By repeating this process from the beginning of the game, no firms merge in the subgame perfect Nash equilibrium.

Finally, the equation given at the end of the proposition is obtained by substituting  $n$  for  $s+r+1$  in the denominator of equation (13). Thus, we have the desired result. ■

**Proof of Proposition 3:** Here, we would like to show that  $\pi_i^n/\pi_j^{n-1}$  is decreasing with respect to  $\theta$ . Using equation (6), we have the following:

$$\begin{aligned} \pi_i^n/\pi_j^{n-1} &= \frac{(1+\beta)(2-\beta)^2}{4} \left[ 1 + \frac{\beta^2(2-\beta-\theta)}{(2-\beta-\theta)[2+\beta(2n+k)-\beta^2n]+\beta\theta} \right]^2, \\ \frac{\partial[\pi_i^n/\pi_j^{n-1}]}{\partial\theta} &= -\frac{(1+\beta)(2-\beta)^2}{2} \left[ 1 + \frac{\beta^2(2-\beta-\theta)}{(2-\beta-\theta)[2+\beta(2n+k)-\beta^2n]+\beta\theta} \right] \\ &\quad \cdot \frac{1}{\{(2-\beta-\theta)[2+\beta(2n+k)-\beta^2n]+\beta\theta\}^2} < 0. \end{aligned}$$

Thus we have the desired result. ■

**Proof of Proposition 4:** (i) Consumer surplus with  $m$  merger pairs is given by  $CS^m = U(\bar{q}) - \sum_{i \in F} p_i q_i$ . We take its first order derivative with respect to  $m$ .

$$\begin{aligned} \frac{\partial CS^m}{\partial m} &= -\frac{b(2-\beta-\theta)(a-c)^2 J}{4[(2-\beta-\theta)[2+\beta(2n+k)-\beta^2m]+\beta\theta]^3}, \\ \text{where } J &= \beta(8-5\beta+\beta^2)(2-\beta-\theta)^2(2n+k) - \beta^2(4+\beta-\beta^2)(2-\beta-\theta)^2 m \\ &\quad + 2(2-\beta)^2(4-3\beta+\beta^2) - (2-\beta)(16-16\beta+5\beta^2-\beta^3)\theta + (4-\beta)(2-\beta)(1-\beta)\theta^2 \end{aligned}$$

Let us minimize  $J$ . Since  $J$  is decreasing in  $m$ , we plug in the largest value of  $m$  available, i.e.  $n$ . After rearranging,  $J$  is increasing in both  $n$  and  $k$ , so we plug in the smallest values for them at  $n=2$  and  $k=0$ , respectively. Then we have the following.

$$J|_{m=n=2, k=0} = (2-\beta-\theta)^2(4+15\beta-6\beta^2-2\beta^3) + (1-\theta)\beta(4\theta+2\beta-\beta\theta-\beta^2)$$

which, given the constraints  $0 \leq \beta \leq 1$  and  $0 \leq \theta \leq 1$ , is minimized at 0 when  $\beta = 1$  and  $\theta = 1$ . Therefore  $J$  is nonnegative, and consumer surplus is nonincreasing in  $m$ . ■

(ii) We simply compare the two welfare levels. That is we show that  $W^0$  is greater than  $W^m$  by deriving the following to be positive.

$$W^0 - W^m = \frac{(a-c)^2}{4} \left[ \frac{F_1}{F_2} - \frac{F_3}{F_4} \right],$$

where  $F_1$  to  $F_4$  are all positive and are available in the online appendix in the author's homepage.<sup>14</sup>

Since the  $F$ 's are positive, the sign remains the same as  $F_1F_4 - F_2F_3 \equiv F_5$ . This can be rewritten as

$$\begin{aligned} F_5 &= F_6 + 2\beta(2-\beta-\theta) \left[ (2-\beta)^2 \{ (1-\theta)(3-\theta-\beta\theta-\beta^2) + (1-\beta)(1+\beta) \} \right. \\ &\quad \left. + (1-\beta)\beta^2(2-\beta-\theta)^2 n \right] k + (1-\beta)\beta^3(2-\beta-\theta)^3 k^2 \geq 0, \text{ since} \\ F_6 &= (2-\beta)^2 \left[ (2-\theta) \{ (1-\beta)(12-2\beta^2-8\theta+\beta^2\theta+\beta\theta^2) + 4(1-\theta)^2 \} \right. \\ &\quad \left. + 2\beta(2-\beta-\theta) \{ 3(1-\theta)^2 + (1-\beta)(5+\beta-2\theta-\theta^2) \} n \right] \geq 0 \end{aligned}$$

for  $\beta \in [0, 1]$  and  $\theta \in [0, 1]$ . Note that equality holds if and only if  $\beta = \theta = 1$ . Thus in the parameter range we are most interested in, the welfare level for the no-merger case is higher than for the complete merger case. ■

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<sup>14</sup><http://www-cc.gakushuin.ac.jp/~20060015/index-e.html>

(iii) We first uniquely derive  $\tilde{m}$ .

$$\frac{\partial W}{\partial m} = \frac{(a-c)^2 \beta (2-\beta-\theta) G_1}{4G_2^3},$$

$$\frac{\partial W}{\partial m} = 0 \iff \tilde{m} = \frac{G_3}{\beta^2(4-\beta)(1-\beta)(2-\beta-\theta)^2},$$

where

$$G_1 = \beta^2(1-\beta)(2-\beta-\theta)^2(2n+k-(4-\beta)m) + G_4,$$

$$G_2 = (2-\beta)(2-\theta) + (2-\theta-\beta)(2n+k-\beta m) > 0,$$

$$G_3 = \beta^2(1-\beta)(2-\beta-\theta)^2(2n+k) + G_4 \geq 0,$$

$$G_4 = (2-\beta)[4(2-\beta-\theta)^2 + \beta(1-\beta)(4-2\beta+\beta\theta+\theta^2)] \geq 0.$$

Since  $G_3$  is nonnegative,  $\tilde{m}$  is also nonnegative (equal to zero if and only if  $\beta = \theta = 1$ ). If  $\tilde{m}$  is between 0 and  $n$  inclusive, then  $W$  takes either the maximum or the minimum here. Otherwise, from (ii),  $W$  is decreasing in  $m$ .

We now show that if  $\tilde{m} \in (0, n)$  and for any  $m$ ,  $\partial W / \partial m \geq 0$ , then  $\partial^2 W / \partial m^2 > 0$ . Then  $\tilde{m}$  is the minimum and we have our result. We have

$$\frac{\partial^2 W}{\partial m^2} = \frac{(a-c)^2 \beta^3 (2-\beta-\theta)^2 G_5}{2G_2^4},$$

where

$$G_5 = \beta(1-\beta)(2-\beta-\theta)^2[2(1-\beta)(2n+k) + (4-\beta)\beta m]$$

$$+ 2(2-\beta)[-3\beta(1-\beta)(2-\beta) + \beta(1-\beta)(4-\beta)(2-\beta-\theta) - (2+2\beta-\beta^2)(2-\beta-\theta)^2].$$

Thus we must show that when  $G_1$  is nonnegative,  $G_5$  is positive. We can show this if  $G_5 - G_1 > 0$ .

$$G_5 - G_1 = (2-\beta)\beta G_6, \quad \text{where}$$

$$G_6 = -2(2-\beta)^2 + (8-5\beta+\beta^2)\theta - (3-\beta)\theta^2 + (1-\beta)(2-\beta-\theta)^2(2n+k).$$

Since the term with  $2n+k$  is positive, we assign  $n = \tilde{m}$  and  $k = 0$  to obtain the smallest  $G_6$ . We

note that we require that  $n$  be large enough, in particular larger than  $\tilde{m}$  (so that a minimum exists).

$$\begin{aligned}
G_6|_{n=\tilde{m}} &= \frac{G_7}{(4-\beta)\beta^2}, \quad \text{where} \\
G_7 &= G_8 + 2(2n+k)(1-\beta)\beta^2(2-\beta-\theta)^2 > 0, \quad \text{since} \\
G_8 &= (2-\beta-\theta)^2\{2+(1-\beta)(14+10\beta-8\beta^2+\beta^3)\} \\
&\quad + \beta(1-\beta)G_9 > 0, \\
G_9 &= 8\beta + (4-\beta)(4-6\beta+\beta^2)(\beta+\theta) \\
&= 2 + (1-\beta)(14+10\beta-8\beta^2+\beta^3) - (4-\beta)(1-\theta)(4-6\beta+\beta^2) \geq 0
\end{aligned}$$

where the inequalities would be equalities if and only if  $\beta = \theta = 1$  (we ignore this case since here  $\tilde{m}$  becomes 0).  $G_9$  can easily be deemed nonnegative using the first equation if  $4-6\beta+\beta^2$  is nonnegative and using the second equation if it is negative. Consequently, we have the desired result. ■

**Proof of Proposition 5:** Taking the derivative of the consumer surplus levels with respect to  $\theta$ , we have the following:

$$\frac{\partial CS}{\partial \theta} = \frac{(2-\beta)(a-c)^2\{2[(1+2\beta n)(2-\beta)^2-2n\beta\theta] + 2\beta[(2-\beta)^2-\theta]k - \beta_2 L_1 m\}}{2[(2-\beta-\theta)(2+\beta(2n+k)-\beta^2 m) + \beta\theta]^3} \quad (14)$$

$$\text{where } L_1 = 4 - 3\beta^2 + \beta^3 - 3\beta\theta + \beta^2\theta = (1-\beta)(4+\beta-\beta^2) + \beta(3-\beta)(1-\theta) \geq 0.$$

The denominator is positive, as is the coefficient on  $k$ . The coefficient on  $m$  is nonpositive. Thus to minimize this, we plug in  $k = 0$  and  $m = n$  into the equation. Examining the numerator of equation (14) we have:

$$\text{Numerator of } \frac{\partial CS}{\partial \theta} \Big|_{k=0, m=n} = (2-\beta)^3(a-c)^2[2+n\beta(4-(1+\beta)(\beta+\theta))] > 0$$

Thus consumer surplus is increasing in  $\theta$ , which is the desired result. ■

**Proof of Proposition 6:** The strategy of the proof is as follows: We initially focus on the range  $\beta \in (0, 1)$ . We examine the cases  $\beta = 0, 1$  at the end of the proof. Then first, we obtain a solution,  $\theta^+$ , that satisfies the first-order condition and show that it is unique. Second, the second-order derivative at the solution is shown to be negative, which implies the solution is a (local) maximum. Third, we examine whether the optimal solution,  $\theta^+$ , is between 0 and 1.

Suppose that  $\beta \in (0, 1)$ . If the first-order derivative equals zero, the solution where the first-order condition is satisfied is unique, and the second-order condition is satisfied at the solution, then this solution has a maximum value. Differentiating  $W(\theta; \cdot)$  with respect to  $\theta$ , we have

$$\frac{\partial W}{\partial \theta} = \frac{(2 - \beta)(a - c)^2 \times H_2}{2 \{(2 - \beta - \theta) [2 + \beta(2n + k) - \beta^2 m] + \beta \theta\}^3} \quad (15)$$

where

$$H_2 = (2 - \beta)^2 [2 - (1 - \beta)\beta^2 m] + \{(4 - \beta)[2 - (1 - \beta)\beta^2 m] + 2\beta[(1 - \beta)(2n + k) - 3 + \beta]\}\theta$$

The optimal privatization level given  $m$ ,  $\theta^*$ , equals 0 if equation (15) is negative, whereas it equals 1 if equation (15) is positive.  $\theta^*$  can take an intermediate solution if equation (15) equals 0. Let us consider a solution satisfying the first-order condition (15)=0. Then we have

$$\theta^+ = \frac{(2 - \beta)^2 [2 - (1 - \beta)\beta^2 m]}{(4 - \beta)[2 - (1 - \beta)\beta^2 m] + 2\beta[(1 - \beta)(2n + k) - 3 + \beta]}. \quad (16)$$

Note that the denominator is increasing in  $n$  and  $k$ , and when  $n = m$  and  $k = 0$  are assigned in order to minimize it, we obtain  $(2 - \beta)(2 + \beta m - \beta^2 m) > 0$ . Therefore the denominator is positive.

Next, we examine whether this value,  $\theta^+$ , is the local maximum. The second-order condition at  $\theta = \theta^+$  yields

$$\left. \frac{\partial^2 W}{\partial \theta^2} \right|_{\theta=\theta^+} = - \frac{(a - c)^2 \{(4 - \beta)[2 - (1 - \beta)\beta^2 m] + 2\beta[(1 - \beta)(2n + k) - 3 + \beta]\}}{2(2 - \beta)^2 H_3^3},$$

where  $H_3 = 2(1 - \beta)\beta(2n + k)[4 + \beta(2n + k - 1 - 2\beta m)] + 2(2 - \beta)^2 - (1 - \beta)\beta^2 m(8 - \beta^2 - 2\beta^2 m)$ .

Note the numerator is  $(a - c)^2$  times the denominator of  $\theta^+$  and is positive. We must show that  $H_3$  is positive. Since it is increasing in  $k$ , we substitute  $k = 0$  to obtain the lowest value of  $H_3$  and we have

$$\begin{aligned} H_3|_{k=0} &= 2(2 - \beta)^2 + 4(1 - \beta)\beta n(4 - \beta + 2\beta n) - (1 - \beta)\beta^2 m[8 + \beta(8n - 2\beta m - \beta)] \\ \frac{\partial H_3|_{k=0}}{\partial m} &= -(1 - \beta)\beta^2[8 + \beta(8n - 4\beta m - \beta)] < 0. \end{aligned}$$

Hence we assign  $m = n$  to minimize  $H_3$ .

$$H_3|_{k=0, m=n} = (2 - \beta)^2 [2 + \beta n(4 + \beta)(1 - \beta) + 2\beta^2 n^2(1 - \beta)] > 0.$$

Thus, if  $\theta^+$  exists and is between 0 and 1, it is always the optimal level of the privatization for the policymaker to maximize social welfare given  $m$ .

Finally, we need to check if the optimal solution  $\theta^+$  given by Equation (16) is in between 0 and 1. Since the sign of the denominator of Equation (16) is positive, we first show that the numerator is always less than the denominator.

$$\begin{aligned} & (2 - \beta)^2 [2 - (1 - \beta)\beta^2 m] - \{(4 - \beta)[2 - (1 - \beta)\beta^2 m] + 2\beta[(1 - \beta)(2n + k) - 3 + \beta]\} \\ & = -(1 - \beta)\beta [4n - (3 - \beta)\beta^2 m + 2k] \leq -(1 - \beta)\beta [\{4 - (3 - \beta)\beta^2\}m + 2k] \quad (\because n \geq m) \\ & < 0. \end{aligned}$$

Thus, we have  $\theta^+ < 1$ . To obtain  $\theta^+ > 0$ , we need its numerator to be positive. Thus when the sign of the second term of Equation (16),  $[2 - (1 - \beta)\beta^2 m]$ , determines the sign of  $\theta^+$ .

Suppose that  $n \leq 2/[(1 - \beta)\beta^2]$ . Then, since the numerator is always positive as follows;

$$2 - (1 - \beta)\beta^2 m > 2 - (1 - \beta)\beta^2 n \geq 2 - (1 - \beta)\beta^2 \frac{2}{(1 - \beta)\beta^2} = 0, \quad (17)$$

the interior solution  $\theta^+$  becomes the optimal solution. Now suppose that  $n > 2/[(1 - \beta)\beta^2]$ . Then, if  $m \in [2(1 - \beta)\beta^2, n)$  for  $\beta \in (0, 1)$  the sign of  $\theta^+$  is positive and the optimal level of privatization  $\theta^*$  given  $m$  equals the interior solution  $\theta^+$ . On the other hand, if  $m \in [0, 2(1 - \beta)\beta^2)$  for  $\beta \in (0, 1)$ ,  $\theta^+$  is negative and  $\theta^*$  equals 0 since the interior solutions are eliminated and  $W(0) > W(1)$ .

To complete the proof, suppose now that  $\beta = 0, 1$ . Substituting  $\beta = 0$  and  $\beta = 1$  into (15), we

have

$$\frac{\partial W}{\partial \theta} \Big|_{\beta=0} = \frac{(1-\theta)(a-c)^2}{(2-\theta)^3}, \quad (18)$$

$$\frac{\partial^2 W}{\partial \theta^2} \Big|_{\beta=0, \theta=1} = -(a-c)^2 < 0, \quad (19)$$

$$\frac{\partial W}{\partial \theta} \Big|_{\beta=1} = \frac{(1-\theta)(a-c)^2}{[(2n+k-m+1)(1-\theta)+1]^3}, \quad (20)$$

$$\frac{\partial^2 W}{\partial \theta^2} \Big|_{\beta=1, \theta=1} = -(a-c)^2 < 0. \quad (21)$$

With respect to both cases  $\beta = 0$  and  $1$ , since the first-order derivatives are positive when  $\theta \in [0, 1)$ , the first-order conditions are satisfied at  $\theta = 1$ , and the second-order derivatives are negative at  $\theta = 1$ , we show that  $W|_{\beta=0}$  is increasing in  $\theta \in [0, 1)$  and has a maximum value at  $\theta = 1$ . Thus,  $\theta^* = 1$  is the optimal value of privatization given  $m$  for the policymaker to maximize the social welfare. ■

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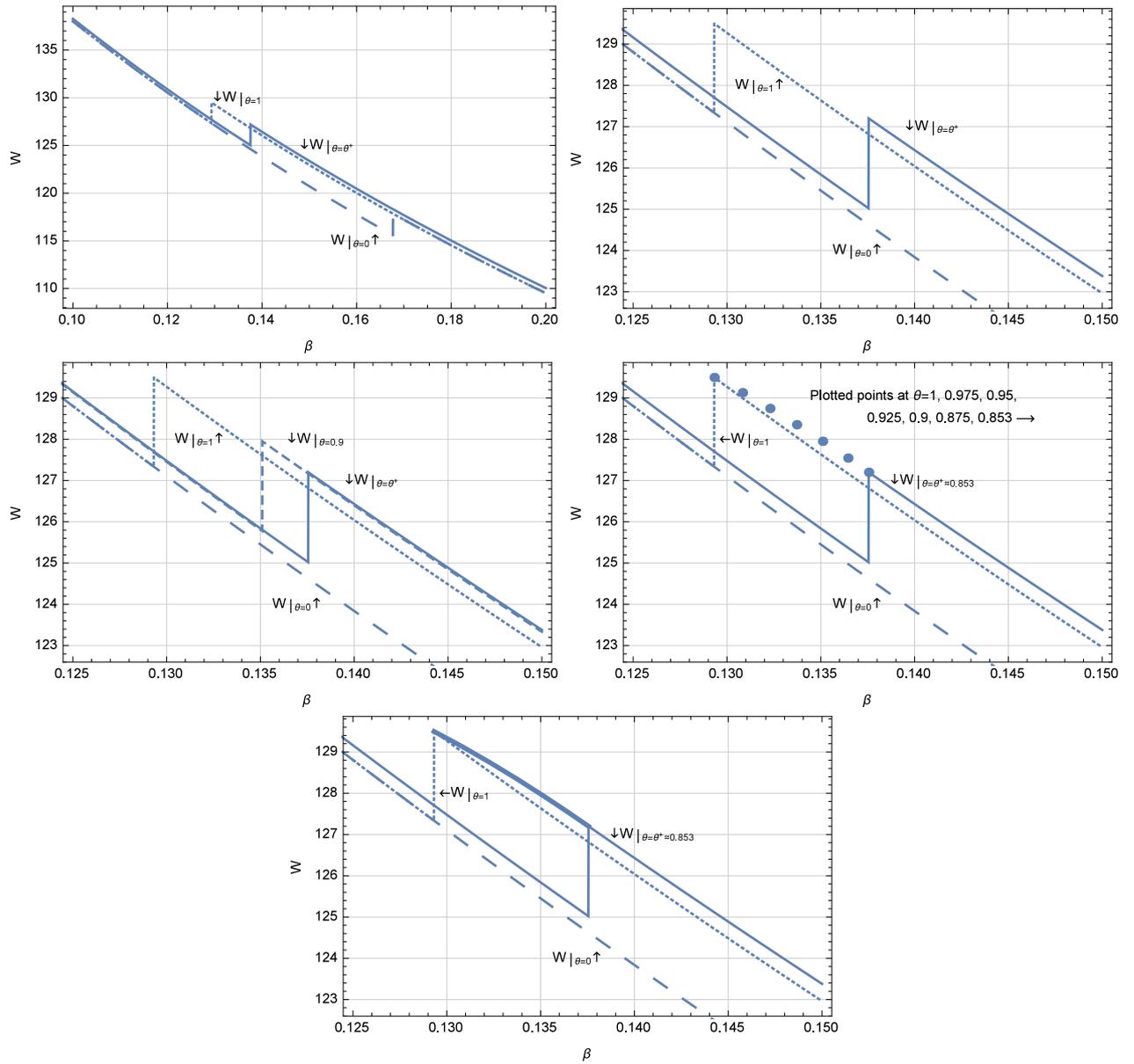


Figure 3: Upper left (a), Upper right (b), Middle left (c), Middle right (d), Lower left (e): The relationship between the degrees of product differentiation  $\beta$  and the welfare under Regime (iii). (a) features welfare for  $\theta$  at 0,  $\theta^*$ , and 1. Dotted line;  $\theta = 1$ . Straight line;  $\theta = \theta^*$ . Broken line;  $\theta = 0$ . (b) is a magnified version of (a). (c) adds the case of  $\theta = 0.9$ . (d) depicts the optimal level of welfare depending on seven levels of  $\beta$ , at leading to optimal  $\theta$  at 1, 0.975, 0.95, 0.925, 0.9, 0.875, and 0.873. (e) describes the optimal welfare level when  $\theta^{**}$  is employed by the policymaker.