Oligopoly Theory (11)
Collusion

Aim of this lecture

(1) To understand the idea of repeated game.
(2) To understand the idea of the stability of collusion.

Outline of the 11th Lecture

11-1 Infinitely Repeated Game
11-2 Stability of Cartel
11-3 Busyness Cycle and the Stability of Cartel
11-4 Vertical Differentiation and Cartel Stability
11-5 Horizontal Differentiation and the Stability of Cartel
11-6 Finitely Repeated Game
11-7 Endogenous Timing and Cartel

Prisoners' Dilemma

2

		C	D
1	C	（3，3）	（0，4）
	D	（4，0）	（1，1）

Nash Equilibrium : (D,D)

Prisoners' Dilemma and Cooperation

In reality, we often observe cooperation even when players seem to face prisoners' dilemma situation. Why?

(1) Players may be irrational

(2) The payoff of each player depends on non-monetary gain → Players does not face prisoners' dilemma situation.

(3) Players face long-run game. They did not maximize short-run profit so as to maximize long-run profit.

→repeated game

(2) ~Altruism, Anti-Inequality

2

		C	D
1	C	（3，3）	（0，2）
	D	（2，0）	（1，1）

Question: Derive Nash equilibria

(3) ~ Repeated Game

The same game is played repeatedly.

→So as to maintain the cooperation and to obtain greater payoff in future, each player dare not to pursue the short-run payoff-maximization.

（finitely repeated game） the number of periods is finite.

（infinitely repeated game）the number of period is infinite.

Finitely Repeated Game

N-period model.

The same stage game is played at each period.

The action chosen in period t is observed at the beginning of period t+1.

The payoff of the game is given by the sum of payoff of each stage game.

Suppose that the stage game is the same as the game in sheet 3 (Prisoner's Dilemma Game).

backward induction

Consider the last period game. (Period N).

Question: Derive Nash equilibria in this subgame.

backward induction

Consider the second to the last period game. (Period N-1).

Question: Derive (subgame perfect) Nash equilibria in this subgame.

backward induction

Consider the third to the last period game. (Period N - 2).

Question: Derive (subgame perfect) Nash equilibria in this subgame.

backward induction

Consider the first period action. (Period 1).

Question: Derive (subgame perfect) Nash equilibria in this subgame.

Is it always impossible to cooperate in repeated game?

It is known that the following situations can yield cooperation

(1) Incomplete information game

(2) There are multiple equilibria in the stage game

→inferior equilibrium is used as the punishment device

(3) Infinitely repeated game

Stage Game with Multiple Equilibria

2

	C	D	E	F
C	(6,6)	(0,7)	(0,0)	(0,0)
D	(7,0)	(3,3)	(0,0)	(0,0)
E	(0,0)	(0,0)	(0,0)	(4,1)
F	(0,0)	(0,0)	(1,4)	(0,0)

1

Cooperation in a Finitely Repeated Game

In period t (t =1, 2, … n-1), (C,C) is played unless at least one player chooses the action other than C before period t. In period n, (D,D) is played unless at least one player chooses the action other than C before period n.

If only player 1 (2) first deviates, (F,E) ((E,F)) is played repeatedly after the deviation. If both simultaneously deviate, (D,D) is played repeatedly after the deviation.

I may be possible to construct credible punishment if the stage game has multiple equilibria. ~ Benoit and Krishna (1985)

Typical Examples of Multiple Equilibria

(a) Endogenous Timing Game.

(b) Bertrand Competition with Supply Obligation.

(c) Location Competition with Strategic Complementarity.

Infinitely Repeated Game

The same stage game is repeated infinitely.

The payoff of each player is the discounted sum of the payoffs at each stage game.

Payoff is payoff in period 1 + δ(payoff in period 2) + δ^2(payoff in period 3) + δ^3(payoff in period 4) +...

$\delta \in (0,1)$：discount factor

Interpretation of discount factor

(1) interest rate $\delta = 1/(1 + r)$ r:interest rate

(2) objective discount rate ~it indicates how patient the player is

(3) the probability that the game continues until the next period.

⇒The probability that the game continue for one million periods is almost zero. However, it the game is played at period t, it continue to the next period with probability δ.

Subgame perfect Nash equilibrium

Consider the prisoner's dilemma game in sheet 3. The following strategies constitute an equilibrium if δ ≧ 1/3

Each player chooses C in period t unless at least one player choses D before period t.

Proof

Suppose that no player takes D before period t.

Given the rival's strategy, if a player follows the strategy above, its payoff is $3/(1 - \delta)$.

If the player deviates form the strategy and takes D, its payoff is $4 + \delta(1 - \delta)$.

$3/(1 - \delta) \geqq 4 + \delta/(1 - \delta) \Leftrightarrow \delta \geqq 1/3$

～If the discount factor is large, each player has an incentive to cooperate.

Infinite Nash Reversion

infinite Nash reversion (grim trigger strategy)

a firm deviates from the collusion

→sever competition (one-shot Nash equilibrium) continues forever

We use this type of strategy

cf Optimal Penal Code

Another equilibrium

The following strategies always constitute an equilibrium

Each player always chooses D

⇒Long-run relationship is not a sufficient condition for cooperation.

Prisoners' Dilemma

2

1

	C	D
C	（4,4）	（0,5）
D	（5,0）	（1,1）

One-Shot Nash Equilibrium : (D,D)

Subgame perfect Nash equilibrium

Consider the prisoner's dilemma game in the previous sheet. The following strategies constitute an equilibrium if $\delta \geqq$ ○

Each player chooses C in period t unless at least one player choses D before period t.

the measure of the difficulty of marinating the collusion

If δ is sufficiently large, collusion is sustainable.

If δ ≧ δ*, then the collusion is sustained in an equilibrium.

→ δ* is a measure of the stability of collusion

The smaller δ* is, the more stable the collusion is.

Grim Trigger Strategy

Let π^D be the one shot profit of the deviator.

Let π^C be the collusive profit of the firm.

Let π^N be the profit of the firm at of one short Nash equilibrium.

If $\pi^C/(1 - \delta) \geqq \pi^D - \pi^C + \delta\, \pi^N/(1 - \delta)$, then the firm has an incentive to maintain collusion.

Thus, $\delta^* =$ ○.

The number of firms and the stability of collusion

Bertrand Oligopoly, n symmetric firms, constant marginal cost

the monopoly price P^M . Under the collusion each firm obtains Π^M/n. Consider the grim trigger strategy.

The collusion is sustainable if and only if $\delta \geqq$ ○.

The number of firms and the stability of collusion

Bertrand Oligopoly, n symmetric firms, constant marginal cost

the monopoly price P^M . Under the collusion each firm obtains Π^M/n. Consider the grim trigger strategy.

$\Pi^M/(n(1 - \delta)) \geqq \Pi^M \Leftrightarrow \delta \geqq (n - 1)/n$

Answer: The larger the number of firms is, the less stable the collusion is.

Neither π^D nor π^N depends on n, while n affects π^C.

The number of firms and the stability of collusion

Cournot Oligopoly, n symmetric firms, constant marginal cost c, the demand is $P = a - Y$, a is sufficiently large.

Question: Derive the monopoly output Q^M and monopoly profit (total profit).

The number of firms and the stability of collusion

Cournot Oligopoly, n symmetric firms, constant marginal cost c, the demand is $P = a - Y$, a is sufficiently large.

Question: Derive the output of the deviator (derive the optimal output of a firm that maximizes one shot profit given that the rivals chooses $q = Q^M/n$).

The number of firms and the stability of collusion

Cournot Oligopoly, n symmetric firms, constant marginal cost c, the demand is $P = a - Y$, a is sufficiently large.

Question: Derive the one shot profit of the deviator given that the rivals chooses $q = Q^M/n$.

The number of firms and the stability of collusion

Cournot Oligopoly, n symmetric firms, constant marginal cost c, the demand is P = a - Y, a is sufficiently large. Consider a grim trigger strategy.

Question: Derive the critical discount factor.

The number of firms and the stability of collusion

An increase of the number of the firms usually increases the deviation incentive（the increase of the profit at one period when it deviates from the collusive behavior)→It instabilizes the collusion

If we consider other models such as Cournot model, an increase of the number of the firms usually reduces the profit at the punishment stage.→It stabilizes the collusion

Usually, the former dominates the latter, so an increase of the number of firms usually (but not always) instabilizes the collusion

The asymmetry between firms and the stability of collusion

Bertrand Duopoly

Collusive price (Monopoly price) is P^M

Under the collusion, firm 1 obtains $\alpha\ \Pi^M$ ($\alpha \geqq 1/2$). If one of two deviates from the collusive pricing, they face Bertrand competition→zero profit.

The conditions under which the collusion is sustainable are

$\alpha\Pi^M/(1 - \delta) \geqq \Pi^M$ and $(1 - \alpha)\Pi^M/(1 - \delta) \geqq \Pi^M \Leftrightarrow \delta \geqq \alpha$

A higher degree of asymmetry instabilizes the collusion.

The asymmetry between firms and the stability of collusion

Symmetric situation → If firm 1 has an incentive to collude, firm 2 also have an incentive to collude.

⇒One condition is sufficient for collusion

Asymmetric situation ⇒ Collusion is sustainable only if both firms have incentive to collude.

Asymmetry usually increases the deviation incentive for one firm and decreases the deviation incentive for another firm→Only the former matters.

⇒Asymmetry instabilizes the collusion.

examples of asymmetry among firms

(1) Unequal distribution of the monopoly profits

(2) Cost difference, capacity difference

(3) Vertical product differentiation

Merger and Stability of Collusion

Merger reduces the number of the firms

→ It stabilizes the collusion.

Merger may increase the asymmetry among firms

→ It may instabilize the collusion.

It is possible the latter effect dominates the former and the merger instabilizes the collusion.

Merger and Stability of Collusion

Before merger

Firm 1~34% market share, Firm 2~33%, Firm 3~33%

After merger

Firm 1'~67% market share, Firm 3~33%

Anti-Trust Department sometimes order to sell some assets to firm 3 so as to reduce the market share of firm 1' and to reduce HHI.

From the viewpoint of preventing the collusion, it is a very bad policy because it reduces the asymmetry of firms and stabilizes the collusion. Compte et al. (2002),

Market Size Expansion and Stability of Collusion

Suppose that the number of firms is given exogenously and it is constant.

Question：The collusion is more stable in (growing, declining) industries.

Market Size Expansion and Stability of Collusion

Suppose that the number of firms is given exogenously and it is constant.

Answer：The collusion is more stable in growing industries.

Future profits is more important in growing industries.

However, in such a market, new entrants will appear

→This instabilizes the collusion.

Business Cycle and Stability of Collusion

The number of firms is constant.

Boom→Recession→Boom→Recession→Boom

Large Demand→Small Demand→ Large Demand →Small Demand → Large Demand

Question: Whether is the collusion more difficult to sustain at larger or at smaller demand period?

Vertical Product Differentiation and Stability of Collusion

Vertical Product Differentiation

～Asymmetry of the firms ~ A further differentiation instabilizes the collusion.

Horizontal Product Differentiation and Stability of Collusion

Horizontal Product Differentiation

Consider the Bertrand Competition

No product differentiation ~ perfect competition

→punishment for the deviation from the collusive behavior is severe.

⇒Product differentiation mitigates competition and instabilizes the collusion because the punishment is less severe？

～This is not always true.

Denekere (1983)

Duopoly, Horizontal Product Differentiation

$P_1 = a - Y_1 - bY_2$ $P_2 = a - Y_2 - bY_1$

If b=1, then two firms produce homogeneous products. A smaller b implies a higher degree of product differentiation.

Cournot: a larger b instabilizes the collusions (higher degree of product differentiation stabilizes the collusion)

Bertrand: Non monotone relationship between b and the stability of collusion.

Chang (1991)

Horizontal Product Differentiation, Hotelling, shopping, duopoly

(1) No product differentiation (Central agglomeration)

→Most severe punishment for deviation is possible

(2) Without product differentiation, a deviator obtains the whole demand by a slight price discount

→Largest deviation incentive

Chang finds that (2) dominates (1)

～the longer the distance between firms is, the more stable the collusion is.

Gupta and Venkatu(2002)

Hotelling, shipping, Bertrand, duopoly

(1) Central agglomeration

→Most severe punishment for deviation is possible

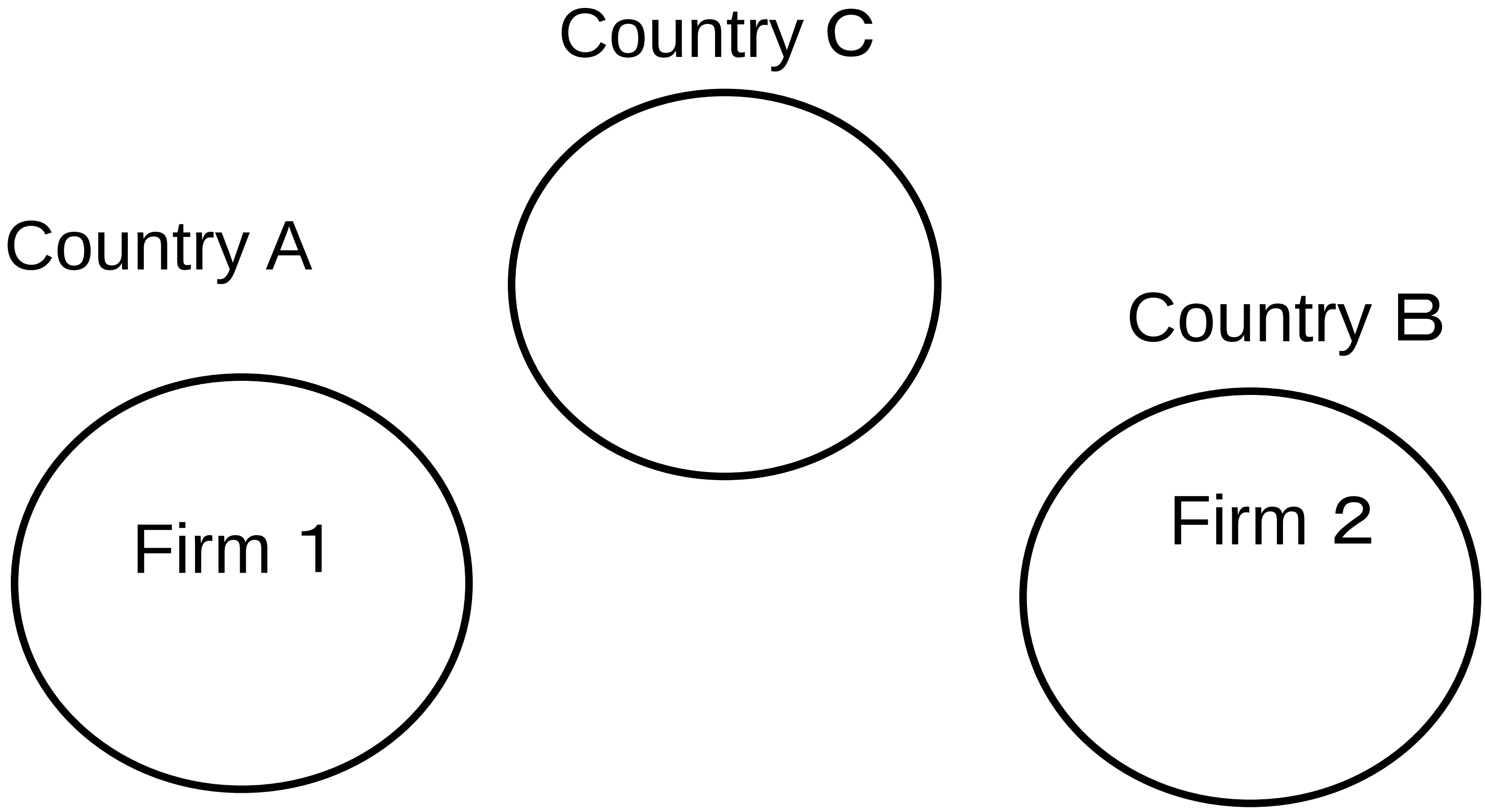
(2) If both firms agglomerate at the central point, a deviator obtains the whole demand and the transport cost is minimized.

→Largest deviation incentive

They find that (1) dominates (2)

～the shorter the distance between firms is , the more stable the collusion is.

Diversification in Duopoly



Country C

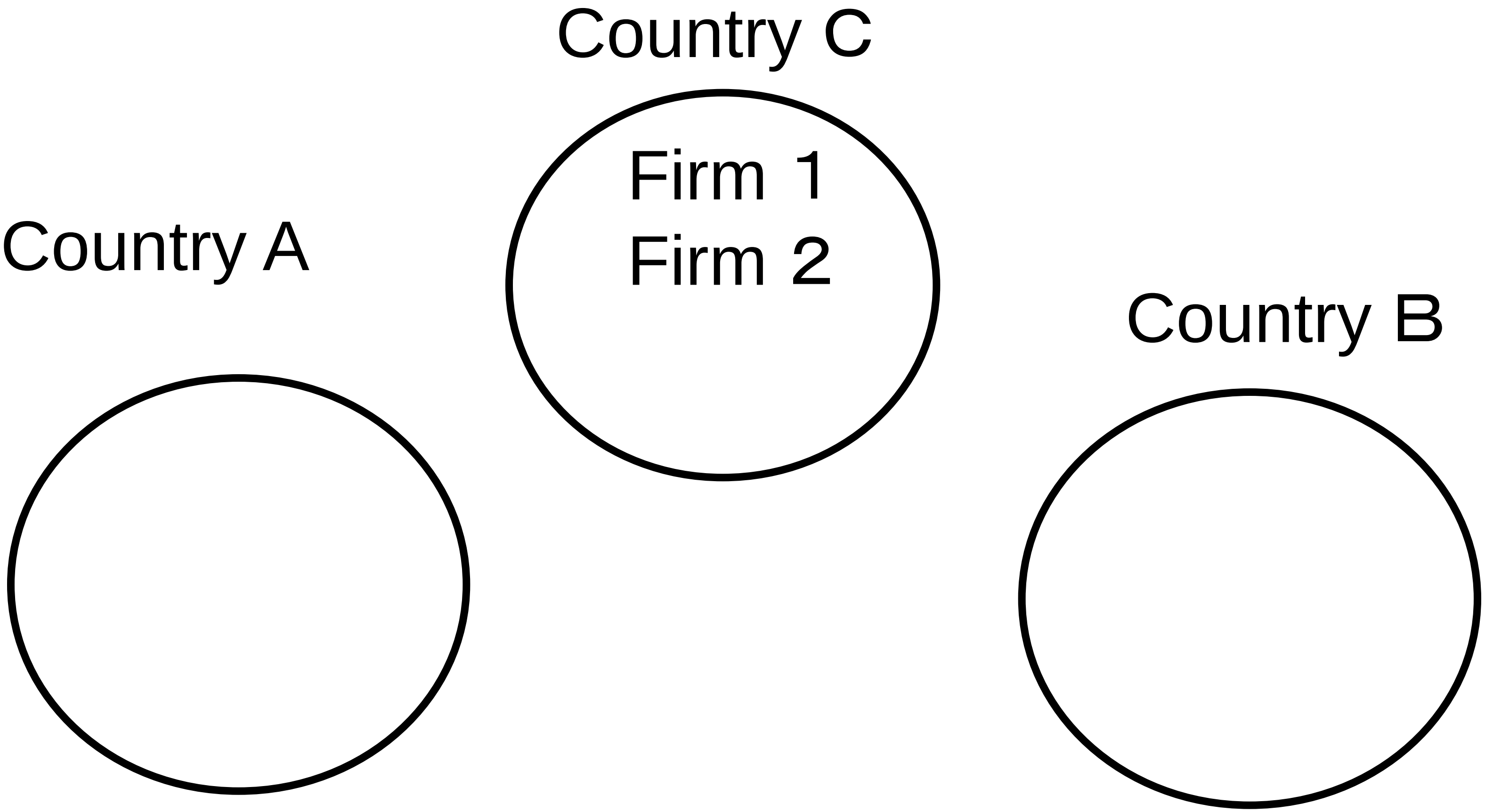
Country A

Country B

Firm 1

Firm 2

Agglomeration in Duopoly



Country C

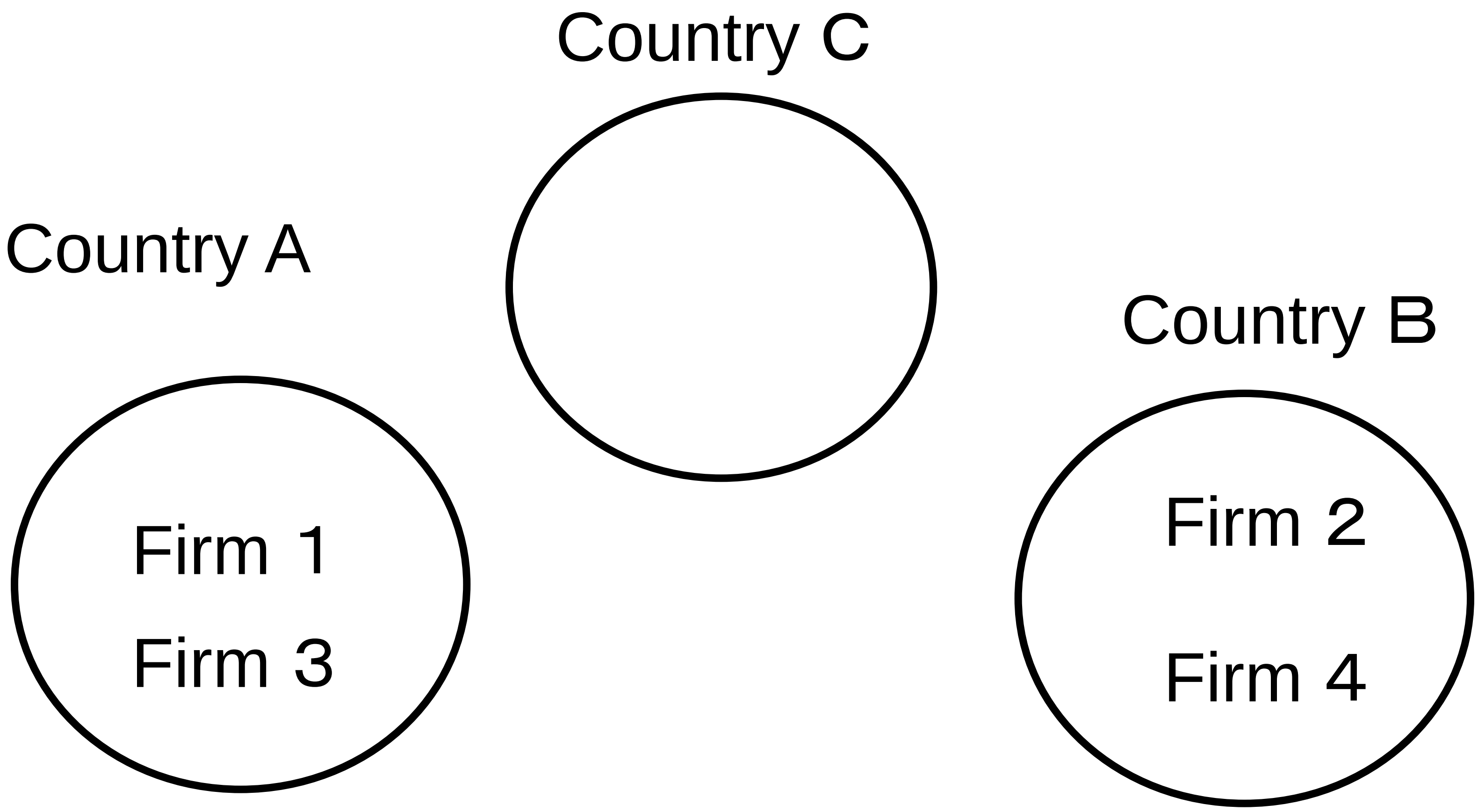
Firm 1

Firm 2

Country A

Country B

Diversification？



Country C

Country A

Firm 1

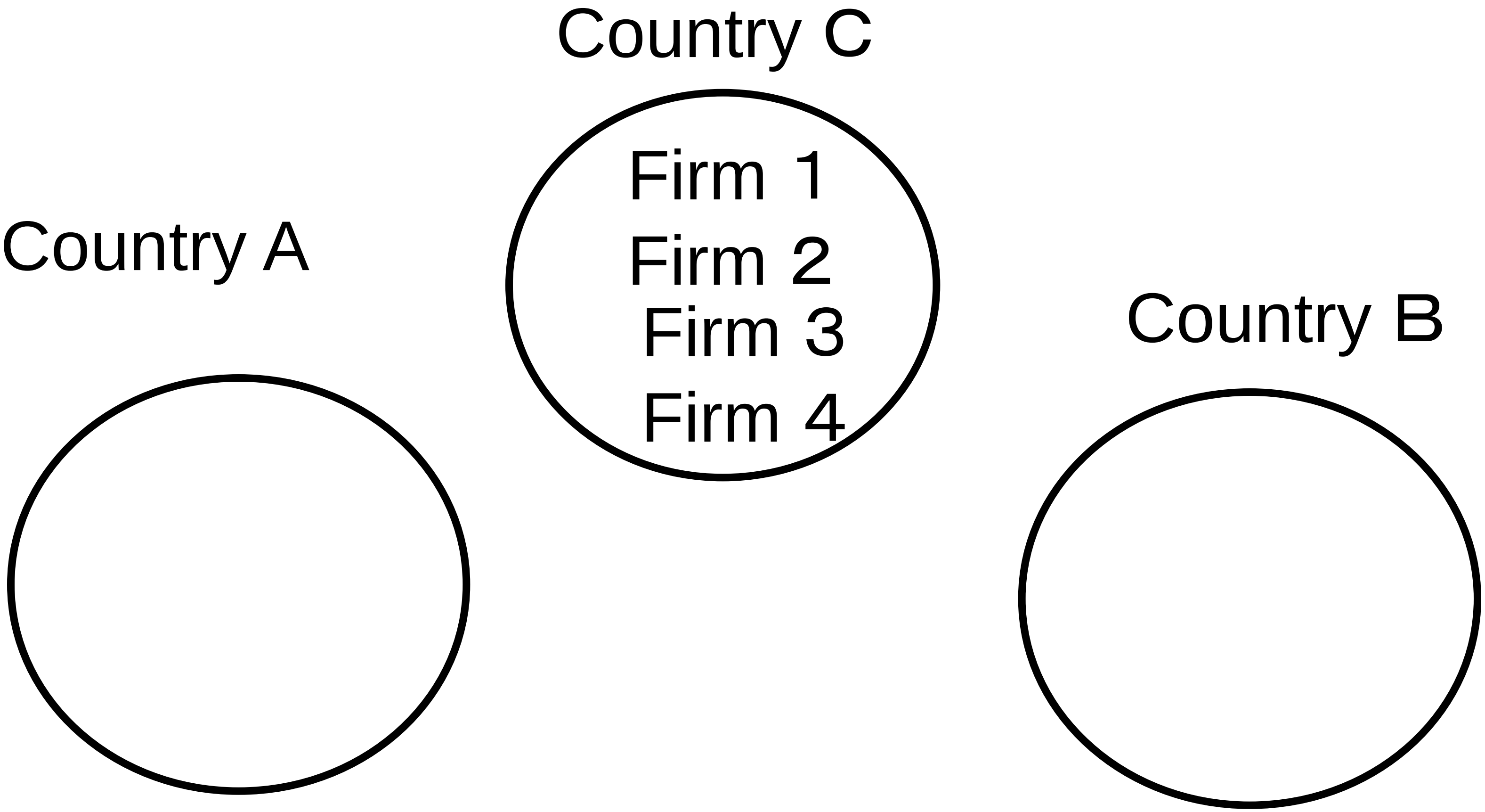
Firm 3

Country B

Firm 2

Firm 4

Agglomeration？



Country C

Firm 1

Firm 2

Firm 3

Firm 4

Country A

Country B

Matsumura and Matsushima(2005)

Hotelling, Salop, shipping, Bertrand

The result that shorter distance stabilizes the collusion holds true only when the number of firms is two or three.

This is because two firms agglomeration is sufficient to strengthen the punishment effect.

Other Trade-Off (1)

Consider a symmetric duopoly in a homogeneous product market. Consider a quantity-setting competition. Suppose that $U_1 = \pi_1 - \alpha\pi_{2.}\ \alpha \in [-1,1]$.

~relative profit maximization approach discussed in 4th lecture.

An increase in α strengthens the punishment effect.

An increase in α increases the deviation incentive.

The latter dominates the former. →An increase in α instabilizes the collusion ~ Matsumura and Matsushima (2012)

Other Trade-Off (2)

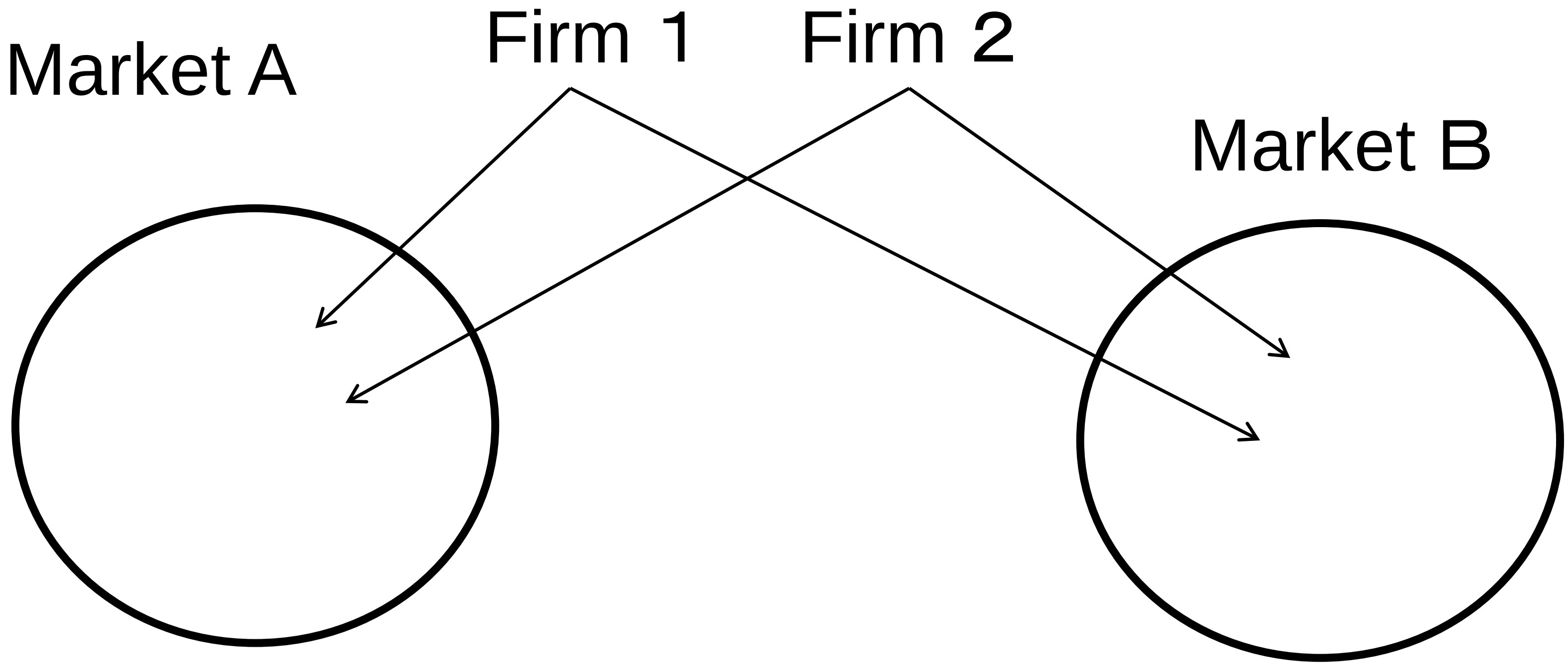
Cross-Licensing strengthens the punishment effect.

Cross-Licensing increases the deviation incentive.

Former dominates (Bertrand）

Latter dominates (Cournot）

Multi-Market Contact



The number of markets and the stability of collusion

Consider the symmetric duopoly. Suppose that two firms compete in n homogeneous markets, where the demand is given by P=f(Y).

Question: Which is correct?

(i) The firm can more easily collude when n is larger.

(ii) The firm can more easily collude when n is smaller.

(iii) n does not affects the stability of collusion.

The number of markets and the stability of collusion

Consider the symmetric duopoly. Suppose that two firms compete in n homogeneous markets, where the demand is given by P=f(Y).

Answer: n does not affects the stability of collusion.

The deviation in one market is punished by the competition in n markets. →an increase in n increases the punishment effect.

The deviator deviates in n market→an increase in n increases the deviation gain.

Two effects are canceled out.

The number of markets and the stability of collusion

If the markets are not homogeneous, it is possible that an increase in n stabilizes the collusion.

Example

(a) One market is in boom, and the other market is in recession.

(b) Firm 1 has an advantage in market a and firm 2 has an advantage in market b.

Bernheim and Whinston (1990)