Full Surplus Extraction and Costless Information Revelation in Dynamic Environments

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Outline

1. **Introduction**
2. Two-Period Example
3. Three-Period Example
4. Model
5. Main Results
6. Conclusion

Short Summary

- In this paper, we study revenue maximizing mechanisms. Especially, we focus on the mechanism which extracts the full surplus.
- We show that full surplus extraction in a dynamic environment is basically '**easier**,' since its complicated structure help the mechanism designer (MD) achieve both efficiency and extraction.

Classical Result

- Under independent value environment, it is impossible to design a mechanism which is
  - incentive compatible,
  - individually rational,
  - extracts the whole surplus.
- On the contrary, we have a chance if the signals agents receive are mutually correlated (not independent).

“Independent Value”

- Agents always share some *common shock factors*.
- Valuations are independent only after these factors are perfectly revealed (Conditional independence).
- If we have conditionally independent values, after revelation of the common shock, we cannot realize full extraction (Myerson 1981, Myerson and Satterthwaite 1988).

Intertemporal Correlation

- However, many real-world allocation problems are dynamic in nature.

- If each agent has a piece of information about *future common shock*, agent i's signal today is correlated with the other agents' signals tomorrow.
  - MD can utilize this feature.
  - cf. Cremer and McLean (1985, 1988)

Closely Related Literature

- DMD: very hot issue! (Licensing problem etc.)
- Efficiency in dynamic environments
  - IPV environments: Athey and Segal (2013), Bergemann and Valimaki (2010)
  - Correlated value environments: Liu (2013)

- This paper's goal: **Efficiency + Extraction**

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Two-Period Example

$t = 1$

f_O θ_1^2

		H	L	
θ_1^1	H	0.25	0.25	0.5
	L	0.25	0.25	0.5
		0.5	0.5	1

$t = 2$

if $\theta_1^1 = \theta_1^2$

f_A θ_2^2

		H	L	
θ_2^1	H	0.64	0.16	0.8
	L	0.16	0.04	0.2
		0.8	0.2	1

otherwise

f_B θ_2^2

		H	L	
θ_2^1	H	0.25	0.25	0.5
	L	0.25	0.25	0.5
		0.5	0.5	1

Periodic Mechanism

- State transition does not depend on allocation.

  → A periodically efficient allocations is also efficient as a whole.

- Distributions are conditionally independent

  → If we divide this problem into two static problems, **surplus extraction is impossible!**

IR and surplus extraction

- Here we simply fix each agent's outside option to zero, but agent can escape from the mechanism **anytime** (*wp-EPIR*).
- The mechanism extracts the full surplus if
  1. allocation is efficient,
  2. the mechanism is individually rational, and
  3. individual rationality binds **in the initial period**. (justified by Bayesian setting and risk-neutrality.)

Intertemporal Correlation

$t = 1$

f_O		θ_1^2		
		H	L	
θ_1^1	H	0.25	0.25	0.5
	L	0.25	0.25	0.5
		0.5	0.5	1

Different!

θ_1^i is informative for $\tilde{\theta}_2^{-i}$!

$t = 2$

if $\theta_1^1 = \theta_1^2$

f_A		θ_2^2		
		H	L	
θ_2^1	H	0.64	0.16	0.8
	L	0.16	0.04	0.2
		0.8	0.2	1

otherwise

f_B		θ_2^2		
		H	L	
θ_2^1	H	0.25	0.25	0.5
	L	0.25	0.25	0.5
		0.5	0.5	1

Intertemporal Correlation

- We can construct a *Cremer-McLean lottery* from this probabilistic structure.
- Assume that $\theta_1^2 = H$ (and agent 1 knows that).
- Then, agent 1's belief over $\tilde{\theta}_2^2$ is

wp-EPIC

		True social state in $t = 2$	
		(H, H)	(L, H)
θ_2^2	H	0.8	0.5
	L	0.2	0.5

Cremer-McLean lottery

- Define $w^1(\theta_1, \theta_2^2)$ as

$w^1((H, H), H) = 1$

$w^1((H, H), L) = -4$

$w^1((L, H), H) = -1$

$w^1((L, H), L) = 1$

True social state in $t = 1$

		(H, H)	(L, H)
θ_1^2	H	0.8	0.5
	L	0.2	0.5

1. Truthful report gives **zero** expected profit.
2. Misreport gives **negative** expected profit.
3. Agent 1's report in $t = 2$ is irrelevant.

ICMM

- Assume that we have private values. Then, **efficiency can be achieved by (iterative) Groves Mechanism** (w/o constant payment).
- Then, agent 1's EPV is equal to expected social welfare $S(\theta_1)$
- Add participation fee to this original mech. Let agent 1's **additional** payment in $t = 1$ be

$$\psi_2^1(\theta_1, \theta_2) = -S(\theta_1) + \alpha \cdot w^1(\theta_1, \theta_2^2)$$

$$\mathbb{E}[w^1(\theta_1, \tilde{\theta}_2^2) | \theta_1] = 0 \quad \mathbb{E}\left[w^1\left((\hat{\theta}_1^1, \theta_1^2), \tilde{\theta}_2^2\right) | \theta_1\right] < 0$$

ICMM (2)

- Taking α sufficiently large, this additional payment does not hurt the incentive structure.
- Total payment = Groves + Additional, so the whole mechanism is also wp-EPIC.
- Moreover, each agent's individual rationality binds in $t = 1 \rightarrow$ Full extraction.
- The remaining part is **individual rationality in $\boldsymbol{t = 2}$**.

Time Inconsistency

- Given θ_1, θ_2, agent 1's on-path payoff from participating this mechanism in period 2 is

$$\underbrace{s(\theta_2) - S(\theta_1)}_{\substack{\| \\ \textbf{Non-negative} \quad s(\theta_1)}} + \underbrace{\alpha \cdot w^1(\theta_1, \theta_2^{-i})}_{\textbf{Possibly negative}}$$

- We give an incentive for truthful report **in $\boldsymbol{t = 1}$** by payment **in $\boldsymbol{t = 2}$** (time inconsistency).
- We must induce a new scheme to prevent agents from escaping.

Deposit Scheme

- As long as the participation constraint in the initial period is satisfied, we can easily keep agents in the mechanism in successive periods.

- The key assumption for this scheme is the **common discount factor**. If MD and agents share the same δ, deposit is irrelevant to both (i) MD's revenue and (ii) agent's EPV.

On-path EPVs

- $t = 1$

cancel out

$$\begin{aligned} & s(\theta_1) - K^i \\ + \;& \mathbb{E}\left[s\left(\tilde{\theta}_2\right) + \alpha \cdot w^1(\theta_1, \tilde{\theta}_2) | \theta_1\right] - S(\theta_1) + K^i \\ = \;& 0 \end{aligned}$$

- $t = 2$

for sufficiently large K^i

$$s(\theta_2) - S(\theta_1) + \alpha \cdot w^1(\theta_1, \theta_2) + K^i > 0$$

- This mechanism satisfies wp-EPIR.

Remarks

- We can also use the Cremer-McLean lottery to achieve efficiency (cf. Liu 2013).
- This mechanism satisfies wp-EPIC and wp-EPIR → More desirable than interim ones!
- wp-EPIC, wp-EPIC → **intratemporal probabilistic structure does not matter!** (We can apply this, but not limited to conditionally independent environments.)

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Rank Condition

- The previous two-period example indicates that a sufficient condition to construct a Cremer-McLean lottery for period t if θ_t^i and θ_{t+1}^{-i} are correlated given for any θ_t^{-i}.

- Indeed, **the rank condition needed in a dynamic environment is much weaker** than this. (See next Three-Period Example!)

Three-Period Example

$t = 1$

θ_1^2

θ_1^1

	H	L	
H	0.25	0.25	0.5
L	0.25	0.25	0.5
	0.5	0.5	1

$t = 2$

if $\theta_1 = (H, H)$ θ_2^2

θ_2^1

	H	L	
H	0.49	0.21	0.7
L	0.21	0.09	0.3
	0.7	0.3	1

if $\theta_1 = (H, L)$ θ_2^2

θ_2^1

	H	L	
H	0.21	0.49	0.7
L	0.09	0.21	0.3
	0.3	0.7	1

if $\theta_1 = (L, H)$ θ_2^2

θ_2^1

	H	L	
H	0.21	0.09	0.3
L	0.49	0.21	0.7
	0.7	0.3	1

if $\theta_1 = (L, L)$ θ_2^2

θ_2^1

	H	L	
H	0.09	0.21	0.3
L	0.21	0.49	0.7
	0.3	0.7	1

$t = 3$

if $\theta_2^1 = \theta_2^2$ θ_3^2

θ_3^1

	H	L	
H	0.64	0.16	0.8
L	0.16	0.04	0.2
	0.8	0.2	1

if $\theta_2^1 \neq \theta_2^1$ θ_3^2

θ_3^1

	H	L	
H	0.25	0.25	0.5
L	0.25	0.25	0.5
	0.5	0.5	1

Three-Period Example (2)

- B/w period 1 and 2, state distributions are truly independent (the distribution of θ_2^i only depends on θ_1^i).
- θ_1^i is not informative for θ_1^{-i} nor θ_2^{-i} → Surplus extraction is impossible if this game ends in period 2.

$t = 1$

Prob. vector of $\tilde{\theta}_1^i$

$$= (0.5, 0.5)$$

$t = 2$

Prob. vector of $\tilde{\theta}_2^i$

$$= \begin{cases} (0.7, 0.3) \text{ if } \theta_1^i = H \\ (0.3, 0.7) \text{ if } \theta_1^i = L \end{cases}$$

$t = 3$

Prob. vector of $\tilde{\theta}_3^i$

$$= \begin{cases} (0.8, 0.2) \text{ if } \theta_2^1 = \theta_2^2 \\ (0.5, 0.5) \text{ if } \theta_2^1 \neq \theta_2^2 \end{cases}$$

Three-Period Example (3)

- However, θ_1^i is correlated with θ_2^i!

- If MD can prevent agent i's **contingent deviation**, it is possible to construct a Cremer-McLean lottery from this structure.

$t = 1$

Prob. vector of $\tilde{\theta}_1^i$

$$= (0.5, 0.5)$$

$t = 2$

Prob. vector of $\tilde{\theta}_2^i$

$$= \begin{cases} (0.7, 0.3) & \text{if } \tilde{\theta}_1^i = H \\ (0.3, 0.7) & \text{if } \tilde{\theta}_1^i = L \end{cases}$$

$t = 3$

Prob. vector of $\tilde{\theta}_3^i$

$$= \begin{cases} (0.8, 0.2) & \text{if } \tilde{\theta}_2^1 = \tilde{\theta}_2^2 \\ (0.5, 0.5) & \text{if } \tilde{\theta}_2^1 \neq \tilde{\theta}_2^2 \end{cases}$$

Insights from Examples

- In order to extract the full surplus, we have to
  1. **implement the efficient allocation**, and
  2. **reveal the private signal in the initial state costlessly**, i.e., construct a Cremer-McLean lottery for the initial report.

Insights from Examples (2)

- We can unveil θ_t^i without leaving information rent if there exists $T = \{t, t+1, \dots, \bar{t}\}$ s.t.

  i. For all $\tau \in T$, given for any θ_τ^{-i}, θ_τ^i is informative for $\theta_{\tau+1} = (\theta_{\tau+1}^i, \theta_{\tau+1}^{-i})$.

  ii. Given for any $\theta_{\bar{t}}^{-i}$, $\theta_{\bar{t}}^i$ is informative for $\theta_{\bar{t}+1}^{-i}$.

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Model (1)

- Finitely many agents $i \in \{1,2,\ldots,I\}$
- Finite or Infinite time horizon $t \in \mathbb{Z}_+$
- Finite and Discrete state space

  $\theta_t^0 \in \Theta_t^0$: public state in period t

  $\theta_t^i \in \Theta_t^i$: i's private state in period t
- $\theta_t = (\theta_t^0, \theta_t^1, \ldots, \theta_t^I)$: state profile in period t
- $x_t \in X_t$ MD's allocation decision in t

Model (2)

- $\mu_{t+1}: X_t \times \Theta_\mathrm{t} \to \Delta(\Theta_\mathrm{t})$ state transition function
  $\mu_{t+1}(x_t, \theta_t)$ denotes θ_{t+1}'s state distribution.
- $v_t^i(x_t, \theta_t)$ agent i's flow valuation function.
- $\delta \in (0,1)$ common discount factor
- $y_t^i \in \mathbb{R}$ monetary transfer to i in t
- Agent payoff is given by

$$\sum_t [\delta^t v_t^i(x_t, \theta_t) + y_t^i]$$

Mechanism

- $\chi_t(\theta_t)$: allocation rule in t

  By Markov formulation, there exists an efficient Markov allocation rule. (hereafter we focus on it.)

- $\psi_t^i(\theta_0, \theta_1, \dots, \theta_t)$: payment rule in t

  We write the history of reports explicitly here.

  $\boldsymbol{\theta_s^t} = (\theta_s, \theta_{s+1}, \dots, \theta_t)$: sequence of reports

  $\psi_t = (\psi_t^1, \dots, \psi_t^I)$

- The mechanism is $(\chi, \psi) = (\chi_t, \psi_t)_{t=0}^{\infty}$

EPVs

- EPV from valuation

$$V_t^i(\theta_t) := \mathbb{E}\left[\sum_{\tau=t}^{\infty} \delta^\tau [v^i(\chi_\tau(\tilde{\theta}_\tau), \tilde{\theta}_\tau)] \middle| \theta_t \right]$$

- EPV from payment

$$\Psi^i(\boldsymbol{\theta}^t) := \mathbb{E}\left[\sum_{\tau=t}^{\infty} \delta^\tau [\psi^i(\tilde{\boldsymbol{\theta}}^\tau)] \middle| \boldsymbol{\theta}^t \right]$$

- Total EPV

$$U_t^i(\boldsymbol{\theta}^t) := V_t^i(\theta_t) + \Psi_t^i(\boldsymbol{\theta}^t)$$

wp-EPIC and wp-EPIR

- (χ, ψ) is wp-EPIC if $\forall i, \forall t, \forall \theta_t, \forall \boldsymbol{\theta}^{t-1}, \forall \hat{\theta}_t^i$

$$U_t^i(\boldsymbol{\theta}^t) \geq v_t^i(\chi(\hat{\theta}_t^i, \theta_t^{-i}), \theta_t) + \psi_t^i(\boldsymbol{\theta}^{t-1}, (\hat{\theta}_t^i, \theta_t^{-i})) \\ + \delta \mathbb{E}\left[U_t^i(\boldsymbol{\theta}^{t-1}, (\hat{\theta}_t^i, \theta_t^{-i}), \tilde{\theta}_{t+1}) \middle| \chi_t(\hat{\theta}_t^i, \theta_t^{-i}), \theta_t \right]$$

($\boldsymbol{\theta}^{t-1}$: seq. of reported state profiles (not true realization))

- $O^i: \Theta \to \mathbb{R}$ agent i's outside option
- (χ, ψ) is wp-EPIR if $\forall i, \forall t, \forall \theta_t, \forall$truthful $\boldsymbol{\theta}^{t-1}$,

$$U_t^i(\boldsymbol{\theta}^t) \geq O_t^i(\theta_t)$$

Revenue

- MD commits a (χ, ψ) ex ante.

  → (χ, ψ) maximizes ex ante expected revenue.

$$R := -\mathbb{E}\left[\sum_{i \in \mathcal{I}} \Psi_0^i(\tilde{\theta}_0)\right]$$

- Full Surplus Extraction iff

  i. χ is the efficient allocation rule,

  ii. $$R = \mathbb{E}\left[\sum_{i \in \mathcal{I}} [V_0^i(\tilde{\theta}_0) - O_0^i(\tilde{\theta}_0)]\right]$$

Outline

(Piecewise) Full Rankness

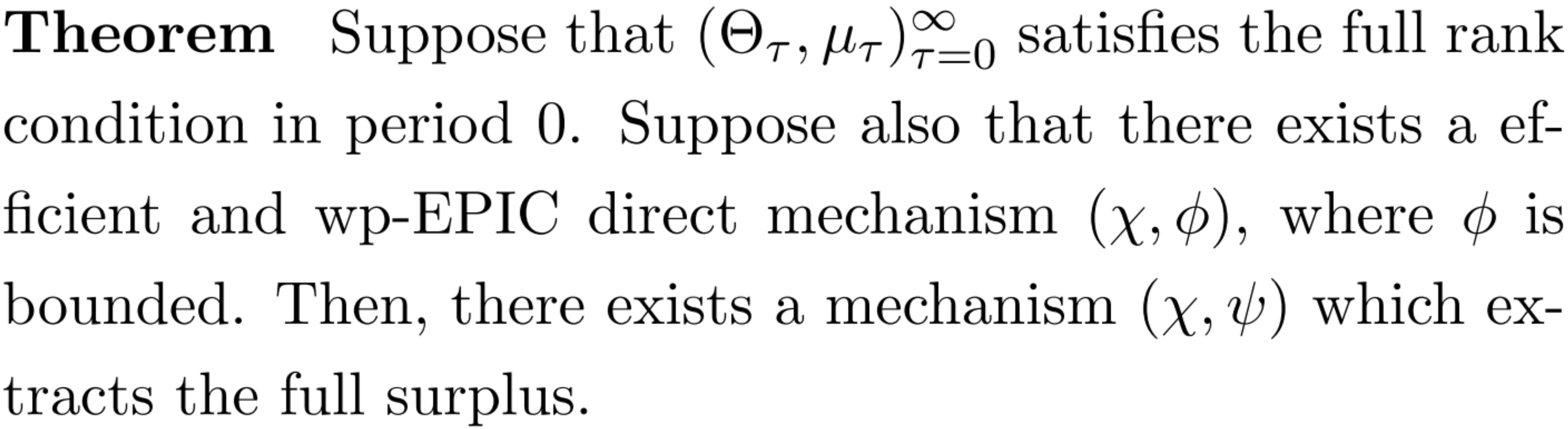
Definition (Intertemporal Full Rankness) $(\Theta_\tau, \mu_\tau)_{\tau=0}^{\infty}$ satisfies the *full rank condition in* t if one of the following conditions are satisfied.

(i) For any i, $x_t \in X_t$, and $\theta_t^{-i} \in \Theta_t^{-i}$, $\{\mu_{t+1}^{-i}(x_t, (\theta_t^i, \theta_t^{-i}))\}_{\theta_t^i \in \Theta_t^i}$ are linearly independent.

(ii) For any i, $x_t \in X_t$, and θ_t^{-i}, $\{\mu_{t+1}(x_t, (\theta_t^i, \theta_t^{-i}))\}_{\theta_t^i \in \Theta_t^i}$ are linearly independent and $(\Theta_\tau, \mu_\tau)_{\tau=0}^{\infty}$ satisfies the full rank condition in $t+1$.

- Parallel to the condition introduced in examples.

Main Result (1)



Theorem Suppose that $(\Theta_\tau, \mu_\tau)_{\tau=0}^{\infty}$ satisfies the full rank condition in period 0. Suppose also that there exists a efficient and wp-EPIC direct mechanism (χ, ϕ), where ϕ is bounded. Then, there exists a mechanism (χ, ψ) which extracts the full surplus.

- Costless revelation of θ_0 is crucial.

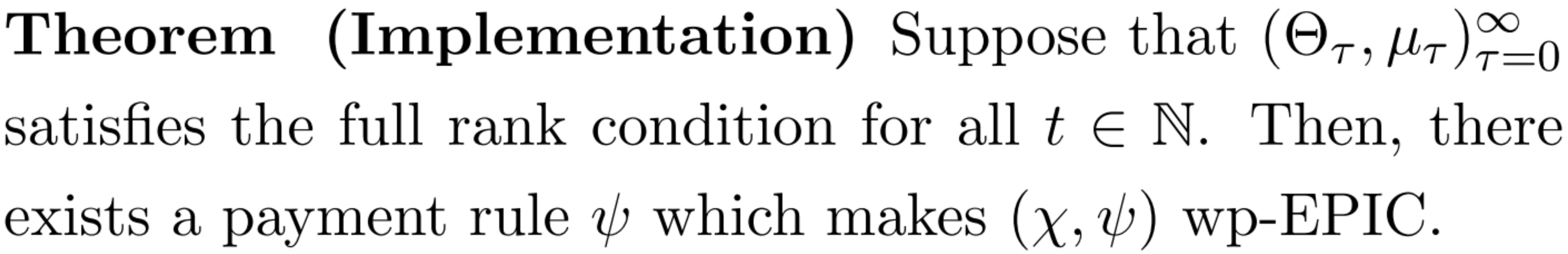
Main Result (2)

Theorem Suppose that $(\Theta_\mathcal{T}, \mu_\mathcal{T})_{\mathcal{T}=0}^{\infty}$ satisfies the full rank condition in period 0. Then, for any $((v_t^i, O_t^i)_{i\in\mathcal{I}})_{t=0}^{\infty}$ s.t. (v_t^i) satisfies private value assumption, there exists a mechanism (χ, ψ) which extracts the full surplus.

- Efficiency: Dynamic Groves mechanism suggested by Athey and Segal (2013) (available if private values)
- Make wp-EPIR binds by Cremer-McLean method.

Main Result (3)

- If the full rank condition is satisfied throughout the time horizon, it also guarantees implementability of the efficient allocation.



Theorem (Implementation) Suppose that $(\Theta_\tau, \mu_\tau)_{\tau=0}^{\infty}$ satisfies the full rank condition for all $t \in \mathbb{N}$. Then, there exists a payment rule ψ which makes (χ, ψ) wp-EPIC.

- However, to sustain wp-EPIR, we need boundedness of the payment rule.

Uniform Full Rankness

Definition (Uniform Intertemporal Full Rankness) Let ρ be the Euclidean distance. $(\Theta_t, \mu_t)_{t=0}^{\infty}$ satisfies the *uniform full rank condition* if there exists $k > 0$ s.t.

(i) there exists $T \in \mathbb{N}$ s.t. for all $t \in \mathbb{N}$, there exists $t \leq t^* \leq T$ s.t. for all $x_{t^*} \in X_{t^*}$ and $\theta_{t^*} \in \Theta_{t^*}$,

$$\rho\left(\mu_{t^*+1}^{-i}(x_{t^*}, \theta_{t^*}), \mathrm{conv}\{\mu_{t^*+1}^{-i}(x_{t^*}, (\hat{\theta}_{t^*}^{i}, \theta_{t^*}^{-i}))\}_{\hat{\theta}_{t^*}^{i} \in \Theta_{t^*}^{i} \setminus \{\theta_{t^*}^{i}\}}\right) > k$$

(ii) for all $t \in \mathbb{N}$, $x_t \in X_t$, $\theta_t \in \Theta_t$

$$\rho\left(\mu_{t+1}(x_t, \theta_t), \mathrm{conv}\{\mu_{t+1}(x_t, (\hat{\theta}_t^{i}, \theta_t^{-i}))\}_{\hat{\theta}_t^{i} \in \Theta_t^{i} \setminus \{\theta_t^{i}\}}\right) > k$$

Main Result (4)

Theorem Suppose that we have the uniform full rank condition. Then full surplus extraction is guaranteed, i.e., for any $((v_t^i)_{i\in\mathcal{I}})_{t=0}^{\infty}$ and $((O_t^i)_{i\in\mathcal{I}})_{t=0}^{\infty}$, there exists a direct mechanism which is wp-EPIC, wp-EPIR and extract the full surplus.

- Uniform full rankness excludes "asymptotically short rank environment."

Outline

Conclusion

- In this paper, we investigate revenue maximizing mechanisms in dynamic environments.

- We show the sufficient condition for full surplus extraction, and construct a mechanism which extracts the whole surplus and works under generic environments.

Conclusion (2)

- This result gives us clear insight about the mechanism design in dynamic environments.

- Even under the environment where static models and mechanisms are applicable, the mechanism designer can increase his revenue by utilizing the problem’s dynamic nature.

Conclusion (3)

- ICMM also has some desirable property.
- It is robust to information leakage (wp-EPIC and wp-EPIR).

- Prediction for “common shock” seems rather realistic compared to the other agents’ valuations at the same time.

Conclusion (4)

- Our setting satisfies Markov property and each agent's belief is affected only by the latest reports.
- However, if the MD can benefit from a relationship between far distant periods.
- This result is quite new and insightful for dynamic mechanism design problems.