

# The Welfare Effects of Third-Degree Price Discrimination in a Differentiated Oligopoly

Takanori Adachi      Noriaki Matsushima  
School of Economics      ISER  
Nagoya University      Osaka University

November 17, 2010

# 1. Introduction

## ■ Examples of 3rd-Degree PD (final products):

- Movie Tickets (discounts to students, senior citizens)
- Pharmaceutical Products (in the international market)
- Computer Software (“academic discounts”)

.....

## ■ Oligopolistic Competition

## ■ Price Discrimination ← Price Control

## ■ One Source: (Horizontal) Product Differentiation

# 1. Introduction

## ■ Research Question:

- Welfare Consequences of 3rd-Degree PD
- Social Welfare under PD  
    > Social Welfare under UP (Uniform Pricing)?
- Under Oligopolistic Competition
- Horizontal Product Differentiation

# 1. Introduction

## ■ Main Results:

- PD can improve SW (especially) if firms' brands are *substitutes* in the “*strong*” market, and are *complements* in the “*weak*” market, but it never improves vice versa.
- Consumer surplus never improves by PD.
- “Prisoners’ Dilemma” may arise if  $\Delta Q < 0$  (conjecture)
- Competition due to strong substitutability  
→ Bad for PD to improve SW

# Road Map

- 2. Related Literature
- 3. Model
- 4. Welfare Analysis

## 2. Related Literature

### ■ *Can 3rd-Degree PD Improve Social Welfare?*

- In the case of Monopoly, well understood:

- *SW never improves* if it doesn't increase aggregate output.  
(Robinson ('33),..., Schmalensee ('81), Varian ('85))

- (inefficiency from interconsumer misallocations should be offset)

- *SW may improve* even if it doesn't increase aggregate output  
in the presence of consumption externalities. (Adachi ('02, '05))

- As Stole ('08, Handbook of IO) points out, less is known in the case of

- Oligopoly,.....

## 2. Related Literature

### ■ Oligopoly

- Holmes ('89, AER)

Symmetry

Decomposition of Equilibrium Price Elasticities

- Corts ('98, RAND)

Asymmetry

Unambiguous cases of welfare improvement

- Dastidar ('06, Manchester)

Symmetry

Not necessarily  $\Delta Q = 0$  even in the case of linear demands

## 2. Related Literature

### ■ Potential Sources in Efficiency

- 1. Aggregate output over all markets is too low if prices exceed marginal cost.
- 2. For a given level of aggregate output, PD typically generates interconsumer misallocations relative to uniform pricing.
  - Aggregate output is not efficiently distributed to the highest-value ends.
- 3. Additional interconsumer misallocations caused by strategic interaction

■ Our model allows a simple and natural result on 3.



# Road Map

- 2. Related Literature
- **3. Model**
- **4. Welfare Analysis**

### 3. Model

- Price-setting firms (not Quantity-setting)

- Categorize:

$$\left\{ \begin{array}{l} \text{“Strong” markets: } \{m | p^u < p_m\} \\ \text{“Weak” markets: } \{m | p^u > p_m\} \end{array} \right.$$

- Restrict attention to the symmetric case:

All firm agree in their ranking of strong markets & weak markets.

- Assume further symmetry across firms:

Everything is symmetric across firms...

### 3. Model

#### ■ Product Differentiation

- (Chamberlin-Robinson approach)
- Representative Consumer's Utility:

$$U_m(q_m^A, q_m^B) \equiv \alpha_m \cdot (q_m^A + q_m^B) - \frac{1}{2} \left( \beta_m [q_m^A]^2 + 2\gamma_m q_m^A q_m^B + \beta_m [q_m^B]^2 \right)$$

- *Substitutes*:  $\gamma_m > 0$
- *Complements*:  $\gamma_m < 0$

### 3. Model

- Demand functions in market  $m$ :

$$\left\{ \begin{array}{l} q_m^A(p_m^A, p_m^B) = \frac{\alpha_m}{\beta_m + \gamma_m} - \frac{\beta_m}{\beta_m^2 - \gamma_m^2} p_m^A + \frac{\gamma_m}{\beta_m^2 - \gamma_m^2} p_m^B \\ q_m^B(p_m^A, p_m^B) = \frac{\alpha_m}{\beta_m + \gamma_m} + \frac{\gamma_m}{\beta_m^2 - \gamma_m^2} p_m^A - \frac{\beta_m}{\beta_m^2 - \gamma_m^2} p_m^B \end{array} \right.$$

- Normalization: common constant marginal cost = 0
- $\alpha_s/\alpha_w \in (\underline{\alpha_s/\alpha_w}, \overline{\alpha_s/\alpha_w})$

(Relative) size of the weak market should be sufficiently *small*

for neither firm to have an incentive to deviate to closing the weak market,

and be also sufficiently *large*

for the weak market to be open under uniform pricing.

# Road Map

■ 2. Related Literature

■ 3. Model

■ **4. Welfare Analysis**

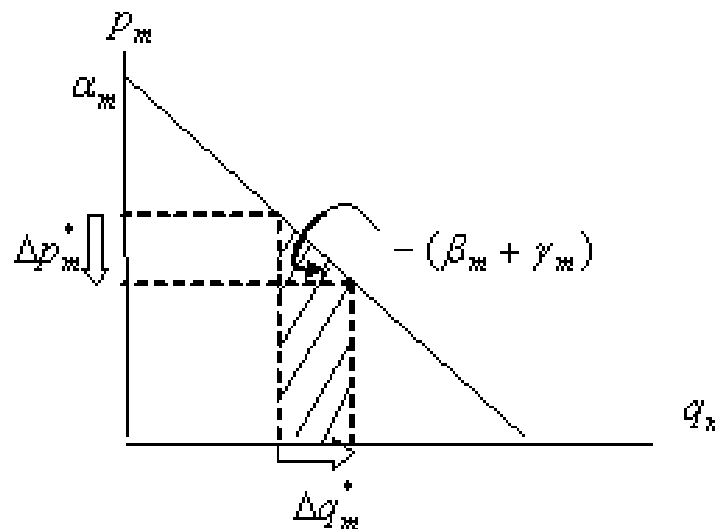
## 4. Welfare Analysis

### ■ Analytical Properties

**Proposition 1.** *Equilibrium differences in social welfare is given by*

$$\Delta SW^*(\gamma, \alpha, \beta) = - \sum_{m \in \{s, w\}} \frac{\Delta p_m^*}{\beta_m + \gamma_m} \cdot (p_m^* + p^*)$$

where  $\Delta p_m^* \equiv p_m^* + p^*$ .



## 4. Welfare Analysis

**Lemma 1.** *Equilibrium price elasticity of demand in market  $m$  in equilibrium is given by*

$$\varepsilon_m(p_m^*) = \underbrace{1}_{\text{market elasticity}} + \underbrace{\left(-\frac{\gamma_m}{\beta_m}\right)}_{\text{cross-price elasticity}}.$$

- Special case of Holmes ('89)
- Product Differentiation → Strategic Interaction
- Cross-price elasticity: how much each firm “steals” from the other firm *in equilibrium*.
- $\varepsilon_m(p_m^*) \underset{>}{\leq} 1$  if and only if  $\gamma_m \underset{<}{\geq} 0$ .

## 4. Welfare Analysis

**Proposition 2.**  $\Delta Q^* \leq 0 \Rightarrow \Delta SW^* < 0$ .

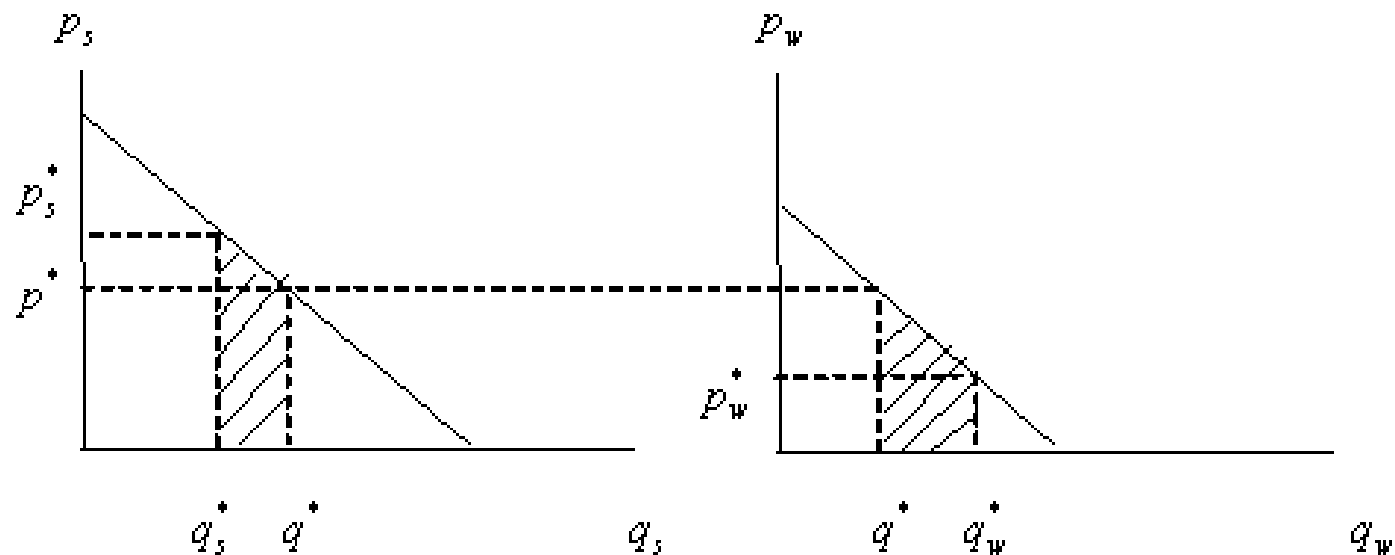
- Same as Monopoly
- $\Delta Q^* \begin{matrix} \geq \\ < \end{matrix} 0 \Leftrightarrow$  if and only if  $\frac{\gamma_s}{\beta_s} \begin{matrix} \geq \\ < \end{matrix} \frac{\gamma_w}{\beta_w}$ .
- Also, a special case of Holmes ('89)



## 4. Welfare Analysis

### ■ Welfare-improving price discrimination

- Reduce the number of parameters:  $\alpha_s = 1 > \alpha_w > 0$  (necessary for social welfare to improve)
- $\Delta SW^* > 0 \Leftrightarrow \Delta q_w^* \cdot (p^* + p_w^*) > \Delta q_s^* \cdot (p^* + p_s^*)$



## 4. Welfare Analysis

### ■ Case of Symmetric Product Differentiation ( $\gamma_s/\beta_s = \gamma_w/\beta_w$ )

**Proposition 3.** *In the case of symmetric product differentiation,*

$$\Delta SW^* < 0.$$

## 4. Welfare Analysis

### ■ Case of Asymmetric Product Differentiation ( $\gamma_s/\beta_s \neq \gamma_w/\beta_w$ )

- (1)  $\gamma_m$  is common:  $\gamma \equiv \gamma_s = \gamma_w$
- (2)  $\beta_m$  is common:  $\beta \equiv \beta_s = \beta_w$

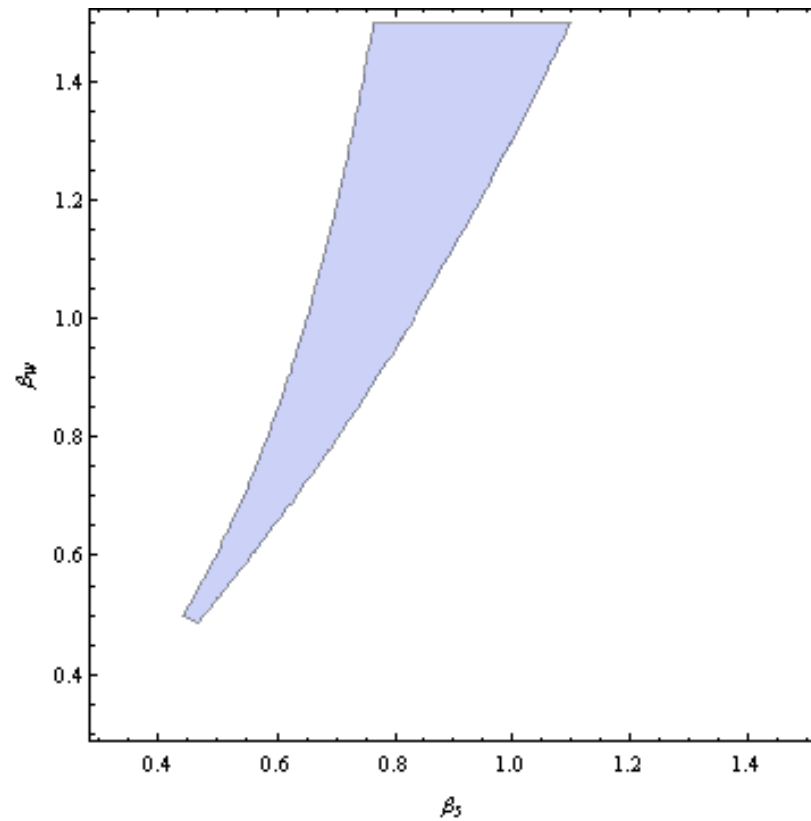
## 4. Welfare Analysis

- (1)  $\gamma_m$  is common ( $\alpha_w = 0.85$ )

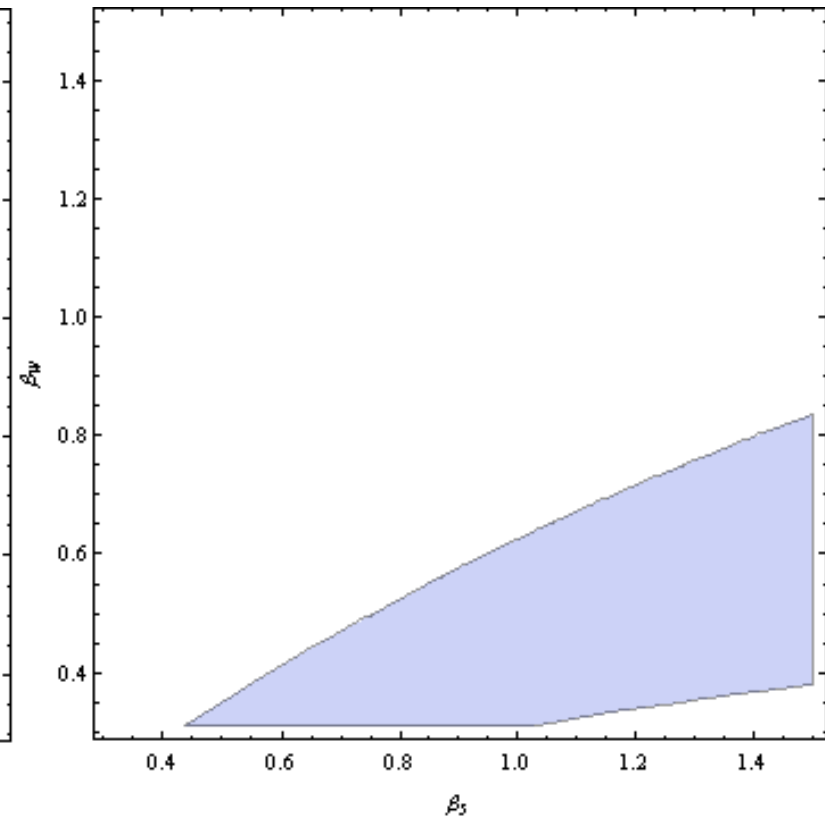
	$(\gamma, \beta_s, \beta_w) =$			
	$(0.3, 1.0, 0.75)$	$(0.3, 0.75, 1.0)$	$(-0.3, 1.0, 0.75)$	$(-0.3, 0.75, 1.0)$
$p^*$	0.3582	0.3644	0.5235	0.5423
$p_s^* (\Delta p_s^*/p^*)$	0.4118 (15%)	0.3750 (3%)	0.5652 (8%)	0.5833 (8%)
$p_w^* (\Delta p_w^*/p^*)$	0.3188 (-11%)	0.3500 (-4%)	0.4958 (-5%)	0.4804 (-11%)
$\Delta q_s^* (\Delta q_s^*/q_s^*(p^*))$	-0.9412 (-8%)	-0.0101 (-2%)	-0.0596 (-9%)	-0.0912 (-9%)
$\Delta q_w^* (\Delta q_w^*/q_w^*(p^*))$	0.0375 (8%)	0.0111 (3%)	0.0615 (8%)	0.0884 (20%)
$\Delta SW^*$	-0.0063	0.0005	-0.0022	-0.0123
$\Delta CS_s^*$	-0.0507	-0.0127	-0.0543	-0.0797
$\Delta CS_w^*$	0.0384	0.0109	0.0419	0.0598
$\Delta \Pi^*$	0.0060	0.0023	0.0102	0.0076
$\Delta Q^*$	-0.0037	0.0009	0.0019	-0.0028

## 4. Welfare Analysis

- (1)  $\gamma_m$  is common ( $\alpha_w = 0.85$ )



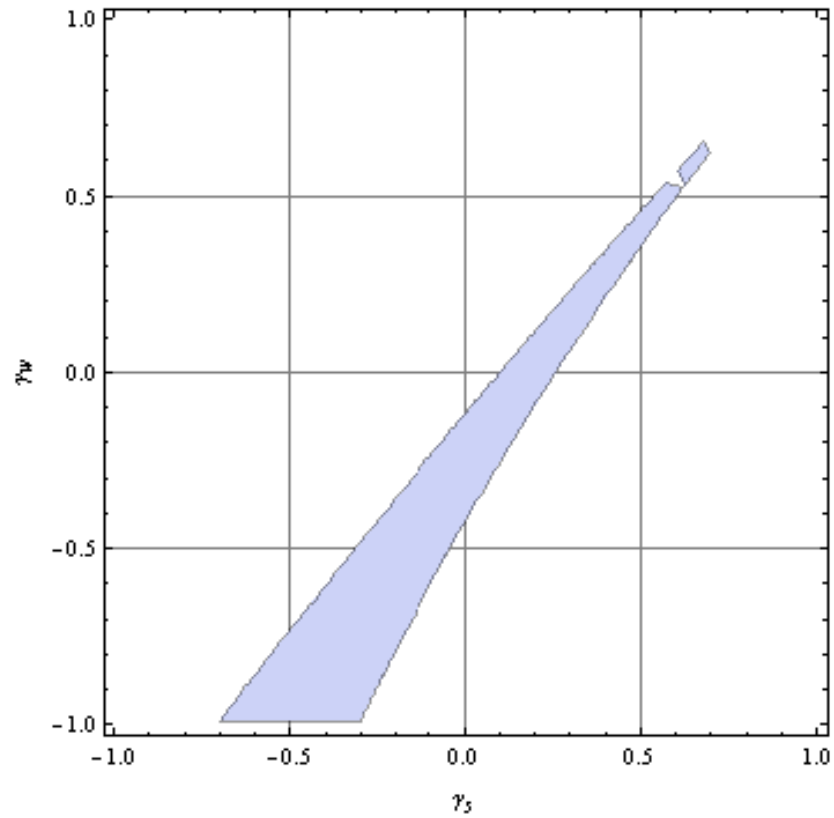
Substitutability ( $\gamma = 0.3$ )



Complementarity ( $\gamma = -0.3$ )

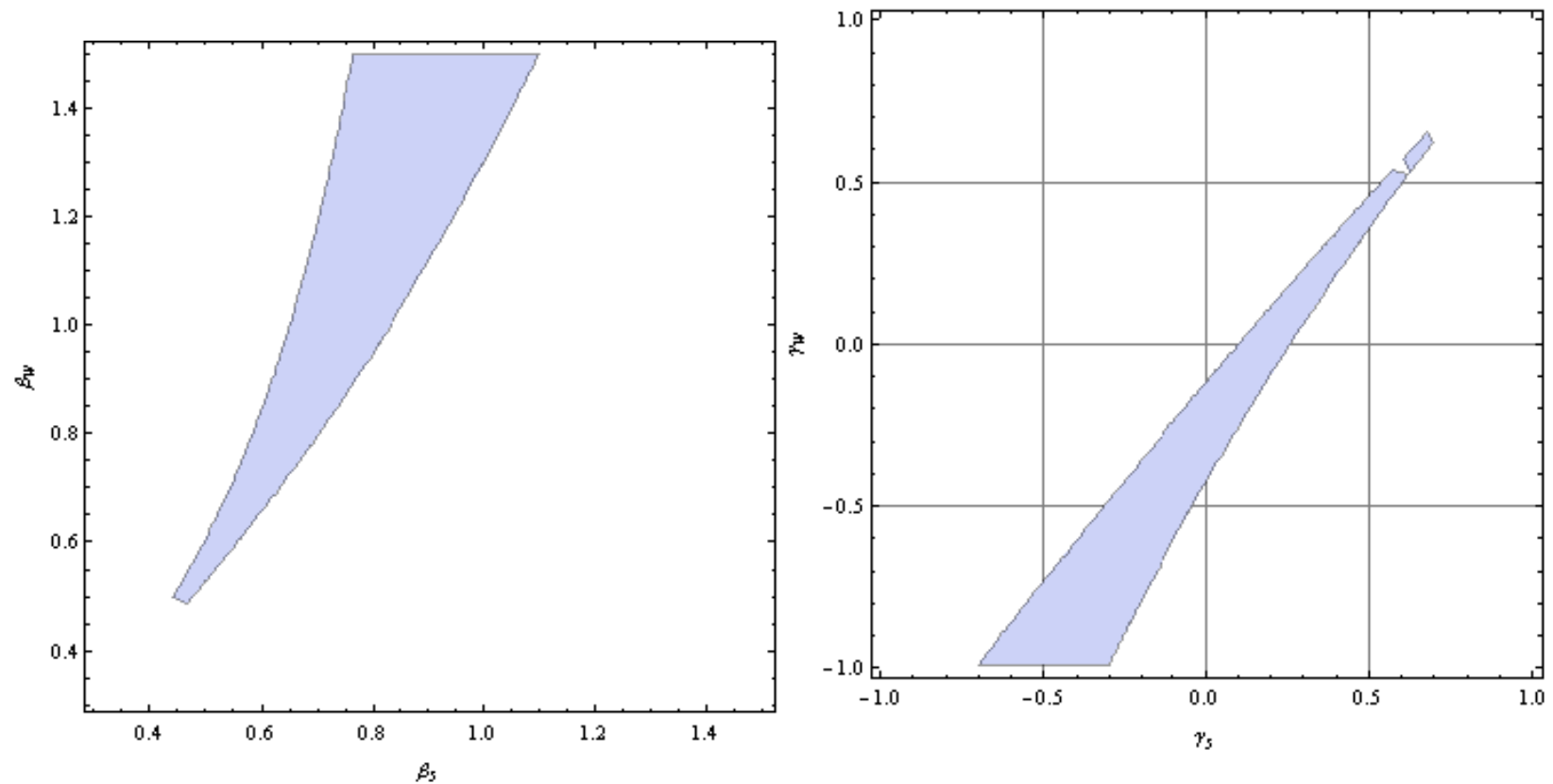
## 4. Welfare Analysis

- (2)  $\beta_m$  is common ( $\alpha_w = 0.85$  and  $\beta = 1.0$ )



## 4. Welfare Analysis

- (2)  $\beta_m$  is common ( $\alpha_w = 0.85$  and  $\beta = 1.0$ )
- Closer to Perfect Substitutes...



## 4. Welfare Analysis

- (2)  $\beta_m$  is common ( $\alpha_w = 0.85$  and  $\beta = 1.0$ )

	$(\gamma_s, \gamma_w) =$	
	$(0.1, -0.1)$	$(-0.1, 0.1)$
$p^*$	0.4588	0.4663
$p_s^* (\Delta p_s^*/p^*)$	0.4737 (3%)	0.5238 (12%)
$p_w^* (\Delta p_w^*/p^*)$	0.4452 (-3%)	0.4026 (-14%)
$\Delta q_s^* (\Delta q_s^*/q_s^*(p^*))$	-0.0615 (-3%)	-0.0640 (-11%)
$\Delta q_w^* (\Delta q_w^*/q_w^*(p^*))$	0.0150 (3%)	0.0578 (17%)
$\Delta SW^*$	0.0009	-0.0131
$\Delta CS_s^*$	-0.0145	-0.0646
$\Delta CS_w^*$	0.0120	0.0481
$\Delta \Pi^*$	0.0034	0.0034
$\Delta Q^*$	0.0014	-0.0061



## 5. Summary

- PD can improve SW (especially) if firms' brands are *substitutes* in the “*strong*” market, and are *complements* in the “*weak*” market, but it never improves vice versa.
- Consumer surplus never improves by PD.
- “Prisoners’ Dilemma” may arise if  $\Delta Q < 0$  (conjecture)
- Competition due to strong substitutability  
→ Bad for PD to improve SW

**(Intentionally Blank)**

## (1. Introduction)

### ■ What is Price Discrimination?

- *Price Discrimination* is present when two or more identical units of the same products or services are sold at *different* prices, either to the same buyer or to different buyers. (Adachi ('07, Encyclopedia))
- It's a marketing technique for a seller to generate higher profits (by extracting some of the consumer surplus).

## (1. Introduction)

### ■ What is Price Discrimination? (cont'd)

- Taxonomy (Pigou ('20); Dupuit (1849) & Tirole ('89))
  - 1st-Degree PD: Each consumer pays her WTP
  - 2nd-Degree PD: Each consumer self-selects into a different price schedule
  - 3rd-Degree PD: Consumers are segmented into groups by unambiguous traits
- This talk concentrates on 3rd-Degree PD.

## (1. Introduction)

### ■ Two conditions for PD to be introduced

- Imperfect competition  
(firms must have some control power over the price)
- No arbitrage  
(no immediate resale among consumers)

# (1. Introduction)

## ■ Methodological Issues

- Evaluation Criteria:
  - Alfred Marshall's Concept of Surplus
  - *Social Welfare* = Profit (of firms) + Surplus (of consumers)
  
- Restrict Attention:
  - Theoretical Analysis
  - Partial Equilibrium Analysis (small income effects)
  - No Uncertainty, No Asymmetric Information
  - Final Products
  - Static Analysis
  - No Entry, No Exit