

Payoff Interdependence and Multi-Store Paradox

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Abstract

We solve the multi-store paradox by introducing interdependent payoff between the firms. We show that firms set up multiple stores unless the degree of payoff interdependence is low. We also show that multiple equilibria, intertwined and neighboring location equilibria, exist if the degree of payoff interdependence is intermediate.

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1 Introduction

Casual observations suggest that a firm often supplies several products that are mutually substitutable. For example, Kellogg, Coca-Cola, and Nisshin supply several varieties of cereals, beverages, and instant noodles. Most major automobile, consumer electronics, and cell phone companies produce various differentiated products. In the spatial context, supermarkets and convenience stores often build multiple stores. However, two influential works suggested that these casual observations are not supported by economic theory. Martinez-Giralt and Neven (1988) formulated a model where duopolists choose to open either one or two stores and then face price competition. They found that firms always choose to open one store in equilibrium, even if the cost of setting up a store is zero. Establishing multiple stores accelerates competition and reduces profits. Therefore, to relax competition, each firm establishes only one store (in the product differentiation context, each firm supplies only one product). In contrast to Schmalensee (1978), Judd (1985) showed that establishing multiple stores does not serve as a commitment to entry deterrence, even when the cost of setting up each store is completely sunk. This problem is called the “multi-store paradox.”

In this study, we introduce interdependent payoff functions into the duopoly model of Martinez-Giralt and Neven (1988) and solve this paradox. We assume that each firm i maximizes $\pi_i - \alpha\pi_j$ where π_k is firm k 's profit ($k = i, j, j \neq i$) and $\alpha \in [0, 1)$ is the degree of interdependence of the payoff (i.e., the relative profit maximization approach). We show that firms set up multiple stores unless the degree of interdependence is too small. We also find that when α is neither too high nor too low, two types of equilibria coexist: the equilibria with intertwined stores (i.e., each firm locates its store between the rival's two stores) and with neighboring stores (i.e., each firm locates its second store next to its first store).

The existence of a neighboring location equilibrium may explain the strategy of Seven-Eleven Japan, the first and the largest convenience store chain in Japan. Seven-Eleven Japan follows an approach called “strategic dominance,” setting up stores in narrow territories (Seven-Eleven Japan Corporate

Profile 2013–2014, pp. 15) instead of establishing nation-wide store networks. The neighboring location equilibrium is consistent with its strategy.

We now discuss the rationale for employing interdependent objective functions in a general context. First, the evaluation of managerial performance is often based on the relative as well as absolute performance of managers.¹ Outperforming managers often obtain good positions in the management job markets. Second, many laboratory (experimental) works have pointed out the importance of relative performance.² Third, as Armstrong and Huck (2010) convincingly discussed, concerns for relative profit are very closely related to imitative behavior among competing firms, and this imitative behavior is observed frequently.³ We believe that it is reasonable to consider that firms are not always profit-maximizers and to apply this approach to the multi-store problem.

We can also interpret that α indicates the measure of the degree of toughness of competition; a higher α indicates tougher competition.⁴ Following this interpretation, our result suggests that tougher competition leads to a multi-store equilibrium, which further accelerates competition.

Some researchers have solved the multi-store paradox. Ishibashi (2003) showed that if there are one incumbent and two or more new entrants, the incumbent may be able to deter entries by establishing two stores. Tabuchi (2012) also showed that a model with three firms solves this problem. The driving force in this study is completely different from any of existing works and our results provide a new story

¹See Gibbons and Murphy (1990) for empirical evidence.

²See Armstrong and Huck (2010). In real world, it is also known that people are concerned with relative performance, and it is not because of the monetary incentives. See Grieco et al. (1993) and Mastanduno (1991) in the context of political science and Ariely (2008) in the context of behavioral economics. The payoff function based on relative wage or relative wealth status has been intensively discussed in the macroeconomics context, as well. Keynes (1936) discussed the rigidity of nominal wage based on relative wage. See also Akerlof and Yellen (1988) and Corneo and Jeanne (1997, 1999). and Futagami and Shibata (1998). We believe that it is because the concern with relative performance is realistic, and thus has been attracted interests of many macroeconomists.

³See Vega-Redondo (1997) for the model formulation of a related evolutionary game. He considered a quantity-setting model in a homogeneous product market and showed that if firms myopically imitate the most profitable firm's strategy, the industry converges to a highly competitive outcome.

⁴For a general discussion of this approach and useful applications, see Matsumura et al. (2013) and Matsumura and Okamura (2015). We can show that given the locations, the ratio between the profit margin (i.e., price minus marginal cost) and the price, known as the Lerner index, is decreasing in α . This index is intensively adopted in the empirical literature as a measure of the intensity of market competition in product markets. Furthermore, Matsumura and Matsushima (2012) showed that collusion is less stable when α is larger under moderate conditions. In this sense, a larger α again indicates a more competitive market.

of multi-store equilibrium.

Murooka's (2013) study is the most closely related to the present study. He also solved the multi-store paradox in a duopoly model.⁵ He extended a model of Judd (1985) and investigated entry deterrence by an incumbent. He showed that the incumbent maintains multiple stores if it can set up five or more stores before the rival's entry. In his model, it is assumed that the new entrant builds one store and the incumbent builds multiple stores. In this study, however, we need not assume any asymmetry between the two firms, and we show that both firms build multiple stores.

2 The Model

There is a circular market of length 1 where infinitely many consumers lie uniformly. Following Martinez-Giralt and Neven (1988), two firms ($K = A, B$) choose their locations on the unit circle and then choose prices. Each firm runs one or two stores and sells homogeneous goods. Let K_1 and K_2 be the locations of the first and second stores of firm K ($K = A, B$), respectively. Without loss of generality, we assume that $K_1 \leq K_2$ and $A_1 = 0$. $K_1 = K_2$ implies that firm K runs one store only. Let p_{Ki} ($K = A, B$, $i = 1, 2$) be the price of the i -th store of firm K .

The game runs as follows: In the first stage, each firm independently chooses the locations of its two stores. In the second stage, after observing the locations of the stores, each firm K independently chooses the prices at the two stores. The payoff of firm K is given by $U_K = \pi_K - \alpha\pi_L$ ($K, L = A, B$, $L \neq K$), where π_K is the profit of firm K and $\alpha \in [0, 1)$. As we discuss in the Introduction, α indicates the degree of interdependence of the payoff function and/or the toughness of competition in the market. We assume that both firms have an identical production cost function and that the marginal production cost is constant. We normalize the marginal cost as zero.

The consumers have unit demands; that is, each consumes one or zero units of the product. Each consumer derives a surplus from consumption (gross of price and transportation costs) equal to v . We

⁵As pointed out by Hendel and Neiva de Figueiredo (1997), in the context of spatial model, the strategic effect of investment in duopoly is much stronger than that in oligopoly ($n \geq 3$) and a duopoly and a triopoly model often yields contrasting results. Thus, it is important to solve this paradox under the strongest strategic effect case (duopoly case) because the multi-product appears under the competition between Coca-Cola and Pepsi or Aderans and Artnature.

assume that v is so large that every consumer consumes one unit of the product. Because the two firms produce the same physical product, a consumer living at point $X \in [0, 1]$ chooses the lowest-cost store that minimizes the sum of the transport cost and the price, $\tau(X - K_i)^2 + p_{K_i}$, where τ is a positive constant. For simplicity, we assume that $\tau = 1$. Let the demand of store K_i ($K = A, B, i = 1, 2$) be D_{K_i} . D_{K_i} depends on the locations and prices of stores, and is derived in the next section.

3 Equilibrium price

The game is solved by backward induction. We first discuss the second-stage game given the locations of two firms. Following Martinez-Giralt and Neven (1988), we discuss the following two cases.

3.1 Intertwined stores

First, we consider the case where $A_1 \leq B_1 \leq A_2 \leq B_2$. Let \bar{X}_{ij} be the consumer whose utility is indifferent to buying from B_i at price p_{B_i} or A_j at price p_{A_j} . We have

$$\bar{X}_{ij} = \frac{p_{B_i} - p_{A_j}}{2\tau(B_i - A_j)} + \frac{B_i + A_j}{2}.$$

The demand of each store is given by

$$D_{A1} = \bar{X}_{11} + (1 - \bar{X}_{21}),$$

$$D_{A2} = \bar{X}_{22} - \bar{X}_{12},$$

$$D_{B1} = \bar{X}_{12} - \bar{X}_{11},$$

$$D_{B2} = \bar{X}_{21} - \bar{X}_{22}.$$

With these demand functions, firms independently choose their prices and maximize their payoffs.

The first-order conditions are

$$\begin{aligned} \frac{2p_{A1}(1 + B_1 - B_2) + P_{B2}B_1(-1 + \alpha) + (1 - B_2)(p_{B1}(-1 + \alpha) - B_1(1 + B_1 - B_2))}{2B_1(-1 + B_2)} &= 0, \\ \frac{2p_{A2}(B_1 - B_2) + P_{B1}(A_2 - B_2)(-1 + \alpha) + (B_1 - A_2)(p_{B2}(-1 + \alpha) + (B_2 - A_2)(B_1 - B_2))}{2(A_2 - B_1)(B_2 - A_2)} &= 0, \\ \frac{2p_{B1}A_2 + P_{A1}(A_2 - B_1)(-1 + \alpha) + B_1(p_{A2}(-1 + \alpha) + A_2(B_1 - A_2))}{2B_1(B_1 - A_2)} &= 0, \\ \frac{2p_{B2}(-1 + A_2) + P_{A2}(-1 + B_2)(-1 + \alpha) + (A_2 - B_2)(p_{A1}(-1 + \alpha) + (1 - A_2)(-1 + B_2))}{2(A_2 - B_2)(-1 + B_2)} &= 0. \end{aligned}$$

We can show that U_K is quasi-concave with respect to prices, and, thus, the second-order conditions are

satisfied. The first-order conditions lead to the following equilibrium prices:

$$\begin{aligned}
p_{A1}^* &= \frac{B_1(-1+B_2)}{Z} \{A_2^2[B_1^2(1+\alpha)^2 + B_2^2(1+\alpha)^2 \\
&\quad -2B_1(1+\alpha)(-1+B_2+B_2\alpha) - 2B_2(1+\alpha) - (-1+\alpha)^2] \\
&\quad +A_2[-B_1^2(1+\alpha)^2 - B_2^2(1+\alpha)^2 + B_1(2B_2(1+\alpha)^2 - 1 - 4\alpha + \alpha^2) + B_2(3+\alpha^2)] \\
&\quad -B_1B_2(-1+\alpha)^2\}, \\
p_{A2}^* &= \frac{(A_2-B_1)(A_2-B_2)}{Z} \{A_2^2(1+B_1-B_2)(1+\alpha)[(B_1-B_2)(1+\alpha) - 1 + \alpha] \\
&\quad -A_2(1+B_1-B_2)(1+\alpha)[(B_1-B_2)(1+\alpha) - 1 + \alpha] \\
&\quad -B_1(-1+B_2)(-1+\alpha)^2\}, \\
p_{B1}^* &= \frac{B_1(-A_2+B_1)}{Z} \{B_1^2(1+\alpha)(-1+A_2)[1-\alpha+A_2+A_2\alpha] \\
&\quad +B_1(1+\alpha)(-2B_2+1)(-1+A_2)[1-\alpha+A_2+A_2\alpha] \\
&\quad +B_2^2(A_2^2(1+\alpha)^2 - 2A_2\alpha(1+\alpha) + 2(-1+\alpha)) \\
&\quad +B_2(-A_2^2(1+\alpha)^2 + A_2(3\alpha^2+1) - 2\alpha+2) - A_2(-1+\alpha)^2\}, \\
p_{B2}^* &= \frac{(-1+B_2)(-A_2+B_2)}{Z} \{B_2^2A_2(1+\alpha)[-2+A_2+A_2\alpha] \\
&\quad -B_2(1+\alpha)A_2(1+2B_2)[-2+A_2+A_2\alpha] \\
&\quad +B_1^2(A_2^2(1+\alpha)^2 - 2A_2\alpha(1+\alpha) - (-1+\alpha)^2) + B_1B_2(A_2(1+\alpha)^2 - 1 - 4\alpha + \alpha^2)\},
\end{aligned}$$

$$\begin{aligned}
\text{where } Z &= (1+\alpha) \{-B_1^2(-1+A_2)(-1+\alpha)^2 - 2B_1A_2(-1+A_2)[B_2(-3-2\alpha+\alpha^2) - (-1-2\alpha+\alpha^2)] \\
&\quad +B_2^2A_2(A_2(-3-2\alpha+\alpha^2) + 4) + 2B_2A_2(A_2(1+2\alpha-\alpha^2) - 2)A_2^2(-1+\alpha)^2\}.
\end{aligned}$$

3.2 Neighboring stores

For the second case, suppose that $A_1 \leq A_2 \leq B_1 \leq B_2$.⁶ Let \bar{X}_K be the consumer whose utility is indifferent to buying from firm K 's first store, located at K_1 , at price p_{K1} or its second store, located at K_2 , at price p_{K2} . We have

$$\bar{X}_K = \frac{p_{K2} - p_{K1}}{2(K_2 - K_1)} + \frac{K_2 + K_1}{2}.$$

⁶Because of the symmetry of the circular market, a similar principle can also apply in the case where $A_1 \leq B_1 \leq B_2 \leq A_2$.

The demand functions for each store are as follows:

$$\begin{aligned}
D_{A1} &= \bar{X}_A + (1 - \bar{X}_{21}), \\
D_{A2} &= \bar{X}_{12} - \bar{X}_A, \\
D_{B1} &= \bar{X}_B - \bar{X}_{12}, \\
D_{B2} &= \bar{X}_{21} - \bar{X}_B.
\end{aligned}$$

Given these demand functions, firms independently choose their prices and maximize their payoffs.

The first-order conditions are

$$\begin{aligned}
\frac{2p_{A1}(1 + A_2 - B_2) + 2p_{A2}(-1 + B_2) + A_2(p_{B2}(-1 + \alpha) + (1 + A_2 - B_2)(-1 + B_2))}{2A_2(-1 + B_2)} &= 0, \\
\frac{2p_{A2}B_1 + 2p_{A1}(A_2 - B_1) + A_2(p_{B1}(-1 + \alpha) + (A_2 - B_1)B_1)}{2A_2(A_2 - B_1)} &= 0, \\
\frac{2p_{B1}(A_2 - B_2) - 2p_{B2}(A_2 - B_1) + (B_1 - B_2)(p_{A2}(-1 + \alpha) - (A_2 - B_1)(A_2 - B_2))}{2(A_2 - B_1)(B_1 - B_2)} &= 0, \\
\frac{2p_{B2}(-1 + B_1) - 2p_{B1}(-1 + B_2) + (B_1 - B_2)(p_{A1}(-1 + \alpha) - (-1 + B_1)(-1 + B_2))}{2(B_1 - B_2)(-1 + B_2)} &= 0.
\end{aligned}$$

We can show that U_K is quasi-concave with respect to prices, and, thus, the second-order conditions

are satisfied. The first-order conditions lead to the following equilibrium prices:

$$\begin{aligned}
p_{A1}^* &= \frac{(1-B_2)}{L} \left\{ -B_1^2(1+\alpha)[A_2(-3-2\alpha+\alpha^2)+4] \right. \\
&\quad + B_1(B_2(1+\alpha)-3+\alpha)(A_2(-3-2\alpha+\alpha^2)+4) \\
&\quad \left. + A_2(-1+\alpha)(B_2(1-\alpha^2)+2A_2(1+\alpha)-6+2\alpha) \right\}, \\
p_{A2}^* &= \frac{(-A_2+B_1)}{L} \left\{ -A_2^2(-3-2\alpha+\alpha^2)[B_1(1+\alpha)-B_2(1+\alpha)+2] \right. \\
&\quad - A_2(1+\alpha)[B_2^2(-3-2\alpha+\alpha^2)+B_2(B_1(3+2\alpha-\alpha^2)+8-4\alpha)+2(-1+\alpha)] \\
&\quad \left. - 4(-1+B_2)((B_2-B_1)(1+\alpha)-3+\alpha) \right\}, \\
p_{B1}^* &= \frac{(-A_2+B_1)}{L} \left\{ B_1^2(-3-2\alpha+\alpha^2)[A_2(1+\alpha)-2] \right. \\
&\quad + B_1[-B_2(1+\alpha)(A_2(-3-2\alpha+\alpha^2)+4)+2(-1+\alpha^2)] \\
&\quad \left. + 2B_2^2(-1+\alpha^2)-2B_2(2A_2(1+\alpha)-7+2\alpha+\alpha^2)+4(A_2(1+\alpha)-3+\alpha) \right\}, \\
p_{B2}^* &= \frac{(1-B_2)}{L} \left\{ B_2^2(-3-2\alpha+\alpha^2)[A_2(1+\alpha)-2] \right. \\
&\quad + B_2(1+\alpha)[-B_1(A_2(-3-2\alpha+\alpha^2)+4)-(A_2^2(-3-2\alpha+\alpha^2)+A_2(5-6\alpha+\alpha^2)+6-2\alpha)] \\
&\quad + 2B_1^2(-1+\alpha^2)+B_1(-3+\alpha)(A_2^2(1+\alpha)^2+A_2(-3-2\alpha+\alpha^2)+2-2\alpha) \\
&\quad \left. - 4A_2(A_2(1+\alpha)-3+\alpha) \right\},
\end{aligned}$$

where $L = (-3-2\alpha+\alpha^2)(B_1(A_2(-3-2\alpha+\alpha^2)+4)+B_2(A_2(3+2\alpha-\alpha^2)-4)-4A_2+4)$.

4 Results

We substitute the equilibrium prices into the payoff functions, and derive the payoff function as a function of location. In the first stage, each firm K independently chooses the locations of its stores.

We investigate how α affects the equilibrium locations of the stores. The following proposition states the relationship between the equilibrium locations of stores and α :

Proposition 1 (i) (*Single-Store Equilibrium*) *If $\alpha \in [0, 3-2\sqrt{2}]$, then $(A_1, A_2, B_1, B_2) = (0, 0, 1/2, 1/2)$ constitutes an equilibrium.*

(ii) (*Neighboring Location Equilibrium*) If $\alpha \in (3 - 2\sqrt{2}, \bar{\alpha}_1]$, then

$$(A_1, A_2, B_1, B_2) = \left(0, \frac{-1 + 6\alpha - \alpha^2}{4 + 4\alpha}, 1/2, 1/2 + \frac{-1 + 6\alpha - \alpha^2}{4 + 4\alpha}\right)$$

constitutes an equilibrium where $\bar{\alpha}_1 \simeq 0.58$.

(iii) (*Intertwined Location Equilibrium*) If $\alpha \in [\bar{\alpha}_2, 1)$, then $(A_1, A_2, B_1, B_2) = (0, 1/2, 1/4, 3/4)$, constitutes an equilibrium where $\bar{\alpha}_2 \simeq 0.19$.

Proof See the Appendix.

Proposition 1(i) implies that the result of Martinez-Giralt and Neven (1988) holds if $\alpha \leq 3 - 2\sqrt{2}$. Martinez-Giralt and Neven (1988) already showed that when $\alpha = 0$, $(A_1, A_2, B_1, B_2) = (0, 0, 1/2, 1/2)$. In other words, neither firm chooses two stores, even if the cost of setting up a store is zero. Setting up two stores accelerates competition and reduces profits. Therefore, each firm avoids building multiple stores in order to mitigate competition if the firms are close to profit-maximizers.

Proposition 1(ii–iii) imply that firms build multiple stores in equilibrium if $\alpha > 3 - 2\sqrt{2}$. Given $A_1 = 0$, a slight increase in A_2 from $A_2 = 0$ accelerates competition between the two firms, reduces prices at both firms, increases (res. decreases) the market share of firm A (res. B), and reduces both firms' profits. It reduces firm B 's profit more significantly because firm B suffers from both lower prices and smaller market share, whereas firm A suffers from lower prices but gains from higher market share. This creates the incentive for establishing two stores.

Proposition 1(ii–iii) imply that multiple equilibria exist if $\alpha \in (\bar{\alpha}_2, \bar{\alpha}_1)$. We explain the intuition as follows: Suppose that $\alpha \in [\bar{\alpha}_2, \bar{\alpha}_1]$. Then, suppose that firm B changes its locations from $(B_1, B_2) = (1/2, 1/2 + (-1 + 6\alpha - \alpha^2)/(4 + 4\alpha))$ to $(B_1, B_2) = (1/4, 3/4)$. This deviation increases the market share of firm B and accelerates competition between the two firms. Given $(B_1, B_2) = (1/4, 3/4)$, firm A 's relocation from $(A_1, A_2) = (0, (-1 + 6\alpha - \alpha^2)/(4 + 4\alpha))$ to $(A_1, A_2) = (0, 1/2)$ increases the market share of firm A and further accelerates competition between the two firms. This competition-accelerating effect (i.e., price-reducing effect) reduces firm B 's profit more significantly because the market share of firm B is larger before the deviation. This may not hold if $(B_1, B_2) = (1/2, 1/2 + (-1 + 6\alpha - \alpha^2)/(4 + 4\alpha))$

because the market share of firm B is the same as that of firm A before the deviation. Therefore, the aforementioned relocation of firm A improves its payoff if $(B_1, B_2) = (1/4, 3/4)$. However, if $(B_1, B_2) = (1/2, 1/2 + (-1 + 6\alpha - \alpha^2)/(4 + 4\alpha))$, firm A's payoff does not improve. This leads to multiple equilibria.

Proposition 1(ii) discusses a neighboring location equilibrium where a store of firm A (res. B) is located next to another store of firm A (res. B). The existence of a neighboring location equilibrium may explain the strategy of Seven-Eleven Japan, the first and the largest convenience store chain in Japan. Seven-Eleven Japan follows a strategy called “strategic dominance,” by which it sets up stores in narrow territories instead of establishing nation-wide networks of stores.⁷ The neighboring location equilibrium is consistent with its strategy. Similar strategies are observed in many Japanese chain stores (Komoto, 1997).

5 Concluding Remarks

In this study, we revisited the multi-store paradox by introducing interdependent payoff functions. We assumed that duopolists are concerned with both their own and their rival's profits. We found that firms set up multiple stores unless the degree of payoff interdependence is low. We also found that multiple equilibria, intertwined and neighboring location equilibria, exist if the degree of payoff interdependence is neither too low nor too high.

If the number of stores of each firm is an exogenous variable, an increase in the number of stores accelerates competition and reduces firms' profits. In this study, we showed that a large degree of interdependence leads to multiple stores in equilibrium. As Matsumura et al. (2013) suggested, we can interpret that the degree of payoff interdependence indicates the degree of toughness of competition. Following this interpretation, our result suggested that tougher competition leads firms to set up multiple stores in equilibrium. This result suggests the possible inverse causality of the traditional view; tougher competition leads to multiple stores, and setting up multiple stores further accelerates competition.

⁷Regarding its store location strategy, Seven-Eleven Japan says “High-Density, Concentrated Store Openings (dominant strategy) are the vital trajectory toward realizing close and convenient stores.” (Seven-Eleven Japan Corporate Profile 2013–2014, pp. 15).

Our result contains another implication. We show that under tough competition, multi-store equilibria appear. This result suggests a possible inverse causality of the traditional view; tougher competition leads to multiple stores, and setting up multiple stores further accelerates competition.

Appendix

Proof of Proposition 1(i)

We show that when $\alpha \in [0, 3 - 2\sqrt{2}]$, given $(A_1, A_2) = (0, 0)$, the best reply for firm B is $(B_1, B_2) = (1/2, 1/2)$.

Because $A_1 = A_2 = 0$, firm B must establish neighboring locations. Let $p_A^* := p_{A_1}^* = p_{A_2}^*$. The payoff of firm B, U_B , is expressed as follows:

$$\begin{aligned} U_B &= \pi_B - \alpha\pi_A \\ &= p_{B_1}^* \left(\frac{p_{B_2}^* - p_{B_1}^*}{2(B_2 - B_1)} + \frac{B_2 + B_1}{2} - \frac{p_{B_1}^* - p_A^*}{2B_1} - \frac{B_1}{2} \right) \\ &\quad + p_{B_2}^* \left(\frac{p_{B_2}^* - p_A^*}{2(B_2 - 1)} + \frac{(B_2 + 1)}{2} - \frac{p_{B_2}^* - p_{B_1}^*}{2(B_2 - B_1)} - \frac{B_2 + B_1}{2} \right) \\ &\quad - \alpha \left\{ p_A^* \left(\frac{p_{B_1}^* - p_A^*}{2(B_1)} + \frac{(B_1)}{2} + 1 - \frac{p_{B_2}^* - p_A^*}{2(B_2 - 1)} - \frac{B_2 + 1}{2} \right) \right\}. \end{aligned}$$

Let $\Theta = B_2 - B_1$ be the distance between stores B_1 and B_2 . The payoff function is rewritten as

$$\begin{aligned} U_B &= \pi_B - \alpha\pi_A \\ &= p_{B_1}^* \left(\frac{p_{B_2}^* - p_{B_1}^*}{2\Theta} + \frac{\Theta + 2B_1}{2} - \frac{p_{B_1}^* - p_A^*}{2B_1} - \frac{B_1}{2} \right) \\ &\quad + p_{B_2}^* \left(\frac{p_{B_2}^* - p_A^*}{2(\Theta + B_1 - 1)} + \frac{(\Theta + B_1 + 1)}{2} - \frac{p_{B_2}^* - p_{B_1}^*}{2\Theta} - \frac{\Theta + 2B_1}{2} \right) \\ &\quad - \alpha \left\{ p_A^* \left(\frac{p_{B_1}^* - p_A^*}{2(B_1)} + \frac{(B_1)}{2} + 1 - \frac{p_{B_2}^* - p_A^*}{2(\Theta + B_1 - 1)} - \frac{\Theta + B_1 + 1}{2} \right) \right\}. \end{aligned}$$

The first-order condition with respect to B_1 is

$$\frac{\partial U_B}{\partial B_1} = \frac{(-1 + \Theta + 2B_1)(-1 + \alpha)(\Theta^2(-1 + \alpha^2) + \Theta(3 + 2\alpha - \alpha^2) - (-3 + \alpha)^2)}{2(-1 + \Theta)(-3 + \alpha)^2(1 + \alpha)} = 0.$$

The second-order condition is satisfied.

We can show that $\Theta^2(-1 + \alpha^2) + \Theta(3 + 2\alpha - \alpha^2) - (-3 + \alpha)^2$ is strictly negative. Thus, the first-order condition is satisfied if and only if $(-1 + \Theta + 2B_1) = 0$ (i.e., $B_1 + B_2 = 1$). This implies that the locations must be symmetric (i.e., $1/2 - B_1 = B_2 - 1/2$). Substituting this condition into the first-order condition

with respect to Θ , we have

$$\frac{\partial U_B}{\partial \Theta} = \frac{-3\Theta^2(-1 + \alpha)^2(1 + \alpha) + 2\Theta(-5 - 5\alpha + \alpha^2 + \alpha^3) - 3 - 19\alpha - 9\alpha^2 + \alpha^3}{8(-3 + \alpha)^2(1 + \alpha)}.$$

We can show that $\partial U_B / \partial \Theta < 0$ for all $\Theta \in [0, 1/2]$ if $\alpha \in [0, 3 - 2\sqrt{2})$. Thus, $\Theta = 0$ is optimal. In addition, $\partial U_B / \partial \Theta = 0$ for $\Theta = 0$ if $\alpha = 3 - 2\sqrt{2}$. These two conditions ($\Theta = 0$ and $-1 + \Theta + 2B_1 = 0$) imply that the best reply for firm B is $(B_1, B_2) = (1/2, 1/2)$ when $\alpha \in [0, 3 - 2\sqrt{2}]$.

By symmetry, given $(B_1, B_2) = (1/2, 1/2)$, $(A_1, A_2) = (0, 0)$ is the best reply for firm A if $\alpha \in [0, 3 - 2\sqrt{2}]$. ■

Proof of Proposition 1(ii)

We show that when $\alpha \in (3 - 2\sqrt{2}, \bar{\alpha}_1]$, given $(A_1, A_2) = (0, (-1 + 6\alpha - \alpha^2)/(4 + 4\alpha))$, the best reply for firm B is $(B_1, B_2) = (1/2, 1/2 + (-1 + 6\alpha - \alpha^2)/(4 + 4\alpha))$. We take the following two steps: First, we show that the above location is firm B's optimal strategy for $\alpha \in (3 - 2\sqrt{2}, 1)$ in Neighboring. Second, we allow firm B to establish intertwined locations and show that this never improves its payoff for $\alpha \in (3 - 2\sqrt{2}, \bar{\alpha}_1]$.

Suppose that firm B establishes neighboring locations. Suppose that $\alpha \in (3 - 2\sqrt{2}, 1)$. Let \bar{A}_2 be $(-1 + 6\alpha - \alpha^2)/(4 + 4\alpha)$. Given $(A_1, A_2) = (0, \bar{A}_2)$, the payoff function of firm B in Neighboring is

$$\begin{aligned} U_B &= \pi_B - \alpha\pi_A \\ &= p_{B1}^* \left(\frac{p_{B2}^* - p_{B1}^*}{2(B_2 - B_1)} + \frac{B_2 + B_1}{2} - \frac{p_{B1}^* - p_{A2}^*}{2(B_1 - \bar{A}_2)} - \frac{B_1 + \bar{A}_2}{2} \right) \\ &\quad + p_{B2}^* \left(\frac{p_{B2}^* - p_{A1}^*}{2(B_2 - 1)} + \frac{(B_2 + 1)}{2} - \frac{p_{B2}^* - p_{B1}^*}{2(B_2 - B_1)} - \frac{B_2 + B_1}{2} \right) \\ &\quad - \alpha \left\{ p_{A1}^* \left(\frac{p_{A2}^* - p_{A1}^*}{\bar{A}_2} + \frac{\bar{A}_2}{2} + 1 - \frac{p_{B2}^* - p_{A1}^*}{2(B_2 - 1)} - \frac{B_2 + 1}{2} \right) \right. \\ &\quad \left. + p_{A2}^* \left(\frac{p_{B1}^* - p_{A2}^*}{2(B_1 - \bar{A}_2)} + \frac{(B_1 + \bar{A}_2)}{2} - \frac{p_{A2}^* - p_{A1}^*}{\bar{A}_2} - \frac{\bar{A}_2}{2} \right) \right\}. \end{aligned}$$

Let $B_2 - B_1$ be Θ . The payoff function is rewritten as

$$\begin{aligned}
U_B &= \pi_B - \alpha\pi_A \\
&= p_{B_1}^* \left(\frac{p_{B_2}^* - p_{B_1}^*}{2\Theta} + \frac{B_2 + B_1}{2} - \frac{p_{B_1}^* - p_{A_2}^*}{2(B_1 - \bar{A}_2)} - \frac{B_1 + \bar{A}_2}{2} \right) \\
&\quad + p_{B_2}^* \left(\frac{p_{B_2}^* - p_{A_1}^*}{2(\Theta + B_1 - 1)} + \frac{(\Theta + B_1 + 1)}{2} - \frac{p_{B_2}^* - p_{B_1}^*}{2\Theta} - \frac{\Theta + 2B_1}{2} \right) \\
&\quad - \alpha \left\{ p_{A_1}^* \left(\frac{p_{A_2}^* - p_{A_1}^*}{\bar{A}_2} + \frac{\bar{A}_2}{2} + 1 - \frac{p_{B_2}^* - p_{A_1}^*}{2(\Theta B_1 - 1)} - \frac{\Theta + B_1 + 1}{2} \right) \right. \\
&\quad \left. + p_{A_2}^* \left(\frac{p_{B_1}^* - p_{A_2}^*}{2(B_1 - \bar{A}_2)} + \frac{(B_1 + \bar{A}_2)}{2} - \frac{p_{A_2}^* - p_{A_1}^*}{\bar{A}_2} - \frac{\bar{A}_2}{2} \right) \right\}.
\end{aligned}$$

The first-order condition with respect to B_1 is

$$\frac{\partial U_B}{\partial B_1} = \frac{(-1 + \Theta - \bar{A}_2 + 2B_1)(-1 + \alpha)g(\bar{A}_2, \Theta, \alpha)}{2(-3 + \alpha)^2(1 + \alpha)(\Theta(\bar{A}_2(-3 - 2\alpha + \alpha^2) + 4)^2)} = 0,$$

$$\begin{aligned}
\text{where } g(\bar{A}_2, \Theta, \alpha) &= \bar{A}_2^3(-1 + \alpha^2)(\Theta(-3 - 2\alpha + \alpha^2) + 4)^2 \\
&\quad + \bar{A}_2^2[\Theta^3(-3 + \alpha)^2(-1 + \alpha)(1 + \alpha)^3 \\
&\quad + 16\Theta^2(1 + \alpha)^2(7 + \alpha - 5\alpha^2 + \alpha^3)(-69 + 68\alpha - 6\alpha^2 - 12\alpha^3 + 3\alpha^4) \\
&\quad + 4\Theta(-41 - 24\alpha + 36\alpha^2 + 6\alpha^3 - 11\alpha^4 + 2\alpha^5)] \\
&\quad + 4\bar{A}_2[2\Theta^3(1 + \alpha)^2(3 - 4\alpha + \alpha^2) - 2\Theta(-41 - 6\alpha + 24\alpha^2 - 10\alpha^3 + \alpha^4) \\
&\quad + \Theta(-32 - 21\alpha + 22\alpha^2 + 4\alpha^3 - 6\alpha^4 + \alpha^5) - 4(15 - 5\alpha - 3\alpha^2 + \alpha^3)] \\
&\quad + 16[\Theta^3(-1 + \alpha^2) - 2\Theta^2(-2 + \alpha)(1 + \alpha) + 4\Theta(-3 + \alpha) + (-3 + \alpha)^2].
\end{aligned}$$

The second-order condition is satisfied. We can show that $g(\bar{A}_2, \Theta, \alpha)$ is strictly negative. Thus, the first-order condition is satisfied if and only if $-1 + \Theta - \bar{A}_2 + 2B_1 = 0$. Taking another first-order condition, $\partial U_B / \partial \Theta = 0$, and substituting the condition $-1 + \Theta - \bar{A}_2 + 2B_1 = 0$ into it, we have

$$\begin{aligned}
\frac{\partial U_B}{\partial \Theta} &= \frac{1}{8(-3 + \alpha)^2(1 + \alpha)} \left\{ \bar{A}_2^2(-1 + \alpha)^2(1 + \alpha) - 3\Theta^2(-1 + \alpha)^2(1 + \alpha) - 2\bar{A}_2(-1 + \alpha)^2(1 + \alpha) \right. \\
&\quad \left. + 2\Theta(1 + \alpha)(\bar{A}_2(-1 + \alpha)^2 + \alpha^2 - 5) - 3 + 19\alpha - 9\alpha^2 + \alpha^3 \right\} = 0.
\end{aligned}$$

The second-order condition is satisfied. The first-order condition leads to the following optimal Θ :

$$\Theta^* = \frac{-1 + 6\alpha - \alpha^2}{4 + 4\alpha}.$$

Therefore, the optimal neighboring location is

$$(B_1^*, B_2^*) = (1/2, 1/2 + \frac{-1 + 6\alpha - \alpha^2}{4 + 4\alpha}).$$

Next, we allow firm B to establish intertwined locations. Given $(A_1, A_2) = (0, \bar{A}_2)$, the payoff function of firm B in Intertwined is

$$\begin{aligned} U_B &= \pi_B - \alpha\pi_A \\ &= p_{B1}^* \left(\frac{p_{B1}^* - p_{A2}^*}{2(B_1 - \bar{A}_2)} + \frac{B_1 + \bar{A}_2}{2} - \frac{p_{B1}^* - p_{A1}^*}{2B_1} - \frac{B_1}{2} \right) \\ &\quad + p_{B2}^* \left(\frac{p_{B2}^* - p_{A1}^*}{2(B_2 - 1)} + \frac{(B_2 + 1)}{2} - \frac{p_{B2}^* - p_{A2}^*}{2(B_2 - \bar{A}_2)} - \frac{B_2 + \bar{A}_2}{2} \right) \\ &\quad - \alpha \left\{ p_{A1}^* \left(\frac{p_{B1}^* - p_{A1}^*}{2B_1} + \frac{B_1}{2} + 1 - \frac{p_{B2}^* - p_{A1}^*}{2(B_2 - 1)} - \frac{B_2 + 1}{2} \right) \right. \\ &\quad \left. + p_{A2}^* \left(\frac{p_{B2}^* - p_{A2}^*}{2(B_2 - \bar{A}_2)} + \frac{B_2 + \bar{A}_2}{2} - \frac{p_{B1}^* - p_{A2}^*}{2(B_1 - \bar{A}_2)} - \frac{B_1 + \bar{A}_2}{2} \right) \right\}. \end{aligned}$$

The two first-order conditions $\partial U_B / \partial B_1 = 0$ and $\partial U_B / \partial B_2 = 0$ are satisfied when $(B_1, B_2) = ((-1 + 6\alpha - \alpha^2)/(8 + 8\alpha), (3 + 10\alpha - \alpha^2)/(8 + 8\alpha))$. The second-order conditions are also satisfied.

Finally, we investigate whether the intertwined or the neighboring location is best for firm B. The neighboring location $(B_1, B_2) = (1/2, 1/2 + (-1 + 6\alpha - \alpha^2)/(4 + 4\alpha))$ is the best reply for firm B if and only if

$$\begin{aligned} U_B(0, \frac{-1 + 6\alpha - \alpha^2}{4 + 4\alpha}, 1/2, 1/2 + \frac{-1 + 6\alpha - \alpha^2}{4 + 4\alpha}) &= \frac{(3 - \alpha)(-1 + \alpha)^2}{16(1 + \alpha)^2} \\ &\geq U_B(0, \frac{-1 + 6\alpha - \alpha^2}{4 + 4\alpha}, \frac{-1 + 6\alpha - \alpha^2}{8 + 8\alpha}, \frac{3 + 10\alpha - \alpha^2}{8 + 8\alpha}) = \frac{2 + 11\alpha - 26\alpha^2 + 20\alpha^3 - 8\alpha^4 + \alpha^5}{64(1 + \alpha)^3}. \end{aligned}$$

This holds true if and only if $\alpha \in (3 - 2\sqrt{2}, \bar{\alpha}_1]$, where $\bar{\alpha}_1$ is a positive solution to the following equation

$$\frac{(3 - \alpha)(-1 + \alpha)^2}{16(1 + \alpha)^2} = \frac{2 + 11\alpha - 26\alpha^2 + 20\alpha^3 - 8\alpha^4 + \alpha^5}{64(1 + \alpha)^3}.$$

By symmetry, given $(B_1, B_2) = (1/2, 1/2 + (-1 + 6\alpha - \alpha^2)/(4 + 4\alpha))$, $(A_1, A_2) = (0, (-1 + 6\alpha - \alpha^2)/(4 + 4\alpha))$ is the best reply for firm A if $\alpha \in (3 - 2\sqrt{2}, \bar{\alpha}_1]$. ■

Proof of Proposition 1(iii)

We show that when $\alpha \in [\bar{\alpha}_2, 1)$, given $(A_1, A_2) = (0, 1/2)$, the best reply for firm B is $(B_1, B_2) = (1/4, 3/4)$. We perform the following two steps: First, we show that the above is the optimal strategy in Intertwined. Second, we allow firm B to set up neighboring locations and show that this never improves its payoff in these α .

Suppose that firm B established intertwined locations. Given $(A_1, A_2) = (0, 1/2)$, the payoff of firm B, U_B , is

$$\begin{aligned}
U_B &= \pi_B - \alpha\pi_A \\
&= p_{B1}^* \left(\frac{p_{B1}^* - p_{A2}^*}{2(B_1 - \frac{1}{2})} + \frac{B_1 + \frac{1}{2}}{2} - \frac{p_{B1}^* - p_{A1}^*}{2B_1} - \frac{B_1}{2} \right) \\
&\quad + p_{B2}^* \left(\frac{p_{B2}^* - p_{A1}^*}{2(B_2 - 1)} + \frac{(B_2 + 1)}{2} - \frac{p_{B2}^* - p_{A2}^*}{2(B_2 - \frac{1}{2})} - \frac{B_2 + \frac{1}{2}}{2} \right) \\
&\quad - \alpha \left\{ p_{A1}^* \left(\frac{p_{B1}^* - p_{A1}^*}{2B_1} + \frac{B_1}{2} + 1 - \frac{p_{B2}^* - p_{A1}^*}{2(B_2 - 1)} - \frac{B_2 + 1}{2} \right) \right. \\
&\quad \left. + p_{A2}^* \left(\frac{p_{B2}^* - p_{A2}^*}{2(B_2 - \frac{1}{2})} + \frac{B_2 + \frac{1}{2}}{2} - \frac{p_{B1}^* - p_{A2}^*}{2(B_1 - \frac{1}{2})} - \frac{B_1 + \frac{1}{2}}{2} \right) \right\}
\end{aligned}$$

The two first-order conditions $\partial U_B / \partial B_1 = 0$ and $\partial U_B / \partial B_2 = 0$ are satisfied when $(B_1, B_2) = (1/4, 3/4)$. The second-order conditions are also satisfied. We now allow firm B to establish neighboring

locations. Given $(A_1, A_2) = (0, 1/2)$, the payoff of firm B is

$$\begin{aligned}
U^B &= \pi_B - \alpha\pi_A \\
&= p_{B1}^* \left(\frac{p_{B2}^* - p_{B1}^*}{2(B_2 - B_1)} + \frac{B_2 + B_1}{2} - \frac{p_{B1}^* - p_{A2}^*}{2(B_1 - \frac{1}{2})} - \frac{B_1 + \frac{1}{2}}{2} \right) \\
&\quad + p_{B2}^* \left(\frac{p_{B2}^* - p_{A1}^*}{2(B_2 - 1)} + \frac{(B_2 + 1)}{2} - \frac{p_{B2}^* - p_{B1}^*}{2(B_2 - B_1)} - \frac{B_2 + B_1}{2} \right) \\
&\quad - \alpha \left\{ p_{A1}^* \left(\frac{p_{A2}^* - p_{A1}^*}{1} + \frac{1}{4} + 1 - \frac{p_{B2}^* - p_{A1}^*}{2(B_2 - 1)} - \frac{B_2 + 1}{2} \right) \right. \\
&\quad \left. + p_{A2}^* \left(\frac{p_{B1}^* - p_{A2}^*}{2(B_1 - \frac{1}{2})} + \frac{(B_1 + \frac{1}{2})}{2} - \frac{p_{A2}^* - p_{A1}^*}{1} - \frac{1}{4} \right) \right\}.
\end{aligned}$$

Let $B_2 - B_1$ be Θ . The payoff function can be expressed by B_1 and Θ .

$$\begin{aligned}
U_B &= \pi_B - \alpha\pi_A \\
&= p_{B1}^* \left(\frac{p_{B2}^* - p_{B1}^*}{2\Theta} + \frac{\Theta + 2B_1}{2} - \frac{p_{B1}^* - p_{A2}^*}{2(B_1 - \frac{1}{2})} - \frac{B_1 + \frac{1}{2}}{2} \right) \\
&\quad + p_{B2}^* \left(\frac{p_{B2}^* - p_{A1}^*}{2(B_1 + \Theta - 1)} + \frac{(B_1 + \Theta + 1)}{2} - \frac{p_{B2}^* - p_{B1}^*}{2\Theta} - \frac{\Theta + 2B_1}{2} \right) \\
&\quad - \alpha \left\{ p_{A1}^* \left(\frac{p_{A2}^* - p_{A1}^*}{1} + \frac{1}{4} + 1 - \frac{p_{B2}^* - p_{A1}^*}{2(B_1 + \Theta - 1)} - \frac{B_1 + \Theta + 1}{2} \right) \right. \\
&\quad \left. + p_{A2}^* \left(\frac{p_{B1}^* - p_{A2}^*}{2(B_1 - \frac{1}{2})} + \frac{(B_1 + \frac{1}{2})}{2} - \frac{p_{A2}^* - p_{A1}^*}{1} - \frac{1}{4} \right) \right\}.
\end{aligned}$$

The first-order condition with respect to B_1 is

$$\frac{\partial U_B}{\partial B_1} = \frac{(-3 + 2\Theta + 4B_1)(-1 + \alpha)f(\Theta, \alpha)}{8(-3 + \alpha)^2(1 + \alpha)(\Theta(5 - 2\alpha + \alpha^2))^2} = 0,$$

$$\begin{aligned}
\text{where } f(\Theta, \alpha) &= 2\Theta^3(-1 + \alpha)(5 - 2\alpha + \alpha^2)^2 + \Theta^2(129 + 48\alpha - 15\alpha^2 - 8\alpha^3 - 45\alpha^4 + 24\alpha^5 - 5\alpha^6) \\
&\quad + 16\Theta(-33 + 33\alpha - 32\alpha^2 + 22\alpha^3 - 7\alpha^4 + \alpha^5) + 16(25 - 26\alpha + 11\alpha^2 - 2\alpha^3).
\end{aligned}$$

The second-order condition is satisfied. We can show that $f(\Theta, \alpha)$ is strictly positive. Thus, the first-order condition is satisfied if and only if $-3 + 2\Theta + 4B_1 = 0$. This implies that the locations must be symmetric (i.e., $3/4 - B_1 = B_2 - 3/4$).

Substituting this condition into $\partial U_B/\partial\Theta$, we have

$$\frac{\partial U_B}{\partial\Theta} = \frac{-12\Theta^2(-1+\alpha)^2(1+\alpha) + 4\Theta(-9-11\alpha+\alpha^2+3\alpha^3) - 15 - 79\alpha - 33\alpha^2 + \alpha^3}{32(-3+\alpha)^2(1+\alpha)}.$$

Let $\bar{\alpha}_4$ be the solution for $15 - 79\alpha + 33\alpha^2 - \alpha^3 = 0$ in $\alpha \in [0, 1)$. We can show that $\partial U_B/\partial\Theta < 0$ for all $\Theta \in [0, 1/2]$ if $\alpha \in [0, \bar{\alpha}_4)$. Thus, $\Theta = 0$ (and, thus, $(B_1, B_2) = (3/4, 3/4)$) is optimal among neighboring locations if $\alpha \in [0, \bar{\alpha}_4)$. If $\alpha \in [\bar{\alpha}_4, 1)$, the first-order condition $\partial U_B/\partial\Theta = 0$ leads to the following optimal Θ :

$$\Theta^* = \frac{-9 - 2\alpha + 3\alpha^2}{6(-1 + \alpha)^2} + \sqrt{\frac{9 + 111\alpha - 158\alpha^2 + 94\alpha^3 - 27\alpha^4 + 3\alpha^5}{9(-1 + \alpha)^4(1 + \alpha)}}.$$

To sum up, given $(A_1, A_2) = (0, 1/2)$, the optimal location among neighboring locations for firm B is

$$(B_1^*, B_2^*) = \begin{cases} (\frac{3}{4}, \frac{3}{4}) & \text{if } 0 \leq \alpha < \bar{\alpha}_4 \\ \left(\frac{(3\alpha^2 - 8\alpha + 9)}{6(\alpha - 1)^2} - \sqrt{\frac{(\alpha - 3)^2(3\alpha^3 - 9\alpha^2 + 13\alpha + 1)}{36(\alpha - 1)^4(\alpha + 1)}}, \frac{\alpha(3\alpha - 5)}{3(\alpha - 1)^2} + \sqrt{\frac{(\alpha - 3)^2(\alpha(3(\alpha - 3)\alpha + 13) + 1)}{36(\alpha - 1)^4(\alpha + 1)}} \right) & \text{if } \bar{\alpha}_4 \leq \alpha < 1. \end{cases}$$

Finally, we investigate whether the intertwined or the neighboring location is best for firm B . The intertwined location $(B_1, B_2) = (1/4, 3/4)$ is the best reply for firm B if and only if $U_B(0, 1/2, 1/4, 3/4) \geq U_B(0, 1/2, B_1^*, B_2^*)$. This holds true if and only if $\alpha \in [\bar{\alpha}_2, 1)$, where $\bar{\alpha}_2$ is a positive solution to the equation

$$U_B(0, 1/2, 1/4, 3/4) = \frac{1 - \alpha}{32(1 + \alpha)} = \frac{25 - 69\alpha + 31\alpha^2 - 3\alpha^3}{64(-3 + \alpha)^2(1 + \alpha)} = U_B(0, 1/2, 3/4, 3/4).$$

By symmetry, given $(B_1, B_2) = (1/4, 3/4)$, $(A_1, A_2) = (0, 1/2)$ is the best reply for firm A if $\alpha \in [\bar{\alpha}_2, 1)$. ■

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