Supply Function Equilibria and Corporate Social Responsibility

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Abstract

We examine *supply function equilibrium*, introduced by Klemperer and Meyer (1989) and accommodate the objective functions to *corporate social responsibility*. Even though there exist many equilibria if there is no uncertainty in a demand function, but under a setting with demand uncertainty, properties of symmetric equilibria are characterized. Furthermore, under a linear demand with uncertainty, we obtain a unique equilibrium with an analytical solution. Using the linear example, we show that supply functions in the equilibrium converges to price contract as slopes of marginal cost functions converges to 0, for any extent of corporate social responsibility. Supply functions in the equilibrium converge to their marginal cost functions as both firms converge to public firms.

Key words: supply function equilibrium, corporate social responsibility, partial privatization

1 Introduction

Since Singh and Vives (1984) pointed out that a quantity contract dominates a price contract, a bunch of researches showed its robustness.[1] Matsumura and Ogawa (2012) showed that, however, if there is a public firm who wants to maximize social welfare, the result is reversed. Ghosh and Mitra (2014) and Matsumura and Ogawa (2014) generalize it to competition among two firms with *cooperate social responsibility* (henceforth, *CSR*), in a sense that they partially care about social welfare. They obtain an intriguing finding that the reversal of price/quantity competitions is not caused by the existence of firms caring about society, but by asymmetry in objective functions. In this paper, we examine whether the robustness of Singh and Vives (1984) in symmetry settings still holds even if firms cannot commit either price or quantity contracts as in the models by Singh and Vives (1984), Matsumura and Ogawa (2012), and Matsumura and Ogawa (2014) but choose arbitrary supply schedules simultaneously. In order to answer this question, we employ an equilibrium concept of *supply function equilibrium* (henceforth, *SFE*), introduced by Klemperer and Meyer (1989) and accommodate the objective functions to CSR. As in Klemperer and Meyer (1989), there exist many equilibria if there is no uncertainty in a demand function, but under a setting with demand uncertainty, properties of symmetric equilibria are characterized. Furthermore, under a linear demand with uncertainty, we obtain a unique equilibrium with an analytical solution. Using the linear example, we show that supply functions in the equilibrium converges to price contract as slopes of marginal cost functions converges to 0, for any extent of CSR. Supply functions in the equilibrium converge to their marginal cost functions as both firms converge to public firms.

2 SFE and CSR

SFE is an equilibrium concept introduced by Klemperer and Meyer (1989). Firms choose their own supply schedule simultaneously, and then, the market is cleared such that total

[1] See Cheng (1985), Tanaka (2001a,b), and Tasnádi (2006).

supply matches to the demand at a certain price. In the model with demand uncertainty, market is cleared after the realization of uncertainty. SFE is defined as the (pure strategy) Nash equilibria in this game. A remarkable feature of SFE is that it is characterized as locus of ex post optimal price-quantity pairs given the other's supply function. That means, each firm guesses the other's (fixed) supply schedule. After the realization of demand uncertainty, combined with the other's supply function, a residual demand function is determined. The firm chooses its ex post optimal price-quantity pair along with the residual demand (there is no incentives to choose price-quantity pair away the residual demand). Ex post optimal points vary according to realizations of uncertainty even though she assumes the other's supply function fixed. Then, locus of ex post optimal points arise as a function from price to quantity. Since she can obtain ex post optimized profit through this supply function, she has no incentive to take other supply functions in the first stage given other's supply function. Thus, locus of ex post optimal points given other's supply function is a best response to the other's supply function. By considering such best responses for each firms, we obtain NE in this game, in other words, SFE.

This equilibrium concept would be valid in cases where firms guess other's strategy as an arbitrary function rather than a price contract with no limit in quantities or a quantity contract with no limit in prices. After Klemperer and Meyer (1989) introduced this concept, SFE is applied mostly in a literature on auctions. For instance, Green and Newbery (1992) and Wolfram (1999) analyze the British electricity market using SFE. Vives (2011) proposes applications in demand schedule competitions in financial markets by reinterpreting SFE. Even though SFE is usually characterized as continuous functions in contrast to auctions in reality, Holmberg et al. (2013) derive conditions that the equilibrium in a discrete model converges to continuous SFE.

The objective function with CSR, in which firms consider a convex combination of its own profit and the social welfare, is introduced by Matsumura (1998). We can literally interpret it as a objective function with CSR or as that of some partially privatized firms. Even though the main measures of CSR would be actions on environment or poverty, we focus on CSR through their own business as in Matsumura and Ogawa (2014) and Ghosh and Mitra (2014). In fact, some firms argue that they set low prices in favor of consumers. It is also possible that partially privatized firms whose markup is regulated would set low prices, or that the government requires large amount of supply trying to supply as many consumers as possible. Thus, trying to improve the welfare through their own business is also common in the society.

In the following section, we generalize the model by Klemperer and Meyer (1989) to accommodate it to CSR, and then, characterize the equilibria.

3 Model

3.1 Without Uncertainty

The demand curve is $Q = D(p)$. We assume that D is twice continuously differentiable, strictly decreasing and concave on $p \in (0, \hat{p})$, where $\hat{p}$ is a price such that $D(\hat{p}) = 0$. The firms have identical cost function C satisfying $C'(p) \geq 0$, and $C''(p) \geq 0$ $\forall p \in [0, \infty)$. A strategy for firm i $(i = 1, 2)$ is a function mapping from price into quantity : $S_i : [0, p) \to (-\infty, \infty)$. The firm i's payoff is defined as $v_i = \theta SW + (1-\theta)\pi_i$, where $\theta \in (0, 1)$, SW is the total social surplus, and π_i is firm i's profit. We denote a positive output pair as $(\bar{q}_1, \bar{q}_2)$ and a market clearing price as $\bar{p} = D^{-1}(\bar{q}_1, \bar{q}_2)$.

We assume that firms 1 and 2 choose supply function simultaneously and we focus on pure strategy Nash equilibria, in which S_i maximizes i's payoff given that j chooses S_j $(i, j = 1, 2, j \neq i)$.

Claim 1 *If there is no uncertainty in the demand function, any pairs of quantities*$(\bar{q}_1, \bar{q}_2)$ *s.t.* $\forall i = 1, 2$ $\frac{(1-\theta)\bar{q}_i + \theta[(\bar{p} - C'(\bar{q}_j))D'(\bar{p})]}{(1-\theta)(\bar{p} - C'(\bar{q}_i)) + \theta(C'(\bar{q}_j) - C'(\bar{q}_i))} \geq 0$ $(j \neq i)$ *are supported as an outcome of supply function equilibria.*

Proof of Claim 1 To support $(\bar{q}_1, \bar{q}_2)$ as an equilibrium outcome, we seek a pair of supply functions S_1 and S_2 passing through $(\bar{p}, \bar{q}_1)$ and $(\bar{p}, \bar{q}_2)$ respectively, and such that $(\bar{p}, \bar{q}_i)$ is a profit maximizing point along i's residual demand curve given other's supply function $S_j (i,\ j = 1,\ 2,\ j \neq i)$. Here, we assume that supply functions are twice continuously differentiable for a moment, and later, we will show that any $(\bar{q}_1, \bar{q}_2)$ satisfying the condition in the claim is supported by such supply functions.

Given S_j, firm i's profit maximization problem is written as follows:

$$\begin{aligned}\max_p \quad & (1-\theta)\left[p\left(D(p)-S_j(p)\right)-C\left(D(p)-S_j(p)\right)\right] \\ + \quad & \theta\left[\int_p^{\hat{p}} D(\dot{p})\,d\dot{p}+p\left(D(p)-S_j(p)\right)-C\left(D(p)-S_j(p)\right)+pS_j(p)-C\left(S_j(p)\right)\right].\end{aligned}$$

The first order condition is

$$\begin{aligned}&(1-\theta)\left[\left(D(p)-S_j(p)\right)+\left(p-C'\left(D(p)-S_j(p)\right)\right)\left(D'(p)-S_j'(p)\right)\right] \\ + \quad & \theta\left[\left(p-C'\left(D(p)-S_j(p)\right)\right)\left(D'(p)-S_j'(p)\right)+\left(p-C'\left(S_j(p)\right)\right)S_j'(p)\right] \quad = 0.\end{aligned}$$

In order for $\bar{p}$ to solve this equation, we must have

$$\begin{aligned}S_j'(\bar{p}) &= \frac{(1-\theta)\left[\bar{q}_i+\left(\bar{p}-C'(\bar{q}_i)\right)D'(\bar{p})\right]+\theta\left[\left(\bar{p}-C'(\bar{q}_i)\right)D'(\bar{p})\right]}{(1-\theta)\left(\bar{p}-C'(\bar{q}_i)\right)+\theta\left(C'(\bar{q}_j)-C'(\bar{q}_i)\right)} \\ &= D'(\bar{p})+\frac{(1-\theta)\,\bar{q}_i+\theta\left[\left(\bar{p}-C'(\bar{q}_j)\right)D'(\bar{p})\right]}{(1-\theta)\left(\bar{p}-C'(\bar{q}_i)\right)+\theta\left(C'(\bar{q}_j)-C'(\bar{q}_i)\right)}. \qquad (1)\end{aligned}$$

The second derivative of i's payoff with respect to p is written as

$$\begin{aligned}\frac{\partial^2 v_i}{\partial p^2} &= \theta[\left(1 - C''\left(D\left(p\right) - S_j\left(p\right)\right)\left(D'\left(p\right) - S'_j\left(p\right)\right)\right)\left(D'\left(p\right) - S'_j\left(p\right)\right) \\ &+ \left(p - C'\left(D\left(p\right) - S_j\left(p\right)\right)\right)\left(D''\left(p\right) - S''_j\left(p\right)\right) \\ &+ \left(1 - C''\left(S_j\left(p\right)\right)S'_j\left(p\right)\right)S'_j\left(p\right) \\ &+ \left(p - C'\left(S_j\left(p\right)\right)\right)S''_j\left(p\right)] \\ &+ \left(1-\theta\right)[2\left(D'\left(p\right) - S'_j\left(p\right)\right) \\ &- C''\left(D\left(p\right) - S_j\left(p\right)\right)\left(D'\left(p\right) - S'_j\left(p\right)\right)^2 \\ &+ \left(p - C'\left(D\left(p\right) - S_j\left(p\right)\right)\right)\left(D''\left(p\right) - S''_j\left(p\right)\right)]\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 v_i}{\partial p^2}\,|_{p=\bar{p}} &= \theta[D'\left(p\right) - C''\left(\bar{q}_i\right)\left(D'\left(\bar{p}\right) - S'_j\left(\bar{p}\right)\right)^2 \\ &+ \left(\bar{p} - C'\left(\bar{q}_i\right)\right)D''\left(\bar{p}\right) \\ &- C''\left(\bar{q}_j\right)\left(S'_j\left(\bar{p}\right)\right)^2 \\ &+ \left(C'\left(\bar{q}_i\right) - C'\left(\bar{q}_j\right)\right)S''_j\left(\bar{p}\right)] \\ &+ \left(1-\theta\right)[2\left(D'\left(\bar{p}\right) - S'_j\left(\bar{p}\right)\right) \\ &- C''\left(\bar{q}_i\right)\left(D'\left(\bar{p}\right) - S'_j\left(\bar{p}\right)\right)^2 \\ &+ \left(\bar{p} - C'\left(\bar{q}_i\right)\right)\left(D''\left(\bar{p}\right) - S''_j\left(\bar{p}\right)\right)]\end{aligned}$$

Therefore, if $\frac{(1-\theta)\bar{q}_i+\theta[(\bar{p}-C'(\bar{q}_j))D'(\bar{p})]}{(1-\theta)(\bar{p}-C'(\bar{q}_i))+\theta(C'(\bar{q}_j)-C'(\bar{q}_i))} \geq 0$, $(\bar{q}_1, \bar{q}_2)$ is supported by a supply function equilibrium with supply functions which satisfy (1) at $\bar{p}$. Global concavity of the objective function is satisfied if other firms take $S_j\left(p\right)$ s.t. $D''\left(p\right) - S''_j\left(p\right) = 0\,\forall p$.**Q.E.D.**

3.2 With Uncertainty

Let demand be subject to an exogenous shock ϵ, where ϵ is a scalar random variable with strictly positive density everywhere on the support $[\underline{\epsilon}, \bar{\epsilon}]$: $Q = D\left(p, \epsilon\right)$, where $\forall\left(p, \epsilon\right)$, $-\infty < D_p\left(p, \epsilon\right) < 0$, $D_{pp}\left(p, \epsilon\right) \leq 0$, and $D_{\epsilon}\left(p, \epsilon\right) > 0$. We assume that $D_{p\epsilon}\left(p, \epsilon\right) =$

$0\,\forall\,(p,\,\epsilon)$. Since $D_\epsilon\,(p,\,\epsilon) > 0$, we can invert the demand curve and write $e\,(Q,\,p)$ for the value of the shock ϵ for which demand is Q at price p, that is, $e\,(Q,\,p)$ satisfies $Q = D\,(p,\,e\,(Q,\,p))$. Since for $\epsilon < e\,(0,\,0)$, there is no point on $D\,(p,\,\epsilon)$ with $p \geq 0$ and $Q \geq 0$, we assume that the support of ϵ is a subset of $[e\,(0,\,0)\,,\,\infty)$. The firms have identical cost function C satisfies $C'\,(p) \geq 0$ and $0 < C''\,(p) < \infty\,\forall q \in [0,\,\infty)$. Without loss of generality, let $C'\,(0) = 0$.

We assume that firms 1 and 2 choose supply function simultaneously before the realization of ϵ. Then, a strategy for firm $i\,(i = 1,\,2)$ is defined as a function mapping from price into quantity: $S_i\,:\,[0,\,p) \rightarrow (-\infty,\,\infty)$. After the realization of ϵ, supply functions are implemented by each firm producing at a point $(p^*\,(\epsilon)\,,\,S_i\,(p^*\,(\epsilon)))$ such that $D\,(p^*\,(\epsilon)) = S_1\,(p^*\,(\epsilon)) + S_2\,(p^*\,(\epsilon))$, that is, demand matches total supply. If there are multiple intersection of total supply and demand, supply functions are implemented at a price which maximizes i's payoff given that other is taking $S_j\,(p)$, provided such a unique maximizer for each firm exists and coincides each other.

We focus on pure strategy Nash equilibria, in which S_i maximizes i's payoff given that j chooses S_j $(i,\,j = 1,\,2,\,j \neq i)$.

Firm 1's residual demand at any price is the difference between demand and quantity that 2 is willing to supply at that price. Thus, if firm 2 is committed to the supply function $S_2\,(p)$, 1's residual demand curve is written as $D\,(p,\,\epsilon) - S_2\,(p)$. Since ϵ is a scalar, the set of payoff maximization points along 1's residual demand curve as ϵ varies is a one-dimensional curve in price-quantity space. If this curve can be described by a supply function $q_i = S_i\,(p)$ that intersects each realization of i's residual demand curve once and only once, then by committing to S_i, firm i can achieve ex post optimal adjustment to the shock. In this case, S_i is clearly i's unique optimal supply function in response to S_j. We assume supply function and show later that under our hypothesis there exist equilibria in which this is indeed this case.

Given S_j, firm i's profit maximization problem is as follows:

$$\max_p \quad (1-\theta)\left[p\left(D\left(p,\,\epsilon\right)-S_j\left(p\right)\right)-C\left(D\left(p,\,\epsilon\right)-S_j\left(p\right)\right)\right] \tag{2}$$
$$+ \quad \theta\left[\int_p^{\hat{p}} D\left(\dot{p},\,\epsilon\right)d\dot{p}+p\left(D\left(p,\,\epsilon\right)-S_j\left(p\right)\right)-C\left(D\left(p,\,\epsilon\right)-S_j\left(p\right)\right)+pS_j\left(p\right)-C\left(S_j\left(p\right)\right)\right].$$

The first order condition is

$$\frac{\partial v_i}{\partial p} = (1-\theta)\left[\left(D\left(p,\,\epsilon\right)-S_j\left(p\right)\right)+\left(p-C'\left(D\left(p,\,\epsilon\right)-S_j\left(p\right)\right)\right)\left(D_p\left(p,\,\epsilon\right)-S_j'\left(p\right)\right)\right] \tag{3}$$
$$+ \quad \theta\left[\left(p-C'\left(D\left(p,\,\epsilon\right)-S_j\left(p\right)\right)\right)\left(D_p\left(p,\,\epsilon\right)-S_j'\left(p\right)\right)+\left(p-C'\left(S_j\left(p\right)\right)\right)S_j'\left(p\right)\right]=0.$$

Let $\{p^n(\epsilon)\}_{n=1}^N$ be prices which satisfy (3) for given ϵ and $S_j\left(p\right)$ and let $D\left(p^n\left(\epsilon\right),\,\epsilon\right)-S_j\left(p^n\left(\epsilon\right)\left(\epsilon\right)\right)=q^n\left(\epsilon\right)$. Suppose each $p^n(\epsilon)$ is invertible, then $S_i\left(p\right)\equiv q^n\left(\left(p^n\right)^{-1}\left(p\right)\right)$. Let us rewrite (3) so that it implicitly defines the function $S_i\left(p\right)$. Replace $q_i^{opt}\left(\epsilon\right)$ by $S_i\left(p\right)$ and use $e\left(Q,\,p\right)$ as defined above to replace $D_p\left(p_i^{opt}\left(\epsilon\right),\,\epsilon\right)$ by $D_p\left(p,\,e\left(S_i\left(p\right)+S_j\left(p\right),\,p\right)\right)$, so (3) becomes

$$\frac{\partial v_i}{\partial p} = (1-\theta)\left[S_i\left(p\right)+\left(p-C'\left(S_i\left(p\right)\right)\right)\left(D_p\left(p,\,e\left(S_i\left(p\right)+S_j\left(p\right),\,p\right)\right)-S_j'\left(p\right)\right)\right] \tag{4}$$
$$+ \quad \theta\left[\left(p-C'\left(S_i\left(p\right)\right)\right)\left(D_p\left(p,\,e\left(S_i\left(p\right)+S_j\left(p\right),\,p\right)\right)-S_j'\left(p\right)\right)+\left(p-C'\left(S_j\left(p\right)\right)\right)S_j'\left(p\right)\right]=0.$$

We suppose symmetric equilibrium, so (4) becomes

$$S_j'\left(p\right)=\frac{(1-\theta)\,S\left(p\right)+\left(p-C'\left(S\left(p\right)\right)\right)D_p\left(p,\,e\left(2S\left(p\right),\,p\right)\right)}{(1-\theta)\left(p-C'\left(S\left(p\right)\right)\right)}. \tag{5}$$

We assume that $D_{p\epsilon}=0\,\forall\left(p,\,\epsilon\right)$. then demand is translated horizontally by the shock ϵ. Then, if we write $D_p\left(p,\,e\left(2S\left(p\right),\,p\right)\right)$ simply as $D_p\left(p\right)$, (5) becomes the differential equation

$$S_j'\left(p\right)=\frac{(1-\theta)\,S+\left(p-C'\left(S\right)\right)D_p\left(p\right)}{(1-\theta)\left(p-C'\left(S\right)\right)}\equiv f\left(p,\,S\right). \tag{6}$$

The non-autonomous first-order differential equation (6) can be written as the system of two-dimensional autonomous differential equation

$$\begin{aligned} S'(t) &= (1-\theta) S + (p - C'(S)) D_p(p), \\ p'(t) &= (1-\theta)(p - C'(S)). \end{aligned}$$

4 Results

We characterize the differential equation (6) by the following series of lemmas.

Lemma 1 *The locus of points satisfying $f(p, S) = 0$ is a continuous, differentiable function $S = S^0(p)$, satisfying*

$$\begin{array}{lc} (i) & S^0(0) = 0, \\ (ii) & S^0(p) < (C')^{-1}(p), \forall p > 0, \\ (iii) & S^{0\prime}(p) \text{ is positive and increasing in } \theta, \forall p \geq 0, \text{ and} \\ (iv) & S^{0\prime}(0) < \frac{1}{C''(0)}. \end{array}$$

Proof of Lemma 1: Differentiation of (6) w.r.t. S yields

$$f_S(p, S) = \frac{p - C'(S) + SC''(S)}{(p - C'(S))^2}$$

so for all $(\bar{p}, \bar{S}) \neq (0, 0)$ such that $f(\bar{p}, \bar{S}) = 0$,

$$\begin{aligned} f_S(\bar{p}, \bar{S}) &= \frac{1}{\bar{p} - C'(\bar{S})} + \frac{\bar{S}}{\bar{p} - C'(\bar{S})} \frac{C''(\bar{S})}{\bar{p} - C'(\bar{S})} \\ &= \frac{1 - \frac{1}{(1-\theta)} D_p(\bar{p}) C''(\bar{S})}{\bar{p} - C'(\bar{S})} \neq 0. \end{aligned}$$

Therefore, by the Implicit Function Theorem, $f(p, S) = 0$ implicitly defines, in the neighborhood of any such $\left(\bar{p}, \bar{S}\right)$, a unique function $S = S^0(p)$, which is continuous and differentiable.

To prove (i) and (ii), observe that $\forall \theta \in [0, 1)$, either $S^0(p)$ and $p - C'(S^0(p))$ are both positive or they are both zero since $-\infty < D_p(p) < 0$ and $f(p, S^0(p)) = 0$. Hence, $p > C'(S^0(p))$ whenever $S^0(p) > 0$. Furthermore, $S^0(0) = 0$ is the unique solution to $f(0, S) = 0$. For all $p > 0$, $S^0(p) > 0$ (otherwise, $S^0(p) = 0$ and $p - C'(S^0(p)) > 0$) and $p > C'(S^0(p))$. Since $C'' > 0$, we can take inverse function of C' and obtain $(C')^{-1}(p) > S^0(p)$ for all $p > 0$. Here, as $p \to 0$, the upper bound of $S^0(p)$ converges to zero and $S^0(p) > 0$ for all $p > 0$. Then, $S^0(p) \to 0$ as $p \to 0$. Thus, $S^0(p)$ is continuous at $p = 0$.

To prove (iii) and (iv), differentiate $f(p, S^0(p)) = 0$ totally with respect to p and substitute using this equation to get

$$S^{0\prime}(p) = -\frac{D_p(p) + D_{pp}(p)(p - C'(S^0(p)))}{(1-\theta) - D_p(p)\, C''(S^0(p))}.$$

Now $\lim_{p \to 0} S^{0\prime}(p)$ exists and equals

$$-\frac{D_p(0)}{(1-\theta) - D_p(0)\, C''(0)} \equiv S^{0\prime}(p),$$

where $0 < S^{0\prime}(p) < \frac{1}{C''(0)}$, so $S^0(p)$ is continuous and differentiable at $p = 0$. **Q.E.D.**

Lemma 2 *The locus of points satisfying $f(p, S) = \infty$ is a continuous, differentiable function, $S = S^\infty(p) \equiv (C')^{-1}(p)$. Hence, $S^\infty(0) = 0$ and $0 < S^{\infty\prime}(p) < \infty \, \forall p \geq 0$.*

Proof of Lemma 2: (same as the proof of claim 2 in KM) From (6), $S^\infty(p)$ solves $f(p, S^\infty(p)) = \infty$ implies $S^\infty(p)$ solves $p - C'(S^\infty(p)) = 0$, so since $C'' > 0$, $S^\infty(p) = (C')^{-1}(p) \; \forall p$. The stated properties of $S^\infty(p)$ follows from the assumptions on $C'(S)$. **Q.E.D.**

Lemma 3 *For all points (p, S) between the $f = 0$ and $f = \infty$ loci, $0 < f(p, S) < \infty$. For all points in the first quadrant above $f = 0$ locus or below the $f = \infty$ locus, $0 > f(p, S) > -\infty$.*

Proof of Lemma 3: (same as the proof of claim 3 in KM) Since $\frac{S}{p - C'(S)}$ is finite and increasing in S as long as $p > C'(S)$, for a given $\bar{p}$, $f(\bar{p}, S)$ is finite and monotonically increasing in S for $S \in [0, (C')^{-1}(p))$. Below the $f = \infty$ locus, $0 > \frac{S}{p - C'(S)} > -\infty$, so since $0 > D_p > -\infty$, $0 > f(p, S) > -\infty$. **Q.E.D.**

Lemma 4 *If $S(p)$ solves (6) and other firm takes $S(p)$, the second derivative of i's payoff with respect to p for a given ϵ evaluated at an intersection of $S(p)$ and residual demand function $D(p) + \epsilon - S(p)$ is written as*

$$\begin{aligned} \frac{\partial^2 v_i(p, \epsilon; S(p))}{\partial p^2}\Big|_{p=p^*} &= (D_p(p^*) - S'(p^*))((1-\theta) + C''(D(p^*) + \epsilon - S(p^*))) \\ &\quad - C''(D(p^*) + \epsilon - S(p^*))(D_p(p^*) - S'(p^*))^2 - (1-\theta) S'(p^*), \end{aligned} \tag{7}$$

where p^* is a price that solves $D(p^*) + \epsilon - 2S(p^*) = 0$.

Proof of Lemma 4: Given that j chooses $S(p)$, the second order derivative of i's payoff with respect to p for a given ϵ is

$$\begin{aligned} \frac{\partial^2 v_i(p, \epsilon; S(p))}{\partial p^2} &= (2-\theta)\{D_p(p) - S'(p)\} - C''(D(p) + \epsilon - S(p))(D_p(p) - S'(p))^2 \\ &\quad + (p - C'(D(p) + \epsilon - S(p)))(D_{pp}(p) - S''(p)) \\ &\quad + \theta S'(p) - \theta C''(S(p))(S'(p))^2 + \theta(p - C'(S(p)))S''(p). \end{aligned} \tag{8}$$

If $S(p)$ solves (6), we can differentiate (6) totally with respect to p to obtain an expression for $S''(p)$:

$$S''(p) = \frac{X_1}{((1-\theta)(p - C'(S(p))))^2}, \tag{9}$$

where

$$
\begin{aligned}
X_1 \equiv\ & \left[(1-\theta) S'(p) + (1 - C''(S(p)) S'(p)) D_p(p) + (p - C'(S(p))) D_{pp}(p)\right]\left[(1-\theta)(p - C'(S(p)))\right] \\
& - \left[(1-\theta) S(p) + (p - C'(S(p))) D_p(p)\right]\left[(1-\theta)(1 - C''(S(p)) S'(p))\right].
\end{aligned}
$$

Using (6) to substitute for $S(p)$ in (9) gives

$$
S''(p) = \frac{(1-\theta) S'(p) + (1 - C''(S(p)) S'(p))(D_p(p) - (1-\theta) S'(p)) + (p - C'(S(p))) D_{pp}(p)}{(1-\theta)(p - C'(S(p)))}, \tag{10}
$$

so when $S(p)$ solves (6), $S''(p)$ in (8) is replaced by (10). Besides, if we evaluate at $p = p^*$ where p^* solves $D(p^*) + \epsilon - 2S(p) = 0$, (8) becomes (7). **Q.E.D.**

By these lemmas, we have the following proposition.

Proposition 1 (Necessity of positive slope) *If ϵ has full support ($\underline{\epsilon} = e(0, 0)$, $\bar{\epsilon} = \infty$) and S is a symmetric SFE tracing through ex post optimal points, then $\forall p \geq 0$, S satisfies (6) and $0 < S'(p) < \infty$.*

Proof of Proposition 1 Satisfaction of (6) $\forall p \geq 0$ is a necessary condition for a supply function defined for all $p \geq 0$ to trace through ex post optimal points when the other firm commits to the same supply function. To show that $0 < S'(p) < \infty \, \forall p \geq 0$ is also a necessary condition, we show that if, for some p, S ever crosses either $f = 0$ from below or $f = \infty$ from the left, then S must eventually violate the global optimality[2].

Once trajectory S cross $f = 0$ from below, S' become and stays negative and, from (A2), S'' also becomes and stays negative. Therefore, the trajectory will eventually intersect the $S = 0$ axis at a point $(p_0, 0)$ with $p_0 > C'(0)$, where $S'(p_0) = f(p_0, 0) = \frac{1}{1-\theta} D_p(p_0)$. Therefore, for $\epsilon = e(0, p_0)$, $Q = D(p_0, \epsilon) = 0$ by definition and then, residual demand

[2]Actually, such a part of S represents ones of multiple intersections for certain ϵ's which results smaller profit than another intersection for the same ϵ.

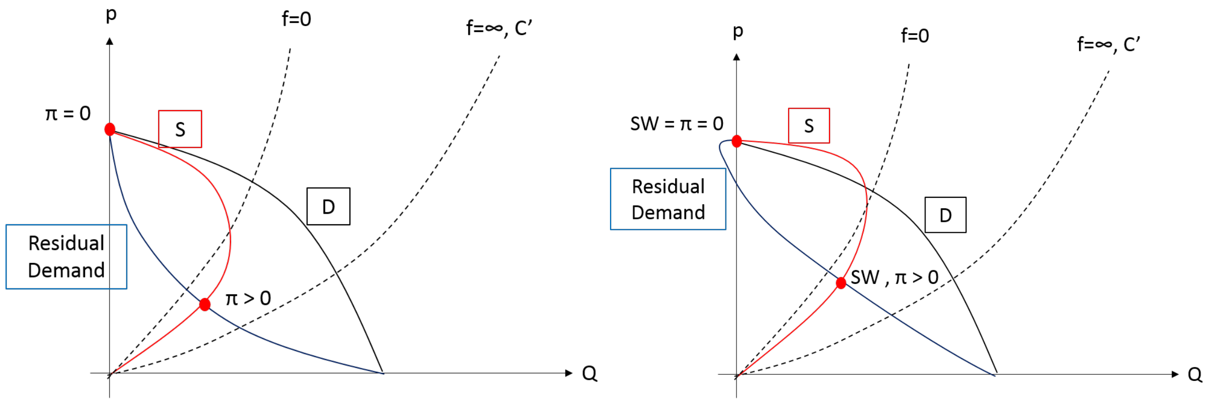

Figure 1: A supply function (satisfying FOC and symmetry) violating global optimality. (Left: $\theta = 0$, Right: $0 < \theta < 1$)

$D(p_0, \epsilon) - S(p_0) = 0$. Then, given firm j takes S, p_0 satisfies the first order condition but that result in $q_i = q_j = 0$ and $SW = \pi_i = v_i = 0$. On the other hand, since $S'(p_0) = \frac{1}{1-\theta} D_p(p_0)$, $S(p)$ and the residual demand $D(p, \epsilon) - S(p)$ for the same $\epsilon = e(0, p_0)$ cross each other at another point (p_1, q_1) where $q_1 > 0$ and $p_1 > C'(q_1)$ (Fig.1). Since SW, π_i, $v_i > 0$ at (p_1, q_1), firm i has an incentive to adjust from p_0 to p_1. Thus, S eventually violates the global optimality. **Q.E.D.**

Lemma 5 (Local optimality of S with positive slope) *If ϵ has full support $(\underline{\epsilon} = e(0, 0), \bar{\epsilon} = \infty)$ and S satisfies (6) and $0 < S'(p) < \infty$, then S is locus of local optimal points given that the other firm is taking S.*

Proof of lemma 5: Since $0 < S'(p) < \infty$, total supply intersects total demand at a unique point for each ϵ. Since S satisfies (6) $\forall p \geq 0$, the first order condition for ex post payoff maximization is satisfied everywhere along S when the other firm commits to the same supply function. The condition on S and S' together imply that $\frac{\partial^2 v_i(p, \epsilon; S(p))}{\partial p^2}|_{p=p^*} < 0 \, \forall \epsilon \geq e(0, 0)$. Therefore, S is locus of local (ex post) optimal points given that the other firm is taking S. **Q.E.D.**

In cases of linear demand and quadratic cost function, we can characterize SFE more clearly, and we can show uniqueness of a symmetric SFE.

4.1 Linear example

In this subsection, we specify cost functions and a demand function to show an example with an analytical solution. The identical cost functions is defined as $C(S) = \frac{c}{2}S^2$ and the total demand function is defined as $D(p, \epsilon) = \epsilon - mp$. Then, the following proposition holds.

Proposition 2 (Uniqueness of symmetric SFE) *In the linear case, if ϵ has full support ($\underline{\epsilon} = e(0, 0), \bar{\epsilon} = \infty$), then S is a symmetric SFE tracing through ex post optimal points if and only if S satisfies (6) and $0 < S'(p) < \infty$. Furthermore, such a S is characterized as a unique and linear supply function.*

Proof of Proposition 2 By proposition 1, for S to be a symmetric SFE, S must satisfy (6) and $0 < S'(p) < \infty$. In the linear case, (6) is rewritten as follows:

$$\begin{aligned} S'(p) &= \frac{(1-\theta)S + \{p - C'(S)\} \cdot D_p(p)}{(1-\theta)\{p - C'(S)\}} \\ &= \frac{(1-\theta)S + \{p - cS\} \cdot (-m)}{(1-\theta)\{p - cS\}}. \end{aligned}$$

In autonomous form,

$$\begin{aligned} \frac{dS}{dt} &= (1-\theta)S + \{p - cS\} \cdot (-m) \\ \frac{dp}{dt} &= (1-\theta)\{p - cS\}. \end{aligned}$$

Then,

$$
\begin{aligned}
\begin{bmatrix} \frac{dS}{dt} \\ \frac{dp}{dt} \end{bmatrix} &= \begin{bmatrix} (1-\theta+mc)\,S - mp \\ -c\,(1-\theta)\,S + (1-\theta)\,p \end{bmatrix} \\
&= \begin{bmatrix} (1-\theta+mc) & -m \\ -c\,(1-\theta) & (1-\theta) \end{bmatrix} \begin{bmatrix} S \\ p \end{bmatrix}.
\end{aligned}
$$

For any eigenvalue r, the following equation must be satisfied:

$$
\begin{aligned}
det\left(\begin{bmatrix} (1-\theta+mc) & -m \\ -c\,(1-\theta) & (1-\theta) \end{bmatrix} - rI\right) &= 0 \\
\Leftrightarrow r &= \frac{2\,(1-\theta) + mc \pm \sqrt{m^2c^2 + 4mc\,(1-\theta)}}{2}.
\end{aligned}
$$

We have different and unequal eigenvalues. For each eigenvalue r_1, r_2, eigenvectors are defined as follows:

$$
\begin{aligned}
\left\{\begin{bmatrix} (1-\theta+mc) & -m \\ -c\,(1-\theta) & (1-\theta) \end{bmatrix} - r_iI\right\} \begin{bmatrix} u_i \\ w_i \end{bmatrix} &= 0 \\
\Leftrightarrow \begin{bmatrix} ((1-\theta+mc) - r_i)\,u_i - mw_i \\ -c\,(1-\theta)\,u_i + ((1-\theta) - r_i)\,w_i \end{bmatrix} &= 0.
\end{aligned}
$$

Then,

$$
\begin{aligned}
\frac{u_i}{w_i} &= \frac{(1-\theta) - r_i}{c\,(1-\theta)} \\
&= \frac{(1-\theta) - (1-\theta) - \frac{mb \pm \sqrt{m^2c^2 + 4mc(1-\theta)}}{2}}{c\,(1-\theta)} \\
&= \frac{-mb \mp \sqrt{m^2c^2 + 4mc\,(1-\theta)}}{2c\,(1-\theta)} \\
&= \frac{-m \mp \sqrt{m^2 + \frac{4m(1-\theta)}{c}}}{2\,(1-\theta)}.
\end{aligned}
$$

Let larger eigenvalue be r_1. Then, $\frac{u_1}{w_1} < 0$ and $\frac{u_2}{w_2} > 0$. Since the eigenvalues are real and unequal, the solution to the differential equation is written as

$$\begin{pmatrix} S \\ p \end{pmatrix} = A_1 e^{\lambda_1 t} \begin{pmatrix} u_1 \\ w_1 \end{pmatrix} + A_2 e^{\lambda_2 t} \begin{pmatrix} u_2 \\ w_2 \end{pmatrix} \tag{11}$$

where A_1 and A_2 are arbitrary constants. Here, if $A_1 \neq 0$,

$$\begin{aligned} \frac{S}{p} &= \frac{A_1 e^{\lambda_1 t} u_1 + A_2 e^{\lambda_2 t} u_2}{A_1 e^{\lambda_1 t} w_1 + A_2 e^{\lambda_2 t} w_2} \\ &= \frac{A_1 u_1 + A_2 \left(e^{(\lambda_2 - \lambda_1)t}\right) u_2}{A_1 w_1 + A_2 \left(e^{(\lambda_2 - \lambda_1)t}\right) w_2} \\ &\to \frac{u_1}{w_1} < 0 \text{ as } t \to \infty, \end{aligned}$$

so all trajectories eventually leave the region between $f = 0$ and $f = \infty$ and their slope become negative. Therefore, only remaining S satisfying the necessary conditions is (11) with $A_1 = 0$:

$$S(p) = \frac{-m + \sqrt{m^2 + \frac{4m(1-\theta)}{c}}}{2(1-\theta)} p \equiv \frac{g(\theta)}{h(\theta)}. \tag{12}$$

Suppose that other firm is taking this linear supply function. Then, local optimality for firm i's payoff function is satisfied along $S(p)$ by lemma 5, and residual demand is also linear since demand function is defined as linear. Since, given ϵ, both of residual demand and marginal cost are linear in p, firm i's profit function π_i is written as a function quadratic in p. On the other hand, since, given ϵ, demand function and industrial marginal costs are linear in p, SW is written as a function quadratic in p. Therefore, the payoff for firm i, which is weighted average of i's profit and SW, is written as a quadratic function. Therefore, the local optimal point given ϵ is actually a unique global maximizer given ϵ. Thus, (12) is a symmetric SFE. **Q.E.D.**

We check the effect of θ. Since $g(1) = h(1) = 0$ by l'Hopital's rule we have

$$\lim_{\theta \to 1} S(p) = \lim_{\theta \to 1} \frac{g(\theta)}{h(\theta)} = \lim_{\theta \to 1} \frac{g'(\theta)}{h'(\theta)} = \lim_{\theta \to 1} \frac{m}{c} \frac{p}{\sqrt{m^2 + \frac{4m(1-\theta)}{c}}} = \frac{p}{c}$$

Thus, θ converges to 1 and supply function converges to marginal production cost.

5 Conclusion

As shown in the previous section, when the level of CSR improve in both firms, welfare would increase since the supply function converges to marginal cost functions. Thus, symmetric improvement in CSR would actually improve social welfare.

In contrast to Matsumura and Ogawa (2014), if firms can choose arbitrary supply schedules, they choose ones close to price contracts under a linear demand with uncertainty and almost linear cost functions. This result bring us a new question: 'What makes the difference? ' Our model is different from the previous research in at least three ways. First, in our model, supply functions can take arbitrary shapes while, in the model by Matsumura and Ogawa (2014), possible price schedules are only horizontal or vertical. Second, in contrast to our model, firms can commit to price/quantity contracts in the first stage of the model in Matsumura and Ogawa (2014). Finally, we assume homogeneous goods rather than differentiated goods. Changing these settings one by one and comparing them would help us understanding what is happening in each model.

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