Cournot-Bertrand Comparison in a Mixed Oligopoly*

Junichi Haraguchi†
Graduate School of Economics, The University of Tokyo
and
Toshihiro Matsumura
Institute of Social Science, The University of Tokyo

June 13, 2015

Abstract

We revisit the classic discussion comparing price and quantity competition, but in a mixed oligopoly in which one state-owned public firm competes against private firms. It has been shown that in a mixed duopoly, price competition yields a larger profit for the private firm. This implies that firms face weaker competition under price competition, which contrasts sharply with the case of a private oligopoly. Here, we adopt a standard differentiated oligopoly with a linear demand. We find that regardless of the number of firms, price competition yields higher welfare. However, the profit ranking depends on the number of private firms. We find that if the number of private firms is greater than or equal to five, it is possible that quantity competition yields a larger profit for each private firm. We also endogenize the price-quantity choice. Here, we find that Bertrand competition can fail to be an equilibrium, unless there is only one private firm.

JEL classification numbers: H42, L13

Key words: Cournot, Bertrand, Mixed Markets, Differentiated Products, Oligopoly

---

*We are indebted to two anonymous referees for their valuable and constructive suggestions. We are grateful to Dan Sasaki and participants of the seminars at The University of Tokyo, Nippon University and annual meeting of JEA 2014 spring for their helpful comments and suggestions. The second author acknowledges the financial support of the Murata Science Foundation and JSPS KAKENHI Grant Number 15k03347. Any remaining errors are our own.

†Corresponding author: Junichi Haraguchi, Graduate School of Economics, The University of Tokyo, 7-3-1, Hongo, Bunkyo-ku, Tokyo 113-0033, Japan. Phone: (81)-5841-4932. Fax: (81)-5841-4905. E-mail: 7299572051@mail.ecc.u-tokyo.ac.jp

1 Introduction

The comparison between price and quantity competition has been discussed extensively in the literature. In oligopolies among private firms, it is well known that price competition is stronger, yielding lower profits than in the case of quantity competition.[1] In related literature, Singh and Vives (1984) endogenized the structure of competition (in terms of price or quantity), finding that firms often choose whether to adopt a price contract or a quantity contract. In a private duopoly in which both firms maximize profits, and assuming linear demand and product differentiation, Singh and Vives (1984) showed that a quantity contract is the dominant strategy for each firm when goods are substitutes. Cheng (1985), Tanaka (2001a,b), and Tasnádi (2006) extended this analysis to asymmetric oligopolies, more general demand and cost conditions, and vertical product differentiation, confirming the robustness of the results. However, these results depend on the assumption that all firms are private and profit-maximizers. Therefore, they may not apply to the increasingly important and popular mixed oligopolies, in which state-owned public firms compete against private firms.

In most countries, there exist state-owned public firms that have substantial influence on their market competitors. Such mixed oligopolies occur in various industries, such as the airline, steel, automobile, railway, natural gas, electricity, postal service, education, hospital, home loan, and banking industries.[2] In addition, we have repeatedly observed how many private enterprises facing financial problems have been nationalized, such as General Motors, Japan Airline, and Tokyo Electric Power Corporation. Studies on mixed oligopolies involving both state-owned public enterprises and private enterprises have recently attracted more attention and have become increasingly popular.

[1]See Shubik and Levitan (1980) and Vives (1985).

[2]Analyses of mixed oligopolies date back to Merrill and Schneider (1966). Their study, and many others in the field, assume that a public firm maximizes welfare (consumer surplus plus firm profits), while private firms maximize profits.

Ghosh and Mitra (2010) revisited the comparison between price and quantity competition in a mixed duopoly. They showed that, in contrast to the case of a private duopoly, quantity competition is stronger than price competition, resulting in a smaller profit for the private firm.[3] Then, Matsumura and Ogawa (2012) examined the endogenous competition structure. In their study of a mixed duopoly, when one of the two firms is public, a price contract is the dominant strategy for both the private and the public firm, regardless of whether goods are substitutes or complements.[4] However, in their analysis, they assume that one public firm competes against one private firm. In this study, we allow for more than one private firm and investigate whether the two aforementioned results hold in an oligopoly.

First, we revisit this price-quantity comparison in mixed oligopolies. We adopt a standard differentiated oligopoly with a linear demand (Dixit, 1979) and show that, regardless of the number of private firms, the Bertrand model always yields higher welfare. However, the profit ranking depends on the number of private firms. If the number of private firms is less than or equal to four, the Bertrand model yields a larger profit for each private firm. However, if the number of private firms is greater than or equal to five, the profit can be larger under Cournot competition. Here, the Bertrand model always yields smaller profits, regardless of the degree of substitutability, if the number of private firms is sufficiently large.

Next, we endogenize the competition structure (i.e., price or quantity) using the model of Singh and Vives (1984). We show that Bertrand competition can fail to be an equilibrium when the number of private firms is greater than or equal to two. These results suggest that, in contrast to the case of private oligopolies, the results of mixed oligopolies depend on the number of the private firms, both in terms of price-quantity comparison and in terms of endogenous competition

[3]See also Nakamura (2013), who include a network externality.

[4]Haraguchi and Matsumura (2014) showed that this result holds, regardless of the nationality of the private firm. Chirco *et al.* (2014) showed that both firms choose a price contract when the organizational structure is endogenized. However, Scrimitore (2013) showed that both firms can choose a quantity contract if a production subsidy is introduced.

structure.

Finally, we investigate a model with multiple public firms. In the literature on mixed oligopolies, it is standard to assume that only one public firm competes against a number of private firms.[5] Recently, studies have begun allowing for multiple public firms, such as Matsumura and Shimizu (2010), Bose and Gupta (2013), Matsumura and Matsushima (2012), and Matsumura and Okumura (2013). A typical example of multiple public firms is the financial market in Japan. In China and Russia, many industries have multiple public firms, such as the banking, energy, and transportation industries.

We consider the case in which the number of public firms is same as that of private firms. With regard to Bertrand-Cournot competition, we find that Bertrand competition always yields a larger total social surplus and profit in private firms, regardless of the number of private firms. This result suggests that the profit ranking does not depend on the number of private firms, but instead depends on how the weight of private firms in the market increases. However, with regard to the endogenous competition structure, the equilibrium competition structure does depend on the number of private firms. When two public firms compete against two private firms, Bertrand competition can fail to be an equilibrium, which is a common result in models with one public and two private firms.

With regard to the Bertrand-Cournot comparison, Scrimitore (2014) established an important contribution. He adopted the partial privatization approach of Matsumura (1998) and considered the optimal degree of privatization. His findings show that under optimal privatization policies, Cournot competition can yield higher profits in private firms than in the case of Bertrand competition. The optimal degree of privatization is lower under Bertrand competition, and a lower degree of privatization leads to stronger competition. Here, the profit ranking can be reversed

[5]See, among others, De Fraja and Delbono (1989), Fjell and Pal (1996), Matsumura and Kanda (2005), Lin and Matsumura (2012), and Ghosh *et al.* (2015). An example of such market is the Japanese overnight delivery market. Here, Japan Post competes against private firms such as Yamato, Sagawa, and Seinou.

under optimal privatization policies. Our study shows that the profit ranking can be reversed even if we do not consider optimal privatization policies.

The remainder of the paper is organized as follows. Section 2 presents the proposed model, and Section 3 compares Bertrand and Cournot competition. Section 4 endogenizes the competition structure (i.e., a price or quantity contract). Then, Section 5 considers a case of multiple public firms. The proofs of the propositions can be found in Appendix B.

2 Model

We adopt a standard differentiated oligopoly with a linear demand (Dixit, 1979). The quasi-linear utility function of the representative consumer is:

$$U(q_0, q_1, q_2, ..., q_n) = \alpha \sum_{i=0}^{n} q_i - \beta(\sum_{i=0}^{n} q_i^2 + \delta \sum_{i=0}^{n} \sum_{i \neq j} q_i q_j)/2 + y,$$

where q_i is the consumption of good i produced by firm i ($i = 0, 1, 2, ..., n$), and y is the consumption of an outside good that is provided competitively (with a unit price). Parameters α and β are positive constants, and $\delta \in (0, 1)$ represents the degree of product differentiation: a smaller δ indicates a larger degree of product differentiation.

Firm i ($i = 0, 1, 2, ..., n$) produces differentiated commodities for which the inverse demand function is given by $p_i = \alpha - \beta q_i - \beta\delta \sum_{i \neq j} q_j$ ($i = 0, 1, 2, ..., n$), where p_i and q_i denote firm i's price and quantity respectively. Here, n is the number of private firms and is a natural number. The marginal production costs are constant. Let c_i denote firm i's marginal cost. We assume that $\alpha > c_i$. Furthermore, we assume that all private firms have the same marginal cost, although we allow for a cost difference between public and private firms. We focus the symmetric equilibrium in which all private firms choose the same price or quantity in equilibrium.

Firm 0 is a state-owned public firm, and its payoff is the social surplus, given by

$$SW = \sum_{i=0}^{n} (p_i - c_i) q_i + \left[\alpha \sum_{i=0}^{n} q_i - \frac{\beta(\sum_{i=0}^{n} q_i^2 + \delta \sum_{i=0}^{n} \sum_{i \neq j} q_i q_j)}{2} - \sum_{i=0}^{n} p_i q_i \right].$$

Firm i ($\neq 0$) is a private firm, and its payoff is its own profit: $\pi_i = (p_i - c_i)q_i$.

3 Bertrand-Cournot Comparison

We assume that the equilibrium quantities of both public and private firms are strictly positive under both Bertrand and Cournot competition. Let $a_i \equiv \alpha - c_i$. This assumption is satisfied if and only if $a_i - \delta a_0 > 0$ and $(1 - \delta + n\delta)a_0 > n\delta a_i$ for $i \neq 0$.[6] Let superscript "C" denote the equilibrium outcome under Cournot competition and "B" denote the equilibrium outcome under Bertrand competition.

3.1 Cournot

First, we discuss the Cournot model in which all firms choose quantities. The first-order conditions for public and private firms are, respectively,

$$\begin{aligned} \frac{\partial SW}{\partial q_0} &= a_0 - \beta q_0 - \beta\delta \sum_{i=1}^{n} q_i = 0, \\ \frac{\partial \pi_i}{\partial q_i} &= a_i - 2\beta q_i - \beta\delta \sum_{j \neq i} q_j = 0 \quad (i \neq 0). \end{aligned}$$

The second-order conditions are satisfied. From the first-order conditions, we obtain the following reaction functions for public and private firms, respectively:

$$\begin{aligned} R_0^C(q_i) &= \frac{a_0 - \beta\delta \sum_{i=1}^{n} q_i}{\beta}, \\ R_i^C(q_j) &= \frac{a_i - \beta\delta \sum_{j \neq i} q_j}{2\beta} \quad (i \neq 0). \end{aligned}$$

These functions lead to the following expression for the equilibrium quantities:

$$\begin{aligned} q_0^C &= \frac{(2 - \delta + n\delta)a_0 - n\delta a_i}{\beta(2 - \delta + n\delta(1 - \delta))}, \\ q_i^C &= \frac{a_i - \delta a_0}{\beta(2 - \delta + n\delta(1 - \delta))} \quad (i \neq 0). \end{aligned}$$

[6]These are satisfied if $a_0 = a_i$. Note that if δ is close to one, $a_i - a_0$ must be close to zero.

Substituting these equilibrium quantities into the demand and payoff functions, we have the following welfare and profit for firm i:

$$SW^C = \frac{H_1}{2\beta(2-\delta+n\delta(1-\delta))^2}, \tag{1}$$

$$\pi_i^C = \frac{(a_i-\delta a_0)^2}{\beta(2-\delta+n\delta(1-\delta))^2} \quad (i \neq 0). \tag{2}$$

Note that the constant H_1 and other constants are described in Appendix A.

3.2 Bertrand

Second, we discuss the Bertrand model in which both firms choose prices. The demand function is given by

$$q_i = \frac{\alpha-\alpha\delta-(1+\delta(n-1))p_i+\delta\sum_{j\neq i}p_j}{\beta(1-\delta)(1+n\delta)}.$$

The first-order conditions for public and private firms are, respectively,

$$\frac{\partial SW}{\partial p_0} = \frac{(1+(n-1)\delta)(c_0-p_0)+\delta\sum_{i=1}^n(p_i-c_i)}{\beta(1+n\delta)(1-\delta)} = 0,$$

$$\frac{\partial \pi_i}{\partial p_i} = \frac{\alpha-\delta\alpha+(1+\delta(n-1))(c_i-2p_i)+\delta\sum_{j\neq i}p_j}{\beta(1+n\delta)(1-\delta)} = 0 \quad (i \neq 0).$$

The second-order conditions are satisfied. From the first-order conditions, we obtain the following reaction functions for public and private firms, respectively:

$$R_0^B(p_i) = \frac{(1+\delta(n-1))c_0+\delta\sum_{i=1}^n(p_i-c_i)}{1+\delta(n-1)},$$

$$R_i^B(p_j) = \frac{\alpha-\alpha\delta+\delta\sum_{j\neq i}p_j+(1+\delta(n-1))c_i}{2(1+\delta(n-1))} \quad (i \neq 0).$$

These functions lead to the following expression for the equilibrium prices:

$$p_0^B = \frac{c_0\delta^2n^2+((-\alpha-2c_0)\delta^2+(\alpha-c_i+3c_0)\delta)n+(1-\delta)(2-\delta)c_0}{\delta^2n^2+3\delta(1-\delta)n+(1-\delta)(2-\delta)},$$

$$p_i^B = \frac{\delta^2c_in^2+(\delta(2-3\delta)c_i+\delta^2c_0+\delta(1-\delta)\alpha)n+(1-\delta)((1-\delta)(\alpha+c_i)+\delta c_0)}{\delta^2n^2+3\delta(1-\delta)n+(1-\delta)(2-\delta)} \quad (i \neq 0).$$

Substituting these equilibrium prices into the demand functions, we have the following equilibrium quantities:

$$\begin{aligned} q_0^B &= \frac{(1-\delta+n\delta)a_0 - n\delta a_i}{\beta(1+n\delta)(1-\delta)}, \\ q_i^B &= \frac{(a_i-\delta a_0)(\delta(n-1)+1)^2}{\beta(1+n\delta)(1-\delta)(\delta^2n^2+3\delta(1-\delta)n+(1-\delta)(2-\delta))} \quad (i \neq 0). \end{aligned}$$

Substituting these equilibrium quantities into the payoff functions, we have the following resulting welfare and profit for firm i:

$$SW^B = \frac{H_2}{2\beta(1+n\delta)(1-\delta)(\delta^2n^2+3\delta(1-\delta)n+(1-\delta)(2-\delta))^2}, \tag{3}$$

$$\pi_i^B = \frac{(a_i-\delta a_0)^2(\delta(n-1)+1)^3}{\beta(1+n\delta)(1-\delta)(\delta^2n^2+3\delta(1-\delta)n+(1-\delta)(2-\delta))^2} \quad (i \neq 0). \tag{4}$$

3.3 Comparison

First, we compare the profit of each private firm in the two games.

Proposition 1 *For $i = 1, 2, ..., n$, (i) $\pi_i^B > \pi_i^C$ for $\delta \in (0,1)$ (i.e., the Cournot model yields a smaller profit for each private firm than does the Bertrand model) if $n \leq 4$, (ii) there exists δ such that $\pi_i^B < \pi_i^C$ if $n \geq 5$, and (iii) $\pi_i^B < \pi_i^C$ for $\delta \in (0,1)$ if n is sufficiently large.*

In a mixed duopoly, the only rival of the private firm is the public firm. In a mixed oligopoly, each private firm competes against both public and private firms, and an increase in the number of private firms increases the importance of competition among private firms. As is well-known in the literature on private oligopolies, the Bertrand model yields stronger competition among private firms than does the Cournot model. Thus, the Bertrand model yields stronger competition when the number of private firms is large.

Figure 1 describes the range for which the profit ranking is reversed. Unless δ is close to one, we can see that the profit ranking is more likely to be reversed when δ is larger. An increase in δ

increases the demand elasticity and, thus, the competition among private firms becomes stronger. Therefore, the Bertrand model yields stronger competition than does the Cournot model. However, when δ is close to one, this property does not hold. Since we assume that $a_i - \delta a_0 > 0$ and $(1 - \delta + n\delta)a_0 > n\delta a_i$, for $i \neq 0$, to ensure the interior solution, a_0 must be very close to a_i when δ is close to one. If $a_0 = a_i$ and $\delta = 1$, the public monopoly leads to the first-best outcome, and is yielded by both the Bertrand and the Cournot model in equilibrium. In other words, the profit in each private firm is zero under both Bertrand and Cournot competition. When δ is close to one, the profit in each private firm is close to zero in both models and the profit ranking becomes unstable. This is why a curious property emerges when δ is very close to one.

(Insert Figure 1 here)

Next, we compare the welfare between the two models.

Proposition 2 *The Bertrand model yields higher welfare than does the Cournot, regardless of δ and n.*

When the number of private firms is small, the Cournot model yields stronger competition and, thus, a higher consumer surplus than does the Bertrand model. However, the Cournot model yields a larger difference between the outputs of public and private firms than does the Bertrand model, which leads to a loss in welfare. The latter effect dominates the former effect (consumer-benefiting effect). Therefore, the Bertrand model yields a higher social surplus than does the Cournot model.

When the number of private firms is large, the Bertrand model yields stronger competition and, thus, a higher consumer surplus. The Cournot model still yields a larger difference between the outputs of public and private firms, thereby leading to a loss in welfare. Based on these two effects, the Bertrand model yields the higher social welfare of the two models.

4 Endogenous competition structure

In this section, we endogenize the competition structure (i.e., as either price or quantity). Here, we follow the standard model formulated by Singh and Vives (1984). The game runs as follows. In the first stage, each firm chooses whether to adopt a price or a quantity contract. In the second stage, after observing the rival's choice in the first stage, each firm simultaneously chooses either p or q, according to the decision in the first stage.

Matsumura and Ogawa (2012) showed that both firms choose the price contract, and thus, Bertrand competition appears in equilibrium when $n = 1$. Proposition 3 suggests that this does not hold when the number of private firms is larger.

Proposition 3 *(i) There exists δ such that Bertrand competition fails to be an equilibrium if $n \geq 2$. (ii) Bertrand competition does not appear in equilibrium for $\delta \in (0, 1)$ if n is sufficiently large. (iii) Cournot competition never appears in equilibrium.*

Again, the number of private firms is important. The competition structure changes when the number of private firms increases. When $n = 1$, all firms choose the price contract regardless of δ. When $n = 2$, the equilibrium outcome is either (i) all firms choose the price contract, or (ii) one private firm chooses the quantity contract and the other firms choose the price contract. Although we fail to solve the general case, we can show that the equilibrium is never Cournot competition, because the public firm chooses the price contract regardless of the number of private firms.

As Singh and Vives (1984) discussed, the demand is more elastic when a firm chooses the price contract. According to Matsumura and Ogawa (2012), the private (res. public) firm is more (res. less) aggressive when the demand is more elastic. Thus, choosing the price contract makes the public (res. private) rival less (res. more) aggressive. Aggressive behavior of private firms reduces the prices and, thus, improves welfare. Therefore, the public firm always chooses the price contract. In contrast, if a private firm chooses the price contract, it makes the public firm less aggressive and other private firms more aggressive. The less aggressive behavior of the public firm is beneficial to the private firm, but the more aggressive behavior of the other private firms is harmful to the private firm. Accordingly, a private firm may have an incentive to choose the quantity contract, unless $n = 1$.

Although we cannot solve the game explicitly in a general case, we present some numerical results. The following three figures shows the relationship between the equilibrium type and the degree of differentiation. Figures 2, 3, and 4 describe the case with 2, 3, and 4 private firms, respectively. In Figure 2, PPP indicates that all firm choose prices, and PPQ indicates that firms 0 and 1 choose the price and firm 2 chooses the quantity, and so on.

(Insert Figures 2-4 here)

5 Multiple public firms

In the previous sections, as well as in most studies on mixed oligopolies, we assume there is only one public firm. However, many economies have more than one public firm. Typical examples include the banking sectors in Japan, Germany, and India, the energy market in the EU, and many sectors in China, Russia, and Malaysia. In the literature, some studies have begun to allow for multiple public firms, such as Matsumura and Shimizu (2010), Bose and Gupta (2013), Matsumura and Matsushima (2012), and Matsumura and Okumura (2013).

In this section, we assume that more than one public firm exists. For simplicity, we assume m public firms and m private firms exist. Let the subscripts i and j denote public and private firms, respectively.

In Bertrand competition, the equilibrium price of each public firm is

$$p_i^B = \frac{\alpha m\delta(1-\delta) - m\delta(\delta m - \delta + 1)c_j + (3\delta m - 3\delta + 2)(\delta m - \delta + 1)c_i)}{2\delta^2 m^2 + \delta(5 - 6\delta)m + (1-\delta)(2-3\delta)},$$

and that of each private firm is

$$p_j^B = \frac{\alpha(1-\delta)(\delta m - \delta + 1) + (\delta^2 m^2 + \delta(3-4\delta)m + 2\delta^2 - 3\delta + 1)c_j + \delta m(\delta m - \delta + 1)c_i}{2\delta^2 m^2 + \delta(5-6\delta)m + (1-\delta)(2-3\delta)}.$$

The resulting profit of each private firm and welfare are respectively,

$$\pi_j^B = \frac{((\delta m - \delta) + 1)^2(2\delta m - 2\delta + 1)(\delta m(a_i - a_j) - (1-\delta)a_j)^2}{\beta(1-\delta)(2\delta m - \delta + 1)(2\delta^2 m^2 - 6\delta^2 m + 5\delta m + (1-\delta)(2-3\delta))^2}, \quad (5)$$

$$SW^B = \frac{mH_3}{(2\beta(1-\delta)(2\delta m - \delta + 1)(2\delta^2 m^2 - 6\delta^2 m + 5\delta m + 3\delta^2 - 5\delta + 2)^2}. \quad (6)$$

In Cournot competition, the equilibrium quantity of each public firm is

$$q_i^C = \frac{\delta m(a_i - a_j) + (2-\delta)a_i}{\beta(\delta m(3-2\delta) + (1-\delta)(2-3\delta))},$$

and that of each private firm is

$$q_j^C = \frac{(1-\delta)a_j + \delta m(a_j - a_i)}{\beta(\delta m(3-2\delta) + (1-\delta)(2-3\delta))}.$$

The resulting profit of each private firm and welfare are respectively,

$$\pi_j^C = \frac{(\delta m(a_i - a_j) - (1-\delta)a_j)^2}{\beta(\delta m(2\delta - 3) - (1-\delta)(2-\delta))^2}, \quad (7)$$

$$SW^C = \frac{mH_4}{2\beta(2\delta^2 m - 3\delta m - \delta^2 + 3\delta - 2)^2}. \quad (8)$$

We obtain the following proposition.

Proposition 4 *The Bertrand model yields higher welfare and a larger profit in each private firm than does the Cournot model, regardless of δ and m.*

This proposition states that Proposition 2 depends on the assumption of one public firm. This suggests that the profit ranking is reversed, not because the number of private firms increases, but because the weight of private firms in the market increases.[7] However, in the context of an

[7] We can show that the Bertrand model yields higher welfare and a larger profit in each private firm than does the Cournot model in the case in which one private firm competes against multiple public firms. This is the opposite case of the basic model.

endogenous competition structure, the number of private firms is crucial. The following proposition states that Bertrand competition can fail to be an equilibrium outcome, even in a four-player model (i.e., two public and two private firms).[8]

Proposition 5 *Suppose that $m = 2$. There exists δ such that Bertrand competition fails to be an equilibrium.*

When Bertrand competition fails to be an equilibrium, the equilibrium occurs when two public firms and one private firm choose the price contract and one private firm chooses the quantity contract. Suppose that all firms chooses the price contract. Suppose that one private firm deviates and chooses the quantity contract. The other private firm becomes less aggressive, which is beneficial to the deviator. This deviation also makes the public firm more aggressive. However, this effect is weak because the other private firm chooses the price contract, and aggressive pricing by the public firms reduces the demand of this private firm, resulting in a loss in welfare. Therefore, the former effect dominates the latter effect.

If both private firms choose the quantity contract, the two public firms become more aggressive, which reduces the profits of the private firms. Thus, one private firm chooses the price contract when the other private firm chooses the quantity contract. Therefore, asymmetric choices by private firms emerge in equilibrium.

Although we cannot solve the game explicitly in a general case, we present some numerical results. The following three figures shows the relationship between the equilibrium type and the degree of differentiation. Figures 5, 6, and 7 describe the case with 2, 3, and 4 public and private firms. In all cases, the public firms choose the price. Thus, we only describe the choice of private firms. In Figure 7, PPPP indicates that all private firms choose the price, PPPQ denote the only one private firm chooses the quantity, and so on.

(Insert Figures 5-7 here)

[8]We can show that Bertrand competition is an equilibrium if two public firms compete against one private firm.

6 Concluding remarks

In this study, we revisit the classic comparison between price and quantity competition, but in a mixed oligopoly setting. Ghosh and Mitra (2010) showed that, in a mixed duopoly, price competition yields a larger profit for the private firm. Nevertheless, price competition yields a higher total social surplus. In this study, we investigate a mixed oligopoly, allowing for more than one private firm. We find that, regardless of the number of private firms, price competition yields higher welfare. However, whether price or quantity competition yields a larger profit for the private firm depends on the number of private firms. When the number of private firms is large, quantity competition yields larger profits. In other words, whether the Cournot or the Bertrand model yields stronger competition depends on the number of private firms in a mixed oligopoly.

We also discuss an endogenous competition structure using the model of Singh and Vives (1984). Matsumura and Ogawa (2012) showed that Bertrand competition appears in a mixed duopoly, in contrast to the case of a private duopoly. We show that this result also depends on the number of private firms. When the number of private firms is large, Bertrand competition fails to appear in equilibrium. That is, the equilibrium competition structure depends on the number of private firms in a mixed oligopoly.

In this study, we assume that the number of firms is given exogenously. In the literature on mixed oligopolies, endogenizing the number of firms by considering free-entry markets is quite popular, and free-entry markets often yield quite different implications for mixed oligopolies.[9] Endogenizing the number of firms and examining the welfare implications under free entry remains as a topic of future research.

In this study, we use the linear demand. Although this demand is popular in the literature,

[9]For discussions on free-entry markets in mixed oligopolies, see Matsumura and Kanda (2005), Brandão and Castro (2007), Fujiwara (2007), Ino and Matsumura (2010), and Wang and Chen (2010). For recent developments in this field, see Cato and Matsumura (2012, 2013), Ghosh *et al.* (2015), Ghosh and Sen (2012), and Wang and Lee (2013).

our results may depend on the linearity of the demand. Natural extension of our analysis is to use a more general or a CES demand function that is discussed in Ghosh and Mitra (2014), but this extension is a very tough work because of the asymmetry of the payoff functions. Extending this direction also remains as a topic of future research.[10]

[10] Our result depends on the linearity of cost function. For example, if we consider a quadratic production cost function, we can show that Cournot competition may yield a larger profit of each private firm than the Bertrand competition even when $n \leq 4$.

Appendix A

$$
\begin{aligned}
H_1 &\equiv \delta(1-\delta)(a_i^2 - 2\delta a_0 a_i + \delta a_0^2)n^2 + ((3-\delta)a_i^2 - 2\delta(3-\delta)a_0 a_i + \delta(\delta^2 - 3\delta + 4)a_0^2)n + a_0^2(2-\delta)^2 \\
H_2 &\equiv \delta^4(a_i - 2\delta a_0 a_i + \delta a_0^2)n^5 + (1-\delta)\delta^3(6a_i^2 - 12\delta a_0 a_i + 7\delta a_0^2)n^4 \\
&- (1-\delta)\delta^2((11\delta - 12)a_i^2 + (-22\delta^2 + 24\delta)a_0 a_i + (18\delta^2 - 19\delta)a_0^2)n^3 \\
&+ (1-\delta)\delta(6\delta^2 - 17\delta + 10)a_i^2 + (-12\delta^3 + 34\delta^2 - 20\delta)a_0 a_i + (18\delta^3 - 44\delta^2 + 25\delta)a_0)n^2 \\
&- (1-\delta)^3((\delta - 3)a_i^2 + (-2\delta^2 + 6\delta)a_0 a_i + (7\delta^2 - 16\delta)a_0^2)n + (1-\delta)^3(2-\delta)^2 a_0^2 \\
H_3 &\equiv 4(a_j - a_i)^2\delta^5 m^5 - 2\delta^4(13a_j^2\delta - 22a_i a_j\delta + 13a_i^2\delta - 11a_j^2 + 18a_i a_j - 11a_i^2)m^4 \\
&+ \delta^3(65a_j^2\delta^2 - 86a_i a_j\delta^2 + 69a_i^2\delta^2 - 110a_j^2\delta + 140a_i a_j\delta - 118a_i^2\delta + 46a_j^2 - 56a_i a_j + 50a_i^2)m^3 \\
&+ (1-\delta)\delta^2(75a_j^2\delta^2 - 64a_i a_j\delta^2 + 83a_i^2\delta^2 - 116a_j^2\delta + 92a_i a_j\delta - 132a_i^2\delta + 44a_j^2 - 32a_i a_j + 52a_i^2)m^2 \\
&+ (1-\delta)^2\delta(40a_j^2\delta^2 - 16a_i a_j\delta^2 + 45a_i^2\delta^2 - 56a_j^2\delta + 20a_i a_j\delta - 66a_i^2\delta + 19a_j^2 - 6a_i a_j + 24a_i^2)m \\
&+ (1-\delta)^3(8a_j^2\delta^2 + 9a_i^2\delta^2 - 10a_j^2\delta - 12a_i^2\delta + 3a_j^2 + 4a_i^2) \\
H_4 &\equiv 2(a_j - a_i)^2(2-\delta)\delta^2 m^2 + \delta(3a_j^2\delta^2 - 2a_i a_j\delta^2 + 3a_i^2\delta^2 - 10a_j^2\delta + 8a_i a_j\delta - 10a_i^2\delta + 7a_j^2 \\
&- 6a_i a_j + 8a_i^2)m + (1-\delta)(a_j^2\delta^2 + a_i^2\delta^2 - 4a_j^2\delta - 4a_i^2\delta + 3a_j^2 + 4a_i^2) \\
H_5 &\equiv -(1-\delta)\delta^4 n^4 + (1-\delta)(3\delta - 2)\delta^3 n^3 + (3\delta^3 - 7\delta^2 + 2\delta + 3)\delta^2 n^2 \\
&+ (1-\delta)(4-\delta)(2-\delta^2)\delta n + (1-\delta)^2(2-\delta)(2+\delta) \\
H_6 &\equiv \delta^3(\delta^2 - 3\delta + 3)n^3 + \delta^2(1-\delta)(\delta^2 - 6\delta + 11)n^2 + \delta(1-\delta)(3\delta^2 - 14\delta + 12)n \\
&- (1-\delta)(2-\delta)(3\delta - 2) \\
H_7 &\equiv 2\delta^3 c_0 n^3 - \delta^2(2c_i(7\delta - 8)c_0 - 2\alpha(1-\delta))n^2 + \delta((3\delta - 2)c_i + (5\delta^2 - 19\delta + 10)c_0 \\
&+ (3\delta^2 - 5\delta + 2)\alpha)n - (\delta(\delta - 2)c_i - (2\delta^3 + 7\delta - 2 - 12\delta + 4)c_0 + \alpha\delta(\delta^2 - 3\delta + 2)) \\
H_8 &\equiv 2\delta^3 c_i n^3 - \delta^2((9\delta - 6)c_i - 2\delta c_0 - 2(1-\delta)\alpha)n^2 + \delta((8\delta^2 - 16\delta + 6)c_i - \delta(5\delta - 4)c_0 \\
&+ (5\delta^2 - 9\delta + 4)\alpha)n + (\delta^3 + 7\delta^2 - 7\delta + 2)c_i + \delta(2\delta^2 - 5\delta + 2)c_0 - (2\delta^3 - 7\delta^2 + 7\delta - 2)\alpha
\end{aligned}
$$

$$
\begin{aligned}
H_9 &\equiv -(\delta^3 n^3 - \delta^3 n^2 + 3\delta^2 n - 3\delta n - \delta^3 - \delta^2 + 4\delta - 2)(4\delta^5 n^5 - 26\delta^5 n^4 + 24\delta^4 n^4 + 57\delta^5 n^3 \\
&- 118\delta^4 n^3 + 56\delta^3 n^3 - 49\delta^5 n^2 + 186\delta^4 n^2 - 198\delta^3 n^2 + 64\delta^2 n^2 + 12\delta^5 n - 103\delta^4 n + 201\delta^3 n \\
&- 146\delta^2 n + 36\delta n + \delta^5 + 13\delta^4 - 54\delta^3 + 72\delta^2 - 40\delta + 8) \\
H_{10} &\equiv (2\delta^3 n^3 - 9\delta^3 n^2 + 8\delta^2 n^2 + 8\delta^3 n - 21\delta^2 n + 10\delta n + \delta^3 + 9\delta^2 - 12\delta + 4)^2 \\
H_{11} &\equiv \delta c_i^2 n^2 - 2\delta^2 c_0 c_i n^2 + 2\alpha\delta^2 c_i n^2 - 2\alpha\delta c_i n^2 + \delta^2 c_0^2 n^2 - \alpha^2\delta^2 n^2 + \alpha^2\delta n^2 + 2\delta c_i^2 n \\
&+ 3c_i^2 n - 4\delta^2 c_0 c_i n - 6\delta c_0 c_i n + 4\alpha\delta^2 c_i n + 2\alpha\delta c_i n - 6\alpha c_i n + \delta^2 c_0^2 n + 4\delta c_0^2 n \\
&+ 2\alpha\delta^2 c_0 n - 2\alpha\delta c_0 n - 3\alpha^2\delta^2 n + 3\alpha^2 n - \delta^3 c_0^2 - 3\delta^2 c_0^2 + 4c_0^2 + 2\alpha\delta^3 c_0 \\
&+ 6\alpha\delta^2 c_0 - 8\alpha c_0 - \alpha^2\delta^3 - 3\alpha^2\delta^2 + 4\alpha^2 \\
H_{12} &\equiv 8\delta^5 m^5 - 16\delta^4 m^5 + 10\delta^3 m^5 - 32\delta^5 m^4 + 76\delta^4 m^4 - 70\delta^3 m^4 + 25\delta^2 m^4 + 50\delta^5 m^3 - 132\delta^4 m^3 \\
&+ 138\delta^3 m^3 - 74\delta^2 m^3 + 18\delta m^3 - 38\delta^5 m^2 + 113\delta^4 m^2 - 126\delta^3 m^2 + 69\delta^2 m^2 - 22\delta m^2 + 4m^2 \\
&+ 14\delta^5 m - 48\delta^4 m + 60\delta^3 m - 32\delta^2 m + 6\delta m - 2\delta^5 + 8\delta^4 - 12\delta^3 + 8\delta^2 - 2\delta \\
H_{13} &\equiv 4\delta^2 m^4 + 8\delta^4 m^3 - 32\delta^3 m^3 + 18\delta^2 m^3 + 2\delta m^3 - 12\delta^4 m^2 + 64\delta^3 m^2 - 77\delta^2 m^2 + 26\delta m^2 \\
&+ 6\delta^4 m - 40\delta^3 m + 68\delta^2 m - 42\delta m + 8m - \delta^4 + 8\delta^3 - 17\delta^2 + 14\delta - 4
\end{aligned}
$$

Appendix B

Proof of Proposition 1 From (4) and (2), we have

$$\pi_i^B - \pi_i^C = \frac{\delta^2 n(a_i - \delta a_0)^2 H_5}{\beta(1+n\delta)(1-\delta)(2-\delta+n\delta(1-\delta))^2(\delta^2 n^2 + 3\delta(1-\delta)n + (1-\delta)(2-\delta))^2}.$$

Here, $\pi_i^B - \pi_i^C$ is positive (res. negative, zero) if $H_5(n,\delta)$ is positive (res. negative, zero). Figure 1 describes the region for $H_5(n,\delta) < 0$. This shows (i) and (ii).[11]

We have $\lim_{n\to\infty} H_5(n,\delta) = -\infty$. This implies (iii). Q.E.D.

Proof of Proposition 2 From (3) and (1), we have

$$SW^B - SW^C = \frac{\delta^2 n^2 (a_1 - \delta a_0)^2 H_6}{2\beta(1+n\delta)(1-\delta)(2-\delta+n\delta(1-\delta))^2(\delta^2 n^2 + 3\delta(1-\delta)n + (1-\delta)(2-\delta))^2}.$$

Here, $SW^B - SW^C$ is positive (res. negative, zero) if $H_6(n,\delta)$ is positive (res. negative, zero). We now show that $H_6(1,\delta) > 0$ and that $H_6(n,\delta)$ is increasing in n for $n \geq 1$.

Substituting $n = 1$ into $H_6(n,\delta)$, we have $H_6(1,\delta) = (2-\delta^2)^2 > 0$. We show that $H_6(n,\delta)$ is increasing in n for $n \geq 1$ if $\delta \in (0,1)$. We have that

$$\frac{\partial H_6(n,\delta)}{\partial n} = 3\delta^3(\delta^2 - 3\delta + 3)n^2 + 2\delta^2(1-\delta)(\delta^2 - 6\delta + 11)n + \delta(1-\delta)(3\delta^2 - 14\delta + 12).$$

This is increasing in n. Substituting in $n = 1$, we have

$$\frac{\partial H_6(n,\delta)}{\partial n}|_{n=1} = \delta^4(2+\delta) + 4\delta(1-\delta)(3+2\delta) > 0.$$

Thus, $\frac{\partial H_6(n,\delta)}{\partial n} > 0$ for $n \geq 1$. Q.E.D.

Proof of Proposition 3 We have already discussed the equilibrium profit of each private firm when all firms choose the price contract (π_i^B). We show that given the contracts of other firms, a private firm has an incentive to choose the quantity contract.

[11] A proof that does not rely on accompanying figure is available upon request.

Consider the subgame in which one private firm chooses the quantity contract and all the other firms choose the price contract. In equilibrium, the public firm names the following price:

$$p_0 = \frac{H_7}{2\delta^3 n^3 + \delta^2(8-9\delta)n^2 + \delta(\delta-2)(8\delta-5)n + (2-3\delta)^2},$$

the private firms that choose the price contract name the following price:

$$p_i = \frac{H_8}{2\delta^3 n^3 + \delta^2(8-9\delta)n^2 + \delta(\delta-2)(8\delta-5)n + (2-3\delta)^2}, \quad (i = 1, 2, ..., n-1),$$

and the private firm that chooses the quantity contract selects the following quantity:

$$q_n = \frac{(a_i - \delta a_0)(1 + \delta(n-2))(2 + \delta(2n-3))}{\beta(2\delta^3(1-\delta)n^3 + \delta^2(9\delta^2 - 17\delta + 8)n^2 - \delta(8\delta^3 - 29\delta^2 + 31\delta - 10)n - (\delta^4 + 8\delta^3 - 21\delta^2 + 16\delta - 4))}.$$

The private firm that chooses the quantity contract obtains the following profit:

$$\pi^{p,\dots,p,q} = \frac{(a_i - \delta a_0)^2(1 + n\delta)((n - 2\delta) + 1)^2((2n-3)\delta + 2)^2}{\beta(1-\delta)(\delta(n-1)+1)(2\delta^3 n^3 + \delta^2(8-9\delta)n^2 + \delta(8\delta^2 - 21\delta + 10)n + (\delta^3 + 9\delta^2 - 12\delta + 4))^2}. \quad (9)$$

From (2) and (9), we have that

$$\pi_i^B - \pi^{p,\dots,p,q} = \frac{\delta^2(a_i - \delta a_0)^2 H_9}{\beta(1-\delta)(\delta n + 1)(\delta n - \delta + 1)(\delta^2 n^2 - 3\delta^2 n + 3\delta n + \delta^2 - 3\delta + 2)^2 H_{10}}.$$

Here, $\pi_i^B - \pi^{p,\dots,p,q}$ is positive (res. negative, zero) if $H_9(n,\delta)$ is positive (res. negative, zero). Figure 8 describes the region for $H_9(n,\delta) < 0$. This shows (i).[12]

(Insert Figure 8 here)

$\lim_{n\to\infty} H_9(n,\delta) = -\infty$. This implies (ii).

We have already discussed the equilibrium welfare when all firms choose the quantity contract (SW^C). We show that given the contracts of all private firms, the public firm has an incentive to choose the price contract, regardless of δ.

[12] A proof that does not rely on accompanying figure is available upon request.

Consider the subgame in which the public firm chooses the price contract and all private firms choose the quantity contract. In equilibrium, the public firm names the following price:

$$p_0 = m_0,$$

and all private firms selects the following quantity:

$$q_i = \frac{a_i - \delta a_0}{\beta(1-\delta)(2+\delta(1+n))} \quad (i = 1, 2, ..., n).$$

Substituting these equilibrium price and quantity into the payoff function, we have the following welfare:

$$SW^{p,q...,q} = \frac{H_{11}}{2\beta(1-\delta)(\delta n+\delta+2)^2}. \tag{10}$$

From (1) and (10), we have that

$$SW^{p,q,...,q} - SW^C = \frac{n\delta^2(a_i-\delta a_0)^2(\delta n(2-\delta^2)+\delta(2-\delta)+4)}{2\beta(1-\delta)(\delta n+\delta+2)^2(\delta^2 n-\delta n+\delta-2)^2} > 0.$$

This implies (iii). Q.E.D.

Proof of Proposition 4 Let π_j^B and π_j^C be the profit of each private firm under Bertrand and Cournot competition, respectively. From (5) and (7), we have that $\pi_j^B - \pi_j^C$ is

$$\frac{\delta^2(\delta m(a_i-a_j)-(1-\delta)a_j)^2 H_{12}}{\beta(1-\delta)(2\delta m-\delta+1)(\delta m(2\delta-3)-(1-\delta)(2-\delta))^2(2\delta^2 m(m-3)+5\delta m+(1-\delta)(2-3\delta))^2}.$$

Here, $\pi_j^B - \pi_j^C$ is positive (res. negative, zero) if $H_{12}(m,\delta)$ is positive (res. negative, zero). We show that $H_{12}(1,\delta)$ is positive, and $H_{12}(m,\delta)$ is increasing in m for $m \geq 1$. Substituting $m = 1$ into $H_{12}(m,\delta)$, we have $H_{12}(1,\delta) = \delta^4 - 4\delta^2 + 4 > 0$. We have

$$\begin{aligned}
\frac{\partial H_{12}(m,\delta)}{\partial m} \;=\;& \delta^3(10m^3-6m^2-2m-2)m + 4\delta^2(1-\delta)(13\delta^2+6(1-\delta)+19(1-\delta)^2)m^3 \\
&+(25(1-\delta)^4+5\delta^3(1-\delta)^3+4\delta^2(1-\delta)^4+(1-\delta)^5+(1-\delta)^6+\delta^2 f_2(\delta))m^2 \\
&+(2(1-\delta)^4+17\delta^3(1-\delta)^4+21\delta^3(1-\delta)^3+15\delta^2(1-\delta)^5+2(1-\delta)^7+\delta^3 f_3(\delta))m \\
&+2\delta(7\delta^4-24\delta^3+30\delta^2-16\delta+3),
\end{aligned}$$

where

$$f_2(\delta) \equiv 8\delta^3 + 21\delta^2 - 57\delta + 29,$$

$$f_3(\delta) \equiv 21\delta^3 + 52\delta^2 - 41\delta + 11.$$

Substituting m=1 into this, we have

$$\frac{\partial H_{12}(m,\delta)}{\partial m}|_{m=1} = 2(2-\delta)(\delta(4-3\delta)+2) > 0.$$

Here, $\frac{\partial H_{12}(m,\delta)}{\partial m}$ is increasing in m for $m \geq 1$ if $f_2(\delta) > 0$ and $f_3(\delta) > 0$. First, we show that $f_2(\delta) > 0$. We have that

$$\frac{df_2(\delta)}{d\delta} = 24\delta^2 + 42\delta - 57.$$

Solving $\frac{df_2(\delta)}{d\delta} = 0$ leads to the following solutions:

$$\delta = \frac{-7+\sqrt{201}}{8}, \delta = \frac{-7-\sqrt{201}}{8}.$$

Thus, $f_2(\delta)$ is minimized when $\delta = \frac{-7+\sqrt{201}}{8}$ for $\delta \in (0,1)$. Since

$$f_2\left(\frac{-7+\sqrt{201}}{8}\right) = \frac{2867 - 201^{3/2}}{8} > 0.$$

$f_2(\delta) > 0$ for $\delta \in (0,1)$.

Next, we show that $f_3(\delta) > 0$. We have that

$$\frac{df_3(\delta)}{d\delta} = -63\delta^2 + 104\delta - 41.$$

Solving $\frac{df_3(\delta)}{d\delta} = 0$ leads to the following solutions:

$$\delta = \frac{41}{63}, \delta = 1.$$

Thus, $f_3(\delta)$ is minimized when $\delta = \frac{41}{63}$ for $\delta \in (0,1)$. Since

$$f_3\left(\frac{41}{63}\right) = \frac{6583}{11907} > 0,$$

$f_3(\delta) > 0$ for $\delta \in (0,1)$.

Therefore, $\frac{\partial H_{12}(m,\delta)}{\partial m} > 0$ for $m \geq 1$.

We now compare the welfare. From (6) and (8), we have that $SW^B - SW^C$ is

$$\frac{\delta^2 m(\delta m - \delta + 1)(\delta m(a_i - a_j) - (1-\delta)a_j)^2 H_{13}}{2\beta(1-\delta)(2\delta m - \delta + 1)(\delta m(2\delta - 3) - (1-\delta)(2-\delta))^2(\delta m(2\delta m - 6\delta + 5) + (\delta - 1)(3\delta - 2))^2}.$$

Here, $SW^B - SW^C$ is positive (res. negative, zero) if $H_{13}(m,\delta)$ is positive (res. negative, zero). We show that $H_{13}(1,\delta)$ is positive, and $H_{13}(m,\delta)$ is increasing in m for $m > 1$. Substituting $m = 1$ into $H_{13}(m,\delta)$ we have $H_{13}(1,\delta) = \delta^4 - 4\delta^2 + 4 > 0$. We show that $H_{13}(m,\delta)$ is increasing in m for $m \geq 1$ if $\delta \in (0,1)$.

We have

$$\begin{aligned}\frac{\partial H_{13}(m,\delta)}{\partial m} &= 4\delta^2(4m^2 - 3m - 1)m + 6\delta(1-\delta)(4\delta(1-\delta) + 8\delta + 1)m^2 + 2\delta f_4(\delta)m \\ &\quad +6\delta^4 - 40\delta^3 + 68\delta^2 - 42\delta + 8,\end{aligned}$$

where

$$f_4(\delta) \equiv -12\delta^3 + 64\delta^2 - 75\delta + 26.$$

Substituting in $m = 1$, we have

$$\frac{\partial H_{13}(m,\delta)}{\partial m}|_{m=1} = 6\delta^4 - 8\delta^3 - 16\delta^2 + 16\delta + 8 > 0.$$

Here, $\frac{\partial H_{13}(m,\delta)}{\partial m}$ is increasing in m for $m \geq 1$ if $f_4(\delta) > 0$. We show $f_4(\delta)$ is positive if $\delta \in (0,1)$. We have that

$$\frac{df_4(\delta)}{d\delta} = -36\delta^2 + 128\delta - 75.$$

Solving $\frac{df_4(\delta)}{d\delta} = 0$ leads to the following solutions:

$$\delta = \frac{32 + \sqrt{349}}{18}, \delta = \frac{32 - \sqrt{349}}{18}.$$

Here, $f_4(\delta)$ is minimized when $\delta = \frac{32-\sqrt{349}}{18}$. Because

$$f_4\left(\frac{32-\sqrt{349}}{18}\right) = \frac{6686 - 349^{3/2}}{243} > 0,$$

$f_4(\delta) > 0$ for $\delta \in (0,1)$. Thus, $\frac{\partial H_{13}(m,\delta)}{\partial m} > 0$ for $m \geq 1$. Q.E.D.

Proof of Proposition 5 We have already discussed the equilibrium profit of each private firm when all firms choose the price contract. Let π_j^B denote this profit. We show that given the contracts of other firms, a private firm has an incentive to deviate and chooses the quantity contract. We consider the subgame in which one private firm (firm 3) chooses the quantity contract and all the other firms (firm 0, firm 1, and firm 2) choose the price contract. In equilibrium , the public firms (firm 0 and firm 1) name the following price:

$$p_i = \frac{(5\delta^3 + 7\delta^2 + 2\delta)m_j + (-13\delta^2 - 16\delta - 4)m_i + 5\alpha\delta^3 - 3\alpha\delta^2 - 2\alpha\delta}{10\delta^3 - 9\delta^2 - 16\delta - 4} \quad (i = 0, 1),$$

the private firm that chooses the price contract (firm 2) names the following price:

$$p_2 = \frac{(10\delta^3 - 4\delta^2 - 9\delta - 2)m_j + (-10\delta^2 - 4\delta)m_i + 5\alpha\delta^2 - 3\alpha\delta - 2\alpha}{10\delta^3 - 9\delta^2 - 16\delta - 4},$$

and the private firm that chooses the quantity contract (firm 3) selects the following quantity:

$$q_3 = \frac{-(3\delta^2 + 5\delta + 2)m_j - (-6\delta^2 - 4\delta)m_i - 3\alpha\delta^2 + \alpha\delta + 2\alpha}{10\beta\delta^4 - 19\beta\delta^3 - 7\beta\delta^2 + 12\beta\delta + 4\beta}.$$

The private firm that chooses the quantity contract (firm 3) obtains the following profit:

$$\pi^{p,p,p,q} = \frac{(1+3\delta)(2+3\delta)^2(\delta(a_j - a_i) + (1-\delta)a_i)^2}{\beta(1-\delta)(1+2\delta)(10\delta^3 - 9\delta^2 - 16\delta - 4)^2}. \tag{11}$$

Substituting $m = 2$ into (5) we have that π_j^B for $m = 2$ is

$$\pi_j^B = \frac{(1+\delta)^2(1+2\delta)(\delta(a_j - a_i) + (1-\delta)a_i)^2}{\beta(1-\delta)(1+3\delta)(\delta^2 - 5\delta - 2)^2} \quad (j = 2, 3). \tag{12}$$

From (12) and (11), we have that $\pi_j^B - \pi^{p,p,p,q}$ is

$$\frac{\delta^2 f_5(\delta)(20\delta^3(1-\delta^2) + 21\delta^3(1-\delta) + 44\delta^3 + 126\delta^2 + 56\delta + 8)(\delta(a_j - a_i) + (1-\delta)a_i)^2}{\beta(1-\delta)(1+2\delta)(1+3\delta)(\delta^2 - 5\delta - 2)^2(10\delta^3 - 9\delta^2 - 16\delta - 4)^2},$$

where $f_5(\delta) = -20\delta^3 - 3\delta^2 + 13\delta + 4$.

Here, $\pi_j^B - \pi^{p,p,p,q}$ is positive (res. negative, zero) if $f_5(\delta)$ is positive (res. negative, zero). $f_5(\delta) < 0$ if $\delta > \delta^* \simeq 0.867$ Q.E.D.

References

Bose A, Gupta B (2013) Mixed markets in bilateral monopoly. Journal of Economics, 110(2):141–164.

Brandão A, Castro S (2007) State-owned enterprises as indirect instruments of entry regulation. Journal of Economics 92(3):263–274.

Cato S, Matsumura T (2012) Long-run effects of foreign penetration on privatization policies. Journal of Institutional and Theoretical Economics 168(3):444–454.

Cato S, Matsumura T (2013) Long-run effects of tax policies in a mixed market. FinanzArchiv 69(2):215–240.

Cheng L (1985) Comparing Bertrand and Cournot equilibria: a geometric approach. RAND Journal of Economics 16(1):146–152.

Chirco A, Colombo C, Scrimitore M (2014) Organizational structure and the choice of price versus quantity in a mixed duopoly. Japanese Economic Review 65(4):521–542.

De Fraja G, Delbono F (1989) Alternative strategies of a public enterprise in oligopoly. Oxford Economic Papers 41(2):302–311

Dixit AK (1979) A model of duopoly suggesting a theory of entry barriers. Bell Journal of Economics 10(1):20–32.

Fjell K, Pal D (1996) A mixed oligopoly in the presence of foreign private firms. Canadian Journal of Economics 29(3):737–743.

Fujiwara K (2007) Partial privatization in a differentiated mixed oligopoly. Journal of Economics 92(1):51–65.

Ghosh A, Mitra M (2010) Comparing Bertrand and Cournot in mixed markets. Economics Letters 109(2):72–74.

Ghosh A, Mitra M (2014). Reversal of Bertrand-Cournot rankings in the presence of welfare concerns. Journal of Institutional and Theoretical Economics 170(3):496–519.

Ghosh A, Mitra M, Saha B (2015) Privatization, underpricing and welfare in the presence of foreign competition. Journal of Public Economic Theory, 17(3):433-460.

Ghosh A, Sen P (2012) Privatization in a small open economy with imperfect competition. Journal of Public Economic Theory 14(3):441–471.

Haraguchi J, Matsumura T (2014) Price versus quantity in a mixed duopoly with foreign penetration. Research in Economics 68(4):338–353.

Ino H, Matsumura T (2010) What role should public enterprises play in free-entry markets? *Journal of Economics* 101(3):213–230.

Lin M H, Matsumura T (2012) Presence of foreign investors in privatized firms and privatization policy. Journal of Economics 107(1):71–80.

Matsumura T (1998) Partial privatization in mixed duopoly. Journal of Public Economics 70(3):473–483.

Matsumura T, Kanda O (2005) Mixed oligopoly at free entry markets. Journal of Economics 84(1):27–48.

Matsumura T, Matsushima N (2012) Airport privatization and international competition. Japanese Economic Review 63(4):431–450.

Matsumura T, Ogawa A (2012) Price versus quantity in a mixed duopoly. Economics Letters 116(2):174–177.

Matsumura T, Okumura Y (2013) Privatization neutrality theorem revisited. Economics Letters 118(2):324–326.

Matsumura T, Shimizu D (2010) Privatization waves. Manchester School 78(6):609–625.

Merrill W C, Schneider N (1966) Government firms in oligopoly industries: A short-run analysis. Quarterly Journal of Economics 80(3):400-412.

Nakamura Y (2013) Social welfare under quantity competition and price competition in a mixed duopoly with network effects: an analysis. Theoretical Economics Letters 3:211–215.

Scrimitore M (2013) Price or quantity?: the strategic choice of subsidized firms in a mixed duopoly. Economics Letters 118(2):337–341.

Scrimitore M (2014) Profitability under commitment in Cournot and Bertrand mixed markets. Journal of Institutional and Theoretical Economics 170(4):684-703.

Singh N, Vives X (1984) Price and quantity competition in a differentiated duopoly. RAND Journal of Economics 15(4):546–554.

Shubik M, Levitan R (1980) Market Structure and Behavior (Harvard University Press, Cambridge, Massachussets, U.S.A.).

Tanaka Y (2001a) Profitability of price and quantity strategies in an oligopoly. Journal of Mathematical Economics 35(3):409–418.

Tanaka Y (2001b) Profitability of price and quantity strategies in a duopoly with vertical product differentiation. Economic Theory 17(3):693–700.

Tasnádi A (2006) Price vs quantity in oligopoly games. International Journal of Industrial Organization 24(3):541–554.

Vives X (1985) On the efficiency of Bertrand and Cournot equilibria with product differentiation. Journal of Economic Theory 36:166–175.

Wang LFS, Chen TL (2010) Do cost efficiency gap and foreign competitors matter concerning optimal privatization policy at the free entry market? Journal of Economics 100(1):33–49.

Wang LFS, Lee JY (2013) Foreign penetration and undesirable competition. Economic Modelling 30(1):729–732

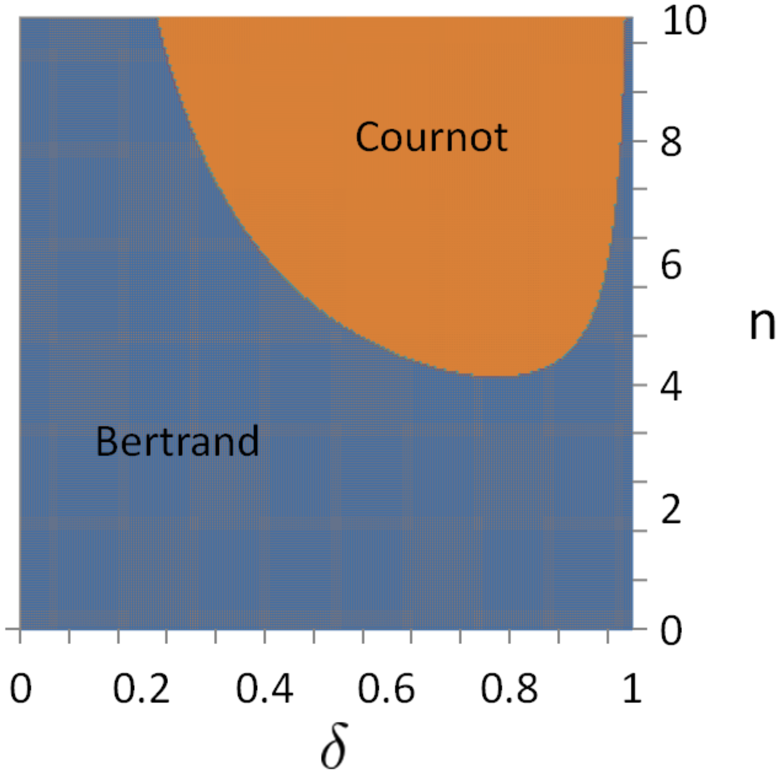

Figure 1

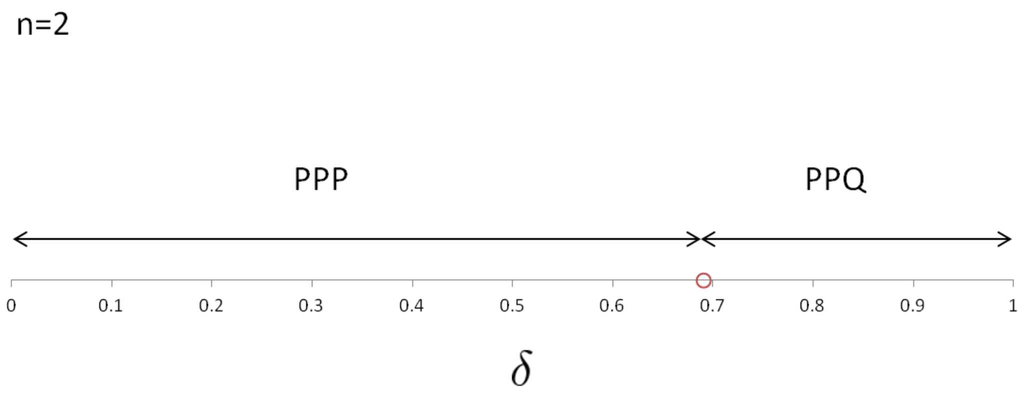

Figure 2

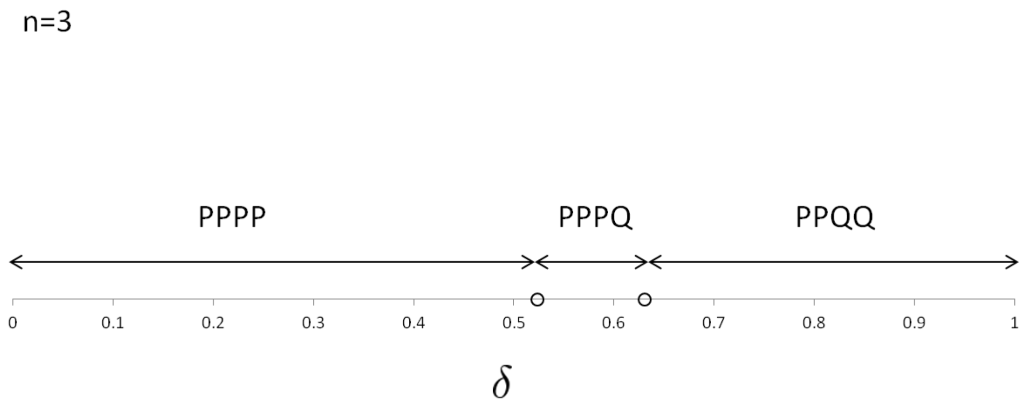

Figure 3

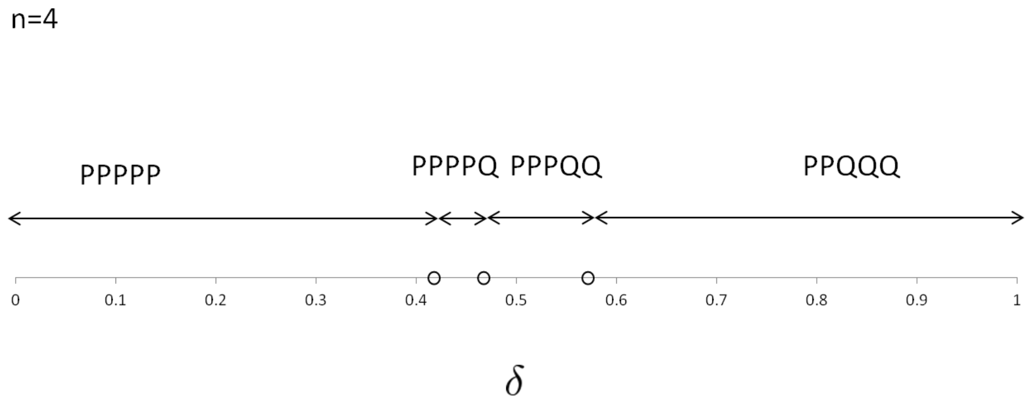

Figure 4

m=2

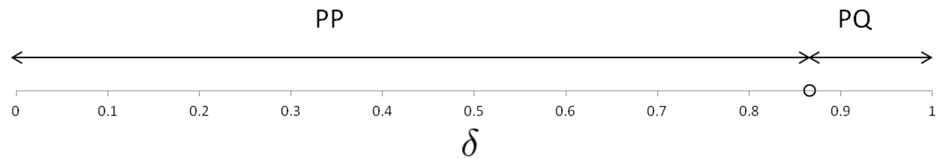

Figure 5

m=3

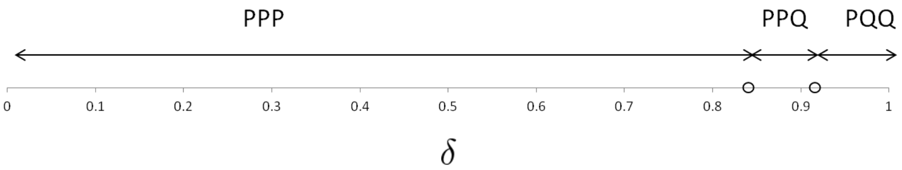

Figure 6

m=4

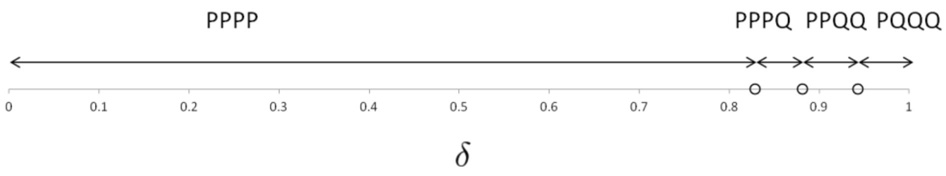

Figure 7

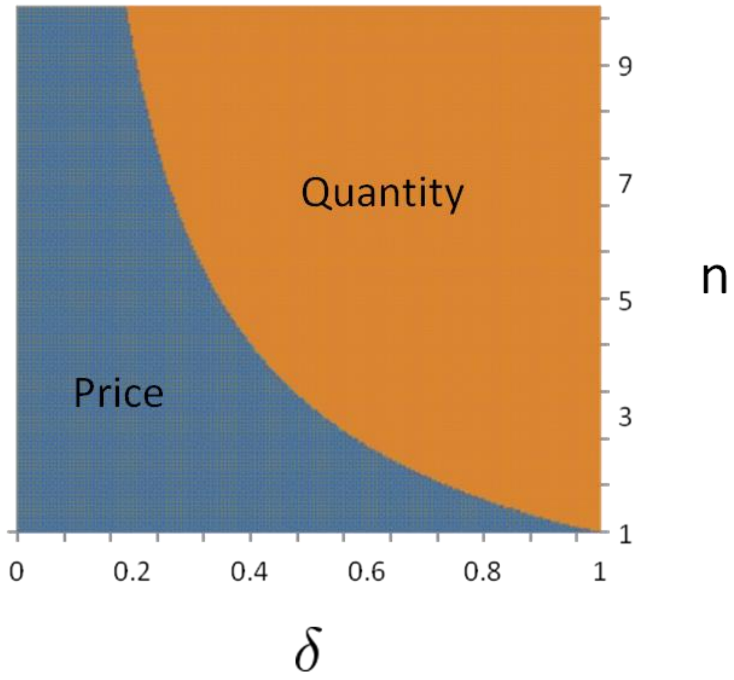

Figure 8