Efficiency, Privatization, and Political Participation

A Theoretical Investigation of Political Optimization in Mixed Duopoly

Cai Dapeng[1] and Li Jie[2]

[1] *Institute for Advanced Research, Nagoya University, Furo-cho, Chikusa-ku, Nagoya, Japan 464-8601, cai@iar.nagoya-u.ac.jp*; [2] *Corresponding Author. Lingnan College, Sun Yat-Sen University, No. 135 Xingangxi Road, Guangzhou, P. R. China, 510275, lijie@lingnan.net*

Abstract

This paper examines the political process behind the allocation of a lump-sum subsidy in a mixed duopolisitic market. We consider a bargaining game in which a partially privatized public firm and a private firm bargain with the government via contributions for the subsidy. We present the equilibrium structure that emerges from the game. We also clarify how changes in the efficiency discrepancy between the two firms, and the state's control over the public firm, can affect the optimal behaviors of the government and individual firms.

***Keywords*:** Public firm, Mixed duopoly, Bargaining

***JEL classification code*:** P20; L32; L13

1. Introduction

Subsidies to industries are universal, with many governments providing industries with export subsidies, and subsidies for R&D, environmental friendly technologies, and many other purposes. Clearly, such subsidies can affect the underlying structure of the game within the targeted industries. There has been a vast literature that considers the effects of such subsidies.[1] However, insufficient attention has been given to the political process through which such subsidies are allocated among firms. In this paper, we consider the political process behind the allocation of a lump-sum subsidy in a partially privatized mixed market. Partially privatized mixed markets, with the government retaining a large ownership share in the privatized assets of the public firm, has been common in developed, developing, and transitional economies (Matsumura, 1998; Maw, 2002; Matsumura and Kanda, 2005).[2] By studying the formation of political equilibrium concerning the allocation of the subsidy, our study aims at shedding lights on the political optimization underlying the endogenous determination of industrial policies in mixed markets.

Quite obviously, firms are likely to be competing with each other for such subsidies, as they are scarce resources. Indeed, “any individual who is affected by government policy has an incentive to influence the policymaker” (Bernheim and Whinston, 1986, p. 3). With conflicting interests, it is not surprising to see that the firms are bargaining with the government by offering rewards (or bribes), in an attempt to influence the formation of policies on subsidies. Naturally, one wonders what characterizes the equilibrium outcome of such a bargaining process. In particular, how

[1] For the basic theoretical argument for export subsidies, see for example, Spencer and Brander (1983).

[2] Note that markets with only private firms can be regarded as special cases of mixed markets.

does the government come to pay attention to the concerns of a particular firm?

There has been a growing literature that examines the relationship between firms' political participation and policy formation. In their seminal work, Shleifer and Vishny (1994) consider the political connection and economic benefits by examining the bargaining between politicians and managers of public firms. Important contributions also include Grossman and Helpman (1994), Konishi *et al.* (1999), and Qiu (2004), which examine the political process in the context of trade policies.

Our simple model is formulated in the setting of a mixed duopolistic market composed of a public firm and a private firm. Our model is related to the large literature on mixed markets, which is receiving increasing attention. The literature dates from the pioneering works of Merrill and Schneifer (1966) and Harris and Wiens (1980), with excellent surveys available in Bös (1986; 1991), Vickers and Yarrow (1988), and De Fraja and Delbono (1990a). Studies on mixed markets are generally carried on the assumption that the public firm is a social welfare maximizer, while the private firm is a pure profit maximizer. Partial privatization is first examined in Matsumura (1998), which explicitly considers the case in which the government holds a non-negligible proportion of shares in privatized firms. As the shareholding structure is a mixture of private and public ownership, the privatized firms must respect the interests of the private shareholders, as well as the political objectives of the state, leaving room for government to control the activities of these firms. Most studies so far, however, have taken the policies as exogenously given, with few considering the political process underlying the formation of policies, which is precisely our focus in this paper. In this paper, we explicitly analyze the role of the political process in the determination of the allocation of a lump-sum subsidy that emerges in the equilibrium.

In our model, we assume that both firms are vying to influence the government's decision by bargaining over the contribution schedules, so as to grab a larger share of the subsidy. By making explicit the process by which the government comes to pay special attention to the concerns of particular firms, we are able to characterize the equilibrium structure that emerges from the bargaining game. We also clarify how changes in the efficiency discrepancy between the two firms, and the state's control over the public firm, can affect the optimal behaviors and bargaining incentives of the government and individual firms.

The rest of the article is organized as follow. In Section 2, we explore a benchmark case, in which the government does not value contributions from industries. In Section 3, we explicitly examine the political interaction between the two firms and the government by extending the two-stage benchmark case into a four-stage bargaining game. We then consider the mechanism and implications of the political equilibrium of the bargaining game by comparing the equilibrium outcomes of the two cases. In Section 4, we consider how changes in the efficiency discrepancy and the state's control over the public firm can affect the optimal behaviors of the government and individual firms by performing a comparative-static analysis. Concluding remarks are presented in Section 6. The proofs of our results are collected in Appendix.

2. The Benchmark Case

In this paper, by considering a bargaining game, we examine how public firms and private firms influence government's policy in mixed duopolistic markets. As in De Fraja and Delbono (1989), we adopt a static, partial equilibrium analysis, and assume complete knowledge on the part of all agents.

We take as a benchmark (labeled as 0) a situation in which the government does not value the contributions from industries. We consider an industry that is composed of two firms: a public firm (*Firm* 1) and a private firm (*Firm* 2). Both firms produce a homogeneous product, with outputs being Q_1 and Q_2. The inverse demand curve is $P(Q_1+Q_2)$, where $P'<0$. We normalize the average cost of the private firm to zero and assume that the public firm has a cost disadvantage: the average cost of which is $k>0$. The government decides how to allocate between the two firms a lump-sum subsidy, Γ ($\Gamma>0$). For the sake of simplicity, we assume that the subsidy can be transformed to a cash payment, γ, for each unit of output produced by the public firm. It is also assumed that in this respect, both firms' objectives are conflicting, as each of them is vying to get more subsidies, at the expense of the other. Hence, the public firm receives γQ_1, whereas the private firm receives $\Gamma-\gamma Q_1$.

The public firm we consider is a joint stock company, jointly owned by both the public and private sectors. Following Matsumura (1998), we assume the public firm maximizes a weighted average of social welfare and its owned profit, with the weight of social welfare determined by the proportion of shares held by the government. The social welfare W is the sum of consumer surplus (CS) and profits of both firm:

$$W = \mathrm{K} + \Pi + CS - \Gamma = \int_0^{Q_1+Q_2} pdq - kQ_1 - \Gamma\ ,$$

where K is the net profit of the public firm, namely $\mathrm{K} = PQ_1 - kQ_1 + \gamma Q_1$, while Π is the net profit of the private firm. The objective function of the public firm is then a linear combination of its profit, $PQ_1 - kQ_1$, the social welfare, W, and the subsidy received, γQ_1:

$$S = (1-\alpha)[PQ_1 - kQ_1] + \gamma Q_1 + \alpha W\ , \tag{1}$$

where α ($\alpha \ge 0$) is an exogenous variable reflecting the state's control over the firm (which is in proportion to the ratio of the state's share), and γ is the subsidy received per unit of output produced by the public firm.[3] On the other hand, the private firm's objective is to maximize its profit plus the subsidy:

$$\Pi = PQ_2 + (\Gamma - \gamma Q_1)\,. \tag{2}$$

The government chooses the level of unit subsidy to the public firm, γ, to maximize social welfare. Throughout the following analysis, we focus on the case in which the public firm has a positive markup, *i.e.*, $P > k$.

The benchmark case is a simple two-stage game. In the first stage, the government decides the level of unit subsidy to public firm, γ; whereas in the second stage, observing the government's choice, both firms engage in quantity competition, *á la* Cournot. Backward induction method is useed to solve this game. The first order conditions are characterized as follows:

$$(1-\alpha)P'Q_1 + P - k + \gamma = 0\,, \tag{3}$$

$$P'Q_2 + P = 0\,. \tag{4}$$

Furthermore, we assume that the standard conditions which ensure the uniqueness and stability for a Cournot game are satisfied (Dixit 1986). The reaction functions for the two firms, denoted by $R_1(Q_2)$ and $R_2(Q_1)$, are implicitly defined by (3) and (4):

$$R_1(Q_2) \equiv \arg\max_{Q_1 \geq 0} S(Q_1, Q_2; \alpha, \gamma, k)\,, \quad R_2(Q_1) \equiv \arg\max_{Q_2 \geq 0} \Pi(Q_1, Q_2; \alpha, \gamma, k)\,.$$

We also assume that one firm's marginal revenue declines when the output of the other firm rises, *i.e.*, the outputs of the two firms are "strategic substitutes:

$$P^{'} + P^{''}Q_2 < 0, \quad P^{'}(1+\alpha) + P^{''}Q_1 < 0.$$

As the national political economy is characterized by parameters, α, γ, and k, we first examine the comparative-static effects of these parameters on the equilibrium

[3] It should be noted that when $\alpha = 1$, equation (1) reduces to the $\gamma Q_1 + W$. It can then be easily shown that maximizing social welfare mandates the government not to subsidize the public firm, and the public firm will set the price at marginal cost, see for example, De Fraja and Delbono (1987).

outputs and price, which are summarized in Lemma 1.

Lemma 1. *Denoting* E_1 *and* E_2 *as the equilibrium outputs of both firms, respectively, we note that:*

(i) *A rise in* α *raises the public firm's equilibrium output, and lowers those of the private firm's output. Moreover, it raises the overall output and lowers the price. Formally,* $\frac{\partial E_1}{\partial \alpha} > 0$, $\frac{\partial E_2}{\partial \alpha} < 0$, $\frac{\partial (E_1 + E_2)}{\partial \alpha} > 0$, $\frac{\partial P}{\partial \alpha} < 0$.

(ii) *A rise in* γ *raises public firm's equilibrium output, and lowers that of the private firm. Moreover, it raises the overall output and lowers the price. Formally,* $\frac{\partial E_1}{\partial \gamma} > 0$,

$\frac{\partial E_2}{\partial \gamma} < 0$, $\frac{\partial (E_1 + E_2)}{\partial \gamma} > 0$, $\frac{\partial P}{\partial \gamma} < 0$.

(iii) *A rise in* k *lowers public firm's equilibrium output and raises that of the private firm. Moreover, it lowers the overall output and raises the price. Formally,* $\frac{\partial E_1}{\partial k} < 0$,

$\frac{\partial E_2}{\partial k} > 0$, $\frac{\partial (E_1 + E_2)}{\partial k} < 0$, $\frac{\partial P}{\partial k} > 0$.

From Lemma 1, we see that due to the strategic substitution effects, the impacts of α, γ, and k are opposite to the two firms, indicating that firms tend to exhibit opposing behaviors towards changes in these parameters. Specifically, we see that the state-ownership and the subsidy to public firm produce a pro-competitive effect, as a larger α or γ induces the public firm to increase its output (Harris and Wiens, 1980). Note that it is this difference in preferences that motivates firms to participate in the political process to influence the government's choices.

Next we move back to the first stage of the game. Taking account of the equilibrium outputs of the second-stage, the government chooses γ to maximize social welfare, under the budget constraint $0 \le \gamma \le \Gamma / E_1$. By differentiating the government's objective function with respect to γ and assuming that the second order condition holds, we obtain the following: when $\gamma = 0$, $\left.\frac{\partial W}{\partial \gamma}\right|_{\gamma=0} = P(1+R_2') - k\Big|_{\gamma=0} < 0$; when $\gamma = \Gamma / E_1$, $\left.\frac{\partial W}{\partial \gamma}\right|_{\gamma=\Gamma/E_1} = P(1+R_2') - k\Big|_{\gamma=\Gamma/E} > 0$; on the other hand, when $0 < \gamma < \Gamma / E_1$, $P + PR_2' - k = 0$. Note that $\gamma = 0$ and $\gamma = \Gamma / E_1$ denotes the two special cases in which the government transfers the subsidy exclusively either to the private firm, or to the public firm, respectively. These two cases are of interest, but we view them as less empirically relevant and, given space constraints, we do not pursue them here. In what follows, our analysis is focused on the case $0 < \gamma < \Gamma / E_1$, *i.e.*, both firms are getting a share of subsidy from the government, which leads to the following lemma:

Lemma 2. *In the benchmark case in which the government does not value the contributions from industries, when the government chooses γ ($0 < \gamma < \Gamma / E_1^0$) to maximizes its payoff W^0, the following condition must be satisfied:*

$$P^0 + P^0 R_2^{0\prime} = k. \tag{5}$$

Lemma 2 implies that the government's optimal choice is to equate the marginal total consumer surplus, $P^0 + P^0 R_2^{0\prime}$, to the efficiency discrepancy between the two firms, k.

3. The Bargaining Process: Methodology and Outcome

Next, we explicitly consider the political interaction between the two firms and the government. There has been a vast literature studying the interaction between special interest groups and the government. Two approaches have been proposed in the literature. The common agency approach, in which special interest groups offer their contribution schedules to government in exchange for favorable policies, was first studied by Bernheim and Whinston (1986), and can also be found in Grossman and Helpman (1994) and Konishi *et al*. (1999). In contrast, Maggi and Rodriguez-Clare (1998) consider the bargaining approach, in which the special interests groups bargain with the government over the amount of contributions. In this paper, following Qiu (2004), we consider the case in which the government and *each* firm bargain over the contribution *schedules*. As commented by Helpman (1997) and Qiu (2004), there is no agreed upon theory of domestic politics, and the selection of the approach should depend on the case in consideration.

In the bargaining game (labeled as *), the two firms contemplate to influence the government's decision on the allocation of the subsidy by bargaining over the schedules of contributions to the government $C_i(\gamma)$, where $i=1, 2$, and $C_i(\gamma)\geq 0$. Given the contribution schedules of the two firms, the government chooses γ ($0<\gamma<\Gamma/E_1$) to maximize its payoff, which is a sum of the social welfare and total contributions:

$$G=W^*+C_1+C_2.\text{[4]} \tag{6}$$

The game we consider is a four-stage one (see Figure 1). In the first stage, each

[4] We assume that the government values a dollar equivalent of social welfare equally with a dollar collected as a contribution from the firms. Our results will remain essentially unchanged even if the government values the social welfare and contributions differently.

firm chooses the bargaining power to be applied when bargaining with the government;[5] in the second stage, the two firms simultaneously bargain with the government over the contribution schedules; in the third stage, the government chooses the unit level of subsidy extended to the public firm; and in the fourth stage, the two firms compete with each other, *á la* Cournot. We use the backward induction approach to solve this game.

In the fourth stage, the Cournot equilibrium is characterized by equations (3) and (4), which is the same as the benchmark case. The comparative-static effects can be referred to Lemma 1.

Next, we consider the third stage of the game. Given the contribution schedules of the two firms, $C_1(\gamma)$ and $C_2(\gamma)$, the government chooses γ to maximize her payoff. Assuming that the second order condition is satisfied, the optimal choice of the government can be reported as the following proposition:

Proposition 1. *Given the contribution schedules $C_1(\gamma)$ and $C_2(\gamma)$, the optimal γ^* chosen by the government that maximizes her payoff G must satisfy:*

$$C_1'(\gamma^*) + C_2'(\gamma^*) = -(P^* - k + P^* R_2^{*\prime}) \frac{\partial E_1^*}{\partial \gamma}. \qquad (7)$$

Proposition 1 immediately leads to the following:

Remark 1. *Around the neighborhood of the equilibrium output levels,*

---

[5] We assume that each firm's bargaining power with the government has an upper bound, $\overline{\eta}_i \, (0 \le \overline{\eta}_i < 1)$, $i = 1, 2$. When bargaining with the government, each firms is able to choose a bargaining power $\eta_i \in [0, \overline{\eta}_i]$

(i) *when* $k = P^* + P^* R_2^{*\prime}$, *firms' contributions have no influence on the government's optimal choice and the unit subsidy is the same as in the benchmark case;*

(ii) *when* $k > P^* + P^* R_2^{*\prime}$, *the unit subsidy received by the public firm is larger than that in the benchmark case;*

(iii) *when* $k < P^* + P^* R_2^{*\prime}$, *the unit subsidy received by the public firm is smaller than that in the benchmark case.*

From Remark 1, we see that around the neighborhood of the equilibrium output levels, if $P^* + P^* R_2^{*\prime} - k \neq 0$, the government's choice of the optimal unit subsidy to the public firm is influenced by both firms' contributions. Specially, the government favors the public firm when it is relative inefficient, $k > P^* + P^* R_2^{*\prime}$; and favors the private firm when the public firm is relatively more efficient, $k < P^* + P^* R_2^{*\prime}$.

Next, we turn to the second stage in which the two firms bargain with the government to determine their respective contribution schedules.

To analyze the bargaining process, there is a need to specify the threat points for the two firms. The threat point of the public firm, S^0, is its payoff in the case in which the private firm bargains with the government to maximize its payoff, whereas the public firm does nothing. Accordingly, the threat point of the private firm, Π^0, is its payoff in the case in which the public firm bargains with the government to maximize its payoff, whereas the private firm does nothing. On the other hand, the government's threat point is its payoff in the case of no contributions from both firms, which is equivalent to its payoff in the benchmark case, W^0. As firms have an incentive to participate in the bargaining process only when they can benefit from doing so (get at least as much payoff as the case of not doing so), we see that the conditions for them to enter the bargaining process are $S(\gamma^*) - C_1(\gamma^*) - S^0 \geq 0$, and $\Pi(\gamma^*) - C_2(\gamma^*) - \Pi^0 \geq 0$, where γ^* is the equilibrium unit subsidy to the public firm, respectively. On the other hand, to ensure that the government has an incentive to participate in the bargaining game, we specify that $W(\gamma^*) + C_1(\gamma^*) + C_2(\gamma^*) - W^0 \geq 0$. Hence, the bargaining game takes place as one of the following four cases: (i) both firms have no incentive to bargain with the government, which is reduced to the benchmark case; (ii) only the public firm has an incentive to bargain, and bargaining takes place only between the public firm and the government; (iii) only the private firm has an incentive to bargain, and bargaining takes place only between the private firm and the government; (iv) both firms have an incentive to bargain with the government. As is shown below, case (ii) and (iii) are special cases of case (iv), in what follows, we will focus on case (iv).

We are now ready to examine the bargaining process. We first consider the bargaining between the public firm and the government. Following the public firm's choice of bargaining power in the first stage, the bargaining power of the government relative to the public firm and that of the public firm relative to the government are β_1 and $(1-\beta_1)$, respectively, with $1-\beta_1 \equiv \eta_1$. Here the net gain for the public firm is $S(\gamma^*) - C_1(\gamma^*) - S^0$, and that for the government is $W(\gamma^*) - W^0 + C_1(\gamma^*) + C_2(\gamma^*)$. The Nash bargaining over $C_1(\gamma^*)$ is represented by

$$\max_{C_1(\gamma^*)} \; [S(\gamma^*) - S^0 - C_1(\gamma^*)]^{1-\beta_1} [W(\gamma^*) - W^0 + C_1(\gamma^*) + C_2(\gamma^*)]^{\beta_1}. \tag{8}$$

Similarly, the bargaining power of the government relative to the private firm and that of the private firm relative to the government are β_2 and $1-\beta_2$ respectively, with $1-\beta_2 \equiv \eta_2$. The net gain for the private firm is $\Pi(\gamma^*) - C_2(\gamma^*) - \Pi^0$, and that for the government is also $W(\gamma^*) - W^0 + C_1(\gamma^*) + C_2(\gamma^*)$. The Nash bargaining over $C_2(\gamma^*)$ is represented by

$$\max_{C_2(\gamma^*)} \; [\Pi(\gamma^*) - \Pi^0 - C_2(\gamma^*)]^{1-\beta_2} [W(\gamma^*) - W^0 + C_1(\gamma^*) + C_2(\gamma^*)]^{\beta_2}. \tag{9}$$

Letting $\Delta S \equiv S(\gamma^*) - S^0$, $\Delta\Pi \equiv \Pi(\gamma^*) - \Pi^0$ and $\Delta W \equiv W(\gamma^*) - W^0$. Solving the optimal problems (8) and (9) simultaneously gives:

$$C_1(\gamma^*) = \frac{\beta_1 \Delta S - (1-\beta_1)\beta_2(\Delta W + \Delta\Pi)}{\beta_1 + \beta_2 - \beta_1\beta_2}, \tag{10}$$

$$C_2(\gamma^*) = \frac{\beta_2 \Delta\Pi - (1-\beta_1)\beta_2(\Delta W + \Delta S)}{\beta_1 + \beta_2 - \beta_1\beta_2} \tag{11}$$

Differentiating equations (10) and (11) with respect to γ^* and making use of the equilibrium results obtained in Result 1 and substituting them into equation (7) yields the optimal unit subsidy to the public firm:

$$\gamma^* = P^*(\theta - \alpha) + k - \frac{k(\theta+\alpha)}{1+R_2^{*\prime}}, \tag{12}$$

where $\theta \equiv \frac{\beta_1 + \beta_2}{\beta_1\beta_2} \geq 2$. Obviously, when $k \leq \frac{(1+R_2^{*\prime})(\theta-\alpha)}{\theta+\alpha-(1+R_2^{*\prime})}P^*$, Nash bargaining does not lead to a deal between the two firms and the government (as in that case $\gamma \leq 0$), and the government follows the optimal choice as determined in Lemma 2. Hence, we have:

Proposition 2. *In the bargaining game, around the neighborhood of the equilibrium, the equilibrium unit subsidy to the public firm is characterized as follows:*[6]

---

[6] When the public firm choose not to lobby the government, *i.e.*, when $\beta_1 = 1$, equation (8) will vanish, with $\theta = (1+\beta_1)/\beta_1$. Similarly, if the private firm choose not to lobby the government, *i.e.*, when $\beta_2 = 1$, equation (9) will vanish, with $\theta = (1+\beta_2)/\beta_2$.

$$\gamma^* = \begin{cases} P^*(\theta-\alpha)+k-\dfrac{k(\theta+\alpha)}{1+R_2^{*\prime}},\ if\ k > \dfrac{(1+R_2^{*\prime})(\theta-\alpha)}{\theta+\alpha-(1+R_2^{*\prime})}P^*, \\ \gamma^0,\ if\ k \le \dfrac{(1+R_2^{*\prime})(\theta-\alpha)}{\theta+\alpha-(1+R_2^{*\prime})}P^*. \end{cases}$$

By explicitly describing the government's optimal choices, proposition 1 and 2 together present the overall outcome of the bargaining process. Proposition 1 shows that it is optimal for the government to equate the marginal reduction in social welfare to the marginal benefit from the firms' contributions, whereas Proposition 2 shows that the government values firms' contributions only when the public firm is sufficiently efficient. In what follows, we consider how the bargaining process, especially firms' bargaining incentives, would be affected by changes in the efficiency discrepancy between the firms, k, and the state's control over the public firm, α.

4. Efficiency, Privatization, and Bargaining Incentives

In this section, we perform a comparative-static analysis and consider how firms' and the government's optimal behaviors are affected by exogenous changes in public firm's efficiency (as a result of the Chinese style state-owned enterprise (SOE) reform, for example), as well as changes in the state's share in the public firm (as a result of an intensified privatization process, for example). We continue to assume that public firms still have a cost disadvantage relative to the non-state sector, and our discussion is focused on the case $k > \dfrac{(1+R_2^{*\prime})(\theta-\alpha)}{\theta+\alpha-(1+R_2^{*\prime})}P$. To simplify the analysis, we also assume a linear market demand function (*i.e.*, R_2' is a constant). Firms choose their bargaining power and decide whether to engage in the bargaining process in the first stage of the game, and we first show how they choose their bargaining powers. Given β_2 (or, $1-\eta_2$), differentiating equation (12) with respect to β_1 yields

$$\frac{\partial \gamma^*}{\partial \beta_1} = \frac{\partial \gamma^*}{\partial \theta}\frac{\partial \theta}{\partial \beta_1} = \frac{[P(1+R_2') - k]}{1+R_2'}(-\frac{1}{\beta_1^2}), \tag{13}$$

Obviously, the sign of the above equation depends on the efficiency discrepancy of the two firms. When $k < P^* + P^* R_2^{*\prime}$, the larger is the bargaining power of the public firm relative to the government, $1-\beta_1$, the larger is the unit subsidy to the public firm, and *vice versa*. Hence, the public firm is induced to choose $\overline{\eta}_1$ when it is relatively more efficient. On the other hand, as

$$\frac{\partial \gamma^*}{\partial \beta_2} = \frac{\partial \gamma^*}{\partial \theta}\frac{\partial \theta}{\partial \beta_2} = \frac{[P^* + P^* R_2^{*\prime} - k]}{1+R_2^{*\prime}}(-\frac{1}{\beta_2^2}), \tag{14}$$

we see that a large bargaining power of the private firm relative to the government is not desirable for the private firm when $k < P^* + P^* R_2^{*\prime}$, since in that case, the larger is the value of $1-\beta_2$, the higher is γ^*. Hence, it would be optimal for the private firm to completely disengage from the bargaining process. In other words, while the public firm is extremely active when she is relatively efficient, the private firm has no incentive to participate into the bargaining process in such a case. We define such a case as the case of *public firm lobbies* (only the public firm bargains with the government). On the other hand, when $k > P + PR_2'$, we see that only the private firm chooses to bargain with the government, with the bargaining power chosen by the private firm being $\overline{\eta}_2$. Accordingly, such a case is defined as *private firm lobbies* (only the private firm bargains with the government).

Letting $\tilde{k} \equiv P^* + P^* R_2^{*\prime}$, $\hat{k} \equiv \dfrac{(1+R_2^{*\prime})(\theta-\alpha)}{\theta+\alpha-(1+R_2^{*\prime})}P^*$, it can be easily shown that when

$\alpha > \frac{1+R_2^{*\prime}}{2}$, $\hat{k} < \tilde{k}$; on the other hand, when $\alpha \le \frac{1+R_2^{*\prime}}{2}$, $\hat{k} \ge \tilde{k}$. Combining this observation with Remark 1 and Propositions 2, we are able to summarize how α is affecting the firms' bargaining incentives:

Proposition 3. *The equilibrium outcome also depends on the state's control over the public firm:*

(i) *if* $\alpha > \frac{1+R_2^{*\prime}}{2}$, *no lobby occurs when* $k < \hat{k}$, *the public firm lobbies when* $\hat{k} < k < \tilde{k}$, *and the private firm lobbies when* $k > \tilde{k}$*;*

(ii) *if* $\alpha \le \frac{1+R_2^{*\prime}}{2}$, *no lobby occurs when* $k < \hat{k}$, *and the private firm lobbies when* $k > \hat{k}$.

Proposition 3 shows that how the privatization process may affect the firms' bargaining incentives. A heuristic description of the theorem is as follows. Under our model formulation, as is shown in Lemma 1(i), the government's control over public firm produces a pro-competitive effect: a rise (fall) in α augments (lowers) the overall output and lowers (increases) the price, resulting in a rise (fall) in consumer surplus. On the other hand, the unit subsidy γ also produces such a pro-competitive effect. Assuming for a moment that the government is free to choose α and γ. To maintain a given level of pro-competitive effect, the government would choose a lower γ when α is high. Of course, the government's choice of γ also depends on the efficiency discrepancy of firms, k. For a given level of α, as a rise (fall) in k lowers (raises) the pro-competitive effect (Lemma 1(iii)), the optimal response of the government would be to extend more (less) subsidy to the public firm. Hence, we can summarize the government behavior as follows. The government compares the current efficiency discrepancy between firms, k, with the two critical points: $\tilde{k}$ and $\hat{k}$. When $k \geq \hat{k}$, it would be optimal for the government to value contributions from firms. As predicted in Result 1, when the government values contributions from firms, it favors the public firm when the public firm is relative inefficient ($k > \tilde{k}$), and the private firm when the public firm is relatively efficient ($k < \tilde{k}$). Hence, the only interval in which the public firm is not favored would be $(\hat{k}, \tilde{k})$, with $\hat{k} < \tilde{k}$, which is only possible when the state's control over the public firm is relatively high. In such a situation, as the public firm becomes worse off as compared with the case in which the government does not value contributions, the public firm is forced to participate into the bargaining process more actively.

Proposition 1, 2, and 3 together describe the Nash bargaining equilibrium outcome. As shown by Proposition 1 and 2, the broader contour of the equilibrium outcomes depends on the efficiency discrepancy of the firms and the government's control over the public firm (Figure 2 and 3). On the other hand, the finer details of the equilibrium outcome, *i.e.*, the extents to which different firms are favored, are exhibited in equations (13) and (14) and Proposition 3.

Figure 2 shows the case for a relatively high α. When the public firm is relatively efficient ($k \in (\hat{k}, \tilde{k})$), it would be optimal for the government to subsidize the public firm less, motivating the public firm to enter politics to raise the subsidy. On the other hand, the optimal response of the government toward an inefficient public firm (when $k > \tilde{k}$) would be to increase the subsidy, so as to maintain the pro-competitive effect. However, as predicted in Lemma 1(ii), the rise in γ^* will lower the payoff of the private firm, motivating the private to bargain with the government.

Figure 3 is about the case for a relatively small α. As a fall in α alleviates the pro-competitive effect and lowers social welfare, to restore the equilibrium level of social welfare, the government is forced to increase the subsidy to induce the public firm to increase its output. However, such a practice aggravates the payoff of the private firm, and as shown in Figure 3, the private firm may be forced to participate into the bargaining process more actively.

5. Concluding Remarks

Undoubtedly, the way we conceptualize and model the relationship among the government, the public firm, and the private firm is not the uniquely best approach. Our approach may not capture all the essential features of a mixed duopolistic industry. For example, we assume that when making decisions concerning the allocation of the lump-sum subsidy, the government only considers the contributions from firms. However, in reality, such decisions are also based on many other factors, such as the firms' locations and R&D capacities. Second, to focus on the impacts of political process, we have assumed that the government does not incur a cost when subsidizing firms, which may lead to oversimplified predictions to certain industries. Also, it would be interesting to compare the social welfare under the two cases. Although for space constraints, such a comparison has not been pursued in this paper, it can be easily performed by specifying contribution schedules and the demand function. Moreover, it would be interesting to consider the case in which both firms engage in price competition, *á la* Bertrand. The results may be reversed as the reaction functions are upward sloping in a price game, rather than downward sloping as in a quantity game. Finally, we have also abstracted from the factors such as the state of market and market-supporting institutions, as well as individual entrepreneurs' political and human capital, which, as pointed in Li *et al*. (2006), may also be important determinants of an entrepreneur's political participation.

The most natural way to advance the current analysis would be to examine an extended model in which these insufficiencies are addressed. It would be interesting to examine whether our conclusions could be carried over to such an extended model.

References

Bartels, L., and Brady, H. (2003): "Economic Behavior in Political Context." *American Economic Review* 93: 156-61.

Bernheim, B. D., and Whinston, M. D. (1986): "Menu Auctions, Resource Allocation, and Economic Influence." *Quarterly Journal of Economics* 101: 1-31.

Bös, D. (1986) *Public Enterprise Economics*. Amsterdam: North-Holland.

Bös, D. (1991): *Privatization: A Theoretical Treatment*. Oxford: Clarendon Press.

De Fraja, G., and Delbono, F. (1987): "Oligopoly, Public Firm, and Welfare Maximization: A Game-theoretic Analysis." *Giornale Degli Economisti e Annali di Economia* 46: 417-435.

De Fraja, G., and Delbono, F. (1990a): "Alternative Strategies of Public Enterprises in Oligopoly." *Oxford Economic Papers* 41: 302-311.

De Fraja, G., and Delbono, F. (1990b): "Game Theoretic Models of Mixed Duopoly." *Journal of Economic Surveys* 4: 1-17.

Dixit, A. (1986): "Comparative Statics for Oligopoly." *International Economic Review* 27: 107-122.

Faccio, M. (2006): "Politically Connected Firms." *American Economic Review* 96: 369-386.

Grossman, G. M., and Helpman, E. (1994): "Protection for Sale." *American Economic Review* 84: 833-850.

Harris, R. G., and Wiens, E. G. (1980): "Government Enterprise: An Instrument for the Internal Regulation of Industry." *Canadian Journal of Economics* 13: 125-132.

Konishi, H., Saggi, K., and Weber, S. (1999): "Endogenous Trade Policy under Foreign Direct Investment." *Journal of International Economics* 49: 289-308.

Li, H., Meng, L., and Zhang, J. (2006): “Why Do Entrepreneurs Enter Politics? Evidence from China.” *Economic Inquiry* 44: 559-578.

Maw, J. (2002): “Partial privatization in transition economies.” Economic System 26: 271-282 .

Matsumura, T. (1998): “Partial Privatization in Mixed Oligopoly.” Journal of Public Economics 70: 473-483.

Matsumura, T., and Kanda, O. (2005): “Mixed Oligopoly at Free Entry Markets.” Journal of Economics 84: 22-48.

Qiu, L. D. (2004): “Lobbying, Multisector Trade, and Sustainability of Free-trade Agreements.” *Canadian Journal of Economics* 37: 1061-1083

Shleifer, A. and Vishny, R. W. (1994): “Politicians and Firms.” *Quarterly Journal of Economics* 109: 995-1025.

Spencer, B. J. and Brander, J. A. (1983): “International R&D Rivalry and Industrial Strategy.” *The Review of Economic Studies* 50: 707-722.

Appendix

Proof of Lemma 1 (i).

By differentiating the first-order conditions (3) and (4) with respect to α and incorporating the above assumptions, we obtain:

$$\frac{\partial E_1}{\partial \alpha} = \frac{P'E_1(P''E_2 + 2P')}{\Delta} > 0\,, \quad \frac{\partial E_2}{\partial \alpha} = -\frac{P'E_1(P''E_2 + P')}{\Delta} < 0\,,$$

$$\frac{\partial(E_1 + E_2)}{\partial \alpha} = \frac{(P')^2 E_1}{\Delta} > 0\,, \quad \frac{\partial P}{\partial \alpha} = P'\left(\frac{\partial(E_1 + E_2)}{\partial \alpha}\right) < 0$$

where $\Delta \equiv \begin{vmatrix} (1-\alpha)P''E_1 + (2-\alpha)P' & (1-\alpha)P''E_1 + P' \\ P''E_2 + P' & P''E_2 + 2P' \end{vmatrix}$. □

Proof of Lemma 1 (ii).

By differentiating the first-order conditions (3) and (4) with respect to γ and incorporating the above assumptions, we get:

$$\frac{\partial E_1}{\partial \gamma} = -\frac{(P''E_2 + 2P')}{\Delta} > 0, \quad \frac{\partial E_2}{\partial \gamma} = \frac{(P''E_2 + P')}{\Delta} < 0\,,$$

$$\frac{\partial(E_1 + E_2)}{\partial \gamma} = -\frac{P'}{\Delta} > 0\,, \quad \frac{\partial P}{\partial \gamma} = P'\left(\frac{\partial(E_1 + E_2)}{\partial \gamma}\right) < 0$$ □

Proof of Lemma 1 (iii).

By differentiating the first-order conditions (3) and (4) with respect to k and incorporating the above assumptions, we obtain:

$$\frac{\partial E_1}{\partial k} = \frac{(P''E_2 + 2P')}{\Delta} < 0\,, \quad \frac{\partial E_2}{\partial k} = -\frac{(P''E_2 + P')}{\Delta} > 0\,,$$

$$\frac{\partial(E_1+E_2)}{\partial k}=\frac{P'}{\Delta}<0\,,\quad \frac{\partial P}{\partial k}=P'\frac{\partial(E_1+E_2)}{\partial k}>0\,.$$ □

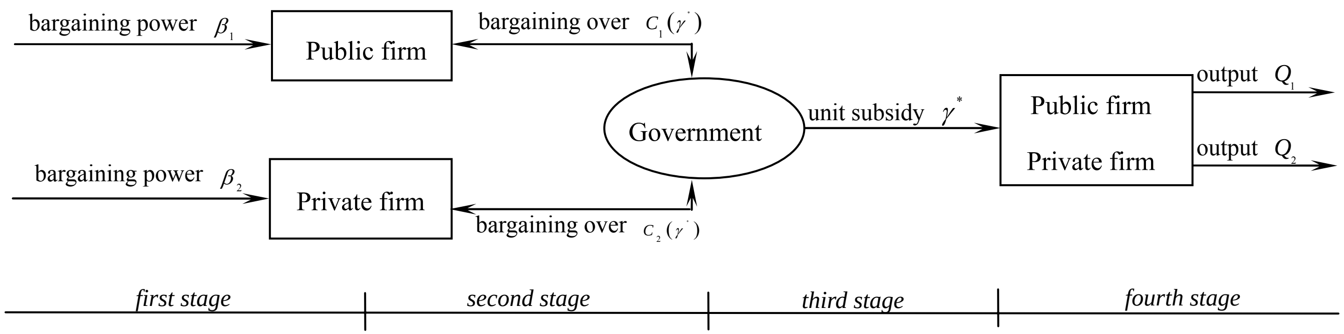

Figure 1. The structure of the game

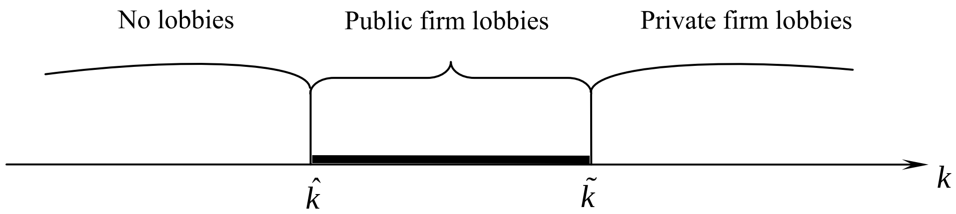

Figure 2. Efficiency discrepancy and the outcome of the bargaining game under a large state share

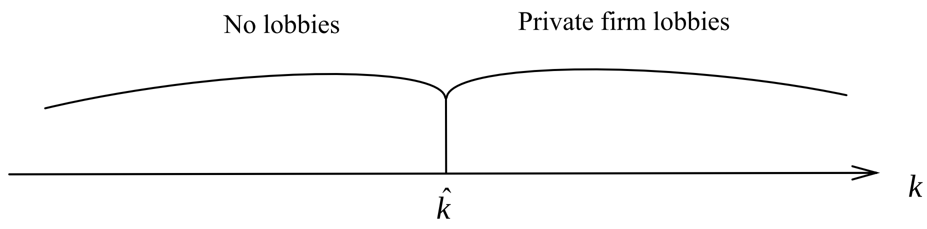

Figure 3. Efficiency discrepancy and the outcome of the bargaining game under a small state share