

# Intellectual Property Right Protection in the Software Market\*

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## Abstract

We capture the differences between patent and copyright by considering the optimal intellectual property right protection scheme in the software market. Patent protects an idea, and therefore a producer can prevent both reverse engineering by rival producers and software duplication by consumers. However, copyright cannot prevent a reverse engineering since copyright does not protect an idea. It is not clear which scheme is socially desirable in the software market. We obtain the following results. First, the number of copy users under the patent protection scheme is larger than that under the copyright protection scheme. Second, government can increase the social welfare by applying the copyright protection when the new technology is innovative enough.

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# 1 Introduction

Patent was also used to reward inventors for their development. In the U.S.A., patent law grants right holders exclusive use only for inventions that are useful, new, and non-obvious. Bessen and Hunt (2004) and Aharonian (2005) report that the United States Patent and Trademark Office (USPTO) grants more than 20,000 software patents a year. The number of software patent is growing rapidly in the U.S.A. On the other hand, the software patent is not granted by the European Patent Office (European Patent Convention Article 52). In July 2005, E.U. rejected the patent proposal, called the Computer Implemented Inventions Directive, and European Patent Office announced clearly that they did not grant the software patent. USPTO gives weight to the software producer's incentive. European Patent Office, by contrast, focuses on the welfare loss by exclusive uses. It is not clear which policy is socially desirable.

Many studies have investigated the promise of patents (Klemperer, 1985; Gallini, 1992; Gilbert and Shapiro, 1990; O'Donoghue, Scotchmer and Thisse, 1998; Tandon, 1982). However, it is difficult to apply such discussions to the software market since they do not consider specific properties of software. Certain kinds of software can be protected by patents if they entail the innovative technologies with regard to enhancing efficiency or productivity. Software is also protected by copyright because it is written by the source code. Although there are many differences between copyright and patent from a legal viewpoint, copyright and patent are treated in the same manner in the economics. Therefore, we have to consider the differences to discuss the software market.

Over the past few years a number of empirical studies have been made on the software patent. For example, Lerner and Zhu (2007) and Mann and Sager (2007) show the impact of software patent to the software development empirically. However, only few attempts have so far been made at theoretical researches. Although there are some papers that consider the software (Church and Gandal, 1992; Ellison and Fudenberg, 2000; Varian, 2000; Banerjee,

2003), they do not take into account differences between patent and copyright.

In this analysis, we capture the two types of copy in the software market. As shown above, patent protects an idea, and therefore a producer can prevent both the reverse engineering by rival producers and the software duplication by consumers. On the other hand, a copyright scheme can not prevent a reverse engineering since copyright does not protect an idea. It is not clear which is socially desirable: the patent protection or the copyright protection.

We obtain the following results. First, the number of copy users under the patent protection scheme is larger than that under the copyright protection scheme. Second, we compare two intellectual property right protection schemes for software market; patent and copyright. When the degree of innovation is small, there are no differences between the two schemes because the rival producer does not steal the new technology. When the new technology is innovative enough, government can increase the all software's quality enough by applying the copyright protection. We show that the effect of improving producer's quality and its subsequent copying on the protection. Recently, the necessity of the software patent has been discussed. We indicated that the government should not protect the software by patent. The government can increase the social welfare to set the appropriate copyright protection to give an enough incentive to producers.

This paper is organized as follows. Section 2 considers the optimal patent protection. Section 3 discusses the optimal copyright protection level against the software duplication. Section 4 then argues that the optimal intellectual property right protection scheme in the software market. Section 5 concludes the discussion. All proofs are provided in the Appendix.

## **2 Patent Protection in the Software Market**

We discuss the optimal patent protection in the software market. In this case, the rival producer can not copy the new technology because of the patent protection against the

reverse engineering. We consider two software producers in the market: producers 1 and 2. Both producers can produce the software with lowest level quality  $q_2 \geq 0$ <sup>1</sup> without innovation. Producer 1 can improve the software quality to  $q_1 = q_2 + \delta$  with the new technology.  $\delta$  means that the degree of innovation. Producer 1 decides whether or not to produce the innovative software with development cost  $F$ . When producer 1 does not develop the new technology, producers will set the zero price and play the Bertrand competition in the software market. We also assume that there are two types of consumers: legal users and illegal users. Legal users decide to purchase software from producer 1, producer 2, or to do nothing. The consumer valuations of the software, each of which is denoted by,  $v_i$  are uniformly distributed on the interval  $[0, 1]$ . Each consumer wants to buy at most one unit. If consumer  $i$  purchases the software at its retail price  $p_j$  ( $j = 1, 2$ ), his utility is given by  $q_j v_i - p_j$ . Illegal users can make a perfect copy of the highest quality software without any cost and their utility is given by  $q_1 v_i$ . The ratio of legal user is  $0 \leq e \leq 1$ . The government can control  $e$  by means of the intellectual property right protection level against software duplication. We present a multi-stage game model to consider the optimal intellectual protection scheme. The four stages of the game have the following rules:

1. Government sets  $e$  to maximize social welfare.
2. Producer 1 decides whether or not to develop the new technology  $\delta$  with the development cost  $F$ .
3. Producers choose the prices  $p_j$  simultaneously.
4. Legal users decide whether they will purchase the software from producer 1 or do nothing. Illegal users make copies of producer 1's software.

The government's goal is to maximize the social surplus, which is defined as the sum of the producers surplus and the consumers surplus. We analyze the sub-game perfect equilibrium

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<sup>1</sup>We do not allow producer 2 to decrease his quality for simplicity. We can obtain the qualitatively same conclusions even if we assume that the producer can decrease  $q_2$ .

by backward induction. First, let us consider consumer behavior.

**Lemma 1**

*Given  $e$  and price  $p_j$ , the optimal choice of legal consumers is not to obtain the good if and only if*

$$v_i < \frac{p_1}{q_1}, v_i < \frac{p_2}{q_2}.$$

*Legal users will purchase the software from producer 2 if and only if*

$$v_i \geq \frac{p_2}{q_2}, v_i < \frac{p_1 - p_2}{q_1 - q_2},$$

*and will purchase the software from producer 1 if and only if*

$$v_i \geq \frac{p_1}{q_1}, v_i \geq \frac{p_1 - p_2}{q_1 - q_2}.$$

*All illegal users will make the copy of producer 1's software.*

A consumer's behavior thus depends on his valuation of the software, quality, and the price. In the first case, legal users ignore software when their valuation of the software is lower than the price of producer 2's software. In the second case, the utility of purchasing producer 2's software is positive and higher than the utility of purchasing producer 1's software. In the third case, consumers prefer producer 1's software to 2's, because the utility of software 1 is positive and higher. Figure 1 shows the consumer behavior when  $p_1q_2 > p_2q_1$ . In this class, consumers with valuations larger than  $(p_1 - p_2)/(q_1 - q_2)$  purchase producer 1's software; those with valuations between  $p_2/q_2$  and  $(p_1 - p_2)/(q_1 - q_2)$  buy the software from producer 2 and those with valuations less than  $p_2/q_2$  do not consume. The legal users' demand for producer 1's software  $D_1$  and the demand for producer 2's software  $D_2$  when  $p_1q_2 > p_2q_1$  are thus given by

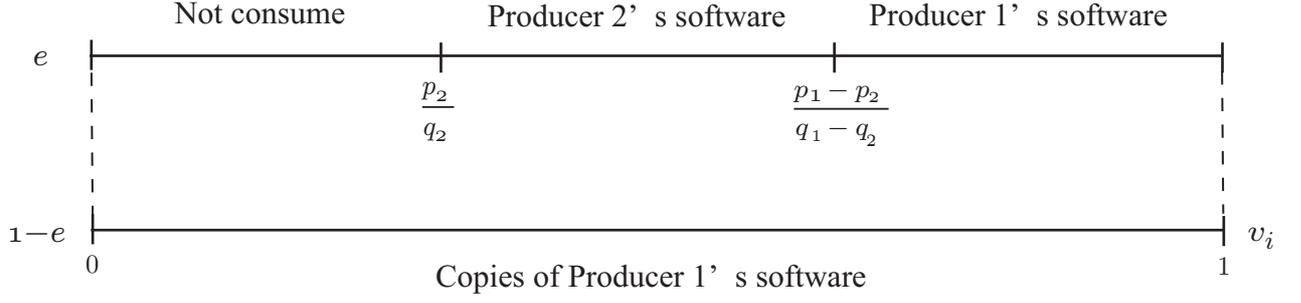


Figure 1: Consumer behavior when  $p_1q_2 > p_2q_1$

$$D_1 = 1 - \frac{p_1 - p_2}{q_1 - q_2}, \quad D_2 = \frac{p_1 - p_2}{q_1 - q_2} - \frac{p_2}{q_2}. \quad (1)$$

From (1), we also obtain

$$\pi_1 = ep_1 \left( 1 - \frac{p_1 - p_2}{q_1 - q_2} \right) - F,$$

$$\pi_2 = ep_2 \left( \frac{p_1 - p_2}{q_1 - q_2} - \frac{p_2}{q_2} \right).$$

Producers choose prices at the third stage. We consider their strategy in the next lemma.

## Lemma 2

(1) If  $0 \leq F < 4eq_1^2(q_1 - q_2)/(4q_1 - q_2)^2$ , then prices of producers are given by

$$p_1^a = \frac{2q_1(q_1 - q_2)}{4q_1 - q_2}, \quad (2)$$

$$p_2^a = \frac{q_2(q_1 - q_2)}{4q_1 - q_2}. \quad (3)$$

The profits of producers are

$$\pi_1^a(q_1, q_2) = \frac{4eq_1^2(q_1 - q_2)}{(4q_1 - q_2)^2} - F, \quad (4)$$

$$\pi_2^a(q_1, q_2) = \frac{eq_1q_2(q_1 - q_2)}{(4q_1 - q_2)^2}. \quad (5)$$

(2) If  $4eq_1^2(q_1 - q_2)/(4q_1 - q_2)^2 \leq F$ , then producer 1 does not develop the new technology and as a result producers set  $p_1^a = p_2^a = 0$ .

We now consider the optimal patent protection level against the software duplication. The government chooses the protection level  $e$  to maximize the social welfare, which is defined as the sum of producer surplus and the consumer surplus. If producer 1 develops the new technology, the social welfare function is given by The first term means the sum of producer surplus and consumer surplus from legal users. The second term represents the consumer surplus due to illegal uses.

$$\begin{aligned} SW^a(e) &= e \left( \int_{\frac{p_1^a - p_2^a}{q_1 - q_2}}^1 q_1 v dv + \int_{\frac{p_2^a}{q_2}}^{\frac{p_1^a - p_2^a}{q_1 - q_2}} q_2 v dv \right) + (1 - e) \int_0^1 q_1 v dv - F \\ &= \frac{eq_1(12q_1^2 - q_1q_2 - 2q_2^2)}{2(4q_1 - q_2)^2} + \frac{q_1(1 - e)}{2} - F \end{aligned} \quad (6)$$

If producer 1 does not develop the new technology, the social welfare is given by

$$\begin{aligned} SW^a(e) &= e \int_0^1 q_2 v dv + (1 - e) \int_0^1 q_2 v dv \\ &= \frac{q_2}{2} \end{aligned}$$

The next lemma shows how changes in the protection affect the social welfare.

### Lemma 3

(1) If  $0 \leq F < 4q_1^2(q_1 - q_2)/(4q_1 - q_2)^2$ , then

$$SW^a(e) = \frac{q_2}{2} \text{ for } 0 \leq e < \frac{F(4q_1 - q_2)^2}{4q_1^2(q_1 - q_2)},$$

$$\frac{\partial SW^a(e)}{\partial e} < 0 \text{ for } e \geq \frac{F(4q_1 - q_2)^2}{4q_1^2(q_1 - q_2)}.$$

(2) If  $4q_1^2(q_1 - q_2)/(4q_1 - q_2)^2 \leq F$ , then producer 1 does not develop the new technology

and as a result  $SW^a(e) = q_2/2$  for all  $e$ .

Implication of this lemma is clear. The social surplus is a decreasing function of the protection level  $e$  since the number consumers who use the software decreases as the protection increases. On the other hand, we can obtain that producer's profit is an increasing function of the protection from equation (4) and (5). Because the number of consumers who purchase the software increases as the protection increases. If the government sets the low protection  $e$ , producer may decide not to develop the new technology because he can not compensate his development cost. In such cases the social surplus will be  $q_2/2$  under the Bertrand competition. The next proposition shows that the optimal patent protection level  $e^a$  against the software duplication.

**Proposition 1**

*The optimal protection level  $e^a$  against the software duplication under the patent scheme is given by*

$$e^a = \frac{F(4q_1 - q_2)^2}{4q_1^2(q_1 - q_2)} \text{ for } 0 \leq F < \frac{4q_1^2(q_1 - q_2)}{(4q_1 - q_2)^2},$$

$$e^a \in [0, 1] \text{ for } F \geq \frac{4q_1^2(q_1 - q_2)}{(4q_1 - q_2)^2}.$$

Lemma 3 shows that government desires to set the protection level  $e$  as low as possible. Producer 1 may decide not to develop the new technology if the protection level is too low since his profit is an increasing function of  $e$ . Figure 2 shows this proposition. In the first case, setting the protection to zero will result in a negative profit for producer 1 with the development. The government sets  $e$  to give an enough incentive to development. The level of protection is set just high enough to result in a non-negative profit after the invention. In this case, producers set the prices as (2) and (3). In the second case, producer 1 will never develop the new technology because the development cost is too high. If producer 1 does not develop the new technology, consumers can use the software without any cost

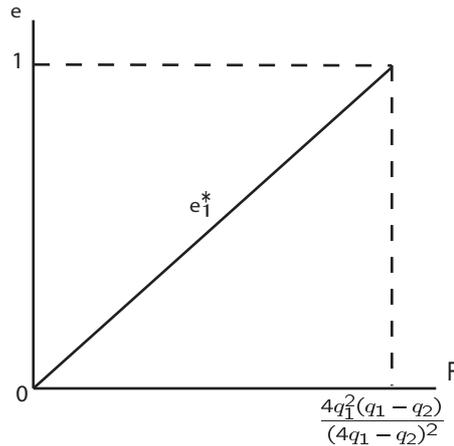


Figure 2: Patent protection against the software duplication

because producers set the zero price and play the Bertrand competition. In this case, the social welfare does not depend on the protection level against the software duplication since all software are provided with zero price. The protection against the software duplication increases as the development cost increases. Next section, we consider the optimal protection level against the software duplication when government applies copyright protection scheme.

### 3 Copyright Protection in the Software Market

We consider how the reverse engineering affects the protection level  $e$  and social welfare since copyright can not prevent the reverse engineering. When producer 1 develops the innovative technology  $\delta$ , producer 2 can decide whether or not to steal that technology by reading source code. We assume that the cost of reverse engineering is zero. The timing of the game is changed as follows.

1. Government sets  $e$  to maximize social welfare.
2. Producer 1 decides whether or not to develop the new technology  $\delta$  at a fixed cost

$F > 0$ . If producer 1 decides to develop, producer 2 chooses his quality to  $q_2 + \gamma$   $0 \leq \gamma \leq \delta$  by reverse engineering.

3. Producers choose prices  $p_j$  simultaneously.
4. Legal consumers decide whether they will purchase the software from producer 1, producer 2, or do nothing. Illegal users make copies of producer 1's software.

In this section, producer 2 can increase his software quality so as to maximize his profit by the reverse engineering. We have to consider how producer 2 applies the new technology. Next lemma shows how changes producers' software quality affect producers surplus.

#### **Lemma 4**

*If producer 1 develops the new technology  $\delta$ , producer 2 decides his strategy as follows;*

- (1) *Producer 2 chooses  $\gamma = (4\delta - 3q_2)/7$  when  $\delta > 3q_2/4$ .*
- (2) *Producer 2 does not improve his quality when  $\delta \leq 3q_2/4$ .*

The relationship between producer 2's quality and his profit depends on the degree of  $\delta$ . When  $\delta$  is large, producer 2 may copy the new technology. Producer 2 can increase his software price and obtain higher profit by the reverse engineering. On the other hand, when the degree of the innovation is small, producer 2 does not have an incentive to steal the technology. In this case, the profit of producer 2 decreases by the reverse engineering because of severe price competition. In this section, we focus on the case  $\delta > 3q_2/4$ . If  $\delta$  is small, the outcomes are same as we discussed in section 2. We can obtain the software producers' strategies in the next lemma.

#### **Lemma 5**

(1) If  $0 \leq F < 7e(q_2 + \delta)/48$ , then the prices of producers are given by

$$p_1^c = \frac{(q_2 + \delta)}{4},$$

$$p_2^c = \frac{(q_2 + \delta)}{14}.$$

The profits of producers are

$$\pi_1^c(q_1, q_2 + \gamma) = \frac{7e(q_2 + \delta)}{48} - F,$$

$$\pi_2^c(q_1, q_2 + \gamma) = \frac{e(q_2 + \delta)}{48}.$$

(2) If  $7e(q_2 + \delta)/48 \leq F$ , then producers set  $p_1^c = p_2^c = 0$ .

The interpretation of this lemma is clear. In the first case, producer 1 develops the new technology and producer 2 copies that to maximize his own profit, because the degree of innovation is large and the development cost is low. When  $7e(q_2 + \delta)/48 \leq F < 4eq_1^2(q_1 - q_2)/(4q_1 - q_2)^2$ , the development cost  $F$  is so large that producer 1 can not obtain an enough incentive to develop the new technology because producer 2's copy decreases producer 1's profit. When  $F > 4eq_1^2(q_1 - q_2)/(4q_1 - q_2)^2$ , producer 1 does not develop the new technology since the development cost is too high. Consequently, producer 1 does not develop the new technology when  $F \geq 7e(q_2 + \delta)/48$ .

When the new technology is innovative enough ( $\delta > 3q_2/4$ ) and the development cost is small ( $0 \leq F < 7e(q_2 + \delta)/48$ ), the social welfare function  $SW^c(e)$  is as follows:

$$\begin{aligned} SW^c(e) &= e \left( \int_{\frac{p_1^c - p_2^c}{q_1 - q_2}}^1 q_1 v dv + \int_{\frac{p_2^c}{q_2}}^{\frac{p_1^c - p_2^c}{q_1 - q_2}} \frac{4(q_2 + \delta)}{7} v dv \right) + (1 - e) \int_0^1 q_1 v dv - F \\ &= \frac{(q_2 + \delta)(12 - e)}{24} - F \end{aligned} \quad (7)$$

The first term means the sum of the consumer surplus from legal buyer and producer surplus. The second term means consumer surplus from illegal uses. In these cases, both producer's qualities are increased by the new technology. The following lemma shows the impact of the protection against the software duplication on the social welfare when there exists the reverse engineering in the market.

**Lemma 6**

(1) If  $0 \leq F < 7(q_2 + \delta)/48$ , then

$$SW^c(e) = \frac{q_2}{2} \text{ for } 0 \leq e < \frac{48F}{7(q_2 + \delta)},$$

$$\frac{\partial SW^c(e)}{\partial e} < 0 \text{ for } \frac{48F}{7(q_2 + \delta)} \leq e.$$

(2) If  $7(q_2 + \delta)/48 \leq F$ , then  $SW^c(e) = q_2/2$  for all  $e$ .

This lemma can be interpreted in the same manner as Lemma 4. The following proposition discusses the optimal protection level against the software duplication  $e_2^*$ .

**Proposition 2**

When  $\delta > 3q_2/4$ , the optimal protection level  $e^c$  is given by

$$e^c = \frac{48F}{7(q_2 + \delta)} \text{ for } 0 \leq F < \frac{7(q_2 + \delta)}{48},$$

$$e^c \in [0, 1] \text{ for } F \geq \frac{7(q_2 + \delta)}{48},$$

This result can be interpreted in the same manner as Proposition 1. Figure 3 shows this proposition. The government wants to set the protection as low as possible to maximize the social surplus. However, the government has to set a high enough protection to prevent

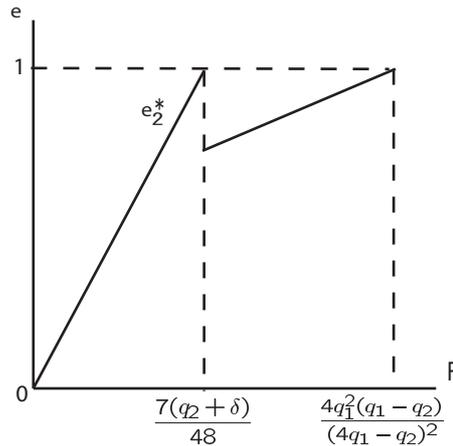


Figure 3: Copyright protection against the software duplication

producer 1's profit from being negative. In the first case, producer 2 applies the new technology to maximize his profit. The government takes into account producer 2's copy to set the protection. In the second case, producer will never develop the new technology because of high development cost and producer 2's copy. When the degree of innovation is not large, producer 2 does not have an incentive to copy the new technology. In this case, the optimal protection level is the same as Proposition 1.

## 4 Patent protection vs. Copyright protection

Thus far, we have considered two intellectual property right protection schemes, one that does not consider producer's reverse engineering and the other that does. The government can decide protection level against the software duplication to maximize the social welfare. However, government also has the option of preventing the reverse engineering by producer 2. As I discussed before, software is prevented by both patent and copyright. Patent can prevent the reverse engineering, however, copyright can not. In this section, we consider which protection scheme is better from the viewpoint of society: copyright protection or patent protection. The next proposition compares the protection levels against the software

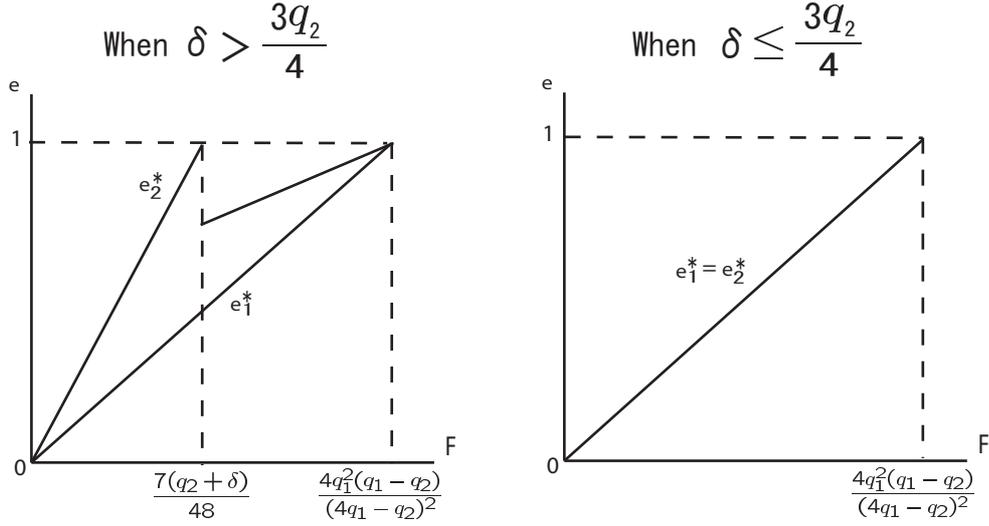


Figure 4: Protection against the software duplication

duplication.

### Proposition 3

*When  $F < 7(q_2 + \delta)/48$ , the number of copy users in the market under the copyright scheme is smaller than that under the patent scheme.*

The intuition of this proposition is clear. When the degree of innovation is not large, the protection level against the software duplication is same in the patent and the copyright because producer 2 does not copy. When the degree of innovation is large and the development cost is small ( $F < 7(q_2 + \delta)/48$ ), the protection levels differ (Figure 4). If the government applies the copyright protection scheme, producer 1's profit may decrease by producer 2's reverse engineering. In this case, government has to set higher protection against the software duplication to give an enough incentive to producer 1. If the development cost is not small ( $7(q_2 + \delta)/48 \leq F$ ), the government can not compensate producer 1's development cost under the copyright protection scheme. The next proposition shows that how we should protect the software.

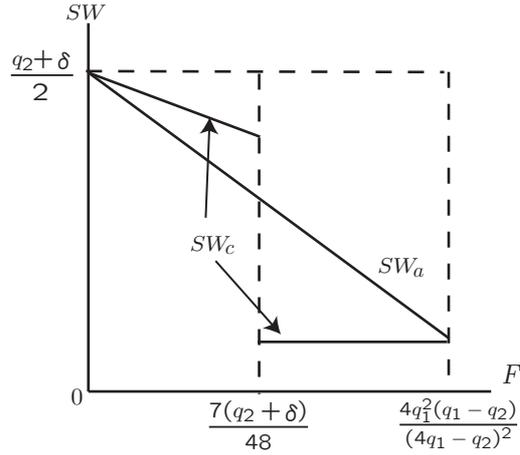


Figure 5: Patent vs. Copyright Protection

**Proposition 4**

*From viewpoint of society, government should apply the copyright protection scheme in the software market when  $F < 7(q_2 + \delta)/48$ . Otherwise, the government should apply the patent protection scheme in the software market.*

Figure 5 compares the social welfare under the both scheme. When the degree of innovation is small, there are no differences between the two schemes, since producer 2 does not steal the new technology. When the new technology is innovative enough and the development cost is low, government can increase all softwares' quality by relaxing the protection to the software duplication. In addition to that, if the government applies the patent protection, producer 1's market power becomes very strong. This is not desirable from viewpoint of social benefit. However, producer 1 does not obtain an enough profit to develop the new technology under the copyright protection scheme when  $F \geq 7(q_2 + \delta)/48$ . In this case, the governments should prevent producer 2's copy by the patent protection scheme. This proposition also shows that the relationship between the protection scheme and the idea. If we define the idea as the combination of the degree of innovation  $\delta$  and the development cost  $F$ , we can show the optimal protection scheme under the idea  $(\delta, F)$  as Figure 6. Figure 6 shows that the innovative idea, for example  $\delta$  is large and  $F$  is small, should be protected by

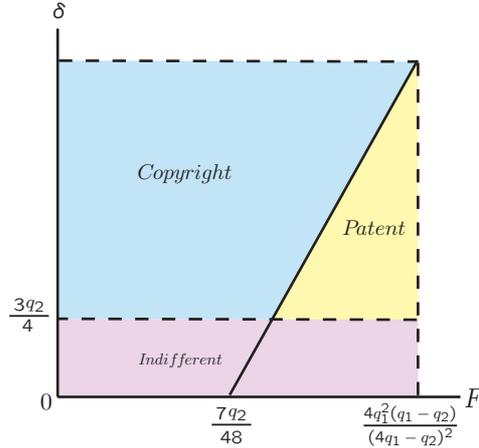


Figure 6: Optimal Protection Scheme

the copyright scheme. Proposition 4 argues that the effect of improving producer's quality and its subsequent copying on the protection. In the U.S.A., there are debates over whether software should be protected by the patent or not. In the software market, the quality of software is improved step by step with update. If the original technology  $q_2$  is large and the degree of innovation  $\delta$  is not large, the range of the development cost where the patent protection scheme is socially desirable becomes small. Therefore, this proposition posits that the government should not set the patent to protect the software, but the government should provide a more stringent protection against the software duplication than it does for other copyright products.

## 5 Conclusion

We have tried to consider that the software should be protected by the patent or not. To discuss this problem, we need to take into account the differences between the patent protection and the copyright protection because the software is protected by both intellectual property rights. In this paper, patent protection and copyright protection is simply distinguished by changing the player who copies. We discussed the intellectual property

right protection scheme under a model wherein (a) the government controls a protection level against the software duplication and that against reverse engineering to maximize social welfare and (b) software company can develop a new technology by incurring certain costs. We obtain the following results.

First, the protection level against the software duplication under the patent scheme is smaller than that under the copyright scheme when the development cost is not large. If there exists the reverse engineering in the market, the protection against the software duplication becomes high. Consequently, the number of copy users under the copyright protection scheme becomes smaller than that under the patent protection scheme. Second, we show the optimal intellectual property right protection scheme in the software market. When the degree of innovation is small, there are no differences between the two schemes because the rival producer does not steal that technology. When the new technology is innovative enough and the development cost is low, government can increase social surplus by adopting the copyright protection to the software duplication. On the other hand, the government has to apply the patent protection scheme when the development cost is high because producer 1 can not obtain enough profit to develop under the copyright protection scheme. We show that the effect of improving producer's quality and its subsequent copying on the protection. Our analysis suggests that changes should be made with regard to the direction of modern copyright and patent policy. We indicated that the importance of copyright protection in the software market.

We close this paper by pointing out some extension of this model. First, we have assumed that the government can prevent producer 2's copy perfectly under the patent protection scheme, and the government does not prevent reverse engineering at all under the copyright protection scheme. However, the government can control the degree of the reverse engineering by setting the software patent breadth in reality. If we consider the case that the government can set the upper limit of the reverse engineering  $\bar{\gamma}$ , we can discuss the optimal intellectual property right protection scheme in the software market. We can obtain the similar result

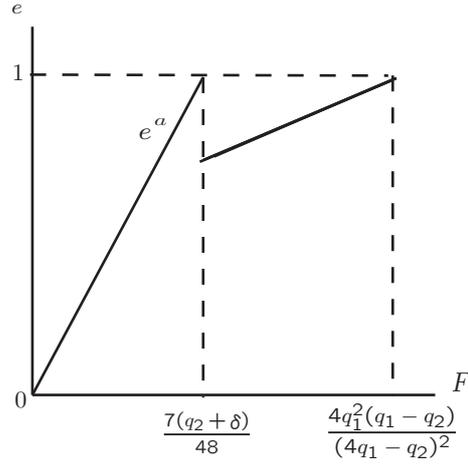


Figure 7: Optimal protection against the software duplication

under this setting. When the development cost is low, the copyright is socially desirable than the patent protection in the software market. The optimal protection level against the software duplication in this case is given by Figure 7. The intuition of this figure is clear. When the development cost is high, the government can decrease the degree of the reverse engineering not to prevent producer 1's development. Consequently, the protection level against the software duplication becomes lower than the first case.

Second, in this paper, government can control the protection level  $e$  directly. However, government controls this protection level through other policies such as the penalty for copyright infringement. It would be interesting to endogenize  $e$ . Third, producer 1 has another option to have an exclusive license for the producer 2. The patent protection may be better than the copyright protection from view point of society. In this case, government does not have to set high protection against the software duplication because producer 1 can obtain the license fee from producer 2. Forth, we assume that the cost of reverse engineering is zero in this model. It is difficult to acquire all information by the reverse engineering in reality. The rival producer has to pay some costs to do that. Considering this cost, the range that the patent protection and copyright protection is indifferent from viewpoint of society becomes large.

In addition to that, we try to capture specific properties of software; however, it is not enough. As Scotchmer (1991) says, cumulative innovation is very important topic to consider the software. For example, software's quality improves sequentially by updating. Ellison and Fudenberg (2000) argues the monopolist's incentives to provide upgraded versions of the software. The arguments presented in the patent protection for sequential innovation (Scotchmer and Green, 1990; O'Donoghue, 1998; O'Donoghue, Scotchmer, and Thisse, 1998; Bessen and Maskin; 2000) are helpful to consider that problems. These extensions will be the basis for future research. We also assume that the quality without the innovation is common to both producers. However, there are differences in technical capabilities among software producers. If we assume that one producer's software quality without the innovation is higher than that of the other producer's, we can obtain the similar result of this paper.

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## 6 Appendix

### Proof of Lemma 1

In this lemma, we consider the optimal consumer behavior. In the first case, consumers choose not to consume the product because the utility of buying is negative. We obtain the equations

$$0 > q_1 v_i - p_1, 0 > q_2 v_i - p_2.$$

In the second case, when consumers use producer 2's software, they obtain a higher utility than when buying producer 1's software or when not using it at all. We therefore obtain the equations

$$q_2 v_i - p_2 > q_1 v_i - p_1, q_2 v_i - p_2 \geq 0.$$

In the third case, when consumers use producer 1's software, they obtain a higher utility than when consuming producer 2's software or not using it at all. We therefore obtain the equations

$$q_1 v_i - p_1 \geq q_2 v_i - p_2, q_1 v_i - p_1 \geq 0.$$

The lemma follows from these equations. Q.E.D.

### Proof of Lemma 2

We define the producers' strategy as  $S = \{(p_1, p_2) \mid p_1 \geq 0, p_2 \geq 0\}$ , where  $p_j (j = 1, 2)$  is the retail price of software. For convenience of analysis, we divide the strategy space  $S$  into two sub-classes:  $S_1 = \{(p_1, p_2) \mid p_1 q_2 \geq p_2 q_1\}$  and  $S_2 = \{(p_1, p_2) \mid p_1 q_2 < p_2 q_1\}$ .

When producers employ strategies in sub-class  $S_1$ , the consumer behavior illustrated in Figure 1. Consumers with valuations larger than  $(p_1 - p_2)/(q_1 - q_2)$  purchase producer 1's software; those with valuations between  $p_2/q_2$  and  $(p_1 - p_2)/(q_1 - q_2)$  buy the software from

producer 2 and those with valuations less than  $p_2/q_2$  do not consume. The legal users' demand for producer 1's software  $D_1$  and the demand for producer 2's software  $D_2$  when  $p_1q_2 > p_2q_1$  are thus given by

$$D_1 = 1 - \frac{p_1 - p_2}{q_1 - q_2}, \quad D_2 = \frac{p_1 - p_2}{q_1 - q_2} - \frac{p_2}{q_2}.$$

From these equations, we also obtain

$$\begin{aligned} \pi_1 &= ep_1 \left(1 - \frac{p_1 - p_2}{q_1 - q_2}\right). \\ \pi_2 &= ep_2 \left(\frac{p_1 - p_2}{q_1 - q_2} - \frac{p_2}{q_2}\right) \end{aligned}$$

Producers decide price to maximize their profit in sub-class  $S_1$  simultaneously. The equilibrium strategies in sub-class  $S_1$  are given by

$$\begin{aligned} p_1^* &= \frac{2q_1(q_1 - q_2)}{4q_1 - q_2}. \\ p_2^* &= \frac{q_2(q_1 - q_2)}{4q_1 - q_2} \end{aligned}$$

The profits are thus given by

$$\begin{aligned} \pi_1^*(q_1, q_2) &= \frac{4eq_1^2(q_1 - q_2)}{(4q_1 - q_2)^2}. \\ \pi_2^*(q_1, q_2) &= \frac{eq_1q_2(q_1 - q_2)}{(4q_1 - q_2)^2}. \end{aligned}$$

When producers employ strategies in sub-class  $S_2$ , producer 2's payoff becomes zero since there are no consumers who purchase producer 2's software in the market. In this class, producer 2 has an incentive to decrease his prices to the level that satisfy  $p_1q_2 \geq p_2q_1$ .

Consequently, the strategies in sub-class  $S_1$  become the equilibrium outcomes over the whole strategy space  $S$ . Q.E.D.

### Proof of Lemma 3

The social welfare when the new technology are developed by producer 1 is given by

$$SW^a(e) = \frac{eq_1(12q_1^2 - q_1q_2 - 2q_2^2)}{2(4q_1 - q_2)^2} + \frac{q_1(1 - e)}{2} - F$$

We thus obtain

$$\frac{\partial SW^a}{\partial e} = -\frac{q_1(q_1 - q_2)(4q_1 - 3q_2)}{2(4q_1 - q_2)^2} < 0$$

The social welfare is a decrease function of the protection  $e$  when the new technology is developed. The technology will not be developed, however, if the profit is negative. The producer's profit depends on the degree of  $\delta$  and  $F$ , and is an increasing function of  $e$ .

In the case of  $0 \leq F < 4q_1^2(q_1 - q_2)/(4q_1 - q_2)^2$ , if the protection is so low that the producer's profit is negative, producer 1 will not develop the new technology and play the Bertrand competition. In the last case, the development cost is larger than the maximum profit of the producer. In this case, the new technology will not be developed for any  $e$ . Q.E.D.

### Proof of Proposition 1

From Lemma 3, the social welfare is a decreasing function of the protection  $e$  if the new technology is developed. In the first case, the protection should be chosen at the minimum level that provides an incentive for the producer to develop, since the new technology is socially desirable in this range of the development cost. In the second case, the producer can not develop the technology for any  $e$ . The government's optimal penalty is therefore unconstrained. Q.E.D.

## Proof of Lemma 4

Producer 2's profit when producer 1 develops the new technology is given by

$$\pi_2^*(q_1, q_2) = \frac{eq_1q_2(q_1 - q_2)}{(4q_1 - q_2)^2}.$$

We can obtain

$$\frac{\partial \pi_2}{\partial q_2} = \frac{eq_1^2(4q_1 - 7q_2)}{(4q_1 - q_2)^3}$$

Producer 2's profit is maximized when  $4q_1 = 7q_2$ . Producer 2 can increase his profit by the reverse engineering when  $\delta$  is larger than  $3q_2/4$ . When  $\delta$  is larger than the limit, producer 2 chooses  $\gamma$  to be  $4(q_2 + \delta) = 7(q_2 + \gamma)$ . Therefore, producer 2 chooses  $\gamma = (4\delta - 3q_2)/7$  to maximize his profit. Q.E.D.

## Proof of Lemma 5

From Lemma 4, producer 2 improves his quality to  $4q_1/7$  when producer 1 develops the technology. We can obtain the following result by substituting  $q_2 = 4q_1/7$  into the results of Lemma 2.

$$p_1^c = \frac{(q_2 + \delta)}{4},$$
$$p_2^c = \frac{(q_2 + \delta)}{14}.$$

Under these strategies, the profits of producers are

$$\pi_1^c \left( q_1, \frac{4(q_2 + \delta)}{7} \right) = \frac{7e(q_2 + \delta)}{48} - F,$$

$$\pi_2^c \left( q_1, \frac{4(q_2 + \delta)}{7} \right) = \frac{e(q_2 + \delta)}{48}.$$

In the second case, producer 1 will not develop the technology because of too high development cost. Q.E.D.

### Proof of Lemma 6

When the new technology is innovative enough ( $\delta > 3q_2/4$ ) and the development cost is smaller than producer 1's profit ( $0 \leq F < 7e(q_2 + \delta)/48$ ), the social welfare when producer 2 steals the new technology that is developed by producer 1 is given by

$$SW^c(e) = \frac{(q_2 + \delta)(12 - e)}{24} - F$$

We obtain

$$\frac{\partial SW^c}{\partial e} = -\frac{(q_2 + \delta)}{24} < 0$$

The social welfare is thus a decreasing function of  $e$  when the technology is developed by producer 1. Producer 1 will not develop the new technology if his profit is negative. The profit depends on the magnitude of  $\delta$  and  $F$ , and is an increasing function of  $e$ . In the first case, producer 1 does not develop the new technology because  $e$  is too small to compensate the development cost with producer 2's copy. When the development cost is large ( $F \geq 7(q_2 + \delta)/48$ ), producer 1 does not develop the new technology because his profit becomes negative even if  $e = 1$ . Therefore, producers play the Bertrand competition. Q.E.D.

### Proof of Proposition 2

From Lemma 6, the social surplus is a decreasing function of  $e$  when the technology is developed. In the first and second cases, the protection is chosen at the minimum level that provides incentive for producer to work. In the last case, the producer can not afford to develop the technology for any  $e$ . The government optimal protection is therefore unconstrained. Q.E.D.

### Proof of Proposition 3

We compare the optimal protection level  $e^a$  and  $e^c$ .

When  $0 \leq F < 7(q_2 + \delta)/48$

$$e^a - e^c = \frac{F(4q_1 - q_2)^2}{4q_1^2(\delta - 1)(q_1 - q_2)} - \frac{48F}{7(q_2 + \delta)} = -\frac{F(21q_2 + 20\delta)(4\delta - 3q_2)}{28(q_2 + \delta)^2\delta} \quad (8)$$

This equation becomes negative when  $\delta > 3q_2/4$ .

From equation (8), the protection level against the software duplication under the copyright protection is more severe than that under the patent protection when the development cost is low. Q.E.D.

#### Proof of Proposition 4

We compare the social welfare under each penalty scheme. We obtain the social surplus when there does not exist the reverse engineering in the market by substituting  $e^a$  into equation (6):

$$SW^a = \frac{4(q_2 + \delta)^2 - F(9q_2 + 12\delta)}{8(q_2 + \delta)} \text{ for } 0 \leq F < \frac{4q_1^2(q_1 - q_2)}{(4q_1 - q_2)^2}, \quad (9)$$

$$SW^a = \frac{q_2}{2} \text{ for } F \geq \frac{4q_1^2(q_1 - q_2)}{(4q_1 - q_2)^2},$$

We then obtain the social welfare when there exists the reverse engineering by substituting  $e^c$  into equation (7):

$$SW^c = \frac{7(q_2 + \delta) - 18F}{14} \text{ for } 0 \leq F < \frac{7(q_2 + \delta)}{48}, \quad (10)$$

$$SW^c = \frac{q_2}{2} \text{ for } F \geq \frac{7(q_2 + \delta)}{48}, \quad (11)$$

We compare equations (9) and (10).

$$\frac{4(q_2 + \delta)^2 - F(9q_2 + 12\delta)}{8(q_2 + \delta)} - \frac{7(q_2 + \delta) - 18F}{14} = -\frac{3F(4\delta - 3q_2)}{36(q_2 + \delta)} \quad (12)$$

This equation becomes negative when  $\delta > 3q_2/4$ .

We compare equations (9) and (11).

$$\frac{4(q_2 + \delta)^2 - F(9q_2 + 12\delta)}{8(q_2 + \delta)} - \frac{q_2}{2} > 0 \quad (13)$$

Therefore, the social welfare under the copyright protection when  $F$  is large is smaller than that under the patent protection. Q.E.D.