Modelling and forecasting realised volatility in the German-Austrian continuous intraday electricity market

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Summary

- Use of high-frequency EPEX continuous electricity market prices to estimate and forecast realised volatility.
- Realised volatility, RV, is a measure of volatility of a time series of length T, during a certain period $t \in T$, for equally spaced observations, M (as $M \to \infty$ then RV converges to IV in probability)
- Estimation of different specifications of Heterogeneous Autoregressive RV models proposed by Corsi (2009).
  - RV decomposed into continuous and jump components.
  - GARCH structures are considered in the innovations.
- Accuracy of the models is measured according to out-of-sample forecast criteria.

Structure of the Seminar

- Motivation
- Aim of the study
- Related literature
- Market design
- Realised volatility
- HAR modellling approach
- Test statistics for jump detection
- Results:
  - RV: continuous, CV, and jump, RV, decomposition.
  - Estimation results
  - Model selection
- Conclusions and further research.

- EPEX includes electricity markets from:
  - France, Germany, Austria and Switzerland.
  - Market coupling with neighbouring markets: APX, NordPool, GME, MIBEL.
- Day-ahead auction:
  - Uniform-price auction mechanism.
  - Beginning April 22, 2005.
  - A single system marginal price is set from the intersection of demand and supply bids.
  - Prices must be between -500 €/MWh and 3000 €/MWh.
  - Hourly settlement, 24 price-quantity pairs, at day $d-1$ to deliver at day d.
  - Takes place at 12.00, 7 days a week.

- Intraday continuous:
  - Pay-as-bid matching algorithm. Anonymous execution of matching orders.
  - Beginning September 25, 2006.
  - Prices must be between -9999.99 and 9999.99.
  - Starts at 15:00.
  - Quarter hour settlement 30 minutes before delivery: 96 (average) price-quantity pairs.
- Intraday auction:
  - Takes place at 15.00, 7 days a week
  - Beginning December 105, 2014.
  - Prices must be between -3000 €/MWh and 3000 €/MWh

Motivation

Prices

- Stylized facts of the prices:
  - Seasonality: hourly, daily, weekly, monthly, quarterly and yearly
  - Mean-reversion
  - High volatility persistence
  - Jumps and short-lived peaks
  - Stationarity.
- Sample ranges from $11 - 18 - 2012$ until $04 - 30 - 2016$. Overall there are $120,596$ quarter-hours.
- Negative prices in $3,805$ quarter hours (3.15%).

Motivation

Approaches to study volatility of electricity prices

- Jump-difusion models: Bierbrauer et al., 2007, Knittel and Roberts, 2005.
- Markov regime-switching models: Huisman and Mahieu, 2003.
- GARCH-type models: Bystrom, 2003, Higgs and Worthington, 2005, Ciarreta and Zarraga, 2015.
- RV models: Chang et al., 2008, Ullrich, 2012, Frommel at al., 2014, Ciarreta and Zarraga, 2016.

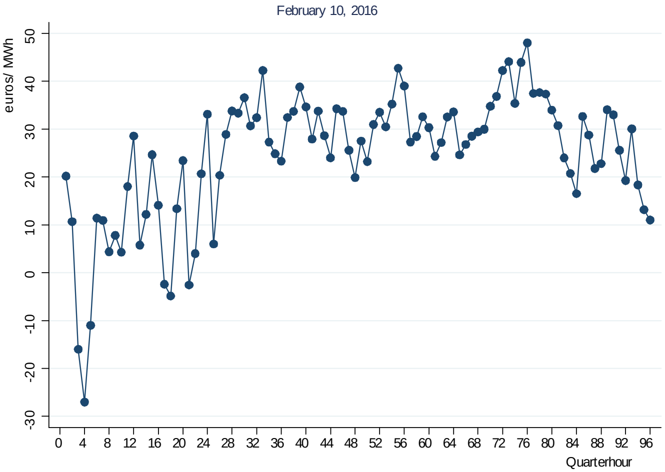
- Estimate different econometric models for the RV obtained from close-to-delivery electricity prices.
- In particular continuous intraday 15-minute blocks of the German-Austrian market.
- These intraday markets are usually small in relation to the spot markets but important to fit buying/selling positions.
- RV is decomposed into jump (JV) and continuous component (CV) using three robust-to-jumps tests:
  - Barndorff-Nielsen and Shephard, 2004: BNS.
  - Corsi, Pirino and Reno, 2010: CPR.
  - Andersen, Dobrev and Schaumburg, 2012: ADS.

- Using RV and (CV, JV) estimate several Heterogeneous Autoregressive Realised Volatility Models:
  - HAR-RV
  - HAR-CV-JV with BNS, CPR and ADS decomposition
  - HAR-RV-GARCH
  - HAR-CV-JV-GARCH with BNS, CPR and ADS decomposition
  - HAR-RV-EGARCH
  - HAR-CV-JV-EGARCH with BNS, CPR and ADS decomposition
- Accuracy using out-of-sample criteria:
  - Mean Absolue Error (MAE)
  - Root Mean Square Error (RMSE)
  - Mean Absolute Percentage Error (MAPE)
  - Rolling MAE, RMSE, MAPE.

Related literature

- Chan et. al (2008): HAR-RV and HAR-CV-JV models in five power markets in **Australia**.
- Frommel et al. (2014): Realized GARCH-type models to estimate the daily price volatility in the **EPEX**.
- Haugom and Ullrich (2012): HAR-RV and HAR-CV-JV on spot and day-ahead forward prices from **Pennsylvania-New Jersey-Maryland** wholesale electricity market.
- Haugom et al. (2011): HAR-RV-EX and HAR-CV-JV-EX models to predict volatility in the **Nord Pool** electricity forward market.
- Ullrich (2012): Estimate RV and the frequency of price jumps using spot price data from **Australia**, **Canada** and the **US**.
- Ciarreta and Zarraga (2016): Simultaneous HAR-type models for the RV from prices of the six intraday sessions of **MIBEL**.

Data: EPEX intraday-continuous



Data

Descriptive statistics of prices

	Spr.	Aut.	Wd.	Su.	All
Min	-117.06	-211.84	-188.91	-135.69	-211.84
Max	240.99	250.00	174.18	140.16	250.00
Mean	30.86	35.83	38.03	22.53	33.44
Median	29.67	35.53	36.01	23.58	32.38
St. Dev.	19.63	22.96	21.06	19.21	20.52
Skewness	0.72^a	-0.66^a	-0.23^a	-0.75^a	0.16^a
Kurtosis (Ex)	7.35^a	7.99^a	9.65^a	3.37^a	6.21^a
Jarque-Bera	67912.6^a	83820.3^a	67265.7^a	9780.7^a	194852.2^a

Realised volatility

- Compute RV as a measure of the unknown volatility using 15-minute prices.
- "Returns" are defined as price differences. $r_{t,j} = p_{t,j} - p_{t,j-1}$.
  - 0.15% of $r_{t,j}$ are zero.
- "Adjusted returns" are $r^*_{t,j} = r_{t,j} - \widehat{r}_{m,d,q}$ where $\widehat{r}_{m,d,q}$ is the median of the month m, day of the week d, and quarter-hour q.
  - 2.89% of $r^*_{t,j}$ are zero.
- Realised volatility: $RV_t = \sum_{j=1}^{96} \left(r^*_{t,j} \right)^2$
- RV_t is decomposed into a continuous, CV_t, and jump, JV_t, components such that $RV_t = CV_t + JV_t$

Test statistics for jump detection

BNS (2006):

Construct the bipower variation (BNS, 2004) as

$$BV_t = 1.57 \frac{M}{M-1} \sum_{j=2}^{M} |r_j| \, |r_{j-1}|$$

It is a consistent estimator of the IV in the absence of jumps.
Test statistic (Huang & Tauchen, 2005):

$$\sqrt{M} \frac{(RV_t - BV_t) / RV_t}{\sqrt{0.61 \max\left(1, TQ_t / BV_t^2\right)}} \sim N(0,1)$$

where TQ_t is the tripower quarticity,

$$TQ_t = 1.74 \frac{M^2}{M-2} \sum_{j=3}^{M} \left(|r_j| \, |r_{j-1}| \, |r_{j-2}|\right)^{4/3}$$

Test statistics for jump detection
CPR (2010)

Consistent and nearly unbiased estimator of IV: Threshold BV:

$$
\begin{aligned}
CTBPV_t &= 1.57 \sum_{j=2}^{M} Z_1\left(r_j, v_j\right) Z_1\left(r_{j-1}, v_{j-1}\right) \\
\text{where } Z_1\left(r_j, v_j\right) &= \left\{ \begin{array}{cc} |r_j| & \text{if} \quad r_j^2 \le c_v^2 \widehat{V}_j \\ 1.094 v_j^{1/2} & \text{if} \quad r_j^2 > c_v^2 \widehat{V}_j \end{array} \right| ,
\end{aligned}
$$

$\widehat{V}_j$ is a local volatility estimate based on an iterative process using a kernel specification. Test:

$$
\sqrt{M} \frac{\left(RV_t - CTBPV_t\right) / RV_t}{\sqrt{0.61 \max\left(1, CTTriPV_t / CTBPV_t^2\right)}} \sim N(0,1)
$$

$$
CTTriPV_t = 1.74 M \sum_{j=3}^{M} \prod_{k=1}^{3} Z_{4/3}\left(r_{j-k+1}, v_{j-k+1}\right)
$$

Test statistics for jump detection
ADS (2012)

Jump-robust estimator of IV using nearest neighbor truncation estimator:

$$MedRV_t = 1.42 \frac{M}{M-2} \sum_{j=2}^{M-1} med\left(|r_{j-1}|\,|r_j|\,|r_{j+1}|\right)^2$$

Test statistic (Huang and Tauchen, 2005):

$$\sqrt{M}\frac{\left(RV_t - MedRV_t\right)/RV_t}{\sqrt{0.96\max\left(1, MedRQ_t / MedRV_t^2\right)}} \sim N(0,1)$$

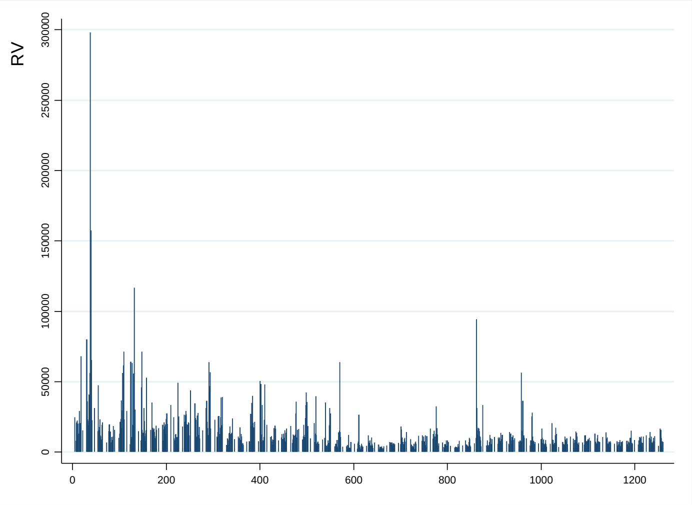
$$MedRQ_t = 0.92 \frac{M^2}{M-2} \sum_{j=2}^{M-1} med\left(|r_{j-1}|\,|r_j|\,|r_{j+1}|\right)^4$$

Test statistics for jump detection: Comments

- BNS:
  - Upward biased in the presence of jumps $\Longrightarrow$ Jump component underestimated.
  - Highly affected by zero returns $\Longrightarrow$ Jump component overestimated.
- CPR:
  - Affected by the same problems than BNS.
  - Detects more jumps than BNS since it imposes a threshold on BNS
- ADS:
  - Sensitive to the presence of adjacent zero returns.

Realised Volatility

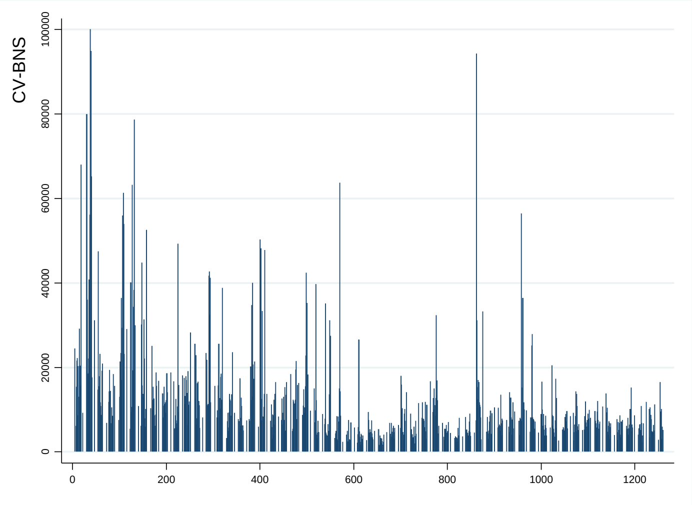
Time series



RV:
Descriptive statistics

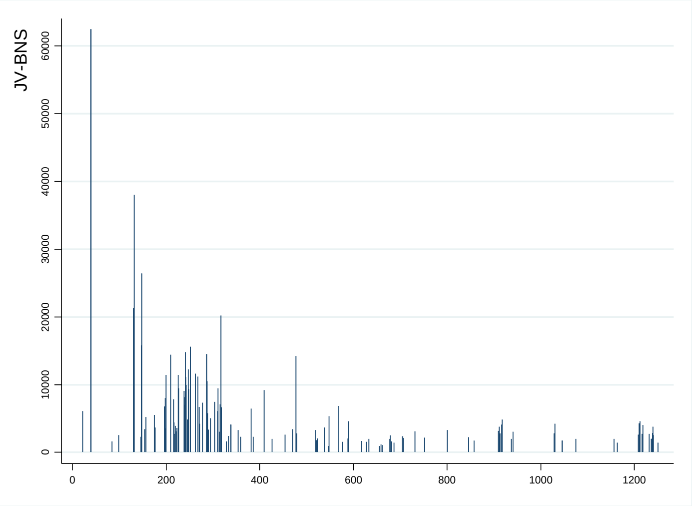
	RV	$\sqrt{\text{RV}}$	**logRV**
Min	1496.00	38.68	7.31
Max	297913.72	545.81	12.60
Mean	13626.28	107.18	9.20
Median	9264.61	96.25	9.13
St. Dev.	16065.86	46.27	0.73
Skewness	7.01[a]	2.38[a]	0.58[a]
Kurtosis (Ex)	90.51[a]	10.90[a]	0.67[a]
Jarque-Bera	440356.40[a]	7420.86[a]	94.14[a]

CV


CV-BNS
0
20000
40000
60000
80000
100000
0
200
400
600
800
1000
1200

	BNS	CPR	Med
Min	1411.30	1397.26	1496.00
Max	297913.72	297913.72	297913.72
Mean	12590.42	10566.52	13611.46
Median	8612.94	7766.97	9205.58
St. Dev.	14791.38	12026.14	16070.91
Skewness	7.71[a]	12.11[a]	7.01[a]
Kurtosis (Ex)	116.85[a]	262.55[a]	90.41[a]
Jarque-Bera	729295.69[a]	3649823.21[a]	439477.03[a]

JV



JV-BNS
60000
50000
40000
30000
20000
10000
0
0
200
400
600
800
1000
1200

	BNS	CPR	Med
Min	794.78	574.81	1046.39
Max	62408.37	88202.70	4622.52
Mean	6098.97	7965.49	3112.90
Median	3699.52	4413.30	3650.04
St. Dev.	7724.16	10546.16	1531.10
Skewness	4.30[a]	3.87[a]	-0.58
Kurtosis (Ex)	23.34[a]	19.64[a]	-1.96
Jarque-Bera	5513.96[a]	8986.13[a]	1.30
Jump days (%)	16.98	38.41	4.80

HAR-RV model

$$\begin{aligned}\sqrt{RV_t} &= \beta_0 + \beta_1 \sqrt{RV_{t-1}} + \beta_2 \sqrt{RV_{w,t-1}} + \beta_3 \sqrt{RV_{m,t-1}} + a_t \\ \log RV_t &= \beta_0 + \beta_1 \log RV_{t-1} + \beta_2 \log RV_{w,t-1} + \beta_3 \log RV_{m,t-1} + a_t\end{aligned}$$

where

$$RV_{w,t} = \frac{1}{7} \sum_{l=1}^{7} RV_{t-l} \text{ and } RV_{m,t} = \frac{1}{30} \sum_{l=1}^{30} RV_{t-l}$$

OLS estimation with heteroscedasticity and autocorrelated consistent standard errors (volatility clustering).

HAR-CV-JV model

$$
\begin{aligned}
\sqrt{RV_t} &= \lambda_0 + \lambda_1 \sqrt{CV_{t-1}} + \lambda_2 \sqrt{CV_{w,t-1}} + \lambda_3 \sqrt{CV_{m,t-1}} \\
&\quad + \theta_1 \sqrt{JV_{t-1}} + \theta_2 \sqrt{JV_{w,t-1}} + \theta_3 \sqrt{JV_{m,t-1}} + a_t \\
\log RV_t &= \lambda_0 + \lambda_1 \log CV_{t-1} + \lambda_2 \log CV_{w,t-1} + \lambda_3 \log CV_{m,t-1} \\
&\quad + \theta_1 \log JV_{t-1} + \theta_2 \log JV_{w,t-1} + \theta_3 \log JV_{m,t-1} + a_t
\end{aligned}
$$

OLS estimation with heteroscedasticity and autocorrelated consistent standard errors.

HAR-RV model estimation results

	$\sqrt{RV_t}$	$\log RV_t$
$\widehat{\beta}_0$	14.90^a	1.15^a
$\widehat{\beta}_1$	0.44^a	0.45^a
$\widehat{\beta}_2$	0.13^b	0.16^a
$\widehat{\beta}_3$	0.27^a	0.26^a
$\overline{R}^2$ (%)	42.03	47.27

^{a} Significance at 1%

^{b} Significance at 5%

HAR-CV-JV model estimation results

	$\sqrt{RV}$			$\log RV$		
	BNS	CPR	Med	BNS	CPR	Med
$\widehat{\lambda}_0$	18.77^a	24.26^a	14.93^a	1.45^a	1.89^a	1.11^a
$\widehat{\lambda}_1$	0.43^a	0.42^a	0.44^a	0.44^a	0.43^a	0.45^a
$\widehat{\lambda}_2$	0.12^b	0.07	0.13^b	0.13^a	0.12^b	0.16^a
$\widehat{\lambda}_3$	0.22^a	0.12	0.27^a	0.25^a	0.06^b	0.27^a
$\widehat{\theta}_1$	0.21^a	0.20^a	0.47^a	0.04^a	0.03^a	0.06^a
$\widehat{\theta}_2$	0.02	0.06	-0.21	$4.1 \cdot 10^{-3}$	0.02^b	0.01
$\widehat{\theta}_3$	0.18^c	0.11^b	0.07	0.01^c	0.09^a	0.01
$\overline{R}^2$ (%)	42.13	42.52	41.98	47.45	47.27	47.26

^{a} Significance at 1%

^{b} Significance at 5%

^{c} Significance at 10%

GARCH(1,1) modelling of the error term

$$a_t = \sigma_t \epsilon_t \text{ where } \epsilon_t \sim iidN$$

where

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \alpha_2 \sigma_{t-1}^2$$

$\alpha_0 > 0, \alpha_1, \alpha_2 \geq 0$ and $\alpha_1 + \alpha_2 < 1$ (positive variance and stationary process)

EGARCH(1,1) modelling of the error term

$$a_t = \sigma_t \epsilon_t \text{ where } \epsilon_t \sim iidN$$

where

$$\log \sigma_t^2 = \delta_0 + \delta_1 \frac{|a_{t-1}|}{\sigma_{t-1}} + \delta_2 \log \sigma_{t-1}^2 + \delta_3 \frac{a_{t-1}}{\sigma_{t-1}}$$

$\delta_3 > 0 (< 0)$ inverse (direct) leverage effect. Positive shocks generate more (less) volatility than negative shocks

HAR-GARCH-RV and HAR-EGARCH-RV model estimation results

According to Ljung-Box test, GARCH structures in innovations are justified only for $\sqrt{RV_t}$ specification.

GARCH		**EGARCH**	
$\widehat{\beta}_0$	14.42^a	$\widehat{\beta}_0$	15.91^a
$\widehat{\beta}_1$	0.45^a	$\widehat{\beta}_1$	0.48^a
$\widehat{\beta}_2$	0.19^a	$\widehat{\beta}_2$	0.16^a
$\widehat{\beta}_3$	0.20^a	$\widehat{\beta}_3$	0.19^a
$\widehat{\alpha}_0$	41.67^a	$\widehat{\delta}_0$	0.21^a
$\widehat{\alpha}_1$	0.09^a	$\widehat{\delta}_1$	0.07^a
$\widehat{\alpha}_2$	0.88^a	$\widehat{\delta}_2$	0.96^a
		$\widehat{\delta}_3$	0.14^a
LogL	-5961.84	LogL	-5936.62

^{a} Significance at 1%

HAR-GARCH-CV-JV model estimation results

	BNS	**CPR**	**Med**
$\widehat{\lambda}_0$	19.24^a	22.91^a	14.22^a
$\widehat{\lambda}_1$	0.43^a	0.41^a	0.44^a
$\widehat{\lambda}_2$	0.16^a	0.13^b	0.20^a
$\widehat{\lambda}_3$	0.16^a	0.09	0.20^a
$\widehat{\theta}_1$	0.20^a	0.21^a	0.47^a
$\widehat{\theta}_2$	0.18^a	0.14^b	-0.26
$\widehat{\theta}_3$	0.07	0.19^b	0.17
$\widehat{\alpha}_0$	61.5^a4	54.08^a	44.08^a
$\widehat{\alpha}_1$	0.13^a	0.10^a	0.09^a
$\widehat{\alpha}_2$	0.83^a	0.85^a	0.87^a
LogL	-5954.9	-5956.2	-5950.4

^{a} Significance at 1%

^{b} Significance at 1%

HAR-EGARCH-CV-JV model estimation results

	BNS	CPR	Med
$\widehat{\lambda}_0$	18.13[a]	19.10[a]	14.55[a]
$\widehat{\lambda}_1$	0.47[a]	0.48[a]	0.48[a]
$\widehat{\lambda}_2$	0.13[a]	0.16[a]	0.15[a]
$\widehat{\lambda}_3$	0.19[a]	0.08	0.21[a]
$\widehat{\theta}_1$	0.22[a]	0.19[a]	0.44[b]
$\widehat{\theta}_2$	0.07	0.03	-0.26
$\widehat{\theta}_3$	0.08	0.22[a]	0.38[b]
$\widehat{\delta}_0$	0.21[a]	0.19[a]	0.19[b]
$\widehat{\delta}_1$	0.08[a]	0.06[a]	0.05[a]
$\widehat{\delta}_2$	0.96[a]	0.97[a]	0.97[a]
$\widehat{\delta}_3$	0.13[a]	0.13[a]	0.14[a]
LogL	-5933.14	-5934.1	-5934.41

[a] Significance at 1%

[b] Significance at 1%

Forecast out-of-sample criteria

$$MAE = \frac{1}{N}\sum_{t=1}^{N}\left|RV_t - \widehat{RV}_t\right|$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}\left(RV_t - \widehat{RV}_t\right)^2}$$

$$MAE = \frac{1}{N}\sum_{t=1}^{N}\frac{\left|RV_t - \widehat{RV}_t\right|}{RV_t}$$

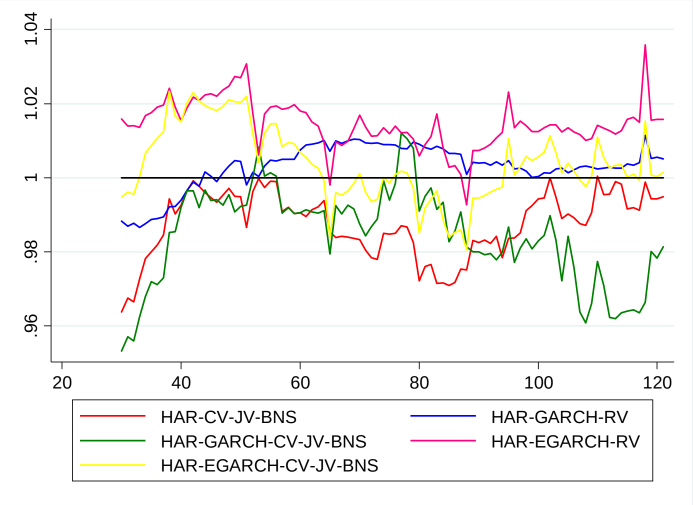
Forecast results standard deviation form:

	MAE	RMSE	MAPE
R	15.0767	19.5739	0.1784
C-J-BNS	14.7714	19.2089	0.1752
C-J-CPR	14.8110	19.4492	0.1756
C-J-MED	15.0610	19.3712	0.1785
G-R	15.0827	19.5883	0.1780
G-C-J-BNS	**14.6218**	**19.0844**	**0.1733**
G-C-J-CPR	14.8349	19.5145	0.1757
G-C-J-MED	15.1113	19.4293	0.1785
EG-R	15.2375	19.8298	0.1814
EG-C-J-BNS	15.0213	19.5349	0.1789
EG-C-J-CPR	14.9806	19.4672	0.1761
EG-C-J-MED	15.2884	19.6601	0.1820

Forecast results logarithmic form:

	MAE	RMSE	MAPE
R	0.3382	0.4189	0.0381
C-J-BNS	**0.3330**	**0.4127**	**0.0375**
C-J-CPR	0.3351	0.4179	0.0377
C-J-MED	0.3370	0.4159	0.0380

Rolling RMSE:



Forecast summary:

- Models with decomposition of RV into CV and JV provide better forecasts.
  - Standard deviation form: All the criteria select HAR-GARCH-CV-JV model using the BNS approach.
  - Logarithmic form: All the criteria select HAR-CV-JV model using the BNS approach.
- Models obtained using the BNS approach provide more accurate forecasts.

Conclusions:

- Decomposition of the total RV is important for (in-sample and out-of-sample) forecasting purposes.
- In-sample forecast chooses:
    - For the logarithmic transformation: HAR-CV-JV with BNS
    - For the square root transformation: HAR-EGARCH-CV-JV with BNS
- Out-of-sample forecast chooses:
    - For the logarithmic transformation: HAR-CV-JV with BNS
    - For the square root transformation: HAR-GARCH-CV-JV with BNS

Further research:

- Realised GARCH-type models: Joint modelling of returns and volatility.
- Interaction with other EPEX markets and neighbouring Non-EPEX markets.
- Jump tests robust to microstructure noise.
- Relationship between markets: Co-jump robust tests.
- Forecast and option pricing valuation.