Product and Process R&D under Asymmetric Demands

Wen-Jung Liang*
Department of Economics
National Dong Hwa University

and

Kuang-Cheng Andy Wang

Department of Industrial and Business Management
Chang Gung University

and

Yi-Jie Wang
Department of Economics
National Dong Hwa University

and

Bo-Yi Lee
Department of Economics
National Dong Hwa University

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* Corresponding Author: Wen-Jung Liang, Department of Economics, National Dong Hwa University, Shoufeng, Hualien County, Taiwan 97401, ROC. Tel: 886-3-863-5538, Fax: 886-3-863-5530, E-mail: wjliang@mail.ndhu.edu.tw

Abstract

This paper indicates that Lin and Saggi (2002) miss analyzing the possibility that product R&D may shrink the extent of horizontal product differentiation, and employs a barbell model with asymmetric demands a la Liang *et al*. (2006) to examine the optimal product and process R&D under Bertrand and Cournot competition. The focus of the paper is on the importance of the influence of spatial barriers and the market-size effect on the determination of product R&D. Several striking results, which are sharply different from those in Lin and Saggi (2002), are derived in the paper.

1. Introduction

Lin and Saggi (2002) studied the equilibrium product and process R&D under Bertrand and Cournot competition by assuming that product R&D *always* expands the extent of horizontal product differentiation and this expansion of product differentiation enhances the demand for both products via the Dixit-type of demand functions. However, it can be found in the real world that many investments of product R&D are undertaken via shrinking rather than enlarging the extent of product differentiation. For instance, HD DVD competed originally with Blu-ray Disc, in which both formats were designed as optical disc standards for storing high definition video and audio. This format war was decided in 2008, when several studios and distributors shifted from HD DVD to Blu-ray disc such that the market of Blu-ray disc became much larger than that of HD DVD. In the end, the main developer of HD DVD, Toshiba, announced to stop the development of the HD DVD players in 2008, conceding the format war to Blu-ray disc.[1] Today, all firms produce DVD players by the same format, Blu-ray disc. As Lin and Saggi (2002) miss analyzing the possibility that product R&D may contract the extent of product differentiation, this gives us an incentive to construct a generalized theoretical model, in the sense that the product R&D may either expand or shrink the extent of horizontal differentiation.

[1] Please refer to the website: http://en.wikipedia.org/wiki/High_definition_optical_disc_format_war.

It can be frequently observed in the real world that the demands for the differentiated products are asymmetric. For instance, Coke had a 25.9 percent share of the soft drink worldwide while Pepsi had just 11.5 percent in 2011;[2] according to research firm IDC, Google's Android mobile operating system captured a 78 percent share of all smart phone users globally, while Apple's iOS owned only 18 percent in the fourth quarter of 2013;[3] and by the Kantar Worldpanel's report, 56% of European smartphone owners switched from a small sized smartphone of 4 or 4.4 inches to a larger sized in 2013, demonstrating that the large sized smartphone is a large market while the small sized is a small market.[4] Based on the above examples, it is reasonable to take into account market asymmetry in the generalized model.

In order to set up a generalized model with market asymmetry, we introduce a barbell model, in which only two asymmetric markets located at the opposite endpoints of the characteristic line, respectively.[5] Each point along the characteristic line represents a distinct characteristic of the horizontally differentiated product. As is common in models involving horizontal differentiation, such as Lederer and Hurter (1986), Anderson and De Palma (1988), De Fraja and Norman(1993), Eaton and

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2 Please refer to the website: http://www.statista.com/statistics/216888/global-market-share-of-coca-cola-and-other-soft-drink-companies-2010/

3 The remainder contains Windows phone, BlackBerry and others. Please refer to the website: http://www.businessinsider.com/iphone-v-android-market-share-2014-5.

4 Please refer to the website: http://www.apple.com/tw/iphone/compare/

5 The barbell model is set up as a monopolistic model by Hwang and Mai (1990), and is extended to a duopolistic model by Liang *et al*. (2006).

Schmit (1994), and Shimizu (2002), consumers incur disutility when they buy a non-preferred product deviated from their ideal product, and this disutility can be measured as the transportation cost between the non-preferred and the ideal products. The advantages of the use of the barbell model contain: first, we can consider the cases where product R&D may either increase or decrease the extent of horizontal differentiation; second, the market asymmetry can be taken into account; and finally, the math is clearer.

Based on the above analysis, the purpose of this paper is to construct a generalized model with market asymmetry by using a barbell model to examine the equilibrium product and process R&D, and to compare with the results derived in Lin and Saggi (2002).

The key differences between this paper and Lin and Saggi (2002) are as follows. First, the product R&D may either expand or shrink the extent of horizontal differentiation in this paper, while Lin and Saggi (2002) only analyzed the former case; and second, increasing horizontal differentiation via enlarging spatial barriers only reduces the degree of competition in the markets keeping the demand curves unchanged and hence raises prices as well as decreases outputs of the products in this paper, while it shifts the demand curves outward and raises prices as well as outputs in Lin and Saggi (2002). As a result, the two-way complementarity between product and process R&D caused by the output effect as proposed in Lin and Saggi (2002) can no longer be valid in this paper.[6]

The main results derived in this paper are as follows. First, when market 1 is sufficiently large and the cost parameter of product R&D is small, the aggregate product R&D under Cournot competition is greater than that under Bertrand competition. Second, provided that the markets are symmetric and the cost parameter of product R&D is large, the aggregate product R&D under Cournot competition is greater than that under Bertrand competition if the transport rate is high while the reverse occurs otherwise. Third, increasing horizontal differentiation will lead the two firms to reducing their optimal process R&D under Bertrand competition. Fourth, suppose that the markets are symmetric and the cost parameter of product R&D is small. The two firms' optimal aggregate process R&D under Bertrand competition is greater than that under Cournot competition if the transport rate is low, while the reverse occurs otherwise. Lastly, suppose that market 1 is not sufficiently large and the cost parameter of product R&D is sufficiently small. the two firms' optimal aggregate process R&D under Bertrand competition is greater than that under Cournot competition if the transport rate is low, while the reverse occurs otherwise.

The remaining related literature includes: Bester and Petrakis (1993) found that

[6] Please refer to Lin and Saggi (2002, p. 202) for the two-way complementarity.

Cournot competition provides a stronger incentive to innovate process R&D than Bertrand competition if the degree of substitutability is low, while the reverse occurs otherwise; Qiu (1997) took into account the externality of process R&D; Chen and Sappington (2010) examined the effect of vertical integration on the process R&D of the upstream firm; Rosenkranz (2003) analyzed simultaneous product and process innovation if demand is characterized by preference for product variety; Lambertini and Mantovani (2009) adopted a dynamic approach to explore the optimal process and product R&D in the long-run steady state for a multiproduct monopolist; Chen and Sappington (2010) studied the effect of vertical integration on the process R&D of the upstream firm; and Ebina and Shimizu (2012) utilized a circular city model to show that the existence of spatial barrier will reduce the optimal product R&D under Cournot competition.

The remainder of the paper is organized as follows. Section 2 sets up a basic model to analyze the optimal product R&D under Bertrand and Cournot competition in the case where firms undertake product R&D only. Section 3 examines the optimal product and process R&D under Bertrand and Cournot competition. The final section concludes the paper.

2. Product R&D Only

Following Hwang and Mai (1990) and Liang *et al*. (2006), we consider a framework, in which there are two firms, denoted as firms A and B, whose characteristics of products are initially assumed to be $\bar{x}_A$ and $\bar{x}_B$ along a characteristic line with unit length, respectively. We further assume that the characteristic of firm A is located at the lefthand side of that of firm B, i.e., $\bar{x}_A \leq \bar{x}_{B,}$ and $\bar{x}_A < (\frac{1}{2})$ and $\bar{x}_B > \left(\frac{1}{2}\right)$. Firm j's product R&D can be denoted as $\Delta x_j = \left|\bar{x}_j - x_j\right|, j = (A, B)$ where x_j is firm j's equilibrium characteristic. The cost function for product R&D is given by $\alpha(\Delta\, x_j)^2/2,$ where $\alpha > 0$ denotes the cost parameter of product R&D. A smaller α represents a superior product R&D. There are two distinct markets, denoted as markets 1 and 2, located at the opposite endpoints of the characteristic line, respectively, as shown in Figure 1. Finally, assume that each firm's marginal production cost is a constant, c.

(Insert Figure 1 here)

In order to exhibit the feature of asymmetric demand structure, we assume that there are n identical consumers whose ideal characteristic is at the site of market 1, one consumer at market 2, and no consumers lie inside the line segment. Thus, the inverse demands facing the two firms at markets 1 and 2 can be expressed as:[7]

$$q_1 = n(1 - p_1), \text{and } \ q_2 = 1 - p_2, \tag{1}$$

[7] Motta and Norman (1996) and Haufler and Wooton (1999) also use these demand curves to exhibit market asymmetry.

where q_i and p_i denotes the quantity demand and delivered price at market i (i = 1, 2), respectively; and n is the number of consumers, whose ideal characteristic is at market 1. Notice that n can be denoted as a measure of relative market size. The two markets are asymmetric (symmetric), when n > (=) 1.

The game in question consists of two stages, when firms undertake product R&D only. In the first stage, firms determine their optimal characteristics to maximize their profits. Each firm's product R&D will then be $\Delta x_j = |\bar{x}_j - x_j|$. In the second stage, given the characteristic decisions, the firms simultaneously choose their quantities (prices) if they engage in Cournot (Bertrand) competition. The sub-game perfect equilibrium of the model is solved by backward induction, beginning with the final stage.

2.1. Bertrand Competition

In the second stage, the two firms simultaneously determine their prices p_{iA} and p_{iB} for market i. Sales for firm j in markets 1 and 2 are then given by:

$$q_{1j}{}^{T} = \begin{cases} 0 \quad if\ p_{1j} > p_{1k}, \\ \frac{n}{2}(1-p_1)\ if\ p_{1j} = p_{1k}, \\ n(1-p_1)\ if\ p_{1j} < p_{1k}, \end{cases} \tag{2.1}$$

$$q_{2j}{}^{T} = \begin{cases} 0 \quad if\ \ p_{2j} > p_{2k}, \\ \frac{1}{2}(1-p_2) if\ p_{2j} = p_{2k}, \quad j,k = A,B, j \neq k, \\ 1-p_2 \qquad if\ p_{2j} < p_{2k}, \end{cases} \tag{2.2}$$

where the superscript "T" denotes variables associated with the case of Bertrand competition.

As have mentioned previously, the consumers will incur disutility when they buy a non-preferred product. This disutility can be represented by $t|x - x_j|$, where t is the per unit output per unit distance transport (marginal disutility) rate, and $|x - x_j|$ is the distance in the characteristic line between the most-preferred product, x, and the product purchase from firm j, x_j. By referring to Liang *et al.* (2006), the firm with a lower marginal cost (marginal production cost plus transportation cost) will undercut the rival's price and takes the whole market. Given the assumption of $x_B \geq x_A$, firm A is closer to market 1, and can use its location advantage to force out its rival from the market. Similarly, firm B can force out firm A in market 2. As a result, the winner's delivered price in its advantageous market under Bertrand competition can be derived as the rival's marginal cost as follows:

$$\begin{cases} p_{1A}^T = t x_B^T + c, \\ p_{2B}^T = t(1 - x_A^T) + c. \end{cases} \tag{3.1}$$

Note that the winner's delivered price will remain the monopoly price for it being the profit-maximizing price, when the transport rate is so high that the rival's marginal cost becomes higher than the monopoly price. This will generate a cap of the transport rate $\bar{t}$ such that the equilibrium (winner's) delivered price remains unchanged, when $t \geq \bar{t}$. Accordingly, we can derive the transport rate cap by equating the monopoly price and p_{iA}^T as follows:

$$\bar{t} = \min.\left[\frac{1-c}{2x_B^T - x_A^T}, \quad \frac{1-c}{1 + x_B^T - 2x_A^T}\right]. \tag{3.2}$$

In stage 1, by use of the equilibrium in the second stage, the profit functions of firms A and B can be specified as follows:

$$\pi_A^T = (p_{1A}^T - c - tx_A^T)q_{1A}^T - \frac{\alpha}{2}|\bar{x}_A - x_A^T|^2, \tag{4.1}$$

$$\pi_B^T = [p_{2B}^T - c - t(1 - x_B^T)]q_{2B}^T - \frac{\alpha}{2}|\bar{x}_B - x_B^T|^2, \tag{4.2}$$

where π_j^T ($j = A, B$) denotes the profit of firm j.

The first term on the right-hand side of the profit functions can be referred to as the operating profit, while the second term as the fixed R&D cost. Differentiating (4) with respect to x_j, respectively, we can derive the profit-maximizing conditions for firms' characteristics as follows:

$$\frac{d\pi_A^T}{dx_A} = \left(\frac{\partial \pi_A^T}{\partial p_{1A}^T}\right)\left(\frac{\partial p_{1A}^T}{\partial x_A^T}\right) + \left(\frac{\partial \pi_A^T}{\partial p_{2B}^T}\right)\left(\frac{\partial p_{2B}^T}{\partial x_A^T}\right) + \frac{\partial \pi_A^T}{\partial x_A^T}$$
$$= -nt(1 - tx_B^T - c) + \alpha(\bar{x}_A - x_A^T), \tag{5.1}$$

$$\frac{d\pi_B^T}{dx_B} = \left(\frac{\partial \pi_B^T}{\partial p_{2B}^T}\right)\left(\frac{\partial p_{2B}^T}{\partial x_B^T}\right) + \left(\frac{\partial \pi_B^T}{\partial p_{1A}^T}\right)\left(\frac{\partial p_{1A}^T}{\partial x_B^T}\right) + \frac{\partial \pi_B^T}{\partial x_B^T}$$
$$= t(1 - t(1 - x_A^T) - c) - \alpha(x_B^T - \bar{x}_B). \tag{5.2}$$

As $\partial p_{1A}^T / \partial x_A^T = 0$ and $\partial \pi_A^T / \partial p_{2B}^T = 0$, we find from (5.1) that firm A's optimal characteristic is determined by the direct effect, which is the third term on the right-hand side of (5.1). The direct effect consists of two opposite parts. When x_A get to be larger, the negative part arises from a decline in the operating profit caused by a larger transportation cost. On the other hand, the positive part emerges because the fixed R&D cost reduces due to a smaller product R&D. Similarly, (5.2) shows that firm B's optimal characteristic is determined by the direct effect denoted by the third term because $\partial p_{2B}^{T}/\partial x_{B}^{T}=0$ and $\partial \pi_{B}^{T}/\partial p_{1A}^{T}=0$. As a result, an endogenous solution of firms' characteristics can be determined by the balance of these two opposite parts.

The second-order condition is always fulfilled, while the stability condition requires:

$$\alpha^2 - nt^4 > 0. \tag{5.3}$$

We can obtain from (5) that given $\alpha^2 > nt^4$, the endogenous solution of firms' optimal characteristics are as follows:

$$x_A^{T^*} = \frac{1}{\alpha^2 - nt^4}\{\alpha^2 \bar{x}_A - nt[(\alpha - t^2)(1-c) - \alpha t \bar{x}_B + t^3]\}, \tag{6.1}$$

$$x_B^{T^*} = \frac{1}{\alpha^2 - nt^4}\{\alpha^2 \bar{x}_B + t[(\alpha - t^2)(1-c) - t(1 - \alpha \bar{x}_A)]\}. \tag{6.2}$$

Next, provided that the cost parameter of product R&D is small, say, $\alpha^2 < nt^4$, the stability condition is violated so that the endogenous solution is no longer valid. The candidate corner solutions are $\left(x_A^{T^*}, x_B^{T^*}\right) = (0,1), (0,0),$ and $(1,1)$. It should be noted that the firms will undercut each other resulting in a zero profit when they agglomerate at the same characteristic such that the agglomeration solutions are infeasible. Thus, the corner solution of firms' characteristics must be $\left(x_A^{T^*}, x_B^{T^*}\right) = (0,1)$.

Based on the above analysis, we can establish:

Proposition 1. *Suppose that the firms undertake product R&D only and engage in Bertrand competition. The firms' optimal characteristics will locate at the opposite endpoints of the line segment when the cost parameter of product R&D is small*, *say*, $\alpha^2 < nt^4$, *while they will locate inside the line segment as shown in* (6) *otherwise*.

2.2. Cournot Competition

As the competition between firms is mild under Cournot competition that firms can co-exist in the same market, the profit functions for the Cournot firms can be expressed as follows:

$$\pi_j^C = q_{1j}^C\left[p_1^C - c - tx_j^C\right] + q_{2j}^C\left[p_2^C - c - t(1 - x_j^C)\right] - \frac{\alpha}{2}\left|\bar{x}_j - x_j^C\right|^2, j = A, B, \quad (7)$$

where the superscript "*C*" denotes variables associated with the case of Cournot competition.

In stage 2, by differentiating the profit function with respect to $q^C{}_{ij}$ (i = 1, 2, j = A, B) and then letting it equal zero, we can solve for the equilibrium outputs as follows:

$$q_{1j}^{C\,*} = \frac{n}{3}\left[1 - c - t\left(2x_j^C - x_k^C\right)\right], j, k = A, B, j \neq k, \quad (8.1)$$

$$q_{2j}^{C\,*} = \frac{1}{3}\left[1 - c - t\left(1 - 2x_j^C + x_k^C\right)\right], j, k = A, B, j \neq k. \quad (8.2)$$

In stage 1, by differentiating the profit function with respect to x_j (j = A, B), we obtain:

$$\frac{d\pi_j^C}{dx_j^C} = \left[\left(\frac{\partial\pi_j^C}{\partial q_{1k}^C}\right)\left(\frac{\partial q_{1k}^C}{\partial x_j^C}\right) + \left(\frac{\partial\pi_j^C}{\partial q_{2k}^C}\right)\left(\frac{\partial q_{2k}^C}{\partial x_j^C}\right)\right] + \frac{\partial\pi_j^C}{\partial x_j^C}$$

$$= \frac{4t}{3}\left(-q_{1j}^{C\,*} + q_{2j}^{C\,*}\right) - \alpha\left|\bar{x}_j - x_j^C\right|\left(\frac{d\left|\bar{x}_j - x_j^C\right|}{dx_j^C}\right), j, k = A, B, j \neq k. \quad (9.1)$$

The first term on the right-hand side of (9.1) can be referred to as the strategic effect. A rise in x_j^C will increase the rival's output in market 1 by increasing its transportation cost to market 1 and then reduce firm *j*'s profit, while will decrease the rival's output in market 2 by the decline in the transportation cost to market 2 and then raise firm *j*'s profit. By manipulating, we find that this strategic effect is negative if firm *j*'s output in market 1 is greater than that in market 2. Next, the second term is the direct effect consisting of the impacts on the operating profit and the R&D cost. A rise in x_j^C will decrease (increase) its operating profit earned from market 1 (2) caused by increasing (decreasing) the transportation cost to market 1 (2). Moreover, a rise in x_A^C (x_B^C) will reduce (raise) the R&D cost, if $x_A^C \leq \bar{x}_A$ and $x_B^C \geq \bar{x}_B$. Thus, firm *j*'s optimal characteristic is determined by the balance of the strategic and direct effects.

The second and stability conditions require:

$$\Delta_1 = \frac{8t^2(n+1)}{9} - \alpha < 0, \text{and } \Delta_2 = \left[\frac{4t^2(n+1)}{3} - \alpha\right]\left[\frac{4t^2(n+1)}{9} - \alpha\right] > 0. \quad (9.2)$$

We find from (9.2) that the endogenous solutions are stable if $\alpha > 4t^2(n+1)/3$. By solving (9.1), we can obtain firm *j*'s optimal characteristic under Cournot competition as follows:

$$x_j^{C^*} = \frac{-27\alpha(\Delta_1)\bar{x}_j - 12\alpha t^2(n+1)\bar{x}_k - 4[3\alpha - 4t^2(n+1)][(n-1)(1-c)+t]}{27\Delta_2},$$

$$j,k = A,B, j \neq k. \qquad (10)$$

Next, given $\alpha < 4\mathrm{t}^2(n+1)/3$, either the second-order or the stability condition is violated so that the endogenous solution is no longer valid. The candidate corner solutions are $\left(x_A^{C^*}, x_B^{C^*}\right) = (0,1), (0,0), \text{and } (1,1)$. Recall that $x_A \leq x_B$. Provided that firm B locates at market 2, we can derive the difference in firm A's profit between firm A locating at market 1 and market 2 as follows:

$$\pi_A^C(0,1) - \pi_A^C(1,1) = \frac{4t}{9}[(n-1)(1-c)+t] + \frac{\alpha}{2}(1-2\bar{x}_A) > 0. \qquad (11.1)$$

Eq. (11.1) shows that firm A would like to choose to locate its characteristic at market 1, if firm B's characteristic locates at market 2.

Similarly, given that firm A locates at market 1, we can derive the difference in firm B's profit between firm B locating at market 1 and market 2 as follows:

$$\pi_B^C(0,0) - \pi_B^C(0,1) = \frac{4t}{9}[(n-1)(1-c) - nt] - \frac{\alpha}{2}(2\bar{x}_B - 1) > (<)0,$$

$$if \;\; n > (<) n^* = \frac{8t(1-c) + 9\alpha(2\bar{x}_B - 1)}{8t(1-c-t)}. \quad (11.2)$$

We find from (11.1) and (11.2) that given $\alpha < 4\mathrm{t}^2(n+1)/3$, the two firms will agglomerate at market 1 if market 1 is sufficiently large, say, $n > n^*$, while they separate at the two endpoints of the line segment if market 1 is not sufficiently large, say, $1 \leq n < n^*$. Accordingly, we obtain the following proposition:

Proposition 2. *Suppose that the firms undertake product R&D only and engage in Cournot competition. We can propose*:

(1) *Provided that* $\alpha > \frac{4t^2(n+1)}{3}$, *the two firms will locate inside the line segment as shown in* (10).

(2) *Given* $\alpha < \frac{4t^2(n+1)}{3}$, *the two firms will agglomerate at market* 1 *if market* 1 *is sufficiently large*, *say*, $n > n^*$, *while they separate at the two endpoints of the line segment otherwise.*

2.3. Optimal Product R&D in Different Competition Modes

Recall that firm j's product R&D is denoted as $\Delta x_j = \left|\bar{x}_j - x_j\right|, j = (A, B)$. By Propositions 1 and 2, provided that the markets are asymmetric and the cost parameter of product R&D is small, i.e., $\alpha^2 < nt^4$, we can derive the following results. First of all, when market 1 is sufficiently large, i.e., $n > n^*$, the optimal characteristic combination under Bertrand competition is $\left(x_A^{T^*}, x_B^{T^*}\right) = (0,1)$ while that under Cournot competition is $\left(x_A^{C^*}, x_B^{C^*}\right) = (0,0)$. It follows that the aggregate product R&D under Bertrand competition is $\Delta x^T = 1 + \bar{x}_A - \bar{x}_B$, while that under Cournot competition is $\Delta x^C = \bar{x}_A + \bar{x}_B$. Recall that $\bar{x}_B > \left(\frac{1}{2}\right)$. We can derive that the aggregate product R&D under Cournot competition is greater than that under Bertrand competition, because $\Delta x^C - \Delta x^T = 2\bar{x}_B - 1 > 0$. Next, when market 1 is not sufficiently large, i.e., $1 < n < n^*$, both the optimal characteristic combinations under Bertrand and Cournot competition locate at the opposite endpoints of the line segment. Thus, the aggregate product R&D under Cournot competition equals that under Bertrand competition.

We proceed to study the situation when $\alpha > 4t^2(n+1)/3$ such that the optimal characteristics under both Cournot and Bertrand competition are interior solutions. In order to simplify the analysis, we assume the markets are symmetric in this case, i.e., $n = 1$.[8] When the markets are symmetric, it is reasonable to have $(\bar{x}_B^h = 1 - \bar{x}_A^h, h = T, C)$ in each competition mode. Thus, by substituting this relationship into (5.1) and (9.1), we can rewrite these profit-maximizing conditions as:

$$\frac{d\pi_A^T}{dx_A^T} = -tq_1^T + \alpha(\bar{x}_A - x_A^T) = -t(1 - t + tx_A^T - c) + \alpha(\bar{x}_A - x_A^T)$$

$$= -MR^T + MC^T, \tag{12.1}$$

$$\frac{d\pi_A^C}{dx_A^C} = \frac{4t}{3}(-q_{1A}^C + q_{2A}^C) + \alpha(\bar{x}_A - x_A^C) = -\frac{4t^2}{3}(1 - 2x_A^C) + \alpha(\bar{x}_A - x_A^C)$$

$$= -MR^C + MC^C, \tag{12.2}$$

where MR^h ($h = T$, C) and MC^h denotes the marginal revenue and the marginal cost of product R&D under competition mode h, respectively.

By subtracting MR^C from MR^T at the level of characteristic $x_A^{T^*}$, we obtain:

$$MR^T\left(x_A^{T^*}\right) - MR^C\left(x_A^{T^*}\right) = \frac{t}{3}\left[3(1-c) - t\left(7 - 11x_A^{T^*}\right)\right]. \tag{13}$$

[8] As the mathematical exposition under the case of asymmetric markets is too tedious to work out an interesting result, we will ignore the analysis of this case in the paper.

By manipulating (13), we obtain $MR^C\left(x_A^{T^*}\right) > (<) \, MR^T\left(x_A^{T^*}\right)$ when $t > (<) \, \hat{t} = \frac{3(1-c)}{7-11x_A^{T^*}}$. Accordingly, we can derive that under symmetric markets, $x_A^{C^*} < (>) \, x_A^{T^*}$ and then the product R&D in Cournot competition is greater (less) than that in Bertrand competition, when the transport rate is higher (lower) than the critical level $\hat{t}$. The intuition behind this result can be stated as follows. Notice that under symmetric markets, the purpose of the firms to undertake product R&D is to reduce the competition in the markets by enhancing the differentiation between firms. As firms engage in price undercutting and each firm becomes a local monopolist in its advantageous market caused by spatial barriers under Bertrand competition, firms can capture the whole demand in the advantageous market and earns a monopoly rent with limit price. Moreover, this monopoly rent becomes larger and the competition between firms is mitigated, when the transport rate, i.e., spatial barriers, gets higher. Thus, the larger the spatial barriers are, the less will be the competition between firms. On the other hand, since firms co-exist and compete in each market under Cournot competition, the impact of spatial barriers on the degree of competition is weaker than that under Bertrand competition. Recall the restriction of the transport rate cap in (3.2). We can obtain that the competition under Bertrand competition will become milder than that under Cournot competition such that the aggregate in product R&D under Cournot competition is greater than that under Bertrand competition, when the transport rate is high, say, $\bar{t} \geq t > \hat{t}$, while the reverse occurs when $t \leq \hat{t}$.

Based on the above analysis, we get:

Proposition 3. *Provided that the firms undertake product R&D only*, *we can propose*:

(*i*) *Suppose that the markets are asymmetric and the cost parameter of product R&D is small*, *i.e.*, $\alpha^2 < nt^4$. *When market* 1 *is sufficiently large*, i.e., $n > n^*$, *the aggregate product R&D under Cournot competition is greater than that under Bertrand competition*, *while the aggregate product R&D is identical in each competition mode otherwise*.

(*ii*) *Suppose that the markets are symmetric and the cost parameter of product R&D is large*, *i.e.*, $\alpha > \frac{4t^2(n+1)}{3}$. *The aggregate product R&D under Cournot competition is greater than that under Bertrand competition*, *when the transport rate is high*, *say*, $\bar{t} \geq t > \hat{t}$, *while the reverse occurs when* $t \leq \hat{t}$.

Proposition 3 is sharply different from the result derived in Lin and Saggi (2002), in which product R&D under Bertrand competition is always greater than that under Cournot competition. The difference arises from the facts that product R&D is capable of making the products less differentiated and meanwhile spatial barriers are taken into account in this paper.

3. The Optimal Investments of Product and Process R&D

In this section, we examine the optimal investments of product and process R&D. There will be an additional stage post the product R&D stage, in which firms simultaneously choose the optimal process R&D. Firm j's process R&D is denoted as ε_j, which can reduce firm j's marginal production cost to $(c - \varepsilon_j)$. The cost function of process R&D can be expressed as $(\frac{\gamma\varepsilon_j^2}{2})$, where $\gamma > 0$ denotes the cost parameter of process R&D.

3.1. Bertrand Competition

Firm j's profit function can be rewritten as follows:

$$\pi_A^T = q_1^T(p_{1A}^T - c + \varepsilon_A^T - tx_A^T) - \frac{\gamma}{2}(\varepsilon_A^T)^2 - \frac{\alpha}{2}|\bar{x}_A - x_A^T|^2, \tag{14.1}$$

$$\pi_B^T = q_2^T[p_{2B}^T - c + \varepsilon_B^T - t(1 - x_B^T)] - \frac{\gamma}{2}(\varepsilon_B^T)^2 - \frac{\alpha}{2}|\bar{x}_B - x_B^T|^2. \tag{14.2}$$

By replacing the marginal production cost c in (3.1) with $(c - \varepsilon_j^T)$, we obtain the equilibrium delivered price under Bertrand competition in stage 3.

In stage 2, by differentiating the profit function with respect to ε_j^T and letting it equal zero, we can solve for the equilibrium process R&D as follows:

$$\varepsilon_A^{T^*} = \frac{n}{\gamma^2 - n}\{(1+\gamma)(1-c) - t[(1 - x_A^T) + \gamma x_B^T]\}, \tag{15.1}$$

$$\varepsilon_B^{T^*} = \frac{1}{\gamma^2 - n}\{(n+\gamma)(1-c) - t[\gamma(1 - x_A^T) + n x_B^T]\}. \tag{15.2}$$

The second and stability conditions require:

$$\gamma^2 - n > 0. \tag{15.3}$$

By differentiating (15.1) and (15.2) with respect to x_A^T and x_B^T, respectively, we obtain the following comparative statics:

$$\frac{\partial \varepsilon_A^{T^*}}{\partial x_A^T} = \frac{nt}{\gamma^2 - n} > 0, \text{and } \frac{\partial \varepsilon_A^{T^*}}{\partial x_B^T} = -\frac{nt\gamma}{\gamma^2 - n} < 0, \tag{16.1}$$

$$\frac{\partial \varepsilon_B^{T^*}}{\partial x_B^T} = -\frac{nt}{\gamma^2 - n} < 0, \frac{\partial \varepsilon_B^{T^*}}{\partial x_A^T} = \frac{t\gamma}{\gamma^2 - n} > 0. \tag{16.2}$$

Eq. (16.1) and (16.2) show that increasing horizontal differentiation between firms (declining x_A^T or increasing x_B^T) will reduce the optimal investments of the two firms' process R&D. Intuitively, increasing horizontal differentiation mitigates the competition in the market. The two firms will increase prices to enlarge their profits, leading to the reduction in two firms' outputs. It follows that the benefit of increasing process R&D declines. As a result, the firms will reduce their process R&D. Accordingly, we obtain:

Proposition 4. *Increasing horizontal differentiation will lead the two firms to reducing their optimal process R&D under Bertrand competition*.

Proposition 4 is significantly different from the result in Lin and Saggi (2002), in which equilibrium process R&D strictly increases with the extent of horizontal differentiation. The difference occurs because increasing horizontal differentiation will make both firms' demand curves shift outward in Lin and Saggi (2002). This will increase both firms' outputs, and then enhances the incentive for process R&D under Bertrand competition.

By substituting (15.1) and (15.2) into (14), and then differentiating (14) with respect to x_i respectively, we have:

$$\frac{d\pi_A^T}{dx_A^T} = \frac{t\gamma}{\gamma^2-n}\left[n\left(tx_B^T - tx_A^T + \varepsilon_A^{T^*} - \varepsilon_B^{T^*}\right) - n\left(1 - tx_B^T - c + \varepsilon_B^{T^*}\right)\right]$$

$$+\left[-nt\left(1 - tx_B^T - c + \varepsilon_B^{T^*}\right) + \alpha(\bar{x}_A - x_A^T)\right], \tag{17.1}$$

$$\frac{d\pi_B^T}{dx_B^T} = \frac{nt\gamma}{\gamma^2 - n}\left\{\left(tx_B^T - tx_A^T + \varepsilon_B^{T^*} - \varepsilon_A^{T^*}\right) - \left[1 - t(1 - x_A^T) - c + \varepsilon_A^{T^*}\right]\right\},$$

$$+\left\{t\left[1 - t(1 - x_A^T) - c + \varepsilon_A^{T^*}\right] - \alpha(x_B^T - \bar{x}_B)\right\}. \tag{17.2}$$

The first term on the right-hand side of (17) can be referred to as the strategic effect, while the second term is the direct effect. We find from (17) that the optimal firms' characteristics under Bertrand competition will locate at the opposite endpoints of the line, respectively, if the cost parameter α is sufficiently small, while they will locate within the line segment otherwise.

3.2. Cournot Competition

We study the case of Cournot competition in this subsection. By replacing the marginal cost c with $(c - \varepsilon_j^C)$, we can solve for the equilibrium outputs in stage 3.

In stage 2, the optimal process R&D under Cournot competition can be derived as follows:

$$\varepsilon_j^{C*} = \frac{4[(n+1)(1-c)-t]}{9\gamma-4(n+1)} - \frac{4t(n-1)\{2x_j[3\gamma-2(n+1)]-3\gamma x_k\}}{[9\gamma-4(n+1)][3\gamma-4(n+1)]}, j,k = A,B, j \neq k, \tag{18}$$

where $3\gamma > 4(n+1)$ by the stability condition.

Differentiating (18) with respect to x_j^C, we obtain the following comparative statics:

$$\frac{\partial \varepsilon_j^{C^*}}{\partial x_j^C} = -\frac{8t(n-1)[3\gamma-2(n+1)]}{\{[9\gamma-4(n+1)][3\gamma-4(n+1)]\}} \leq 0, j = A,B, \tag{19.1}$$

$$\frac{\partial \varepsilon_k^{C^*}}{\partial x_j^C} = \frac{12\gamma t(n-1)}{\{[9\gamma-4(n+1)][3\gamma-4(n+1)]\}} \geq 0, j,k = A,B, j \neq k, \tag{19.2}$$

$$\frac{\partial \varepsilon_j^{C^*}}{\partial x_j^C} + \frac{\partial \varepsilon_k^{C^*}}{\partial x_j^C} = -\frac{4t(n-1)(3\gamma-4(n+1))}{\{[9\gamma-4(n+1)][3\gamma-4(n+1)]\}} \leq 0, j,k = A,B, j \neq k. \tag{19.3}$$

Recall that $3\gamma > 4(n+1)$. We find from (19) that when the markets are asymmetric, i.e., $n > 1$, a rise in horizontal differentiation (declining x_A or increasing x_B) will increase the optimal investment of firm A's process R&D, while will reduce that of firm B. Moreover, the aggregate process R&D enhances (decreases), when the rise in horizontal differentiation is caused by declining x_A^C (increasing x_B^C). Intuitively, declining x_A^C will increase (decrease) firm A's output in the large (small) market while has opposite effect to firm B via reducing (increasing) firm A's transportation cost to the large (small) market. Since the rise in the output in the large market is greater than the decline in the small market, the net output of firm A will increase and then raise the returns from process R&D. Thus, firm A's process R&D rises. On the contrary, firm B's process R&D declines. However, the former outweighs the latter so that the aggregate process R&D enhances. The same intuition applies to the case where the rise in horizontal differentiation is caused by increasing x_B^C. Next, when the markets are symmetric, i.e., n = 1, a rise in horizontal differentiation generates no effect to each firm's process R&D. This result occurs because markets are symmetric such that the increase in the output of market 1 will offset by the decrease in the output of market 2, resulting in zero effect on firms' process R&D. Accordingly, we obtain:

Proposition 5. *Increasing horizontal differentiation will increase firm A's optimal process R&D while decrease that of firm B under Cournot competition, when the markets are asymmetric. Moreover, the aggregate process R&D enhances (reduces), when the rise in horizontal differentiation is caused by declining* x_A *(increasing* x_B*). However, it generates no effect to each firm's as well as the aggregate optimal process R&D, when the markets are symmetric.*

Proposition 5 is sharply different from the result derived in Lin and Saggi (2002), in which a rise in horizontal differentiation decreases the optimal aggregate process R&D under Cournot competition when the products are more differentiated, say, the extent of product differentiation $s < 2/3$, while increases the optimal aggregate process R&D otherwise.

3.3. The Aggregate Process R&D under Different Modes

In order to simplify the analysis, in what follows we shall discuss the case where the cost parameter of product R&D, α, is sufficiently small such that the optimal product characteristics will locate at the endpoints of the line.

Similarly, we can derive that the optimal product characteristics under Cournot competition are $\left(x_A^{C^*}, x_B^{C^*}\right) = (0,0)$ if market 1 is sufficiently large, while they locate at $\left(x_A^{C^*}, x_B^{C^*}\right) = (0,1)$ otherwise. On the other hand, the optimal firms' characteristics under Bertrand competition will locate at $\left(x_A^{T^*}, x_B^{T^*}\right) = (0,1)$, if the cost parameter α is sufficiently small.

3.3.1. The Case Where *n* = 1 and α Is Sufficiently Small

By substituting $\left(x_A^{T^*}, x_B^{T^*}\right) = \left(x_A^{C^*}, x_B^{C^*}\right) = (0,1)$ and $n = 1$ into (15.1), (15.2) and (18), we obtain:

$$\left(\varepsilon_A^{T^*} + \varepsilon_B^{T^*}\right) - \left(\varepsilon_A^{C^*} + \varepsilon_B^{C^*}\right) = \frac{2\gamma(1-c-t)(1+\gamma)-8t(\gamma^2-1)}{(\gamma^2-1)(9\gamma-8)}. \tag{20}$$

Similar to the analysis in previous section, the equilibrium delivered price will remain the monopoly price, when the transport rate is so large that p_{iA}^T is higher than the monopoly price. This will generate a ceiling of the transport rate $\bar{t}^F$ as follows:[9]

$$\bar{t}^F = \frac{\gamma(1-c)(\gamma+1)}{2\gamma^2 - n + \gamma}. \tag{21}$$

Recall (9.2) and (9.3) that $\gamma > 8/3$ when n = 1, and (21) that the ceiling of the

[9] $\bar{t}^F$ is derived by equating the monopoly price and p_{1A}^T in the large market.

transport rate $t \leq \bar{t}^F$. We find from (20) that the two firms' optimal aggregate process R&D under Bertrand competition is greater than that under Cournot competition if $t < t^* = \left[\frac{\gamma(1+\gamma)(1-c)}{5\gamma^2+\gamma-4}\right]$, while the reverse occurs if $t^* < t \leq \bar{t}^F$. Recall (15.3) that $\gamma^2 > n = 1$. It follows that $t^* > 0$. The intuition behind the above result is as follows. Note that the winner's delivered price in its advantageous market under Bertrand competition is the rival's marginal production cost plus the transport rate, leading to the result that the higher the transport rate is, the larger will be the equilibrium delivered price. Thus, the equilibrium delivered prices under Bertrand competition will be lower than those under Cournot competition such that the aggregate outputs and the returns from process R&D under Bertrand competition will be greater than those under Cournot competition, when the transport rate is lower than t^*. As a result, the optimal aggregate process R&D under Bertrand competition will be greater than those under Cournot competition when $t < t^*$, while the reverse occurs when $t^* < t \leq \bar{t}^F$.

Based on the above analysis, we can propose:

Proposition 6. *Suppose that the markets are symmetric and the cost parameter of product R&D is small such that the two firms' optimal product characteristics separate at the endpoints of the line, respectively, regardless of the competition mode.*

The two firms' optimal aggregate process R&D under Bertrand competition is greater than that under Cournot competition if $t < t^* = \left[\frac{\gamma(1+\gamma)(1-c)}{5\gamma^2+\gamma-4}\right]$*, while the reverse occurs if* $t^* < t \leq \bar{t}^F$.

Proposition 6 is significantly different from the result derived in Lin and Saggi (2002), in which the equilibrium process R&D is higher under Cournot competition than that under Bertrand competition given the same extent of product differentiation in each competition mode. The difference arises because when the transport rate is low, the competition between firms gets severe such that the spatial rent extracted by the local monopolist under Bertrand competition is smaller than the profit earned by firms under Cournot competition. Consequently, the aggregate outputs and then the returns from process R&D are higher under Bertrand competition than those under Cournot competition.

3.3.2. The Case Where *n* Is Sufficiently Large and α Is Sufficiently Small

By substituting $\left(x_A^{T^*}, x_B^{T^*}\right) = (0,1)$ and $\left(x_A^{C^*}, x_B^{C^*}\right) = (0,0)$ into (15.1), (15.2) and (18), we get:

$$\left(\varepsilon_A^{T^*} + \varepsilon_B^{T^*}\right) - \left(\varepsilon_A^{C^*} + \varepsilon_B^{C^*}\right) = \frac{(1-c-t)Z - 8nt(\gamma^2-n)}{(\gamma^2-n)[9\gamma-4(n+1)]}, \tag{22}$$

where $Z = \gamma[10n + (n+1)\gamma - 4n^2 - 4]$.

Recall $[\gamma > \frac{4(n+1)}{3}]$ and $(\gamma^2 - n > 0)$. It follows that the denominator in (22) is positive. By manipulating, we derive from (22) that $Z < (>) 0$ if $n > (<) n^{**} = \left(10 + \gamma + \sqrt{\gamma^2 + 36\gamma + 36}\,\right)$. Thus, the two firms' optimal aggregate process R&D under Bertrand competition is less than that under Cournot competition if $n > n^{**}$. The intuition behind the result can be stated as follows. When market 1 is sufficiently large, the two firms' characteristics will agglomerate at market 1 under Cournot competition, while they separate at the opposite endpoints of the line, respectively, under Bertrand competition. The competition in the large market becomes more severe under Cournot competition than under Bertrand competition. This will result in a larger aggregate output and higher returns from process R&D under Cournot than under Bertrand competition, if the market is sufficiently large, say, $n > n^{**}$.

Next, provided that $n < n^{**}$ such that $Z > 0$, we can calculate that when the transport rate is low, say, $t < t^{**} = Z(1 - \mathrm{c})/[8n(\gamma^2 - n) + Z]$, the two firms' optimal aggregate process R&D under Bertrand competition is greater than that under Cournot competition, while the reverse occurs otherwise. The same intuition in Proposition 6 carries over to this result. Thus, we get:

Proposition 7. *Suppose that market* 1 *is sufficiently large and the cost parameter of product R&D is sufficiently small such that* $\left(x_A^{T^*}, x_B^{T^*}\right) = (0,1)$ and$\left(x_A^{C^*}, x_B^{C^*}\right) = (0,0)$. *the two firms' optimal aggregate process R&D under Bertrand competition is* *less than that under Cournot competition* $if\ \ n > n^{**}$. *Next, provided that* $n < n^{**}$, *the two firms' optimal aggregate process R&D under Bertrand competition is greater than that under Cournot competition if* $t < t^{**} = Z(1-\mathrm{c})/[8n(\gamma^2 - n) + Z]$, *while the reverse occurs otherwise*.

3.4. Optimal Product R&D in Different Competition Modes

Suppose that the markets are symmetric, i.e., n = 1, and the cost parameter of product R&D is large such that the optimal characteristics lie within the characteristic line segment. By the same procedures as those in Section 2.3, we can derive:

$$\begin{aligned}\frac{d\pi_A^T}{dx_A^T} &= -MR^T + MC^T \\ &= -\frac{t\gamma}{(\gamma-1)^2(\gamma+1)}\left\{\left[1-c-t\left(1-x_A^T\right)\right]\gamma^2 + (\gamma-1)\left[(1-c)-t\left(2-3x_A^T\right)\right]\right\} + \alpha\left(\overline{x}_A - x_A^T\right),\end{aligned} \tag{23}$$

$$\frac{d\pi_A^C}{dx_A^C} = -MR^C + MC^C = -\frac{4}{3}t^2\left(1-2x_A^C\right) + \alpha\left(\overline{x}_A - x_A^C\right). \tag{24}$$

By subtracting (24) from (23), we obtain:

$$\begin{aligned}& MR^T\left(x_A^{C*}\right) - MR^C\left(x_A^{C*}\right) \\ &= \frac{t\left\{3\gamma(1-c)\left(\gamma^2+\gamma-1\right) - t\left[\gamma^3\left(7-11x_A^{C*}\right)+\gamma^2\left(2-x_A^{C*}\right)-\gamma\left(10-17x_A^{C*}\right)+4\left(1-2x_A^{C*}\right)\right]\right\}}{3(\gamma-1)^2(\gamma+1)} \\ &> (<) 0 \text{ if } \mathrm{t} < (>) \hat{t}^F = \frac{3\gamma(1-c)\left(\gamma^2+\gamma-1\right)}{\gamma^3\left(7-11x_A^{C*}\right)+\gamma^2\left(2-x_A^{C*}\right)-\gamma\left(10-17x_A^{C*}\right)+4\left(1-2x_A^{C*}\right)}.\end{aligned} \tag{25}$$

Similarly, we can derive the ceiling of the transport rate by equating the monopoly price and p_{iA}^T as follows:

$$t \le \bar{t}^F = \frac{\gamma(1-c)}{\gamma\left(2-3x_A^{T*}\right)-\left(1-2x_A^{T*}\right)}. \tag{26}$$

By comparing (25) with (26), we can derive that:[10]

$$\bar{t}^F - \hat{t}^F > 0 \text{ if } x_A^{C*} < x_A^{T*} \le \frac{1}{2}. \tag{27}$$

By appealing to (25) – (27), we obtain that the competition under Bertrand competition will become milder than that under Cournot competition such that the aggregate investment of product R&D under Cournot competition is greater than that under Bertrand competition, when the transport rate is high, say, $\bar{t}^F \ge t > \hat{t}^F$, while the reverse occurs when $t \le \hat{t}^F$. This result is similar to part (ii) of Proposition 3. Similarly, the results of the case, where the markets are symmetric and the cost parameter of product R&D is small such that the optimal characteristics locate at the opposite endpoints of the characteristic line segment, are similar to part (i) of Proposition 3. Thus, we skip the procedures to save the space.

4. Concluding Remarks

This paper has indicated that Lin and Saggi (2002) miss analyzing the possibility that product R&D may contract the extent of product differentiation, and has employed

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[10] We find from (25) and (26) that $\bar{t}^F - \hat{t}^F > 0 \text{ if } \Delta > 0,$ where $\Delta = \gamma^3\left(1-11x_A^{C*}+9x_A^{T*}\right)-\gamma^2\left(1+x_A^{C*}-3x_A^{T*}\right)-\gamma\left(1-17x_A^{C*}+15x_A^{T*}\right)+\left(1-8x_A^{C*}+6x_A^{T*}\right).$ It follows that $\Delta = 0$ if $x_A^{C*} = x_A^{T*} = 1/2$. Moreover, by differentiating Δ with respect to x_A^{C*} and x_A^{T*}, respectively, we can obtain that $\partial\Delta/\partial x_A^{C*} = -11\gamma^3-\gamma^2+17\gamma-8<0$ and $\partial\Delta/\partial x_A^{T*} = 9\gamma^3+3\gamma^2-15\gamma+6>0$. Thus, we can show that $\Delta > 0$ if $x_A^{C*} < x_A^{T*} \le 1/2$.

the barbell model with asymmetric demands a la Liang *et al*. (2006) to examine the optimal product and process R&D under Bertrand and Cournot competition. The focus of the paper is on the importance of the influence of spatial barriers and the market-size effect on the decision of product R&D. Several striking results are derived in the paper, which are sharply different from those in Lin and Saggi (2002).

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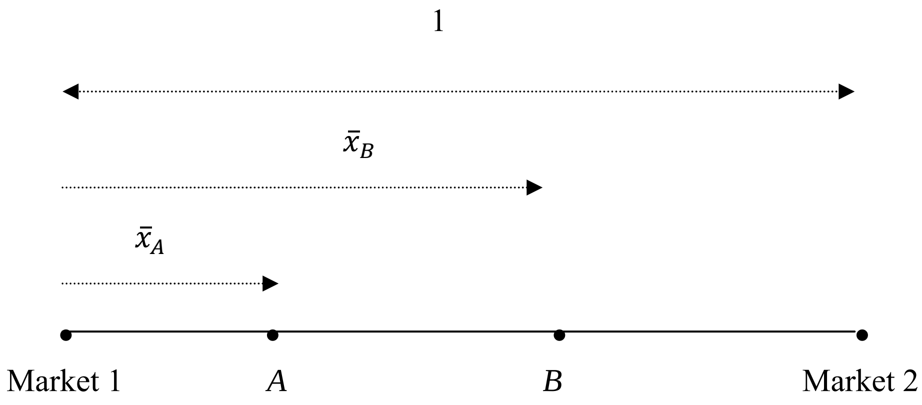

Figure 1. The Characteristic Line