Welfare and Profit Comparison between Quantity and Price Competition in Stackelberg Mixed Duopolies

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Abstract

We compare welfare and profits under price and quantity competition in mixed duopolies, in which a state-owned public firm competes against a private firm. It has been shown that price competition yields larger profit for the private firm and greater welfare if the two firms move simultaneously, regardless of whether the private firm is domestic or foreign. We investigate Stackelberg competition. Under public leadership, the profit and welfare rankings have common features with the simultaneous-move game, regardless of the nationality of private firms. By contrast, under private leadership, the result depends on the nationality of the private firm. When the private firm is domestic, welfare is greater under quantity competition, while the result is reversed when the private firm is foreign. However, regardless of the nationality of the private firm, the private firm earns more under price competition. These results indicate that profit ranking is fairly robust to time structure in mixed Stackelberg duopolies, but welfare ranking is not.

JEL classification numbers: H42, H44, L13, L32

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1 Introduction

The comparison between price and quantity competition has been discussed extensively in the literature. In oligopolies comprising private firms, it is well known that price competition is stronger, yielding lower profits and greater welfare than in the case of quantity competition.[1] Ghosh and Mitra (2010) revisited the comparison between price and quantity competition in a mixed duopoly in which a welfare-maximizing public firm competes against a profit-maximizing private firm.[2] They showed that price competition yields larger profit in the private firm and greater welfare than quantity competition. In other words, welfare ranking is common with private duopolies but profit ranking is the opposite. Haraguchi and Matsumura (2014) showed that this result holds regardless of the nationality of the private firm.

In the aforementioned studies, firms are assumed to play the games simultaneously. In this study, we consider sequential-move games and revisit the price–quantity comparison. First, we analyze the model in which the private firm (public firm) is the leader (follower). In the context of mixed oligopolies, it is known that the public firm's acceptance of the role of the follower often improves welfare (Pal, 1998; Matsumura, 2003; Ino and Matsumura, 2010). It is often discussed that the public firm should play a complementary role to the private firms and the role of the follower is adequate from this viewpoint (Ino and Matsumura, 2010). In this study, we show that when the public firm is the follower, quantity competition is stronger than price competition, resulting in a smaller profit for the private firm. This result is in accordance with that in the simultaneous-move game. However, the welfare ranking can be reversed. When the private firm is domestic (foreign), quantity (price) competition yields greater welfare than price (quantity) competition. In other words, welfare ranking is crucially dependent on the nationality of the private firm.[3]

[1] See Shubik and Levitan (1980) and Vives (1985).

[2] In most countries, there are state-owned public firms that have substantial influence on their market competitors. Such mixed oligopolies occur in various industries, such as the airline, steel, automobile, railway, natural gas, electricity, postal service, education, hospital, home loan, and banking industries. Analyses of mixed oligopolies date back to Merrill and Schneider (1966). Their study and many others in the field assume that a public firm maximizes welfare (consumer surplus plus firm profits), while private firms maximize profits.

[3] It is known that the nationality of private firms is often crucial in shaping mixed oligopolies. Refer to the literature starting with Corneo and Jeanne (1994) and Fjell and Pal (1996). For recent developments in this field, refer to Han and Ogawa (2008), Lin and Matsumura (2012), and Cato and Matsumura (2015).

Next, we analyze the model in which the public firm (private firm) is the leader (follower). We find that both profit and welfare rankings are common with the simultaneous-move game, that is, price competition yields larger profit of the private firm and greater welfare, regardless of the nationality of the private firm.

With regard to the Bertrand–Cournot comparison, Scrimitore (2014) established an important contribution. She adopted the partial privatization approach of Matsumura (1998) and considered the optimal degree of privatization. Her findings showed that under optimal privatization policies, Cournot competition could yield higher profits in private firms than in the case of Bertrand competition. The optimal degree of privatization is lower under Bertrand competition, and a lower degree of privatization leads to stronger competition. Here, the profit ranking can be reversed under optimal privatization policies while welfare ranking is not (Bertrand yields greater welfare). In our Stackelberg models, the leader undertakes strategic commitment, while in the model of Scrimitore (2014), the public firm undertakes strategic commitment through privatization. Thus, both are closely related. Nevertheless, there are contrasting results for the two. Welfare ranking, not profit ranking, can be reversed in Stackelberg models.[4]

2 Model

We adopt a standard duopoly model with differentiated goods and linear demand (Dixit, 1979).[5] The quasi-linear utility function of the representative consumer is

$$U(q_0, q_1, y) = \alpha(q_0 + q_1) - \frac{\beta}{2}(q_0^2 + 2\delta q_0 q_1 + q_1^2) + y, \tag{1}$$

where q_0 is the consumption of good 0 produced by the public firm, q_1 is the consumption of good 1 produced by the private firm, and y is the consumption of an outside good provided competitively (with a unitary price). Parameters α and β are positive constants and $\delta \in (0, 1)$ represents the degree

[4]For another viewpoint on reverted profit ranking but not revertedwelfare ranking, see Haraguchi and Matsumura (2016).

[5]This demand function is popular in the literature on mixed oligopolies. Refer to Bárcena-Ruiz (2007), Fujiwara (2007), Ishida and Matsushima (2009), Matsumura and Shimizu (2010), and Nakamura (2013).

of product differentiation, where a smaller δ indicates a larger degree of product differentiation. The inverse demand functions for goods $i = 0, 1$ with $i \neq j$ are

$$p_i = \alpha - \beta q_i - \beta\delta q_j, \tag{2}$$

where p_i is the price of firm i.

The marginal cost of production is constant for both firms. Let us denote with c_i the marginal cost of firm i, assuming $\alpha > c_0 \geq c_1$. In addition, we assume that α is sufficiently large to assure the interior solutions in the following games. Firm 0 is a state-owned public firm whose payoff is the domestic social surplus (welfare). This is given by

$$SW = (p_0 - c_0)q_0 + (1-\theta)(p_1 - c_1)q_1 + \left[\alpha(q_0 + q_1) - \frac{\beta(q_0^2 + 2\delta q_0 q_1 + q_1^2)}{2} - p_0 q_0 - p_1 q_1\right], \tag{3}$$

where $\theta \in [0, 1]$ is the ownership share of foreign investors in firm 1, which potentially is affected by policymakers acting on capital liberalization. Firm 1 is a private firm and its payoff is its own profit:

$$\pi_1 = (p_1 - c_1)q_1. \tag{4}$$

3 Private leadership (game in which the public firm is the Stackelberg follower)

In this section, we analyze the model in which firm 0 (1) is the follower (leader). First, we discuss quantity competition. Firm 1 chooses its quantities and then firm 0 moves after observing q_1. The first-order condition for firm 0 is given by[6]

$$\frac{\partial SW}{\partial q_0} = \alpha - c_0 - \beta(q_0 + (1-\theta)\delta q_1) = 0. \tag{5}$$

From (5), we obtain the following reaction function of firm 0.

$$R_0(q_1) = \frac{\alpha - c_0 - \beta(1-\theta)\delta q_1}{\beta}. \tag{6}$$

[6]All of the second-order conditions in this study are satisfied.

Firm 1 maximizes its profit, $\pi_1(q_1, R_0(q_1))$, with respect to q_1. The first-order condition for firm 1 is

$$\alpha - c_1 - \delta(\alpha - c_0) - 2\beta(1 - (1-\theta)\delta^2)q_1 = 0. \tag{7}$$

Let the superscript "FQ" denote the equilibrium outcome of this game, where "F" means public followership (private leadership) and "Q" means quantity competition. From (7), we obtain

$$q_1^{FQ} = \frac{\alpha - c_0}{\beta} - \frac{\delta(\alpha - c_1 - (\alpha - c_0)\delta)}{2\beta(1 - (1-\theta)\delta^2)}. \tag{8}$$

Substituting it into (6), we obtain

$$q_0^{FQ} = \frac{\alpha - c_1 - (\alpha - c_0)\delta}{2\beta(1 - (1-\theta)\delta^2)}. \tag{9}$$

Substituting these into firm 1's profit and domestic welfare functions, we obtain

$$\pi_1^{FQ} = \frac{(\alpha - c_1 - (\alpha - c_0)\delta)^2(1 - (1-2\theta)\delta^2)}{4\beta(1 - (1-\theta)\delta^2)^2}, \tag{10}$$

$$SW^{FQ} = \frac{H_1}{8\beta(1 - (1-\theta)\delta^2)^2}, \tag{11}$$

where H_1 and other constants are reported in the Appendix.

Next, we discuss price competition. The direct demand for goods i is given by

$$q_i = \frac{\alpha - \alpha\delta - p_i + \delta p_j}{\beta(1-\delta^2)} \ (i = 1, 2, i \neq j). \tag{12}$$

After observing p_1, the follower, firm 0, chooses p_0. The first-order condition is

$$\frac{\partial SW}{\partial p_0} = \frac{c_0 - (1-\theta)\delta c_1 - p_0 - \theta\delta p_1}{\beta(1-\theta)} = 0. \tag{13}$$

From (13), we obtain the following reaction function of firm 0.

$$R_0(p_1) = c_0 - (1-\theta)\delta c_1 + (1-\theta)\delta p_1. \tag{14}$$

The leader, firm 1, maximizes its profit, $\pi_1(p_1, R_0(p_1))$. The first-order condition of firm 1 is

$$\frac{\alpha(1-\delta) + \delta c_0 + (1 - 2(1-\theta)\delta^2)c_1 - 2(1 - \delta^2 + \theta\delta^2)p_1}{\beta(1-\theta)} = 0. \tag{15}$$

Let the superscript "FP" denote the equilibrium outcome of this game, where "P" means price competition. From (15), we obtain

$$p_1^{FP} = \frac{\alpha(1-\delta) + \delta c_0 + (1 - 2(1-\theta)\delta^2)c_1}{2 - 2(1-\theta)\delta^2}. \tag{16}$$

Substituting it into (14), we obtain

$$p_0^{FP} = \frac{\alpha(1-\delta)\delta - \delta c_1 + (2 - (1-2\theta)\delta^2)c_0}{2 - 2(1-\theta)\delta^2}. \tag{17}$$

Substituting these into firm 1's profit and domestic welfare functions, we obtain

$$\pi_1^{FP} = \frac{(\alpha(1-\delta) - c_1 + \delta c_0)^2(1 - (1-2\theta)\delta^2)}{4\beta(1-\delta^2)(1-(1-\theta)\delta^2)^2}, \tag{18}$$

$$SW^{FP} = \frac{H_2}{8\beta(1-(1-\theta)\delta^2)^2}. \tag{19}$$

We now compare welfare and profit levels in these two games. We address how the leader–follower structure affects profit ranking and welfare ranking in mixed duopoly.

Proposition 1 *Consider Stackelberg competition with private leadership. (i) Price competition yields greater welfare than quantity competition if and only if the foreign ownership share in the private firm is smaller than the threshold value $\bar{\theta} \in (0,1)$. (ii) The private firm obtains greater profit under price competition than under quantity competition regardless of θ.*

Proof. Comparing social surplus under price competition and quantity competition, we obtain

$$SW^{FP} - SW^{FQ} = \frac{(-1+2\theta)\delta^2(\alpha - c_1 - (\alpha - c_0)\delta)^2}{8\beta(1-(1-\theta)\delta^2)^2}.$$

This shows that $SW^P > SW^Q$ holds if $0 < \theta < \bar{\theta} = 1/2$. Similarly, straightforward computations show

$$\pi_1^{FP} - \pi_1^{FQ} = \frac{\delta^2(\alpha - c_1 - (\alpha - c_0)\delta)^2(1-(1-2\theta)\delta^2)}{4\beta(1-\delta^2)(1-(1-\theta)\delta^2)^2} > 0. \blacksquare$$

As Haraguchi and Matsumura (2014) showed, price competition yields greater welfare and profit of the private firm than quantity competition, regardless of the nationality of the private firm in the simultaneous-move game. Proposition 1(i) is in sharp contrast to the result of the simultaneous-move game, whereas Proposition 1(ii) suggests that profit ranking is in accordance with it. We explain the intuition behind Proposition 1(i).

As the Stackelberg leader, the private firm has an incentive to make the public firm less aggressive (choosing smaller output or higher price) to increase its profit. Thus, under quantity (price) competition, the private firm chooses a larger output (higher price) than in the simultaneous-move case. The larger output (higher price) of the private firm improves (reduces) the consumer surplus. Therefore, welfare is greater under quantity competition than under price competition when the private firm is domestic.

However, when θ is positive, both the larger output and higher price of the private firm increase the profit of the private firm, resulting in an increase of the outflow to foreign investors, and the above-mentioned welfare gain is canceled. This effect is more significant when the ownership share by foreign investors in the private firm is larger. This is because the welfare ranking is again reversed when the foreign ownership share in the private firm is large.

4 Public leadership (game in which the public firm is the Stackelberg leader)

In this section, we analyze the Stackelberg model in which firm 0 (1) is the leader (follower). First, we discuss quantity competition. Firm 1 maximizes its own profit given q_0. The first-order condition for firm 1 is given by

$$\frac{\pi_1}{\partial q_1} = \alpha - c_1 - \beta(\delta q_0 + 2q_1) = 0. \tag{20}$$

From (20), we obtain the following reaction function of firm 1.

$$R_1(q_0) = \frac{\alpha - c_1 - \beta\delta q_0}{2\beta}. \tag{21}$$

Taking into account the reaction function, $R_1(q_0)$, firm 0 maximizes domestic welfare. The first-order condition for firm 0 is

$$\frac{1}{4}(4(\alpha - c_0) - (\alpha - c_1)(3 - 2\theta)\delta - \beta(4 - 3\delta^2 + 2\theta\delta^2)q_0) = 0. \tag{22}$$

Let the superscript "LQ" denote the equilibrium outcome of this game, where "L" means public leadership (private followership) and "Q" means quantity competition. From (22), we obtain

$$q_0^{LQ} = \frac{4(\alpha - c_0) - (\alpha - c_1)(3 - 2\theta)\delta}{\beta(4 - (3 - 2\theta)\delta^2)}. \tag{23}$$

Substituting it into (21), we obtain

$$q_1^{LQ} = \frac{2((\alpha - c_1) - \delta(\alpha - c_0))}{\beta(4 - (3 - 2\theta)\delta^2)}. \tag{24}$$

Substituting these equilibrium quantities into firm 1's profit and domestic welfare, we obtain

$$\pi_1^{LQ} = \frac{4((\alpha - c_1) - \delta(\alpha - c_0))^2}{\beta(4 - (3 - 2\theta)\delta^2)^2}, \tag{25}$$

$$SW^{LQ} = \frac{H_3}{2\beta\left(\delta^2(2\theta - 3) + 4\right)}. \tag{26}$$

Next, we discuss price competition. After observing p_0, firm 1 chooses p_1 to maximize its own profit. The first-order condition for firm 1 is

$$\frac{\partial \pi_1}{\partial p_1} = \frac{\alpha(1 - \delta) + c_1 + \delta p_0 - 2p_1}{\beta(1 - \delta^2)} = 0. \tag{27}$$

From (27), we obtain the following reaction function of firm 1.

$$R_1(p_0) = \frac{\alpha(1 - \delta) + c_1 + \delta p_0}{2)}. \tag{28}$$

The leader, firm 0, maximizes domestic welfare with respect to p_0. The first-order condition for firm 0 is

$$\frac{\alpha\delta(2\delta\theta - \delta - 2\theta + 1) - 2c_0\left(\delta^2 - 2\right) + c_1\delta(2\theta - 1) - (2\delta^2\theta - 3\delta^2 + 4)p_0}{\beta\left(\delta^2 - 1\right)} = 0. \tag{29}$$

Let the superscript "LP" denote the equilibrium outcome of this game. From (29), we obtain

$$p_0^{LP} = \frac{\delta(1 - 2\theta)(\alpha(1 - \delta) - c_1) + 2c_0\left(2 - \delta^2\right)}{\delta^2(2\theta - 3) + 4}. \tag{30}$$

Substituting it into (28), we obtain

$$p_1^{LP} = \frac{\left(2 - \delta^2\right)\left(\alpha(1 - \delta) + c_0\delta\right) + 2c_1\left(1 - \delta^2(1 - \theta)\right)}{\delta^2(2\theta - 3) + 4}. \tag{31}$$

Substituting these equilibrium prices into firm 1's profit and domestic welfare, we obtain

$$\pi_1^{LP} = \frac{\left(2-\delta^2\right)^2 (\alpha(1-\delta)+c_0\delta-c_1)^2}{\beta(1-\delta)(\delta+1)\left(\delta^2(2\theta-3)+4\right)^2}, \tag{32}$$

$$SW^{LP} = \frac{H_4}{\beta(1-\delta)(1+\delta)\left(\delta^2(2\theta-3)+4\right)^2}. \tag{33}$$

We now compare welfare and profit levels in these two games.

Proposition 2 *Consider Stackelberg competition with public leadership. (i) Price competition yields greater welfare than quantity competition. (ii) The private firm obtains greater profit under price competition than under quantity competition.*

Proof. Comparing social surplus and the private profit under both types of competition, we obtain

$$SW^{LP} - SW^{LQ} = \frac{\delta^2(\alpha(1-\delta)+c_0\delta-c_1)^2}{2\beta(1-\delta)(1+\delta)\left(\delta^2(2\theta-3)+4\right)} > 0$$

$$\pi_1^{LP} - \pi_1^{LQ} = \frac{\delta^4(\alpha(\delta-1)-c_0\delta+c_1)^2}{\beta(1-\delta)(1+\delta)\left(\delta^2(2\theta-3)+4\right)^2} > 0. \quad \blacksquare$$

As the Stackelberg leader, the public firm has an incentive to make the private firm more aggressive (choosing larger output or lower price) to improve welfare. Thus, under quantity (price) competition, the public firm chooses a smaller output (lower price) than in the simultaneous-move case. Therefore, the private firm's profit is larger (smaller) than in the simultaneous-move case under quantity (price) competition. Thus, the strategic behavior of the public firm as the Stackelberg leader reduces the profit advantage of price competition. Nevertheless, profit ranking is not reversed. A lower price of the public firm reduces the resulting output of the private firm, and thereby, reduces welfare. Therefore, the public firm still sets a higher price under price competition than the resulting price under quantity competition, and thus, price competition yields larger profit of the private firm.

We now explain the intuition of Proposition 2(i). Under quantity competition, a larger output of the private firm improves welfare, while (given the output of the private firm) a smaller output of the public firm reduces welfare. Under price competition, lower prices of both the public and private firms improve welfare. Thus, the welfare-improving effect of the strategic behavior of the public firm is stronger under price competition than under quantity competition. Therefore, welfare ranking is not reversed.

5 Concluding remarks

In this study, we revisit welfare and profit comparison between price and quantity competition in mixed duopolies. We consider sequential-move games. We find that welfare ranking can be reversed when the public firm is the follower, while price competition is always better for welfare when the public firm is the leader. In addition, we find that foreign ownership share plays an important role when the public firm is the follower.

In this study, we do not consider any strategic policies by the government, such as tax-subsidy, privatization, and trade policies. These policies are intensively discussed in the literature and incorporating these into our analysis remains for future research.[7]

[7] For recent developments in studies of these policies in mixed markets, see Cato and Matsumura (2015) and works cited therein.

Appendix

$H_1 = (\alpha - c_0)^2(\delta^4(1-2\theta) + \delta^2(6\theta-5) + 4) + 2(\alpha - c_0)(\alpha - c_1)\delta(4\delta^2\theta^2 - 6\delta^2\theta + 3\delta^2 + 2\theta - 3) + (\alpha - c_1)^2(3 - 4\delta^2\theta^2 - (2 - 6\delta^2)\theta - 3\delta^2)$,

$H_2 = \alpha^2(8\delta^3(1-\theta)^2 - \delta^2(4\theta^2 - 14\theta + 9) + \delta(4\theta - 6) - 2\theta + 7) + c_0^2(\delta^2(6\theta-5)+4) - 2\alpha c_0(4\delta^3(1-\theta)^2 + \delta^2(6\theta-5) + \delta(2\theta-3) + 4) + 2c_1(4\delta^2(1-\theta)^2 + 2\theta - 3)(\alpha(1-\delta) + c_0\delta) + c_1^2(3 - 4\delta^2(1-\theta)^2 - 2\theta)$,

$H_3 = \alpha^2(\delta(4\theta-6) - 2\theta + 7) + 4c_0^2 - 2\alpha c_0(\delta(2\theta-3)+4) - 2c_1(3-2\theta)(\alpha(1-\delta) + c_0\delta) + c_1^2(3-2\theta)$,

$H_4 = \alpha^2(1-\delta)(-\delta^3 - \delta^2(5-4\theta) + 2\delta\theta + \delta - 2\theta + 7) + c_0^2(2-\delta^2)^2 - 2\alpha c_0(\delta^4 + 2\delta^3(1-\theta) - 4\delta^2 - \delta(3 - 2\theta) + 4) - 2c_1(3 - 2\delta^2(1-\theta) - 2\theta)(\alpha(1-\delta) + c_0\delta) + c_1^2(3 - 2\delta^2(1-\theta) - 2\theta)$,

References

Bárcena-Ruiz, J.C. 2007. "Endogenous Timing in a Mixed Duopoly: Price Competition." *Journal of Economics* 91(3):263–272.

Cato, S., Matsumura, T. 2015. "Optimal Privatization and Trade Policies with Endogenous Market Structure." *Economic Record* 91:309–323.

Corneo, G., Jeanne, O. 1994. "Oligopole Mixte Dans un Marche Commun," *Annales d'Economie et de Statistique*, 33:73–90.

Dixit, A.K., 1979. "A Model of Duopoly Suggesting a Theory of Entry Barriers." *Bell Journal of Economics* 10(1):20–32.

Fjell, K., Pal, D. 1996. "A Mixed Oligopoly in the Presence of Foreign Private Firms." *Canadian Journal of Economics* 29(3):737–743.

Fujiwara, K. (2007), "Partial Privatization in a Differentiated Mixed Oligopoly," *Journal of Economics* 92(1):51–65.

Han, L., Ogawa, H. 2008. "Economic Integration and Strategic Privatization in an International Mixed Oligopoly." *FinanzArchiv* 64(3):352–363.

Ghosh, A., Mitra, M. 2010. "Comparing Bertrand and Cournot in Mixed Markets." *Economics Letters* 109(2):72–74.

Haraguchi, J., Matsumura, T. 2014. "Price versus Quantity in a Mixed Duopoly with Foreign Penetration." *Research in Economics* 68(4):338–353.

Haraguchi, J., Matsumura, T. 2016. "Cournot-Bertrand Comparison in a Mixed Oligopoly." forthcoming in *Journal of Economics.*

Ino, H., Matsumura, T. 2010. "What Role Should Public Enterprises Play in Free-Entry Markets?" *Journal of Economics* 101(3):213–230.

Ishida, J., Matsushima, N. 2009. "Should Civil Servants Be Restricted in Wage Bargaining? A Mixed-Duopoly Approach." *Journal of Public Economics*:93(3–4), 634–646.

Lin, M.H., Matsumura, T. 2012. "Presence of Foreign Investor in Privatized Firms and Privatization Policy", *Journal of Economics* 107(1):71–80.

Matsumura, T. 1998. "Partial Privatization in Mixed Duopoly." *Journal of Public Economics* 70(3):473–483.

Matsumura, T. 2003. "Endogenous Role in Mixed Markets: A Two-Production Period Model." *Southern Economic Journal* 70(2):403–413.

Matsumura, T., Ogawa, A. 2012. "Price versus Quantity in a Mixed Duopoly." *Economics Letters* 116(2):174–177.

Matsumura, Toshihiro, Shimizu, Daisuke, 2010. Privatization waves. Manchester School 78(6), 609–625.

Nakamura, Y. 2013. "Social Welfare under Quantity Competition and Price Competition in a Mixed Duopoly with Network Effects: An Analysis." *Theoretical Economics Letters* 3:211–215.

Pal, D. 1998. "Endogenous Timing in a Mixed Oligopoly." *Economics Letters* 61(2):181–185.

Scrimitore, M. 2014. "Profitability under Commitment in Cournot and Bertrand Mixed Markets." *Journal of Institutional and Theoretical Economics* 170(4):684–703.

Shubik, M., Levitan, R. 1980. *Market Structure and Behavior* (Harvard University Press, Cambridge, Massachussets, U.S.A.).

Vives, X. 1985. "On the Efficiency of Bertrand and Cournot Equilibria with Product Differentiation." *Journal of Economic Theory* 36:166–175.