Price versus Quantity in a Mixed Duopoly under Uncertainty*

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Abstract

We characterize the endogenous competition structure (in price or quantity) in a differentiated mixed duopoly under demand uncertainty. We find that price competition yields higher welfare and private firm's profit under one dimensional uncertainty. We also endogenize the price-quantity choice. Here, we find that Bertrand competition appears in equilibrium. However, the ranking of welfare and profit for private firm can be reversed if there exists two dimensional uncertainty. We also show that Cournot competition can be the endogenous competition structure under two dimensional uncertainty.

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*Any remaining errors are our own.

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1 Introduction

The comparison between price and quantity competition has been discussed extensively in the literature. In oligopolies among private firms, it is well known that price competition is stronger, yielding lower profits than in the case of quantity competition.[1] In related literature, Singh and Vives (1984) endogenized the structure of competition (in terms of price or quantity), finding that firms often choose whether to adopt a price contract or a quantity contract. In a private duopoly in which both firms maximize profits, and assuming linear demand and product differentiation, Singh and Vives (1984) showed that a quantity contract is the dominant strategy for each firm when goods are substitutes. However, a price contract is the dominant strategy when goods are complements. Cheng (1985), Tanaka (2001a,b), and Tasnádi (2006) extended this analysis to asymmetric oligopolies, more general demand and cost conditions, and vertical product differentiation, confirming the robustness of the results. However, these results depend on the assumption that all firms are private and profit-maximizers. Therefore, they may not apply to the increasingly important and popular mixed oligopolies, in which state-owned public firms compete against private firms.

Ghosh and Mitra (2010) revisited the comparison between price and quantity competition in a mixed duopoly. They showed that, in contrast to the case of a private duopoly, quantity competition is stronger than price competition, resulting in a smaller profit for the private firm.[2] Then, Matsumura and Ogawa (2012) examined the endogenous competition structure. In their study of a mixed duopoly, when one of the two firms is public, a price contract is the dominant strategy for both the private and the public firm, regardless of whether goods are substitutes or complements.[3] However, in these analysis, they assume that demand is certain. In other words the effect of demand

[1] See Shubik and Levitan (1980) and Vives (1985).

[2] See also Nakamura (2013) and Haraguchi and Matsumura(2015)

[3] Haraguchi and Matsumura (2014) showed that this result holds, regardless of the nationality of the private firm. Chirco *et al.* (2014) showed that both firms choose a price contract when the organizational structure is endogenized. However, Scrimitore (2013) showed that both firms can choose a quantity contract if a production subsidy is introduced.

shock is ignored.

In Resinger and Ressener(2009), they showed that in a private duopoly market, Bertrand competition can appear in equilibrium if demand uncertainty is high relative to the degree of substitutability. Their results implies that the demand uncertainty affects the firm's strategy. However they did not considers the existence of the public firm. In this study, we investigate the effect of demand shock in a mixed duopoly market.

First, we revisit this price-quantity comparison in mixed duopoly with the exogenous demand shock. We adopt a standard differentiated oligopoly with a linear demand (Dixit, 1979) and show that, regardless of the existence of demand shock, the Bertrand model always yields higher welfare and private firm's profit.

Next, we endogenize the competition structure (i.e., price or quantity) using the model of Singh and Vives (1984). We show that Bertrand competition appears in the equilibrium regardless of the existence of demand shock.

Finally, we consider two dimensional demand shock. We show that Cournot competition can be an endogenous competition structure and Cournot model can yields higher welfare and private firm's profit.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 endogenizes the competition structure (i.e., a price or quantity contract) in case of a shock affecting the demand. Section 4 presents our main result. Section 5 considers two dimensional uncertainty. Section 6 concludes.

2 Model

We adopt a standard differentiated oligopoly with a linear demand (Dixit, 1979). The quasi-linear utility function of the representative consumer is:

$$U(q_0, q_1) = \alpha(q_0 + q_1) - \beta(q_0^2 + 2\delta q_0 q_1 + q_1^2)/2 + y,$$

where q_i is the consumption of good i produced by firm i $(i = 0, 1)$, and y is the consumption of an outside good that is provided competitively (with a unit price). Parameters α and β are positive constants, and $\delta \in (0, 1)$ represents the degree of product differentiation: a smaller δ indicates a larger degree of product differentiation.

Firm 0 and firm 1 produce differentiated commodities for which the inverse demand function is given by

$$p_i = \alpha - \frac{\beta}{\theta} q_i - \frac{\beta}{\theta} \delta q_j \ (i = 0, 1, i \neq j), \tag{1}$$

where p_i and q_i are firm i's price and quantity respectively. θ is a random variable with $E[\theta] = 1$ and $Var(\theta) = \sigma_\theta^2$. We denote $E[\frac{1}{\theta}] = z$. By Jensen's inequality, $z > 1$ and it increases in σ_θ^2. The marginal production costs are constant. Let c_i denote firm i's marginal cost. We assume that $\alpha > c_i$.

Firm 0 is a state-owned public firm, and its payoff is the social surplus, given by

$$SW = (p_0 - c_0)q_0 + (p_1 - c_1)q_1 + \left[\alpha(q_0 + q_1) - \frac{\beta(q_0^2 + 2\delta q_0 q_1 + q_1^2)}{2} - p_0 q_0 - p_1 q_1 \right].$$

Firm 1 is a private firm, and its payoff is its own profit: $\pi_1 = (p_1 - c_1)q_i$.
The game runs as follows. In the first stage, each firm chooses whether to adopt a price contract or a quantity contract. In the second stage, after observing the rival's choice in the first stage, each firm simultaneously chooses its own strategy, according to the decision in the first stage. Thereafter, the shock realizes, market clear, and welfare and profit accrue.

3 Second-stage games

First we discuss four possible subgames: both firms choose quantity contract(q-q game), both firms choose price contract(p-p game), only firm 0 chooses the quantity contract(q-p game), or only firm 0 chooses price contract(p-q game). We assume that the solutions in all the following games are interior, that is, equilibrium prices and quantities of both firms are strictly positive. Let us define $a_i \equiv \alpha - c_i$, and let us adopt the superscript $'ij'$ to denote the equilibrium outcome when firm 0 chooses $i \in p, q$ and firm 1 chooses $j \in p, q$.

3.1 Cournot model (q-q game)

First, we discuss the Cournot model (q-q game) in which both firms choose quantities. Because $E[\frac{1}{\theta}] = z$, expected inverse demand is given by $p_i = \alpha - z\beta q_i - z\beta\delta q_j$ $(i = 0, 1, i \neq j)$. The first-order conditions for public firm and private firm are, respectively,

$$\begin{aligned} \frac{\partial SW}{\partial q_0} &= a_0 - \beta q_0 - \beta\delta q_1 = 0, \\ \frac{\partial \pi_1}{\partial q_i} &= a_1 - 2\beta q_1 z - \beta\delta q_0 z = 0. \end{aligned}$$

The second-order conditions are satisfied. From the first-order conditions, we obtain the following reaction functions for firm 0 and firm 1, respectively:

$$\begin{aligned} R_0^{qq}(q_1) &= \frac{a_0 - \beta\delta q_1}{\beta}, \\ R_1^{qq}(q_0) &= \frac{a_1 - \beta\delta q_0 z}{2\beta z}. \end{aligned}$$

These functions lead to the following expression for the equilibrium quantities:

$$\begin{aligned} q_0^{qq} &= \frac{2a_0 z - \delta a_1}{\beta(2-\delta^2)z}, \\ q_1^{qq} &= \frac{a_1 - \delta a_0 z}{\beta(2-\delta^2)z} \quad . \end{aligned}$$

Substituting these equilibrium quantities into the demand and payoff functions, we have the following expected welfare and expected profit for firm 1:

$$SW^{qq} = \frac{((4-\delta^2)a_0 - 2\delta(2-\delta^2)a_1)a_0 z^2 + 2((2-\delta^2)a_1 - \delta a_0)z - (1-\delta^2)a_1^2}{2\beta(2-\delta^2)^2 z^2}, \tag{2}$$

$$\pi_1^{qq} = \frac{(a_1 - \delta a_0 z)^2}{\beta(2-\delta^2)^2 z}. \tag{3}$$

3.2 Bertrand model (p-p game)

We now characterize the Bertrand model (p-p game) in which both firms choose prices. By (1) the direct demand function is given by

$$q_i = \frac{\theta(\alpha - \alpha\delta - p_i + \delta p_j)}{\beta(1-\delta^2)}, \quad (i = 0, 1, i \neq j).$$

As $E[\theta] = 1$, expected direct demand is given by $\frac{\alpha-\alpha\delta-p_i+\delta p_j}{\beta(1-\delta^2)}$ $(i = 0, 1, i \neq j)$. The first-order conditions for public and private firms are, respectively,

$$\begin{aligned}\frac{\partial SW}{\partial p_0} &= \frac{c_0 - p_0 - \delta c_1 + \delta p_1}{\beta(1-\delta^2)} = 0,\\ \frac{\partial \pi_1}{\partial p_1} &= \frac{c_1 - 2p_1 + \alpha + \delta p_0 - \delta\alpha}{\beta(1-\delta^2)} = 0.\end{aligned}$$

The second-order conditions are satisfied. From the first-order conditions, we obtain the following reaction functions for public and private firms, respectively:

$$\begin{aligned}R_0^{pp}(p_1) &= c_0 + \delta(p_1 - c_1),\\ R_1^{pp}(p_0) &= \frac{c_1 + \alpha + p_0\delta - \alpha\delta}{2}.\end{aligned}$$

These functions lead to the following expression for the equilibrium prices:

$$\begin{aligned}p_0^{pp} &= \frac{\alpha\delta - \alpha\delta^2 + 2c_0 - \delta c_1}{2-\delta^2},\\ p_1^{pp} &= \frac{\alpha - \alpha\delta + c_1 + \delta c_0 - \delta^2 c_1}{2-\delta^2}.\end{aligned}$$

Substituting these equilibrium quantities into the payoff functions, we have the following resulting expected welfare and expected profit for firm 1:

$$SW^{pp} = \frac{(4-5\delta^2+2\delta^4)a_0^2 + (3-3\delta^2+\delta^4)a_1^2 - 2\delta(3-3\delta^2+\delta^4)a_0a_1}{2\beta(1-\delta^2)(2-\delta^2)^2}, \tag{4}$$

$$\pi_1^{pp} = \frac{(a_1 - \delta a_0)^2}{\beta(1-\delta^2)(2-\delta^2)^2}. \tag{5}$$

3.3 p-q game

We discuss the situation in which firm 0 chooses the price contract and firm 1 chooses the quantity contract. The first-order conditions for firms 0 and 1 are, respectively,

$$\begin{aligned}\frac{\partial SW}{\partial p_0} &= \frac{c_0 - p_0}{\beta} = 0,\\ \frac{\partial \pi_1}{\partial q_1} &= \alpha - \delta\alpha - c_1 + \delta p_0 - 2\beta(1-\delta^2)q_1 z = 0.\end{aligned}$$

The second-order conditions are satisfied. From the first-order conditions, we obtain the following reaction functions for public and private firms, respectively:

$$\begin{aligned}R_0^{pq}(q_1) &= c_0,\\ R_1^{pq}(p_0) &= \frac{\alpha - \delta\alpha - c_1 + \delta p_0}{2\beta(1-\delta^2)z}.\end{aligned}$$

These functions lead to the following expression for the equilibrium price and quantity:

$$\begin{aligned}p_0^{pq} &= c_0,\\ q_1^{pq} &= \frac{a_1 - \delta a_0}{2\beta(1-\delta^2)z}.\end{aligned}$$

Substituting these equilibrium price and quantity into the payoff functions, we have the following resulting expected welfare and expected profit for firm 1:

$$SW^{pq} = \frac{4(1-\delta^2)a_0^2 z^2 + 4(a_1 - \delta a_0)^2 z - (a_1 - \delta a_0)^2}{8\beta(1-\delta^2)z^2}, \tag{6}$$

$$\pi_1^{pq} = \frac{(a_1 - \delta a_0)^2}{4\beta(1-\delta^2)z}. \tag{7}$$

3.4 q-p game

We now discuss the situation in which firm 0 chooses the quantity contract and firm 1 chooses the price contract. The first-order conditions for firms 0 and 1 are, respectively,

$$\begin{aligned}\frac{\partial SW}{\partial q_0} &= a_0 - \delta a_1 - \beta(1-\delta^2)q_0 = 0,\\ \frac{\partial \pi_1}{\partial p_1} &= \frac{c_1 - 2p_1 + \alpha - \beta\delta q_0}{\beta} = 0.\end{aligned}$$

The second-order conditions are satisfied. From the first-order conditions, we obtain the following reaction functions for public and private firms, respectively:

$$\begin{aligned} R_0^{qp}(p_1) &= \frac{a_0 - \delta a_1}{\beta(1-\delta^2)}, \\ R_1^{qp}(q_0) &= \frac{\alpha + c_1 - \beta\delta q_0}{2}. \end{aligned}$$

These functions lead to the following expression for the equilibrium quantity and price:

$$\begin{aligned} q_0^{qp} &= \frac{a_0 - \delta a_1}{\beta(1-\delta^2)}, \\ p_1^{qp} &= \frac{c_1 + \alpha + \delta c_0 - \delta\alpha - 2\delta^2 c_1}{2(1-\delta^2)}. \end{aligned}$$

Substituting these equilibrium quantity and price into the payoff functions, we have the following resulting expected welfare and expected profit for firm 1:

$$SW^{qp} = \frac{(4-5\delta^2)a_0^2 + (3-4\delta^2)a_1^2 - 2\delta(3-4\delta^2)a_0 a_1}{8\beta(1-\delta^2)^2}, \tag{8}$$

$$\pi_1^{qp} = \frac{(a_1 - \delta a_0)^2}{4\beta(1-\delta^2)^2}. \tag{9}$$

4 Result

We now discuss the choice at the first stage.

Lemma 1 *(i) $SW^{pq} > SW^{qq}$, (ii)$SW^{pp} > SW^{qp}$, (iii) $\pi_1^{pp} > \pi_1^{pq}$, and (iv) $\pi_1^{qp} > \pi_1^{qq}$*

Proof (i)From (6) and (2), we have

$$SW^{pq} - SW^{qq} = \frac{\delta((1-\delta^2)(2z-1)a_0 + \delta a_1)H_1(\delta, z)}{8\beta(1-\delta^2)(2-\delta^2)^2 z},$$

where $H_1(\delta, z) \equiv (2(1-\delta)(2a_1 + \delta(a_1 - \delta a_0)) + 4(a_1 - \delta^2 a_0))z + (2-\delta^3)a_0 - (4-3\delta^2)a_1$. $SW^{pq} - SW^{qq}$ is positive(res. negative, zero) if $H_1(\delta, z)$ is positive(res. negative, zero). Obviously $H_1(\delta, z)$ is increasing in z for $z \geq 1$. We now show that $H_1(\delta, 1) > 0$. Substituting $z = 1$ into $H_1(\delta, z)$, we have $H_1(\delta, 1) = 2(1-\delta)a_0 + (4-\delta^2)(a_1 - \delta a_0) > 0$. Thus $SW^{pq} - SW^{qq}$ is positive for $z > 1$.

(ii)From (4) and (8), we have

$$SW^{pp} - SW^{qp} = \frac{\delta^2(a_1 - \delta a_0)^2(4 - 3\delta^2)}{8\beta(1-\delta^2)^2(2-\delta^2)^2}.$$

This is positive under the assumption of interior solution.

(iii)From (5) and (7), we have

$$\pi_1^{pp} - \pi_1^{pq} = \frac{(a_1 - \delta a_0)^2(4z - (2-\delta^2)^2)}{4\beta(1-\delta^2)(2-\delta^2)^2 z}.$$

This is positive under the assumption of interior solution.

(iv)From (9) and (3)

$$\pi_1^{qp} - \pi_1^{qq} = \frac{-4\delta^2(1-\delta^2)^2 a_0^2 z^2 + (4(a_1^2 + \delta^2 a_0^2 + \delta^5 a_0 a_1) - \delta^2(4-\delta^2)(a_1 + \delta a_0)^2)z - 4(1-\delta^2)^2 a_1^2}{4\beta(1-\delta^2)^2(2-\delta^2)^2 z} \equiv f_1(\delta, z).$$

We now show that $f_1(\delta, 1) > 0$ and that $f_1(\delta, z)$ is increasing in z under the assumption of the interior solution. Substituting $z = 1$ into $f_1(\delta, z)$, we have $f_1(\delta, 1) = \frac{\delta^2(4-3\delta^2)(a_1-\delta a_0)^2}{4\beta(1-\delta^2)^2(2-\delta^2)^2} > 0$. We show that $f_1(\delta, z)$ is increasing in z under the assumption of the interior solution. We have that

$$\frac{\partial f_1(\delta, z)}{\partial z} = \frac{(a_1 - \delta a_0 z)(a_1 + \delta a_0 z)}{\beta(2-\delta^2)^2 z^2}.$$

This is positive under the assumption of the interior solution. Q.E.D.

We now present our main result:

Proposition 1 *Bertrand competition is the endogenous competition structure for any degree of demand shock.*

Proof Lemma 3(i) and Lemma 3(ii) imply that choosing p is the dominant strategy for firm 0. Lemma 3(iii) and Lemma 3(iv) imply that choosing p is dominant strategy for firm 1. Q.E.D.

We explain the intuition why one dimensional demand uncertainty dose not change the competition structure. First we check the private firm's incentive. Suppose that public firm chooses the price contract. $R_0^{pq} = c_0$ indicates that firm 0 engages in marginal cost pricing regardless of private firm's output. Since the quantity of private firm is given, marginal cost pricing is best for welfare. From $R_0^{pp}(p_1) = c_0 + \delta(p_1 - c_1)$, public firm chooses higher price than its marginal cost, responding to private firm's pricing, when private firm chooses the price contract. If private firm chooses the price contract its output depends on public firm's price and a lower public firm pricing reduces output of private firm and a smaller private firm's output reduces social welfare. Thus, public firm chooses higher price than its marginal cost to reduce this welfare loss. This higher price is beneficial for private firm. In addition, since $\frac{\partial R_1^{pq}(q_0)}{\partial z} = -\frac{\alpha - \delta\alpha - c_1 + \delta p_0}{(2\beta(1-\delta^2)z^2} < 0$, an increasing in z decreases private firm's output. This reduces private firm's profit in p-q game and price contract is more attractive for private firm.

Suppose that public firm chooses the quantity contract. Suppose that private firm choose quantity contract. Substituting $R_0^{qq}(q_1) = \frac{a_0 - \beta\delta q_1}{\beta}$ into the expected inverse demand function of public firm, we have that public firm chooses output such that $p_0(q_0, q_1) = c_0 - (z-1)\alpha \leq c_0$. Suppose that private firm chooses the price contract. Substituting $R_0^{qp}(p_1) = \frac{a_0 - \delta a_1}{\beta(1-\delta^2)}$ into the expected demand function of public firm, we have that public firm chooses output such that $p_0(q_0, p_1) > c_0$. If private firm chooses the price contract, its output depends on public firm's output. A lager out put of public firm reduces the quantity of private firm, and a smaller private firm's output reduces welfare. Therefore, public firm chooses a smaller quantity than that when private firm chooses the quantity contract, and this smaller quantity of public firm is beneficial for private firm. Additionally, since $\frac{\partial R_1^{qq}(q_0)}{\partial z} = -\frac{2\beta a_1}{(2\beta z)^2} < 0$, an increasing z decreases the output of private firm in q-q game and decreases private firm's profit. Thus private firm prefers the price contract to the quantity contract.

Second, we check the public firm's incentive. As Singh and Vives(1984) discussed, the demand elasticity of private firm is higher when public firm chooses price contract than quantity contract. Thus private firm becomes more aggressive when public firm chooses price contract rather than the quantity contract, improving welfare. In addition as discussed above, the existence of uncertainty decreases private firm's quantity in q-q and p-q game in these games. This reduction of private firm's output reduces social welfare. Thus, public firm has a strict incentive to choose the price contract under uncertainty. We show that the result of Matsumura and Ogawa(2012) is robust under one dimensional demand uncertainty.

We discuss the Cournot Bertrand comparison in a mixed duopoly.

Proposition 2 *The Bertrand model yields higher welfare and private firm's profit than does the Cournot, regardless of the degree of demand uncertainty.*

Proof From (4) and (2), we have

$$SW^{pp} - SW^{qq} = \frac{H_2(\delta, z)}{2\beta(1-\delta^2)(2-\delta^2)^2 z},$$

where $H_2 \equiv ((a_1 - \delta a_0)((1-\delta^2)a_1 + (a_1 - \delta^3 a_0)) + (1-\delta^2)a_1^2)z^2 - 2(1-\delta^2)a_1((2-\delta^2)a_1 - \delta a_0)z + (1-\delta^2)^2 a_1^2$. $SW^{pp} - SW^{qq}$ is positive(res. negative, zero) if $H_2(\delta, z)$ is positive(res. negative, zero). We now show that $H_2(\delta, 1) > 0$ and that $H_2(\delta, z)$ is increasing in z for $z \geq 1$. Substituting $z = 1$ into $H_2(\delta, z)$, we have $H_2(\delta, 1) = \delta^2(a_1 - \delta a_0)^2 > 0$. We show that $H_2(\delta, z)$ is increasing in z for $z \geq 1$ if $\delta \in (0, 1)$. We have that

$$\frac{\partial H_2(\delta, z)}{\partial z} = 2((a_1 - \delta a_0)((1-\delta^2)a_1 + (a_1 - \delta^3 a_0)) + (1-\delta^2)a_1^2)z - 2(1-\delta^2)a_1((2-\delta^2)a_1 - \delta a_0).$$

This is increasing in z. Substituting in $z = 1$, we have

$$\frac{\partial H_2(\delta, z)}{\partial z}|_{z=1} = a_1(a_1 - \delta a_0) > 0.$$

Thus, $\frac{\partial H_2(\delta,z)}{\partial z} > 0$ for $z \geq 1$. From (5) and (3), we have

$$\pi_1^{pp} - \pi_1^{qq} = \frac{-\delta^2(1-\delta^2)a_0^2 z^2 + (a_1^2 + \delta^2 a_0^2 - 2\delta^3 a_0 a_1)z - (1-\delta^2)a_1^2}{\beta(1-\delta^2)(2-\delta^2)^2 z} \equiv f_2(\delta, z).$$

We now show that $f_2(\delta, 1) > 0$ and that $f_2(\delta, z)$ is increasing in z under the assumption of the interior solution. Substituting $z = 1$ into $f_2(\delta, z)$, we have $f_2(\delta, 1) = \frac{\delta^2(a_1-\delta a_0)^2}{\beta(1-\delta^2)(2-\delta^2)^2} > 0$. We show that $f_2(\delta, z)$ is increasing in z under the assumption of the interior solution. We have that

$$\frac{\partial f_2(\delta, z)}{\partial z} = \frac{(a_1 - \delta a_0 z)(a_1 + \delta a_0 z)}{\beta(2 - \delta^2)^2 z^2}.$$

This is positive under the assumption of the interior solution. Q.E.D.

As Gosh and Mitra(2010) discussed, in a mixed duopolies the Cournot model yields stronger competition among firms than does the Bertrand model. Since $\frac{\partial R_1^{qq}(q_0)}{\partial z} = -\frac{2\beta a_1}{(2\beta z)^2} < 0$, given public firm's output an increasing z decreases private firm's quantity in Cournot competition and the output of public firm is not affected by z. This decreases firm 1's profit and social welfare in Cournot competition. Additionally, as discussed in Resinger and Ressener(2009), a demand shock which affects the slope dose not change the equilibrium outcome in the Bertrand model. Therefore, an increasing z makes the Bertrand model is more attractive than the Cournot model for both public and private firm. We show that the result of Gosh and Mitra(2010) is robust under one dimensional demand uncertainty.

5 Extension

This section extends the model. In this section we considers the two dimensional demand uncertainty. Until now we have only assumed a demand shock which affects the slope. In this section we additionally consider a shock to the intercept. The inverse demand is then given by

$$p_i = \alpha + \epsilon - \frac{\beta}{\theta} q_i - \frac{\beta}{\theta} \delta q_j \ (i = 0, 1, i \neq j), \tag{10}$$

where ϵ is a random variable. Without loss of generality, we assume $E[\epsilon] = 0$ and $Var(\epsilon) = \sigma_\epsilon^2 > 0$. We denote the covariance between the shocks by $\sigma_{\theta\epsilon}$.

Under qq-game, we obtain the following reaction functions for firm 0 and firm 1, respectively:

$$\begin{aligned} R_0^{qq}(q_1) &= \frac{a_0 - \beta\delta q_1}{\beta}, \\ R_1^{qq}(q_0) &= \frac{a_1 - \beta\delta q_0 z}{2\beta z}. \end{aligned}$$

The equilibrium quantity of the public firm can be derived as

$$q_0^{qq} = \frac{2a_0 z - \delta a_1}{\beta(2-\delta^2)z},$$

and that of the private firm is

$$q_1^{qq} = \frac{a_1 - \delta a_0 z}{\beta(2-\delta^2)z}.$$

Substituting these equilibrium quantities in to the demand and payoff functions we have the following expected welfare and firm 1's expected profit:

$$SW^{qq} = \frac{((4-\delta^2)a_0 - 2\delta(2-\delta^2)a_1)a_0 z^2 + 2((2-\delta^2)a_1 - \delta a_0)z - (1-\delta^2)a_1^2}{2\beta(2-\delta^2)^2 z^2}, \tag{11}$$

$$\pi_1^{qq} = \frac{(a_1 - \delta a_0 z)^2}{\beta(2-\delta^2)^2 z}. \tag{12}$$

Under pp-game, we obtain the following reaction functions for public and private firms, respectively:

$$\begin{aligned} R_0^{pp}(p_1) &= c_0 + \delta(p_1 - c_1) + \sigma_{\theta\epsilon}(1-\delta), \\ R_1^{pp}(p_0) &= \frac{c_1 + \alpha + p_0\delta - \alpha\delta + \sigma_{\theta\epsilon}(1-\delta)}{2}. \end{aligned}$$

The equilibrium price of the public firm can be derived as

$$p_0^{pp} = \frac{\alpha\delta - \alpha\delta^2 + 2c_0 - \delta c_1 + (2-\delta-\delta^2)\sigma_{\theta\epsilon}}{2-\delta^2},$$

and that of the private firm is

$$p_1^{pp} = \frac{\alpha - \delta\alpha + c_1 - \delta c_0 + (1-\delta^2)\sigma_{\theta\epsilon}}{2-\delta^2}.$$

Substituting these equilibrium prices in to the demand and payoff functions we have the following expected welfare and firm 1's expected profit:

$$SW^{pp} = \frac{H_3}{2\beta(1-\delta^2)(2-\delta^2)^2}, \tag{13}$$

$$\pi_1^{pp} = \frac{(a_1 - \delta a_0 + \sigma_{\theta\epsilon}(1+\delta^2))^2}{\beta(1-\delta^2)(2-\delta^2)^2}, \tag{14}$$

where $H_3 \equiv -(1-\delta^2)\sigma_{\theta\epsilon}^2 + (2a_0\delta^3 - 2a_1\delta^2 + 2(a_1 - \delta a_0))\sigma_{\theta\epsilon} - 2a_0a_1\delta^5 + (a_1^2 + 2a_0^2)\delta^4 + 6a_0a_1\delta^3 - (3a_1^2 + 5a_0^2)\delta^2 - 6a_0a_1\delta + 3a_1^2 + 4a_0^2$.

Under pq-game, we obtain the following reaction functions for public and private firms, respectively:

$$R_0^{pq}(q_1) = c_0 + \sigma_{\theta\epsilon},$$

$$R_1^{pq}(p_0) = \frac{\alpha - \delta\alpha - c_1 + \delta p_0}{2\beta(1-\delta^2)z}.$$

The equilibrium price of the public firm can be derived as

$$p_0^{pq} = c_0 + \sigma_{\theta\epsilon},$$

and the equilibrium quantity of the private firm can be derived as

$$q_1^{pq} = \frac{a_1 - \delta a_0 + \delta\sigma_{\theta\epsilon}}{2\beta(1-\delta^2)z}.$$

Substituting these equilibrium price and quantity in to the demand and payoff functions we have the following expected welfare and firm 1's expected profit:

$$SW^{pq} = \frac{H_4}{8\beta(1-\delta^2)z^2}, \tag{15}$$

$$\pi_1^{pq} = \frac{(a_1 - \delta a_0 + \sigma_{\theta\epsilon}\delta)^2}{4\beta(1-\delta^2)z}, \tag{16}$$

where $H_4 \equiv 4a_0^2(1-\delta^2)z^2 + 4(a_1 - \delta a_0)(a_1 - \delta a_0 + \delta\sigma_{\theta\epsilon}) - \delta^2\sigma_{\theta\epsilon} - 2\delta(a_1 - \delta a_0)\sigma_{\theta\epsilon} - a_0^2\delta^2 + 2a_0a_1\delta - a_1^2$

Under qp-game, we obtain the following reaction functions for public and private firms, respectively:

$$
\begin{aligned}
R_0^{qp}(p_1) &= \frac{a_0 - \delta a_1}{\beta(1-\delta^2)}, \\
R_1^{qp}(q_0) &= \frac{\alpha + c_1 - \beta\delta q_0 + \sigma_{\theta\epsilon}}{2}.
\end{aligned}
$$

The equilibrium quantity of the public firm can be derived as

$$
q_0^{qp} = \frac{a_0 - \delta a_1}{\beta(1-\delta^2)},
$$

and the equilibrium price of the private firm can be derived as

$$
p_1^{qp} = \frac{(1-\delta^2)\sigma_{\theta\epsilon} - 2c_1\delta^2 - (\alpha - c_0)\delta + c_1 + \alpha}{2(1-\delta^2)}.
$$

Substituting these equilibrium quantity and price in to the demand and payoff functions we have the following expected welfare and firm 1's expected profit:

$$
\begin{aligned}
SW^{qp} &= \frac{H_5}{8\beta(1-\delta^2)^2}, && (17) \\
\pi_1^{qp} &= \frac{(a_1 - \delta a_0 + \sigma_{\theta\epsilon}(1+\delta^2))^2}{4\beta(1-\delta^2)^2}, && (18)
\end{aligned}
$$

where $H_5 \equiv -(1-\delta^2)^2\sigma_{\theta\epsilon}^2 + 2(1-\delta^2)(a_1 - \delta a_0)\sigma_{\theta\epsilon} + 8a_0a_1\delta^3 - \delta^2(4a_1^2 + 5a_0^2) - 6a_0a_1\delta + 3a_1^a + 4a_0^2$

We now present our main result:

Proposition 3 *Cournot competition can be the endogenous competition structure if there exists two dimensional demand shock.*

Proof From (15) and (11), we have

$$
SW^{pq} - SW^{qq} = \frac{H_6}{8\beta(1-\delta^2)(2-\delta^2)^2 z}, \quad (19)
$$

where $H_6 \equiv \delta(-(1-\delta^2 2a_0z - (2-\delta^2)\sigma_{\theta\epsilon} - a_0\delta^2 - a_1\delta + 2a_0)((-2a_0\delta^3 + 4a_1\delta^2 + 6a_0\delta - 8a_1)z + (2-\delta^2)\delta\sigma_{\theta\epsilon} + a_0\delta^3 - 3a_1\delta^2 - 2a_0\delta + 4a_1)$.

From (13) and (17), we have

$$SW^{pp} - SW^{qp} = \frac{\delta^2(a_1 - \delta a_0 + \sigma_{\theta\epsilon}(1+\delta^2))(-(1-\delta^2)(4-\delta^2) + (4-3\delta^2)(a_1 - \delta a_0))}{8\beta(1-\delta^2)^2(2-\delta^2)^2}. \quad (20)$$

From (14) and (16), we have

$$\pi_1^{pp} - \pi_1^{pq} = \frac{H_7}{4\beta(1-\delta^2)(2-\delta^2)^2 z}. \quad (21)$$

where $H_7 \equiv 4(a_1 - \delta a_0 + \sigma_{\theta\epsilon}(1+\delta^2)^2)z - \delta^2(2-\delta^2)^2\sigma_{\theta\epsilon}^2 - 2\delta(2-\delta^2)^2(a_1 - \delta a_0)\sigma_{\theta\epsilon} - a_0^2\delta^6 + 2a_0a_1\delta^5 + \delta^4(4a_0^2 - a_1^2) - 8\delta^3 a_0 a_1 + 4\delta^2(a_1^2 - a_0^2) + 8\delta a_0 a_1 - 4a_1^2$.

From (18) and (12) we have

$$\pi_1^{qp} - \pi_1^{qq} = \frac{H_8}{4\beta(1-\delta^2)^2(2-\delta^2)^2 z}, \quad (22)$$

where $H_8 \equiv -4(1-\delta^2)^2\delta^2 a_0^2 z^2 + ((1-\delta^2)^2(2-\delta^2)^2\sigma_{\theta\epsilon}^2 + 2(1-\delta^2)(2-\delta^2)^2(a_1 - \delta a_0)\sigma_{\theta\epsilon} + \delta^6 a_0^2 + 6\delta^5 a_0 a_1 + \delta^4(a_1 - 4a_0^2) - 8\delta^3 a_0 a_1 + 4\delta^2(a_0^2 - a_1^2) + 4a_1^2)z - 4(1-\delta^2)^2 a_1^2$.

Substituting $\alpha = 10, \beta = 1, c_0 = 9, c_1 = 8, \delta = 0.3, z = 3, \sigma_{\theta\epsilon} = 1.5$ into $(19), (20), (21)$, and (22) we have $SW^{qp} - SW^{qq} \approx 0.289, SW^{pp} - SW^{qp} \approx 0.011, \pi_1^{pp} - \pi_1^{pq} = 2.406$, and $\pi_1^{qp} - \pi_1^{qq} \approx 2.726$. Then Bertrand competition can be the endogenous competition structure. Substituting $\alpha = 10, \beta = 1, c_0 = 9, c_1 = 8, \delta = 0.2, z = 1.5, \sigma_{\theta\epsilon} = -1.5$ into $(19), (20), (21)$, and (22), we have $SW^{pq} - SW^{qq} \approx -0.073, SW^{pp} - SW^{qp} \approx 0.006, \pi_1^{pp} - \pi_1^{pq} = -0.355$, and $\pi_1^{qp} - \pi_1^{qq} \approx -0.466$ Then, quantity contract is dominant strategy for firm 1. Given firm 1's quantity contract, choosing quantity contract makes lager welfare than price contract. Therefore Cournot can appear in the equilibrium. Q.E.D.

We explain the intuition why two dimensional demand uncertainty can change the competition structure. First we explain why private firm has incentive to deviate from the p-p game. The reducing $\sigma_{\theta\epsilon}$ induces both public and private firm's lower pricing. In other words the reducing $\sigma_{\theta\epsilon}$ makes severe competition in the p-p game. This reduces profit for private firm. Similarly, the reducing $\sigma_{\theta\epsilon}$ induces public firm's aggressive behavior in p-q game. However, the decreasing $\sigma_{\theta\epsilon}$ dose not reduces private firm's quantity directly in p-q game. Therefore, reduction of profit for private firm by the effect of decreasing $\sigma_{\theta\epsilon}$ is lager p-q game than p-p game. Then private firm can earn lager profit in p-q game for some $\sigma_{\theta\epsilon}$.

Suppose that public firm chooses the quantity contract. Suppose that private firm choose quantity contract. In q-q game, $\sigma_{\theta\epsilon}$ dose not matter. Suppose that private firm chooses the price contract. In q-p game, the decreasing $\sigma_{\theta\epsilon}$ induces private firm's lower pricing and this is harmful for profit for private firm. Therefore, $\sigma_{\theta\epsilon}$ is negative and private firm prefers the quantity contract to the price contract.

Second, we check the public dose not have a incentive to deviate from q-q game if $\sigma_{\theta\epsilon}$ is negative. As argued above, $\sigma_{\theta\epsilon}$ dose not matter in q-q game. However, in p-q game reducing $\sigma_{\theta\epsilon}$ decreases price for public firm and quantity of private firm. The former effect gains welfare and latter effect makes welfare loss. Then, the latter effect dominates the former effect and welfare loss occurs. Thus decreasing $\sigma_{\theta\epsilon}$ can achieve lager welfare in the q-q game than p-q game.

Proposition 4 *If there exists two dimensional uncertainty, both* $SW^{pp} > SW^{qq}$ *and* $SW^{pp} < SW^{qq} < 0$ *are possible.*

Proof From (13) and (11), we have

$$SW^{pp} - SW^{qq} = \frac{H_9}{2\beta(1-\delta^2)(2-\delta^2)^2 z}, \tag{23}$$

where $H_9 \equiv (-(1-\delta^2)\sigma_{\theta\epsilon}^2 + 2(1-\delta^2)(a_1 - \delta a_0)\sigma_{\theta\epsilon} + \delta^4(a_0^2 + a_1^2) - 3\delta^2 a_1^2 - 2\delta a_0 a_1 + 3a_1^2)z^2 - 2(1-\delta^2)a_1(2a_1 - \delta a_0 - \delta a_1)z + (1-\delta^2)^2 a_1^2$

Substituting $\alpha = 10, \beta = 1, c_0 = 9, c_1 = 8, \delta = 0.3, z = 3, \sigma_{\theta\epsilon} = 3$ into (23), we have $SW^{pp} - SW^{qq} \approx 1.047$. Substituting $\alpha = 10, \beta = 1, c_0 = 9, c_1 = 8, \delta = 0.3, z = 3, \sigma_{\theta\epsilon} = 5$ into (23), we have $SW^{pp} - SW^{qq} \approx -0.214$. Q.E.D.

Under two dimensional uncertainty, an increasing the $\sigma_{\theta\epsilon}$ makes both public and private firm's higher pricing in Bertrand model and $\sigma_{\theta\epsilon}$ dose not affect the equilibrium outcome in Cournot model. Since the higher pricing reduces social welfare, and then the Cournot model can yield higher welfare than the Bertrand model if $\sigma_{\theta\epsilon}$ is sufficiently large.

Proposition 5 *If there exists two dimensional uncertainty, both $\pi_1^{pp} > \pi_1^{qq}$ and $\pi_1^{pp} < \pi_1^{qq}$ are possible.*

Proof From (14) and (12), we have

$$\pi_1^{pp} - \pi_1^{qq} = \frac{-(1-\delta^2)\delta^2 a_0^2 z^2 + ((1-\delta^2)^2\sigma_{\theta\epsilon}^2 + 2(1-\delta^2)(a_1\delta a_0)\sigma_{\theta\epsilon} - 2a_0a_1\delta^3 + a_0^2\delta^2 + a_1^2)z + \delta^2 a_1^2 - a_1^2}{\beta(1-\delta^2)(2-\delta^2)^2 z}. \tag{24}$$

Substituting $\alpha = 10, \beta = 1, c_0 = 9, c_1 = 8, \delta = 0.3, z = 3, \sigma_{\theta\epsilon} = -1$ into (24), we have $\pi_1^{pp} - \pi_1^{qq} \approx 0.077$. Substituting $\alpha = 10, \beta = 1, c_0 = 9, c_1 = 8, \delta = 0.3, z = 3, \sigma_{\theta\epsilon} = -1.5$ into (24), we have $\pi_1^{pp} - \pi_1^{qq} \approx -0.077$. Q.E.D.

Under two dimensional uncertainty, the negative $\sigma_{\theta\epsilon}$ induces both public and private firm's aggressive pricing in the Bertrand model. This aggressive pricing decreases private firm's profit in the Bertrand model. The existence of $\sigma_{\theta\epsilon}$ does not matter in the Cournot model. Therefore Cournot model can yield higher profit for private firm if $\sigma_{\theta\epsilon}$ takes negative value.

6 Conclusion

In this study, we revisit the classic discussion of the comparison between price and quantity competition. but in a mixed duopoly under demand uncertainty. Ghosh and Mitra(2010) considered the certain demand and showed that in a mixed duopoly, price competition yields higher welfare and a lager profit for private firm. We show that regardless of the degree of demand uncertainty, price competition yields higher welfare and a lager profit for private firm. We also endogenize the choice of price or quantity contract. Matsumura and Ogawa(2012) considered the certain demand and showed that choosing a price contract is the dominant strategy for both firms. We find that both firms choose price contracts in the unique equilibrium regardless of the degree of demand shock. We also show that Cournot model can yield higher welfare and profit for private firm and quantity competition appears in the equilibrium if there are two dimensional demand uncertainty.

References

Bose A, Gupta B (2013) Mixed markets in bilateral monopoly. Journal of Economics, 110(2):141–164.

Brandão A, Castro S (2007) State-owned enterprises as indirect instruments of entry regulation. Journal of Economics 92(3):263–274.

Cato S, Matsumura T (2012) Long-run effects of foreign penetration on privatization policies. Journal of Institutional and Theoretical Economics 168(3):444–454.

Cato S, Matsumura T (2013) Long-run effects of tax policies in a mixed market. FinanzArchiv 69(2):215–240.

Cheng L (1985) Comparing Bertrand and Cournot equilibria: a geometric approach. RAND Journal of Economics 16(1):146–152.

Chirco A, Colombo C, Scrimitore M (2014) Organizational structure and the choice of price versus quantity in a mixed duopoly. Japanese Economic Review 65(4):521–542.

De Fraja G, Delbono F (1989) Alternative strategies of a public enterprise in oligopoly. Oxford Economic Papers 41(2):302–311

Dixit AK (1979) A model of duopoly suggesting a theory of entry barriers. Bell Journal of Economics 10(1):20–32.

Fjell K, Pal D (1996) A mixed oligopoly in the presence of foreign private firms. Canadian Journal of Economics 29(3):737–743.

Fujiwara K (2007) Partial privatization in a differentiated mixed oligopoly. Journal of Economics 92(1):51–65.

Ghosh A, Mitra M (2010) Comparing Bertrand and Cournot in mixed markets. Economics Letters 109(2):72–74.

Ghosh A, Mitra M, Saha B (2013) Privatization, underpricing and welfare in the presence of foreign competition. Journal of Public Economic Theory, forthcoming.

Ghosh A, Sen P (2012) Privatization in a small open economy with imperfect competition. Journal of Public Economic Theory 14(3):441–471.

Haraguchi J, Matsumura T (2014) Price versus quantity in a mixed duopoly with foreign penetration. Research in Economics 68(4):338–353.

Haraguchi J, Matsumura T (2015) Cournot-Bertrand Comparison in a Mixed Oligopoly. Journal of Economics forthcoming

Ino H, Matsumura T (2010) What role should public enterprises play in free-entry markets? *Journal of Economics* 101(3):213–230.

Klemperer P, Meyer M (1986) Price Competition vs. Quantity Competition: The Role of Uncertainty RAND Journal of Economics 17(4):618–638.

Lin M H, Matsumura T (2012) Presence of foreign investors in privatized firms and privatization policy. Journal of Economics 107(1):71–80.

Matsumura T (1998) Partial privatization in mixed duopoly. Journal of Public Economics 70(3):473–483.

Matsumura T, Kanda O (2005) Mixed oligopoly at free entry markets. Journal of Economics 84(1):27–48.

Matsumura T, Matsushima N (2012) Airport privatization and international competition. Japanese Economic Review 63(4):431–450.

Matsumura T, Ogawa A (2012) Price versus quantity in a mixed duopoly. Economics Letters 116(2):174–177.

Matsumura T, Okumura Y (2013) Privatization neutrality theorem revisited. Economics Letters 118(2):324–326.

Matsumura T, Shimizu D (2010) Privatization waves. Manchester School 78(6):609–625.

Merrill W C, Schneider N (1966) Government firms in oligopoly industries: A short-run analysis. Quarterly Journal of Economics 80(3):400-412.

Nakamura Y (2013) Social welfare under quantity competition and price competition in a mixed duopoly with network effects: an analysis. Theoretical Economics Letters 3:211–215.

Reisinger M and Ressener L (2009) The Choice of Price versus Quantities under Uncertainty. Journal of Economics and Management Strategy 18(4):1155–1177

Scrimitore M (2013) Price or quantity?: the strategic choice of subsidized firms in a mixed duopoly. Economics Letters 118(2):337–341.

Scrimitore M (2014) Profitability under commitment in Cournot and Bertrand mixed markets. Journal of Institutional and Theoretical Economics 170(4):684–703.

Singh N, Vives X (1984) Price and quantity competition in a differentiated duopoly. RAND Journal of Economics 15(4):546–554.

Shubik M, Levitan R (1980) Market Structure and Behavior (Harvard University Press, Cambridge, Massachussets, U.S.A.).

Tanaka Y (2001a) Profitability of price and quantity strategies in an oligopoly. Journal of Mathematical Economics 35(3):409–418.

Tanaka Y (2001b) Profitability of price and quantity strategies in a duopoly with vertical product differentiation. Economic Theory 17(3):693–700.

Tasnádi A (2006) Price vs quantity in oligopoly games. International Journal of Industrial Organization 24(3):541–554.

Vives X (1985) On the efficiency of Bertrand and Cournot equilibria with product differentiation. Journal of Economic Theory 36:166–175.

Wang LFS, Chen TL (2010) Do cost efficiency gap and foreign competitors matter concerning optimal privatization policy at free entry market? Journal of Economics 100(1):33–49.

Wang LFS, Lee JY (2013) Foreign penetration and undesirable competition. Economic Modelling 30(1):729–732